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Phenomenology of n-$\bar{n}$ oscillations revisited

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Abstract

We revisit the phenomenology of n-$\bar{n}$ oscillations in the presence of external magnetic fields, highlighting the role of spin. We show, contrary to long-held belief, that the n-$\bar{n}$ transition rate need not be suppressed, opening new opportunities for its empirical study.

1. Introduction. Searches for processes that violate standard model (SM) symmetries are of particular interest because their discovery would serve as unequivocal evidence for dynamics beyond the SM. The gauge symmetry and known particle content of the SM implies that its Lagrangian conserves baryon number $\mathcal{B}$ and lepton number $\mathcal{L}$, though it is the combination $\mathcal{B}-\mathcal{L}$ that survives at the quantum level. Thus the observation of neutron-antineutron (n-$\bar{n}$) oscillations, a $|\Delta\mathcal{B}| = 2$ process, would show that $\mathcal{B} - \mathcal{L}$ symmetry is broken and ergo that dynamics beyond the SM exists. The current constraints on $|\mathcal{B}| = 1$ operators from the non-observation of nucleon decay are severe, with the strongest limits coming from searches for proton decay to final states that respect $\mathcal{B}-\mathcal{L}$ symmetry, such as $p \to e^+\pi^0$, for which the partial half-life exceeds 8.2×10^{33} years at 90% C.L. [1]. Although particular $|\Delta\mathcal{B}| = 1$ operators, such as those that mediate $n \to e^-\pi^+$, e.g., can also give rise to n-$\bar{n}$ oscillations, Mohapatra and others have emphasized that the origin of nucleon decay and n-$\bar{n}$ oscillations can be completely different [2–8]. Recently, moreover, simple models that give rise to n-$\bar{n}$ oscillations but not nucleon decay have been enumerated [7].

The seminal papers on free n-$\bar{n}$ oscillations have employed a 2×2 effective Hamiltonian matrix [9, 10], familiar from the analysis of meson mixing [11], though this choice explicitly suppresses the role of spin — unlike neutral mesons and neutrinos, the neutron and antineutron each have a significant magnetic moment. We note the neutron and antineutron are themselves distinguished by the sign of the lepton charge in semileptonic decay, and their respective interactions with atomic nuclei are strikingly different as well [12, 13]. The n-$\bar{n}$ system thus has four physical degrees of freedom because the spin projection of a neutron or an antineutron can either be parallel or antiparallel to a quantization axis. In this paper we develop a suitable 4×4 effective Hamiltonian framework for its study. Since previous studies of n-$\bar{n}$ oscillations have been realized in the context of a 2×2 effective Hamiltonian matrix, we discuss this framework before turning to our generalization. The neutron magnetic moment is empirically well-known, yielding an interaction with an external magnetic field $\mathbf{B}$ of form $-\mu_n \mathbf{S}_n \cdot \mathbf{B}/S_n$, where μ_n is the magnitude of the magnetic moment and $\mathbf{S}_n$ is the neutron spin. Supposing the neutron spin to be in the direction of the applied $\mathbf{B}$-field and employing **CPT** invariance, the mass matrix $\mathcal{M}$ takes the form [9]

$$\mathcal{M} = \begin{pmatrix} M_n - \mu_n B & \delta \\ \delta & M_n + \mu_n B \end{pmatrix} , \tag{1}$$

where **CPT** invariance guarantees not only that the neutron and antineutron masses are equal but also that the projections of the neutron and antineutron magnetic moments on $\mathbf{B}$ are equal in magnitude and of opposite sign. We work in units $\hbar = c = 1$ and ignore the finite neutron and antineutron lifetimes throughout. Diagonalizing $\mathcal{M}$ yields the mass eigenstates $|u_i\rangle$, namely,

$$\begin{aligned} |u_1\rangle &= \cos\theta|n\rangle + \sin\theta|\overline{n}\rangle \,, \\ |u_2\rangle &= -\sin\theta|n\rangle + \cos\theta|\overline{n}\rangle \,. \end{aligned} \tag{2}$$

Since the energy scale $\mu_n B$ naturally dwarfs that of δ, we note that the eigenvalue difference is $\Delta E \simeq 2\mu_n B$ and that θ is small: $\theta \simeq \delta/\Delta E$. The n-$\bar{n}$ transition probability becomes [14]

$$P_{\overline{n}}(t) \simeq 2\theta^2 \left[1 - \cos\left(\Delta E t\right)\right] \,. \tag{3}$$

This result can be considered in two different limits: either (a) $\Delta E t \gg 1$ or (b) $\Delta E t \ll 1$. In case (a) the second term oscillates to zero, yielding $P_{\overline{n}}(t) \simeq 2(\delta/\Delta E)^2$ whereas in case (b),

$$P_{\overline{n}}(t) \simeq \left(\frac{\delta}{\Delta E}\right)^2 (\Delta E t)^2 = (\delta t)^2 \,. \tag{4}$$

Evidently unless $t \ll 1/\Delta E$, the energy splitting of the neutron and antineutron in a magnetic field "quenches" the appearance of n-$\bar{n}$ oscillations. Thus the strategy in past and proposed searches for n-$\bar{n}$ oscillations has been to minimize the magnetic field [14–16], so that $t \ll 1/\Delta E$, as well as to maintain a vacuum in the neutron flight volume [10], so that the neutrons are quasifree over the neutron observation time t.

Motivated by the realization that a neutron and an antineutron of opposite spin projection have the same energy in a magnetic field, we consider the spin dependence of n-$\bar{n}$ oscillations explicitly and thus develop a 4×4 effective Hamiltonian framework for its analysis. Spin dependence can arise from effects either within or beyond the SM. As long known from the theory of magnetic resonance, applied magnetic fields can mitigate, or even remove, the energy splitting of spin states in a static magnetic field, note, e.g., Ref. [17, 18]. In this paper we show that such SM effects can remove the magnetic field "quenching" noted in the usual 2×2 Hamiltonian framework and yield new experimental possibilities for the study of n-$\bar{n}$ mixing. It is also possible to have new, spin-dependent $\mathcal{B} - \mathcal{L}$ violating operators, yielding a "new physics" mechanism to evade the magnetic field quenching we have noted. Although we consider both of these distinct possibilities in this paper, our primary focus is the role of spin-dependent SM effects in mediating n-$\bar{n}$ oscillations.

2. Effective Hamiltonian for n-$\bar{n}$ transitions with spin. To realize the most general form of a low-energy, phenomenological Hamiltonian for n-$\bar{n}$ oscillations with spin, we develop a mass matrix $\mathcal{M}$ to this purpose. Its entries $\mathcal{M}_{ij}$ with $i, j = 1, \ldots 4$ correspond to bras and kets containing $n(\mathbf{p}, +)$, $\bar{n}(\mathbf{p}, +)$, $n(\mathbf{p}, -)$, and $\bar{n}(\mathbf{p}, -)$, respectively, with $+\,(-)$ denoting a spin-up (down) state, relative to a quantization axis $\mathbf{z}$. We impose the constraint of Hermiticity, as well as those of charge-conjugation–parity ($\mathbf{CP}$) and time-reversal ($\mathbf{T}$) invariance, on the resulting mass matrix, to determine its model-independent form under these assumptions.

We can implement the discrete symmetry transformations in relativistic quantum field theory and translate them to quantum mechanics by noting [11]

$$\mathbf{b}^\dagger(\mathbf{p}, s)|0\rangle = |n(\mathbf{p}, s)\rangle \quad ; \quad \mathbf{d}^\dagger(\mathbf{p}, s)|0\rangle = |\bar{n}(\mathbf{p}, s)\rangle \,, \tag{5}$$

where $\mathbf{b}[\mathbf{b}^\dagger](\mathbf{p}, s)$ and $\mathbf{d}[\mathbf{d}^\dagger](\mathbf{p}, s)$ denote annihilation [creation] operators for neutrons [antineutrons] of momentum $\mathbf{p}$ and spin projection s, for which $s = \pm 1 \equiv \pm$ with respect to the quantization axis $\mathbf{z}$. We determine the transformation properties of these operators under $\mathbf{CP}$ and $\mathbf{T}$ as follows. We work in the Dirac-Pauli representation for the γ^μ matrices and note that the Dirac field operator $\psi(x)$ has a plane-wave expansion of form

$$\psi(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}\sqrt{2E}} \sum_{s=\pm} \left\{ b(\mathbf{p}, s) u(\mathbf{p}, s) e^{-ip\cdot x} + d^\dagger(\mathbf{p}, s) v(\mathbf{p}, s) e^{ip\cdot x} \right\} \,, \tag{6}$$

with spinors defined as

$$u(\mathbf{p}, s) = \mathcal{N} \begin{pmatrix} \chi^{(s)} \\ \frac{\sigma\cdot\mathbf{p}}{E+M}\chi^{(s)} \end{pmatrix} \quad ; \quad v(\mathbf{p}, s) = \mathcal{N} \begin{pmatrix} \frac{\sigma\cdot\mathbf{p}}{E+M}\chi'^{(s)} \\ \chi'^{(s)} \end{pmatrix} \,, \tag{7}$$

noting $\chi'^{(s)} = -i\sigma^2\chi^{(s)}$, $\chi^+ = \left(\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right)$, $\chi^- = \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right)$, and $\mathcal{N} = \sqrt{E+M}$. This yields

$$\mathbf{CP}\,\mathbf{b}(\mathbf{p}, s)\,(\mathbf{CP})^\dagger = \mathbf{d}(-\mathbf{p}, s) \quad ; \quad \mathbf{CP}\,\mathbf{d}(\mathbf{p}, s)\,(\mathbf{CP})^\dagger = -\mathbf{b}(-\mathbf{p}, s) \tag{8}$$

and

$$\mathbf{T}\,\mathbf{b}(\mathbf{p}, s)\,(\mathbf{T})^{-1} = s\mathbf{b}(-\mathbf{p}, -s) \quad ; \quad \mathbf{T}\,\mathbf{d}(\mathbf{p}, s)\,(\mathbf{T})^{-1} = s\mathbf{d}(-\mathbf{p}, -s) \tag{9}$$

for the transformation properties under $\mathbf{CP}$ and $\mathbf{T}$, respectively[1]. In what follows we assume that the ground (vacuum) state remains invariant under $\mathbf{CP}$ and $\mathbf{T}$: $\mathbf{CP}|0\rangle = |0\rangle$ and $\mathbf{T}|0\rangle = |0\rangle$.

[1] These results differ from those in Ref. [11] because that work uses a different choice of antiparticle spinor.

Under an assumption of **CP** and **T** invariance relationships between the matrix elements of $\mathcal{M}$ follow. For example, under **CPT** invariance we have

$$\langle n(\mathbf{p}, s_1)|H|n(\mathbf{p}, s_2)\rangle = s_1 s_2 \langle \bar{n}(\mathbf{p}, -s_2)|H|\bar{n}(\mathbf{p}, -s_1)\rangle \,, \tag{10}$$

noting H is the Hamiltonian and **T** is an anti-unitary operator. Thus under **CPT** and Hermiticity we find $\mathcal{M}$ has ten parameters, and it is of form

$$\begin{pmatrix} A_1 & \delta & M_1 & \varepsilon_1 \\ \delta^* & A_2 & \varepsilon_2 & -M_1 \\ M_1^* & \varepsilon_2^* & A_2 & -\delta \\ \varepsilon_1^* & -M_1^* & -\delta^* & A_1 \end{pmatrix} , \tag{11}$$

where A_1 and A_2 are real constants. Under **CP** invariance we have, e.g.: $\langle n(\mathbf{p}, s_1)|H|n(\mathbf{p}, s_2)\rangle = \langle \bar{n}(-\mathbf{p}, s_1)|H|\bar{n}(-\mathbf{p}, s_2)\rangle$, yielding relationships between M_{ij} in the low-energy limit, i.e., as $|\mathbf{p}| \to 0$. Thus under Hermiticity and **CP** and **CPT** invariance we have in this case

$$\begin{pmatrix} A_1 & i\delta & 0 & \varepsilon_1 \\ -i\delta & A_1 & -\varepsilon_1 & 0 \\ 0 & -\varepsilon_1^* & A_1 & -i\delta \\ \varepsilon_1^* & 0 & i\delta & A_1 \end{pmatrix} , \tag{12}$$

where both A_1 and δ are real — and only four parameters suffice to characterize the mass matrix. In Eq. (12), two distinct n-$\bar{n}$ transition operators appear: δ that describes the transition between states of the same spin, $n(s) \leftrightarrow \bar{n}(s)$ and ε_1 that describes the transition between states of opposite spin, $n(s) \leftrightarrow \bar{n}(-s)$. Note that since the neutron and antineutron are of opposite intrinsic parity, we have under **CP**, $\langle n(\mathbf{p}, s_1)|H|\bar{n}(\mathbf{p}, s_2)\rangle = -\langle \bar{n}(-\mathbf{p}, s_1)|H|n(-\mathbf{p}, s_2)\rangle$, yielding, e.g., terms in $\pm i\delta$. If, rather, the relevant piece of H is odd under **CP**, the δ terms become real, as chosen in Eq. (1). Previous analyses [14] have only considered the possibility of $n(s) \leftrightarrow \bar{n}(s)$. We will show that the second process can occur through the application of magnetic fields, both within and beyond the SM. The parameters δ and ε_1, however, characterize n-$\bar{n}$ mixing *en vacuo*. Since we have chosen the antiparticle spinors in a manner consistent with Dirac hole theory, the underlying two-component spinor of a particle with spin s has the same orientation as that of an antiparticle with spin $-s$; in the presence of baryon-number violation it would seem that both pathways could occur. Indeed there are two Lorentz-invariant, leading-mass-dimension n-$\bar{n}$

operators: $in^T Cn$ and $n^T\gamma_5 Cn$, where $C = i\gamma^2\gamma^0$ and T denotes transpose. The latter operator, $n^T\gamma_5 Cn$, can potentially yield a spin flip. The leading-mass-dimension operators that yield n-$\bar{n}$ transitions have been analyzed in QCD [19, 20], and they entrain both possibilities at the quark level. Our detailed analysis of their n-$\bar{n}$ matrix elements reveals, however, that $n(s) \leftrightarrow \bar{n}(-s)$ does not occur (at zero momentum transfer) [21], as one might expect from angular momentum conservation. Indeed only the $n(s) \to \bar{n}(s)$ transition occurs for a free neutron in vacuum. The associated n-$\bar{n}$ matrix elements have been computed in models [19, 22] and in lattice QCD [23]. Thus we set $\varepsilon_1 = 0$ henceforth, though such could be nonzero in the presence of a hidden U(1) sector with a "dark photon" and an associated magnetic field $\mathbf{B}_{\text{hidden}}$. Returning to the operators $in^T Cn$ and $n^T\gamma_5 Cn$, the first is **CP** odd, whereas the second is **CP** even — and both are **CPT** invariant. We assumed the second case in determining Eq. (12), and this will prove useful in what follows. However, since n-$\bar{n}$ transitions in the absence of a magnetic field are, in effect, mediated by $in^T Cn$, we use

$$\begin{pmatrix} A_1 & \delta & 0 & 0 \\ \delta & A_1 & 0 & 0 \\ 0 & 0 & A_1 & -\delta \\ 0 & 0 & -\delta & A_1 \end{pmatrix}, \tag{13}$$

with δ real for our Hamiltonian matrix in this case.

These parametrizations also allow us to generalize our effective Hamiltonian framework to include external magnetic fields. For example, the interaction of an electrically neutral particle with an electromagnetic field is characterized at low energies by $-\mu\cdot\mathbf{B}$ if $\mathbf{T}$ and $\mathbf{P}$ are not broken; this comes from the nonrelativistic limit of $\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$, where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the usual electromagnetic field strength tensor. Under **CP** or **T** the fermion bilinear $\bar{\psi}\sigma^{\mu\nu}\psi$ transforms to $-\bar{\psi}\sigma_{\mu\nu}\psi$, and $F_{\mu\nu}$ transforms to $-F^{\mu\nu}$. Thus their scalar product is itself both **CP** and **T** invariant. However, the explicit **CPT** and **CP** constraints we have investigated operate on the fermion and antifermion degrees of freedom only; the terms in H resulting from the overall minus sign associated with $F_{\mu\nu}$ under **CP** are revealed by comparing the parametrizations under Hermiticity and **CPT** with and without a **CP** constraint, Eqs. (11) and (12). We can also combine magnetic-field interactions with n-$\bar{n}$ oscillations through the operator $\psi^T\sigma^{\mu\nu}C\psi F_{\mu\nu}$ and its Hermitian conjugate; this operator is even under **CP** and **T**. Thus through these comparisons we see how $F_{\mu\nu}$ terms, i.e., those with external magnetic fields, can enter both within and beyond the SM. We now turn to concrete expressions for these terms.

3. Effective Hamiltonian for n-$\bar{n}$ transitions in external magnetic fields. The operator $\psi^T\sigma^{\mu\nu}C\psi F_{\mu\nu}$ and its Hermitian conjugate yield $n\to\bar{n}$ and $\bar{n}\to n$ transitions, respectively. Computing these matrix elements using the free Dirac field operator of Eq. (6) yields

$$\langle\bar{n}(\mathbf{0},s')|\psi^T\sigma^{\mu\nu}C\psi F_{\mu\nu}|n(\mathbf{0},s)\rangle = -\chi'^{(s')\dagger}2\sigma\cdot\mathbf{B}\chi'^{(s)} - \chi^{(s)\dagger}2\sigma\cdot\mathbf{B}\chi^{(s)}\,, \tag{14}$$

where we recall $\chi'^{(s)} = -i\sigma^2\chi^{(s)}$, and

$$\langle n(\mathbf{0},s')| - \psi^{*T}C(\sigma^{\mu\nu})^\dagger\psi^* F_{\mu\nu}|\bar{n}(\mathbf{0},s)\rangle = -\chi'^{(s')\dagger}2\sigma\cdot\mathbf{B}\chi'^{(s)} - \chi^{(s)\dagger}2\sigma\cdot\mathbf{B}\chi^{(s)}\,. \tag{15}$$

Although these expressions vanish for elementary fermions, we note that since both n and $\bar{n}$ possess anomalous magnetic moments compositeness could make these matrix elements nonzero if operators of form $\psi^TC\psi$ exist. We leave a detailed study to a subsequent publication [21]. Nevertheless, these expressions correspond to nonrelativistic operators containing n-$\bar{n}$ transition magnetic moments. Thus we suppose the n and $\bar{n}$ interactions in the presence of external magnetic fields, under **CPT** invariance, to be of form

$$H_B = -\mu_n\frac{\mathbf{S}_n}{S_n}\cdot\mathbf{B} + \mu_n\frac{\mathbf{S}_{\bar{n}}}{S_{\bar{n}}}\cdot\mathbf{B} - \mu^*_{n\bar{n}}\frac{\mathbf{S}_{\bar{n}n}}{S_{\bar{n}n}}\cdot\mathbf{B} - \mu_{n\bar{n}}\frac{\mathbf{S}_{n\bar{n}}}{S_{n\bar{n}}}\cdot\mathbf{B}\,, \tag{16}$$

where μ_n is the neutron magnetic moment, the first two terms being the usual neutron and antineutron interactions in a magnetic field, and $\mu_{n\bar{n}}$ is the n-$\bar{n}$ transition magnetic moment. The last two terms correspond to Eqs. (14) and (15), respectively. The spin operators each act in a 2×2 subspace. With $(S_n)_{i,j}$ such that $(i,j)\in(n(+),n(-))$, we choose $(S_{\bar{n}})_{i,j}$ with $(i,j)\in(\bar{n}(+),\bar{n}(-))$, as well as $(S_{n\bar{n}})_{ij}$ and $(S_{\bar{n}n})_{ji}$ with $i\in n(+),n(-)$ and $j\in\bar{n}(+),\bar{n}(-)$. Within a given subspace, we compute $\mathbf{S}\cdot\mathbf{B}/S = \sigma\cdot\mathbf{B}$. We also suppose that magnetic fields both longitudinal and transverse to the quantization axis exist, and we introduce $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ and $\mathbf{B}_1 = B_1\hat{\mathbf{x}}$, respectively. Defining $\omega_0 \equiv -\mu_n B_0$, $\omega_1 \equiv -\mu_n B_1$, $\delta_0 \equiv -\mu_{n\bar{n}}B_0$, $\delta_1 \equiv -\mu_{n\bar{n}}B_1$, and employing the usual Pauli matrices, we find that the matrix $\mathcal{H}_B$ corresponding to Eq. (16) is

$$\mathcal{H}_B = \begin{pmatrix} \omega_0 & \delta_0 & \omega_1 & \delta_1 \\ \delta_0^* & -\omega_0 & \delta_1^* & -\omega_1 \\ \omega_1 & \delta_1 & -\omega_0 & -\delta_0 \\ \delta_1^* & -\omega_1 & -\delta_0^* & \omega_0 \end{pmatrix}\,, \tag{17}$$

a form consistent with the comparison of Eq. (11) and Eq. (12). Moreover, we see that **CPT** invariance guarantees that a neutron and an antineutron of opposite spin in vacuum are always degenerate irrespective of the size of the magnetic field: the presence of external magnetic fields cannot quench transitions between these states.

Additional constraints on the form factors follow because in the presence of n-$\bar{n}$ oscillations the weak interaction eigenstates can be expressed in terms of Majorana states. A Majorana state $|\Psi_M\rangle$ transforms into itself under $\mathbf{C}$, up to a global phase. Since $\mathbf{C}\,\mathbf{b}(\mathbf{p},s)\,\mathbf{C}^\dagger = \mathbf{d}(\mathbf{p},s)$,

$$|\Psi_M^\pm(\mathbf{p},s)\rangle = \frac{1}{\sqrt{2}}\left(|\bar{n}(\mathbf{p},s)\rangle \pm |n(\mathbf{p},s)\rangle\right) . \tag{18}$$

As we have noted, the neutron and antineutron are distinguished by the sign of the lepton charge upon semileptonic decay, so that the Majorana basis has four degrees of freedom. There are no γ^μ, $\sigma^{\mu\nu}$, or $\sigma^{\mu\nu}\gamma_5$ form factors associated with a Majorana state [24–29]; thus the constraint $\langle\Psi_M^\pm(\mathbf{p},s')\rangle|H_B|\Psi_M^\pm(\mathbf{p},s)\rangle = 0$ or, equivalently, $\eta^T\mathcal{H}_B\eta = 0$, where $\eta = \{a,a,b,b\}$ and a and b are arbitrary constants, yields $\mathrm{Re}(\delta_0) = 0$ and $\mathrm{Re}(\delta_1) = 0$. With these supplemental constraints, Eq. (17) becomes

$$\mathcal{H}_B = \begin{pmatrix} \omega_0 & i\delta_0 & \omega_1 & i\delta_1 \\ -i\delta_0 & -\omega_0 & -i\delta_1 & -\omega_1 \\ \omega_1 & i\delta_1 & -\omega_0 & -i\delta_0 \\ -i\delta_1 & -\omega_1 & i\delta_0 & \omega_0 \end{pmatrix} , \tag{19}$$

where δ_0 and δ_1 are real constants. This bears comparison to studies of resonant spin-flavor neutrino precession in matter, such as in the Sun [30–32], though the neutrino transition magnetic moment in that work is associated with the transverse magnetic field and is flavor-changing. The final Hamiltonian matrix $\mathcal{M}$ for low-energy, n-$\bar{n}$ oscillations in applied magnetic fields thus takes the form

$$\mathcal{H} = \begin{pmatrix} M+\omega_0 & (\delta+i\delta_0) & \omega_1 & i\delta_1 \\ (\delta-i\delta_0) & M-\omega_0 & -i\delta_1 & -\omega_1 \\ \omega_1 & i\delta_1 & M-\omega_0 & -(\delta+i\delta_0) \\ -i\delta_1 & -\omega_1 & -(\delta-i\delta_0) & M+\omega_0 \end{pmatrix} . \tag{20}$$

The transition magnetic moment terms δ_0 and δ_1 are of higher mass dimension and ought be much smaller in effect than δ, despite the appearance of an external magnetic field. This follows because the energy scales associated with magnetic fields are naturally so small — note that $|\mu_n| \approx 60\,\mathrm{neV/T}$. We employ naive dimensional analysis to flesh out our assessment. That is, we estimate the n-$\bar{n}$ matrix element associated with the leading operator, of mass dimension nine, as $\kappa\Lambda_{\rm QCD}^6/M_{n\bar{n}}^5$ [14], where κ is a dimensionless constant presumably of $\mathcal{O}(1)$, $M_{n\bar{n}}$ is the scale of n-$\bar{n}$ mixing, and $\Lambda_{\rm QCD} \sim 200$ MeV. Writing $\mu_{n\bar{n}}B = (\mu_{n\bar{n}}/|\mu_n|)|\mu_n|B$, noting $\mu_{n\bar{n}}/|\mu_n| \sim \kappa'(\Lambda_{\rm QCD}/M_{n\bar{n}})^7$ with κ' a dimensionless constant, we estimate $\mu_{n\bar{n}}B/\delta \sim (\kappa'/\kappa)\Lambda_{\rm QCD}|\mu_n|B/M_{n\bar{n}}^2$. Even in the environment of a pulsar, for which $B \sim 10^8$ T is possible, we see that $|\mu_n|B$ is many orders of magnitude smaller than $\Lambda_{\rm QCD}$ — so that $\mu_{n\bar{n}}B$ is negligible relative to δ if we assume $\kappa'/\kappa \sim \mathcal{O}(1)$.

Before closing this section we note that it is also possible to have a n-$\bar{n}$ transition electric dipole moment as well, though this would certainly require an additional new physics mechanism to generate an appreciable effect. The n-$\bar{n}$ matrix elements of $\psi^T\gamma_5\sigma^{\mu\nu}C\psi F_{\mu\nu}$ and its Hermitian conjugate yields terms of the form given in Eqs. (14) and (15), but with $-\mathbf{B}$ replaced with $i\mathbf{E}$. These operators are **CP** and **T** even but **P** odd.

4. Examples. In what follows we consider concrete examples of how applied magnetic fields can be used to evade the quenching of n-$\bar{n}$ oscillations found in earlier work [9, 10]. We consider the leading n-$\bar{n}$ transition operator matrix element exclusively, so that we rely on SM effects to realize this. To compute the transition probabilities, we must first find the normalized eigenvectors of the Hamiltonian matrix in terms of our chosen $\{|n+\rangle, |n-\rangle, |\bar{n}+\rangle, |\bar{n}-\rangle\}$ basis; we denote a state of the latter by $|n_i\rangle$ and a normalized eigenvector by $|u_i\rangle$ with associated eigenvalue λ_i, noting $i \in 1, \ldots, 4$. The time evolution of a state of the Hamiltonian is thus given by

$$|\psi(t)\rangle = \sum_{i=1}^{4} e^{-i\lambda t}\langle u_i|\psi(0)\rangle\,|u_i\rangle\,. \tag{21}$$

Letting $|\psi(0)\rangle = |n_k\rangle$ and defining $a_{ij} \equiv \langle n_j|u_i\rangle$, we find

$$\mathcal{P}_{n_k\to n_j} = \left|\sum_{i=1}^{4} e^{-i\lambda_i t}a_{ij}a_{ik}^*\right|^2\,. \tag{22}$$

For reference, we find in the absence of magnetic fields that $\mathcal{P}_{n\to\bar{n}} = \sin^2(\delta t)$, identical to that found using Eq. (1) [9].

As a first example, we consider a system with a static magnetic field $\mathbf{B}_0$, serving as the quantization axis, to which a static transverse field $\mathbf{B}_1$ is suddenly applied at $t=0$. For $t>0$ the mass matrix has the form of Eq. (20) with $\delta_0 = \delta_1 = 0$. Noting that $|\delta| \ll |\omega_0|\,, |\omega_1|$, we find that the probability of a neutron in a $s=+$ state transforming to $\bar{n}$ of fixed spin is

$$
\begin{aligned}
\mathcal{P}_{n+\to\bar{n}+}(t) = \delta^2 \Bigg[& \frac{\omega_1^4 t^2}{(\omega_0^2+\omega_1^2)^2} \cos^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) + \frac{\omega_0^4}{(\omega_0^2+\omega_1^2)^3} \sin^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) \\
& + \frac{\omega_0^2\omega_1^2 t}{(\omega_0^2+\omega_1^2)^{5/2}} \Bigg] + \mathcal{O}(\delta^3); \qquad (23) \\
\mathcal{P}_{n+\to\bar{n}-}(t) = \delta^2 \Bigg[& \frac{\omega_1^2 t^2}{\omega_0^2+\omega_1^2} - \frac{\omega_1^4 t^2}{(\omega_0^2+\omega_1^2)^2} \cos^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) \\
& + \frac{\omega_0^2\omega_1^2}{(\omega_0^2+\omega_1^2)^3} \sin^2\left(t\sqrt{\omega_0^2+\omega_1^2}\right) - \frac{\omega_0^2\omega_1^2 t}{(\omega_0^2+\omega_1^2)^{5/2}} \sin\left(2t\sqrt{\omega_0^2+\omega_1^2}\right) \Bigg] \\
& + \mathcal{O}(\delta^3) \,. \qquad (24)
\end{aligned}
$$

If $|\omega_0| \sim |\omega_1|$, we see that the last two terms of Eqs. (23) and (24) are of $\mathcal{O}(\delta^2/\omega_0^2)$ and $\mathcal{O}(t\delta^2/\omega_0)$, respectively, so that they are indeed quenched in a magnetic field. The other terms, however, are of $\mathcal{O}(1)$. We note that $\mathcal{P}_{n+\to\bar{n}-}(t)$ is larger, since $\omega_1^2/(\omega_0^2+\omega_1^2) > (\omega_1^2/(\omega_0^2+\omega_1^2))^2$ in this limit — we had anticipated this because the two states are of the same energy. We note that $\mathcal{P}_{n+\to\bar{n}-}(t) = \mathcal{P}_{n-\to\bar{n}+}(t)$ and $\mathcal{P}_{n+\to\bar{n}+}(t) = \mathcal{P}_{n-\to\bar{n}-}(t)$, so that the unpolarized transition probability is

$$
\begin{aligned}
\mathcal{P}_{n\to\bar{n}}(t) = \delta^2 \Bigg[& \frac{\omega_1^2 t^2}{\omega_0^2+\omega_1^2} + \frac{\omega_0^2}{(\omega_0^2+\omega_1^2)^2} \sin^2(t\sqrt{\omega_0^2+\omega_1^2}) \\
& + \frac{\omega_0^2\omega_1^2 t}{(\omega_0^2+\omega_1^2)^{5/2}} \left(1 - \sin\left(2t\sqrt{\omega_0^2+\omega_1^2}\right)\right) \Bigg] + \mathcal{O}(\delta^3) \,, \qquad (25)
\end{aligned}
$$

— and the first term is of $\mathcal{O}(1)$. For reference, $\mathcal{P}_{n+\to n-}(t) = (\omega_1^2/(\omega_0^2+\omega_1^2)) \sin(t\sqrt{\omega_0^2+\omega_1^2}) + \mathcal{O}(\delta^2)$. The exact eigenvalues and eigenstates for $t > 0$ are

$$
\begin{aligned}
E_1 &= M_1 - \sqrt{\omega_0^2 + (\delta - \omega_1)^2} \,, \\
E_2 &= M_1 + \sqrt{\omega_0^2 + (\delta - \omega_1)^2} \,, \\
E_3 &= M_1 - \sqrt{\omega_0^2 + (\delta + \omega_1)^2} \,, \\
E_4 &= M_1 + \sqrt{\omega_0^2 + (\delta + \omega_1)^2} \qquad (26)
\end{aligned}
$$

and

$$
\begin{aligned}
u_1 &= \frac{1}{\sqrt{N_1}}\left\{1, \frac{(\delta-\omega_1)}{\omega_0-\sqrt{\omega_0^2+(\delta-\omega_1)^2}}, \frac{-(\delta-\omega_1)}{\omega_0-\sqrt{\omega_0^2+(\delta-\omega_1)^2}}, 1\right\}, \\
u_2 &= \frac{1}{\sqrt{N_2}}\left\{1, \frac{(\delta-\omega_1)}{\omega_0+\sqrt{\omega_0^2+(\delta-\omega_1)^2}}, \frac{-(\delta-\omega_1)}{\omega_0+\sqrt{\omega_0^2+(\delta-\omega_1)^2}}, 1\right\}, \\
u_3 &= \frac{1}{\sqrt{N_3}}\left\{-1, \frac{-(\delta+\omega_1)}{\omega_0-\sqrt{\omega_0^2+(\delta+\omega_1)^2}}, \frac{-(\delta+\omega_1)}{\omega_0-\sqrt{\omega_0^2+(\delta+\omega_1)^2}}, 1\right\}, \\
u_4 &= \frac{1}{\sqrt{N_4}}\left\{-1, \frac{-(\delta+\omega_1)}{\omega_0+\sqrt{\omega_0^2+(\delta+\omega_1)^2}}, \frac{-(\delta+\omega_1)}{\omega_0+\sqrt{\omega_0^2+(\delta+\omega_1)^2}}, 1\right\},
\end{aligned}
\tag{27}
$$

with

$$
\begin{aligned}
N_{\frac{1}{2}} &= 2\left[1+\frac{(\delta-\omega_1)^2}{\omega_0\mp\sqrt{\omega_0^2+(\delta-\omega_1)^2}}\right], \\
N_{\frac{3}{4}} &= 2\left[1+\frac{(\delta+\omega_1)^2}{\omega_0\mp\sqrt{\omega_0^2+(\delta+\omega_1)^2}}\right].
\end{aligned}
\tag{28}
$$

If $\delta = 0$ *or* $\omega_0 = \omega_1 = 0$, we see that $E_1 = E_3$ and $E_2 = E_4$. In the former case, $u_1 + u_3$ and $u_2 + u_4$ yield linear combinations of $\bar{n}(+)$ and $\bar{n}(-)$, and $u_1 - u_3$ and $u_2 - u_4$ yield linear combinations of $n(+)$ and $n(-)$. In contrast, in the latter case, we find Majorana states; that is, $u_1 \pm u_3 \propto \Psi_M^{\pm}(\mp)$ and $u_2 \pm u_4 \propto \Psi_M^{\mp}(\mp)$.

As long known, the spin of a macroscopic sample of fermions can be made to flip through the use of magnetic resonance techniques. Indeed, supposing the spins are aligned (or anti-aligned) with a static magnetic field, and an oscillatory magnetic field is applied transverse to it, we can tune the frequency of the transverse field in such a way that the probability of flipping the neutron spin is of $\mathcal{O}(1)$ irrespective of the size of the applied magnetic fields — this is the famous Rabi formula [17, 18]. Thus as a second example we study n-$\bar{n}$ oscillations in such a magnetic field arrangement [18], replacing B_1 with a time-dependent magnetic field $B_1(t)$, so that the SM Hamiltonian for a neutron becomes $H(t) = \omega_0\,\sigma_z + \omega_1(\cos\omega t\,\sigma_x + \sin\omega t\,\sigma_y)$. The resulting n-$\bar{n}$ Hamiltonian matrix is of form

$$
\mathcal{H}(t) = \begin{pmatrix} M+\omega_0 & \delta & \omega_1 e^{-i\omega t} & 0 \\ \delta & M-\omega_0 & 0 & -\omega_1 e^{-i\omega t} \\ \omega_1 e^{i\omega t} & 0 & M-\omega_0 & -\delta \\ 0 & -\omega_1 e^{i\omega t} & -\delta & M+\omega_0 \end{pmatrix}.
\tag{29}
$$

To compute the transition probabilities in this case, we solve the time-dependent Schrödinger equation $i\partial_t\psi = \mathcal{H}\psi$ with $\psi = \{a_+(t), \bar{a}_+(t), a_-(t), \bar{a}_-(t)\}$ through the change of variable $\overset{(-)}{a}_{\pm} = \overset{(-)}{b}_{\pm} \exp(\mp i\omega t/2)$. This yields $i\partial_t\tilde{\psi} = \tilde{\mathcal{H}}\tilde{\psi}$ with $\tilde{\psi} = \{b_+(t), \bar{b}_+(t), b_-(t), \bar{b}_-(t)\}$ and

$$\tilde{\mathcal{H}} = \begin{pmatrix} M - \Delta\omega_- & \delta & \omega_1 & 0 \\ \delta & M - \Delta\omega_+ & 0 & -\omega_1 \\ \omega_1 & 0 & M + \Delta\omega_- & -\delta \\ 0 & -\omega_1 & -\delta & M + \Delta\omega_+ \end{pmatrix} \tag{30}$$

with $\Delta\omega_\pm \equiv \omega/2 \pm \omega_0$, noting that the transition probabilities of interest follow immediately from its solution because $\mid \overset{(-)}{a}_{\pm} \mid^2 = \mid \overset{(-)}{b}_{\pm} \mid^2$. The oscillatory transverse field needed for magnetic resonance experiments is typically realized, however, through the application of a radio frequency (rf) field with linear polarization, so that if $\Delta\omega_+ = 0$, then $\Delta\omega_- = 0$ also. Thus under usual experimental conditions the largest contributions have $\Delta\omega_+ = -\Delta\omega_-$, and the n-$\bar{n}$ transition probabilities can be estimated from Eqs. (23) and (24) upon the replacement $\omega_0 \to \Delta\omega_+$. On resonance, for which $\Delta\omega_\pm = 0$, we have

$$\mathcal{P}_{n+\to\bar{n}+}(t) \approx \delta^2 t^2 \cos^2\left(t\sqrt{\omega_0^2 + \omega_1^2}\right) + \mathcal{O}(\delta^3); \tag{31}$$

$$\mathcal{P}_{n+\to\bar{n}-}(t) \approx \delta^2 t^2 \sin^2\left(t\sqrt{\omega_0^2 + \omega_1^2}\right) + \mathcal{O}(\delta^3)\,, \tag{32}$$

where we have neglected contributions controlled by $|\omega|/2 + \omega_0$ as per standard practice [33]. Finally, we find, similarly, that the unpolarized transition probability is $\mathcal{P}_{n\to\bar{n}}(t) \approx \delta^2 t^2 + \mathcal{O}(\delta^3)$.

6. New Experimental Prospects. We have shown through explicit example that the removal of magnetic fields is not necessary for the observation of n-$\bar{n}$ oscillations; this opens new possibilities for their experimental discovery. For example, it becomes possible to study n-$\bar{n}$ oscillations by confining neutrons in magnetic traps, or bottles; such are under development for improved measurements of the neutron lifetime [34–36]. In a gravitomagnetic trap a single spin state is confined; we suppose, in addition, that a transverse rf field at resonance is applied. If the spin-flip time is short compared to the time for a confined neutron to be lost from the trap, we suppose that the storage time determined under these conditions can be used to set a limit on n-$\bar{n}$ oscillations. That is, an experimental limit on n-$\bar{n}$ oscillations can be defined by writing the transition probability as $\mathcal{P}_{n\to\bar{n}} \simeq (t/\tau_{n\bar{n}})^2$ and bounding $\tau_{n\bar{n}}$. A crude estimate of the oscillation lifetime is given by $(\tau_{n\bar{n}})_{\rm bottle} \sim \left(N_{\rm fill} N_{\rm trial} \langle t^2\rangle / \bar{N}\right)^{1/2}$, where $N_{\rm fill}$ is the number of neutrons (i.e., nV with n the neutron number density and V the volume of the trap) added to the bottle at one time, $N_{\rm trial}$ is the number of times the trap is filled, $\bar{N}$ is the limit on the number of antineutrons detected, and $\langle t^2\rangle^{1/2}$ is the storage time in the trap. Estimating $N_{\rm fill} \sim 10^7$, $N_{\rm trial} \sim 10^5$, and $\langle t^2\rangle^{1/2} \sim 400\,{\rm s}$ and using $\bar{N} \leq 2.3$ at 90% C.L. [15] yields $\tau_{n\bar{n}} \sim 2 \times 10^8\,{\rm s}$, so that the gain seems modest over the existing limit of $\tau_{n\bar{n}} \geq 0.86 \times 10^8\,{\rm s}$ at 90% C.L. [15], though one can expect further improvements with bettered ultracold neutron sources.

7. Summary. As long recognized, the discovery of $\mathcal{B} - \mathcal{L}$ violation would speak to the existence of Majorana dynamics in Nature. This would not imply, however, that the neutron is its own antiparticle, but, rather, that the weak interaction eigenstates of the n-$\bar{n}$ system in vacuum transform into themselves under the charge conjugation operator $\mathbf{C}$. Although many authors [37–41] have studied the impact of external magnetic fields on n-$\bar{n}$ oscillations within the context of the 2×2 phenomenological framework [9], our work is the first to incorporate spin in a fundamental way. The results that emerge are remarkably different from earlier studies — in particular, magnetic field mitigation is not required to observe n-$\bar{n}$ mixing, as had been previously thought [14, 16].

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