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The Role of the Electron Mass in Damping Chiral Plasma Instability in Supernova and Neutron Stars

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We show that the nonzero electron mass plays a critical role in determining the magnetic properties of neutron stars by suppressing the generation of the chiral charge density needed to trigger a strong Chiral Plasma Instability during the core collapse of supernovae. This instability has been proposed as a plausible mechanism for generating extremely large helical magnetic fields in neutron stars at their birth; the mechanism relies on the generation of a large non-equilibrium chiral charge density via electron capture reactions that selectively deplete left-handed electrons during core-collapse and the early evolution of the protoneutron star. Our calculation shows that the electron chirality violation rate induced by Rutherford scattering, despite being suppressed by the smallness of the electron mass relative to the electron chemical potential, is still fast compared to the weak interaction electron capture rate. The resulting asymmetry between right and left-handed electron densities is therefore unlikely to attain an astrophysically relevant magnitude.

The inference of extreme surface magnetic fields $B_S \simeq 10^{14} - 10^{15}$ G from observations of a class of neutron stars called magnetars [1] raises many questions about how and when such fields are generated. In the conventional scenario, they are expected to arise either due to strong hydrodynamical or magnetohydrodynamic instabilities during core-collapse supernova, or during the early evolution of the proto-neutron star [2–4]. Other mechanisms, which rely on a spontaneous magnetization of the ground state of strongly interacting matter at extreme density, have also been proposed; these remain speculative due to large theoretical uncertainties. Recently in [5], it was suggested that Chiral Plasma Instability (CPI) [6] could be used to generate large fields. In this intriguing scenario, a net chiral charge is produced during core collapse of the progenitor star. As matter is compressed during core collapse, left-handed electrons are captured by protons due to the weak interaction, which results in an imbalance between the Fermi energies of left-handed and right-handed electrons. This imbalance triggers an instability that equilibrates the two chiralities and the released energy drives the growth of a coherent magnetic field. Key to this analysis is the assumption that the electron mass, which explicitly violates chirality, can be neglected [5]. The authors argue that this is a reasonable approximation because the electron mass $m_e = 0.51$ MeV is much smaller than the typical electron Fermi momentum $p_{\rm Fe} \simeq 100$ MeV encountered in supernova and neutron stars. We revisit this assumption here and claim that in fact the electron mass cannot be neglected as it leads to chiral charge equilibration much faster than the weak interactions can create an asymmetry, and that therefore this mechanism does not lead to astrophysical interesting magnetic fields.

We start by reviewing the Chiral Plasma Instability for massless electrons with only electromagnetic interactions. In this case, chiral symmetry is only violated by quantum effects (the anomaly), and at the classical level left and right handed electron numbers are separately conserved. Absent the chiral anomaly, inverse beta decay during core collapse of the neutron star progenitor leads to a net chiral charge in the resultant neutron star. Already in 1980, Vilenkin had realized that a net chiral charge density in the plasma would result in an anomalous current of the form

$$\vec{J} = \frac{2\alpha}{\pi}\mu_5 \vec{B}\,, \tag{1}$$

where $\mu_5 = \mu_R - \mu_L$ is the chemical potential associated with the chiral charge density, and μ_R and μ_L are the chemical potentials associated with the right and left handed massless particles, $\vec{B}$ is the magnetic field, and $\alpha = e^2/4\pi$ is the fine structure constant [6]. Vilenkin, and soon afterwards, Rubakov and Tavkhelidze also recognized that the ground state with a chiral charge density would be unstable, and that this instability would resolve by generating a magnetic field [6–8]. More recently, there has been renewed interest in similar phenomena in the context of relativistic heavy-ion collisions, and is now widely known as the Chiral Magnetic Effect [9].

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We will refer to Eq. 1 as the chiral magnetic current; it can be derived from a parity violating effective action for the gauge fields (in the plasma rest frame) of the form

$$\int d^4x\, d^4y\, g(x-y)\epsilon^{0ijk} A_i(y)\partial_j A_k(x)$$

where $g(x-y)$ is in general nonlocal and proportional to μ_5 with the chiral plasma current arising from the leading term in a derivative expansion of g. [8] The origin of the chiral magnetic current is easy to understand: in a constant magnetic field electrons occupy Landau levels, where each Landau level can be viewed as a 1+1 dimensional Dirac fermion traveling along the direction of the magnetic field; the excited levels contain electrons of both spin polarizations, while the lowest Landau level only contains electrons with spin anti-aligned with the field. At nonzero μ_5 it follows that there is a difference between the density of particles in the lowest Landau level moving parallel to the magnetic field (LH chirality) versus antiparallel (RH chirality), and hence there exists an electric current in the direction of the magnetic field, $\vec{B}$. It is given by the $1+1$ dimensional current density in the magnetic field direction, $(e\mu_5/2\pi)$, times the transverse density of the lowest Landau orbits, (eB/π) (see derivation in [10], for example). Nonzero μ_5 also forces a chiral asymmetry in the excited Landau levels, but as these levels contain electrons of both polarizations they do not contribute to the electric current. No mention has been made of the anomaly, but the Landau level picture of the anomaly in 3+1 dimensions shows that the two are intimately related [1].

When modified to incorporate the chiral magnetic current, Eq. (1), and finite electrical conductivity, Maxwell's equations read

$$\frac{\partial \vec{B}}{\partial t} = -\nabla\times\vec{E} \tag{2}$$

$$\nabla\times\vec{B} - \frac{\partial \vec{E}}{\partial t} = \sigma\ \vec{E} + \frac{2\alpha}{\pi}\mu_5\ \vec{B} \equiv \vec{J}\,, \tag{3}$$

where σ is the electrical conductivity, scaling as μ_e/α. The normal component of the electric current is given by $J = \sigma\ \vec{E}$ because as we shall show below the electron mean free path is short compared to other the length scales in the problem. Assuming constant σ and μ_5, and ignoring the $\partial\vec{E}/\partial t$ term (justified below) we combine the above equations to obtain

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\sigma}\ \nabla^2\vec{B} + \frac{2\alpha}{\pi\sigma}\ \mu_5\nabla\times\vec{B}\,, \tag{4}$$

which describes the time evolution of $\vec{B}$ in the presence of the Chiral Plasma current. The unstable modes are characterized by the vector potential

$$\vec{A}_\pm = (\hat{x} \pm i\hat{y})\ e^{(ikz-i\omega t)}\ , \tag{5}$$

which corresponds to electric fields $\vec{E}_\pm = i\omega\vec{A}_\pm$ and magnetic fields $\vec{B}_\pm = \pm k\vec{A}_\pm$, where the $\pm$ subscript denotes the helicity of the fields for positive k. The wavenumber k and the frequency $\Re[\omega]$ are constants. Eq. (3) has exponentially growing solutions, whose helicity depend on the sign of μ_5, with amplitude

$$B_k(t) = B_k(0)e^{tk(2k_\star - k)/\sigma}\ , \qquad k_\star = \frac{\alpha\mu_5}{\pi} \tag{6}$$

for $0 < k < 2k_\star$, where $B_k(0)$ is the initial magnetic field – either a thermal fluctuation, or the field inherited from the progenitor star. Note that the terms kept in Eq. (4) are proportional to $k_\star^2/\sigma$, while the term $\partial\vec{E}/\partial t$ neglected in going from Eq. (3) to Eq. (4) is smaller by a factor of $\omega/k = k_\star/\sigma = O(\alpha^2)$; similarly the neglected plasma frequency of the photon has a negligible effect on the growing mode solution. The maximally unstable mode occurs for $k = k_\star$, with that mode growing

[1] See J. Preskill's lecture notes on Quantum Chromodynamics at http://www.theory.caltech.edu/~preskill/notes.html, pp. 3.43-3.45, or else the derivation in [10].

as

$$B_\star(t) = B_\star(0)\exp\left(\Gamma_{\rm CPI}\, t\right)\,, \qquad \Gamma_{\rm CPI} = \frac{k_\star^2}{\sigma} = \frac{\alpha^2\mu_5^2}{\pi^2\sigma}\,. \tag{7}$$

For a recent discussion of this instability in the context of the high temperature plasmas and matter encountered in the early universe see Ref. [11–13].

The local evolution of the chiral charge density is described by the anomaly equation

$$\partial_\mu j_5^\mu = -\frac{\alpha}{2\pi}F_{\alpha\beta}\tilde{F}^{\alpha\beta} = -\partial_\mu K^\mu\,, \qquad K^\mu = \frac{\alpha}{\pi}\epsilon^{\mu\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma\,. \tag{8}$$

Integrating over space and assuming fields vanish at spatial infinity yields the conservation law

$$\frac{d}{dt}\left(n_5 + \frac{\alpha}{\pi}H\right) = 0\,, \qquad H = \frac{1}{V}\int d^3x\,\vec{A}\cdot\vec{B}\,, \tag{9}$$

where $n_5 = N_5/V$ is the average chiral charge density, V is the volume, and H is the gauge invariant "helicity density". Note that a time-dependent helicity implies a nonzero electric field, and thus the above equation can be simply understood as the conventional effect of an electric field changing the momenta of electrons in the lowest Landau level.

Since the field Eq. (5) has nonzero helicity, the growth of the unstable mode converts electron chiral charge density n_5 into electromagnetic helicity H at a rate

$$\frac{\partial n_5}{\partial t} = -\frac{\alpha}{\pi}\frac{dH}{dt} = -\frac{2\alpha\Gamma_{\rm CPI}}{\pi k_\star}\,B_\star(t)^2 = -\frac{2\alpha^2\mu_5}{\pi^2\sigma}\,B_\star(t)^2 \equiv -\Gamma_B n_5\,, \tag{10}$$

where $B_\star(t)$ is given in eq. (7). The free energy in the magnetic field is supplied by the imbalance of Fermi energy between left and right handed electrons. In time, μ_5 is driven to zero locally, and a global helical magnetic field that spontaneously breaks rotational symmetry is generated. As we elaborate on below, this is the phenomena essential to the proposed mechanism for generating large magnetic fields during the supernova in Ref. [5]. However, one immediately sees a problem with using the CPI to directly generate large coherent magnetic fields on astrophysical scales: for long wavelength magnetic fields, $k_\star$ must be exceedingly small compared to μ_e, as must to a lesser extent $\mu_5 = \pi k_\star/\alpha$. This in turn implies both that the growth rate $\Gamma_{\rm CPI}$ would be very slow and that the total amount of electron energy available for conversion to magnetic field energy would be very small. For example, for $k_\star \sim (100\text{ m})^{-1}$ one finds $\mu_5 \sim 10^{-6}$ eV and $\Gamma_{\rm CPI} \sim (1\text{ yr})^{-1}$. While a large value of μ_5 can produce a large field with short wavelength, a subsequent mechanism is needed to convert this into a coherent field on a macroscopic length scale. Although such a mechanism, called the inverse cascade, has been proposed in Ref. [14], in what follows we show that in the supernova large values of μ_5 are unlikely.

In order to find out what actually happens, we need to estimate how large μ_5 gets in a core collapse supernova, and to do this we need to consider massive electrons. Now the anomaly equation, eq. (8), is modified to include explicit chiral symmetry breaking due to the electron mass

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma_5\psi - \frac{\alpha}{2\pi}F_{\alpha\beta}\tilde{F}^{\alpha\beta}\,. \tag{11}$$

It is not particularly simple to use this equation directly to compute rates in a plasma, since single particle asymptotic states are no longer eigenstates of chirality. Instead it is useful to discuss electron helicity eigenstates, as helicity is exactly conserved for any electron mass in the absence of interactions. For free massive electrons in a multi-electron state of definite helicity $|h\rangle$, the expectation value $\langle h|n_5|h\rangle$ is time-independent since $|h\rangle$ is a stationary state, despite n_5 not commuting with the Hamiltonian. In fact, this expectation value is given by the sum of helicity times the magnitude of the velocity ($|p|/E$) for each electron — a result that goes smoothly to the $m = 0$ limit, since in that limit all electrons have $|p|/E = 1$ and helicity becomes synonymous with chirality. We can now turn on interactions and see how the evolution of $|h\rangle$ due to electron helicity flipping interactions leads to a time dependence of the expectation value of n_5, where

$$n_5(t) = \int\frac{d^3k}{(2\pi)^3}\left[f_+(k,t) - f_-(k,t)\right]\frac{|\mathbf{k}|}{\omega_{\mathbf{k}}}\,, \tag{12}$$

$f_\pm(k,t)$ being the electron occupation number in a state with momentum k and $\pm$ helicity.

We make the assumption that deviations of $f_\pm(k,t)$ from equilibrium are small, and use linear response with

$$f_\pm(k,t) = f(k) \pm \frac{\partial f(k)}{\partial \mu}\delta\mu_5(k,t) \simeq f(k) \pm \frac{\partial f(k)}{\partial \mu}\delta\bar{\mu}_5(t) \ , \tag{13}$$

where $\delta\bar{\mu}_5(t)$ is k-independent and $f(k)$ is the equilibrium Fermi-Dirac distribution

$$f(k) = \frac{1}{1+e^{-\beta(\omega_k-\mu_e)}} \ , \qquad \omega_k = \sqrt{k^2+m_e^2} \ . \tag{14}$$

For the first part of eq. (13) we simply assumed $|\delta\mu_5(k,t)| \ll \mu_e$ for all k, an approximation which will be seen to be self-consistent, as the equilibration of n_5 to zero due to electron helicity changing scattering is found to be much faster than the rate of change of n_5 arising from either the CPI or the weak interactions. For the second part of eq. (13) we used the fact that $\partial f/\partial\mu$ only has support for $|k-\mu_e| \lesssim T$, and assumed that $\delta\mu_5(k,t)$ was roughly independent of k in this region, allowing the replacement $\delta\mu_5(k,t) \to \delta\bar{\mu}_5(t)$. This latter assumption is justified by the fact that helicity preserving scattering will be fast (not suppressed by the small electron mass) and so the positive and negative helicity electrons will each be in independent approximate quasi-static thermal equilibrium. Given eq. (13) we can express n_5 as

$$n_5(t) \simeq 2\delta\bar{\mu}_5(t)\int \frac{d^3k}{(2\pi)^3}\frac{\partial f(k)}{\partial\mu}\frac{|\mathbf{k}|}{\omega_\mathbf{k}} \simeq \frac{\delta\bar{\mu}_5(t)\mu_e^2}{\pi^2}, \qquad \Longrightarrow \qquad \frac{\dot{n}_5}{n_5} \simeq \frac{\delta\dot{\bar{\mu}}_5}{\delta\bar{\mu}_5} \ , \tag{15}$$

where again we made use of the fact that $\partial f(k)/\partial\mu$ is sharply peaked at $|k-\mu_e| \lesssim T$, with $m_e, T \ll \mu_e$. We will refer to the contribution to $\dot{n}_5/n_5$ arising from electron helicity changing scattering as $-\Gamma_m$, since these contributions must vanish at zero electron mass.

We find that helicity changing Rutherford scattering of electrons off the ambient protons to be the dominant contribution to Γ_m. Other contributions come from electron-electron scattering and Compton scattering, but the former is expected to be suppressed relative to Rutherford scattering due to the fact that electrons are far more degenerate than protons, while the latter is relatively suppressed since the proton density scales as μ_e^3, while the ambient photon density scales as T^3, where T is the temperature and $T/\mu_e \lesssim 1/10$ during the core collapse. From the Boltzmann equation, in the approximation of eq. (13), we find

$$\frac{\partial_t\,\delta\mu_5(k,t)}{\delta\mu_5(k,t)} = -2\frac{1}{2\omega_\mathbf{k}}\int\frac{d^3k'}{(2\pi)^3 2\omega_{\mathbf{k}'}}\frac{1+e^{\beta(\omega_\mathbf{k}-\mu_e)}}{1+e^{\beta(\omega_{\mathbf{k}'}-\mu_e)}}\,W(k,k')_{+-} \tag{16}$$

where

$$W(k,k')_{hh'} = \int\frac{d^3p\,d^3p'}{(2\pi)^3 2\omega_\mathbf{p}(2\pi)^3 2\omega_{\mathbf{p}'}}|\mathcal{M}_{hh'}|^2(2\pi)^4\delta^4(p+k-p'-k')f(p')\,(1-f(p)) \tag{17}$$

for electron scattering with incoming and outgoing momentum and helicity $(\mathbf{k},h)$ and $(\mathbf{k}',h')$ respectively, where p, p' are the proton momenta, and $\mathcal{M}_{hh'}$ is the Rutherford scattering amplitude averaged and summed over incoming and outgoing proton spins. Neglecting proton recoil (suppressed by μ_e/M_p) one finds

$$|\mathcal{M}|^2_{+-} = 128\pi^2\alpha^2\frac{E_\mathbf{p}^2 m^2\,(1-\cos\theta)}{\left(2k^2(1-\cos\theta)+q_D^2\right)^2} \tag{18}$$

where θ is the scattering angle, E_p is the proton energy, and the inverse Debye screening length q_D provides an infrared cutoff to the scattering process. Inserting this expression into eq. (16) and evaluating at $k=\mu_e$, where $\partial f/\partial\mu$ is peaked, we find

$$\Gamma_m = -\left(\frac{\delta\dot{\bar{\mu}}_5}{\delta\bar{\mu}_5}\right)_{\text{Ruth.}} = -\left(\frac{\partial_t\,\delta\mu_5(k,t)}{\delta\mu_5(k,t)}\right)_{\text{Ruth.}}\Bigg|_{k=\mu_e} \simeq \frac{\alpha^2 m_e^2}{3\pi\mu_e}\left[\ln\frac{4}{x}-1\right] \ , \qquad x \equiv \frac{q_D^2}{\mu_e^2} \ . \tag{19}$$

Because proton degeneracy and recoil can be neglected, this result coincides with the simpler expression $\Gamma_m = n_p\sigma_\mathrm{R}(\mu_e)$, where n_p is the proton density and $\sigma_{\rm R}(\mu_e)$ is the Rutherford cross section for electrons on the Fermi surface. Noting that $q_D^2 = 4\pi\alpha\ \partial^2\Omega/\partial\mu_e^2$, where Ω is the total free energy of the plasma, and that at the fiducial density and temperature characteristic of the supernova, the electrons can be treated as degenerate and protons as non-degenerate, we find that $x = 4\alpha(1+(\mu_e/3T))/\pi$. For $\mu_e = 100$ MeV and $T = 30$ MeV we find $\Gamma_m \simeq 6\times 10^{-8}$ MeV $\simeq 10^{14}/$ s.

The equation for the local evolution of the net helicity density including helicity flipping and electron capture rates is given by

$$\frac{1}{n_5}\frac{\partial n_5}{\partial t} = -\Gamma_B - \Gamma_m + \frac{n_e}{n_5}\Gamma_w \tag{20}$$

where Γ_w is the rate of depletion of the electron fraction Y_e due to electron capture via charged current interactions during core collapse. Although Γ_w is density and temperature dependent, and thus governed by complex supernova dynamics, a nearly model independent upper bound can be be derived by noting that the total change during core collapse $\delta Y_e \simeq 0.4$ occurs on a time scale that is greater than the free-fall timescale $t_{\text{free-fall}} \simeq 100$ ms. Therefore,

$$\Gamma_w = \frac{\dot{Y}_e}{Y_e} < 10\ \ \mathrm{s}^{-1}\,. \tag{21}$$

However, simulations indicate that the typical value is $\Gamma_w \simeq 1\ \mathrm{s}^{-1}$ [15] and we use this to make numerical estimates in the following calculations. Γ_m is the equilibration rate of n_5 due to explicit chiral symmetry breaking by the electron mass, given above in eq. (19), and Γ_B is the anomalous depletion rate of n_5 due the conversion of n_5 into magnetic field via the CPI. We derived a formula for Γ_B in the massless electron limit in eq. (10), in the presence of a chemical potential μ_5. In the realistic case with nonzero electron mass, chirality is only approximately conserved, and there is no chemical potential for chirality. Instead there is the effective $\delta\bar{\mu}_5(t)$ computed in eq. (15). However, simply substituting this into eq. (10) is not valid in general, since the growing mode solution eq. (7) was derived assuming a constant μ_5, which can be thought of as allowing the heat bath to provide an infinite source of energy for the growing magnetic field.

The case where it is approximately valid to use eq. (10) with the substitution $\mu_5 \to \delta\bar{\mu}_5(t)$ is when the CPI effect has a negligible effect on the background chiral density n_5. We will investigate this regime and show that it is in fact a self-consistent solution during core collapse. We first neglect Γ_B in eq. (20), in which case a fixed point solution is found where the slow production of n_5 from the weak interactions is balanced against the rapid equilibration of n_5 due to the nonzero electron mass:

$$n_5 = \frac{\Gamma_w}{\Gamma_m} n_e\ \sim 10^{-14} n_e\,. \tag{22}$$

Using eq. (15), this steady-state density corresponds to a very small time-independent effective chemical potential

$$\delta\bar{\mu}_5 = \frac{\pi^2 n_5}{\mu_e^2} = \frac{\pi^2 n_e \Gamma_w}{\mu_e^2\Gamma_m} \simeq \frac{\mu_e}{3}\frac{\Gamma_w}{\Gamma_m} \sim \frac{1}{3}10^{-14}\mu_e \tag{23}$$

We can now use this steady state value to compute the rate of magnetic field production, Γ_B, as well as the length scale of the unstable mode, $k_\star$. We find that $k_\star^{-1} = \pi/(\alpha\delta\bar{\mu}_5) \sim 250$ m for $\mu_e \simeq 100$ MeV which is astrophysically interesting. Since $k_\star^{-1}$ is a macroscopic length scale it is much larger than the electron mean free path and justifies our use of Eq. 3 to calculate the growth of the CPI. Using the above expression for $\delta\bar{\mu}_5$ in eq. (10) and eq. (7) to compute Γ_B, we find

$$\Gamma_B(t) = \frac{2\alpha^2}{\pi^2\sigma}\frac{\delta\bar{\mu}_5}{n_5} B_\star(t)^2 = \frac{2\alpha^2}{\sigma\mu_e^2} B_\star(t)^2 = \frac{2\alpha^2}{\sigma\mu_e^2} B_\star(0)^2 e^{2t\Gamma_{\rm CPI}}\ , \qquad \Gamma_{\rm CPI} = \frac{\alpha^2\delta\bar{\mu}_5^2}{\pi^2\sigma}\ . \tag{24}$$

The above expression for Γ_B is only valid to the extent that $\Gamma_B \ll \Gamma_w$, or else the fixed point solution eq. (22) – obtained by ignoring the effects of Γ_B – will not be correct. Such an inequality will break down eventually due to the exponential growth of $B_\star(t)$ if the prefactor proportional to the seed field $B_\star(0)^2$ at wavenumber $k_\star$ is sufficiently large and the time scale $\Gamma_{\rm CPI}^{-1}$ is sufficiently short compared to the duration of core collapse and the concomitant electron capture.

Both the prefactor and the exponential growth rate depend on the electrical conductivity σ, which is quite high in the supernova plasma. By assuming that protons are non-degenerate and uncorrelated we derive a lower bound

$$\sigma \gtrsim \sigma_{\min} = \frac{\mu_e}{4\alpha}\left[\ln\frac{4}{x} - 1\right]^{-1}, \qquad x \equiv \frac{q_D^2}{\mu_e^2}\ . \tag{25}$$

which implies that

$$\Gamma_B(0) = \frac{2\alpha^2}{\sigma\mu_e^2}B_\star(0)^2 \lesssim \frac{8\alpha^3}{\mu_e^3}\left[\ln\frac{4}{x} - 1\right]B_\star(0)^2 \sim \Gamma_m \times \left(\frac{B_\star(0)}{5\times 10^{14}\ \mathrm{G}}\right)^2 \tag{26}$$

and

$$\Gamma_{\mathrm{CPI}} = \frac{\alpha^2\delta\bar{\mu}_5^2}{\pi^2\sigma} \lesssim \frac{4\alpha^3}{9\pi^2}\left[\ln\frac{4}{x} - 1\right]\left(\frac{\Gamma_w}{\Gamma_m}\right)^2\mu_e \sim 6\times 10^{-34}\ \mathrm{MeV} \sim 10^{-12}\ \mathrm{s}^{-1}\ . \tag{27}$$

We see that at the beginning of the collapse, our assumption that Γ_B may be neglected compared to Γ_m is justified unless the initial seed field $B_\star(0)$ is already very large, around 10^{15} G. Furthermore, for more moderate initial magnetic fields, the extremely slow growth rate means that no exponential enhancement of the magnetic field occurs during the few seconds of core collapse. Finally it should be noted that if Γ_B was ever large compared to Γ_m, that would only serve to drive n_5 smaller, slowing the process down and driving it to smaller wave number $k_\star$. It is remarkable that the relatively large value obtained for Γ_m – which is proportional to m_e^2 – is responsible for damping out the Chiral Plasma Instability . To our knowledge, this is the first time that the fact that the electron is not massless has been shown to play a critical role in the structure and evolution of neutron stars.

In closing we comment on the idea that a permanent instability could persist in cold neutron matter due to the the neutral current interaction between electrons and neutrons, proportional to $G_F(\bar{e}\gamma^\mu\gamma_5 e)(\bar{n}\gamma_\mu n)$. It has been observed that in mean field theory, this term gives an effective contribution to the electron dispersion relation that resembles a chiral chemical potential, $(G_F n)(\bar{e}\gamma^0\gamma_5 e)$, where n is the neutron density. That such a term could lead to a magnetic instability was proposed in ref. [16], and considered but discarded much earlier by Vilenkin [17], who also considered the effects of rotation. While an attractive idea for generating the large magnetic fields observed in magnetars, we note the absence of an energy source for the growing magnetic field in this scenario, making Vilenkin's conclusion that such a mechanism does not work more intuitively plausible. This is to be contrasted with the scenario considered in this paper, where the growth of the helical magnetic field is powered by gravitational energy released during core collapse and temporarily stored in fermi energy of the left handed and right hand electrons, which are temporarily out of thermal equilibrium with each other; a mechanism that fails because the electron mass does not allow them to depart very far from equilibrium. Apparently what is needed to explain magnetars is a more efficient mechanism for transferring the gravitational energy released during collapse into electromagnetic energy.

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