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Hans Günter Dosch, Guy F. de Téramond, and Stanley J. Brodsky

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Superconformal Baryon-Meson Symmetry and Light-Front Holographic QCD

Hans Günter Dosch

Institut für Theoretische Physik, Philosophenweg 16, D-69120 Heidelberg, Germany[*]

Guy F. de Téramond

Universidad de Costa Rica, 11501 San Pedro de Montes de Oca, Costa Rica[†]

Stanley J. Brodsky

SLAC National Accelerator Laboratory,

Stanford University, Stanford, California 94309, USA[‡]

Abstract

We construct an effective QCD light-front Hamiltonian for both mesons and baryons in the chiral limit based on the generalized supercharges of a superconformal algebra. The superconformal construction is shown to be equivalent to a semi-classical approximation to light-front QCD and its embedding in AdS space. The specific breaking of conformal invariance inside the superconformal algebra uniquely determines the effective confinement potential. The generalized supercharges connect the baryon and meson spectra to each other in a remarkable manner. In particular, the π/b_1 Regge trajectory is identified as the superpartner of the nucleon trajectory. However, the lowest-lying state on this trajectory, the π-meson is massless in the chiral limit and has no supersymmetric partner.

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[*] h.g.dosch@thphys.uni-heidelberg.de

[†] gdt@asterix.crnet.cr

[‡] sjbth@slac.stanford.edu

I. INTRODUCTION

Light-front holographic QCD has brought important insights into hadron dynamics, especially to the color confinement problem. In Refs. [1, 2] a remarkable equivalence between the bound-state equations of the light-front Hamiltonian in 3+1 physical space-time [3] and those obtained in five-dimensional anti-de Sitter space (AdS_5) has been observed: The holographic coordinate z in AdS_5 space can be identified with the boost-invariant light front (LF) separation ζ between constituents [4]. This holographic equivalence allows one to relate the effective light-front potential for bosons to the symmetry-breaking factor introduced in AdS_5. In the case of integer spin fields, the breaking of the conformal isometries of AdS space can be done by introducing an additional model-dependent factor into the AdS action – a dilaton term $e^{\varphi(\zeta)}$. The specific form of the symmetry-breaking factor, however, is not fixed *a priori*, but it can be deduced from the comparison with the experimentally observed spectra. Linear Regge trajectories demand for $\varphi(\zeta)$ the form $\varphi(\zeta) = \lambda_M \, \zeta^2$ [5, 6]. The resulting LF effective potential is harmonic and confining, and it also includes a J-dependent constant term. This extra term is a consequence of the separation between kinematical and dynamical quantities for arbitrary spin [7], prescribed by the light-front mapping of AdS bound-state equations. The extra constant term has important phenomenological consequences; in particular, it leads in the chiral limit to a massless pion.

A large step forward in understanding why the effective potential must have the form of a confining harmonic potential was made by applying a method developed in conformal quantum mechanics by de Alfaro, Fubini and Furlan [8] (dAFF) to the light-front bound-state equations [9]. Starting from a conformally invariant action, a new Hamiltonian can be constructed as a superposition of the generators of the conformal algebra. Remarkably, the action remains conformally invariant, and the form of the resulting confining potential is uniquely fixed [9]. It has the form of a harmonic oscillator and corresponds to the quadratic dilaton term previously introduced by purely phenomenological arguments [5, 6]. However, the J-dependent constant term, referred to above, cannot be derived from the dAFF procedure. Furthermore, for half-integer spin a dilaton term in the AdS action does not lead to confinement [10], and therefore an additional Yukawa-like interaction term $\bar{\psi}\rho(\zeta)\psi$ has to be added to the fermionic action. This interaction term in the action leads to a potential $V(\zeta)$ in the corresponding Dirac equation, and again has to be determined phenomenologically –

one finds that the linear baryon Regge trajectories, with equal spacing in the orbital and radial excitations, as observed phenomenologically, requires the form $V(\zeta) = \lambda_B \, \zeta$ [11, 12].

Recently, we have shown [13] that a comparison of the half-integer LF bound-state equations with the Hamiltonian equations of superconformal quantum mechanics fixes the form of the LF potential in full agreement with the phenomenologically deduced form $V(\zeta) = \lambda_B \, \zeta$. This procedure, originally developed by Fubini and Rabinovici (FR) [14], is the superconformal extension of the procedure applied by dAFF [8]. In brief: a new evolution Hamiltonian can be constructed using a generalized supercharge which is a superposition of the original supercharge together with a spinor operator which occurs only in the superconformal algebra. The resulting superconformal quantum mechanics applied to the fermionic light front bound-state equations is completely dual to LF holographic QCD; this is in contrast to conformal quantum mechanics without supersymmetry, which is dual to the bosonic sector of LF holographic QCD only up to a constant term, which in turn is fixed only by embedding the LF wave equations for arbitrary integer spin into LF holographic QCD.

As we shall discuss in this paper, superconformal quantum mechanics applied to LF bound-state equations also implies striking similarities between the meson and baryon spectra. In fact, as we shall show, the holographic QCD light-front Hamiltonians for the states on the pion and proton trajectories are identical if one shifts the internal angular momentum of the meson (L_M) by one unit with respect to that of the baryon (L_B), $L_M = L_B + 1$. The baryon and meson trajectories are actually observed to be linear in the squared masses $M^2 \propto (n + L)$, as predicted by LF holographic QCD, a feature not obvious for states satisfying effective bound-state equations (Dirac or generalized Rarita-Schwinger). The slope of the trajectories in the principal quantum number n and the orbital angular momentum L are also very similar. In fact, the best fits to the numerical values for the Regge slopes agree within $\pm 10\%$ for all hadrons, mesons and baryons; this leads to a near-degeneracy of meson and baryon levels in the model.

The idea to apply supersymmetry to hadron physics is certainly not new [15–17]. In [15] mesons and baryons are grouped together in a big supermultiplet, a representation of $U_{6/21}$. In [16] the supersymmetry results of Miyazawa [15] are recovered in a QCD framework, provided that a diquark configuration emerges through an effective string interaction. This approach relies heavily on the fact that in $SU(3)_C$ a diquark can be in the same color representation as an antiquark, namely a $\bar{3}$. A meson is formed by a quark-antiquark pair and a baryon by a quark and a diquark, which remains color singlet. It is plausible to assume that the color force between a quark and a diquark is approximately equal to that between a quark and an antiquark; and from this, an effective supersymmetry between mesons and baryons follows. An apparent difficulty in this approach is that the pion and the nucleon would have the same mass and thus, supersymmetry would be badly broken [17]. In fact, in the chiral limit – the limit of massless quarks – the pion is massless, and this state has no obvious supersymmetric partner: there is no (nearly) massless baryonic state. In the direct diquark approach [15–17] there is no natural way to take into account the special role of the pion.

In certain aspects, our approach is similar to the diquark picture described above. The light-front clustering decomposition used here divides the baryon constituents into a special constituent, the active quark, and the rest, the spectator cluster, which could be identified with a diquark. However, in contrast to the direct diquark picture, the problem of a baryonic partner of the pion does not occur in our approach. It yields a massless pion, but the supercharge, which transforms meson into baryon wave functions, annihilates the pion wave function and therefore it has no baryonic partner. The details of this mechanism, which only occurs for a massless pion, are explained in Sec. IV A.

The approach described here, in contrast to the direct diquark picture of Refs. [15–17], is by no means restricted to a special number of colors. Indeed, in this effective theory the color quantum number does not appear explicitly. However, since it is an offspring of the Maldacena AdS/CFT correspondence [18], it is reminiscent of an $N_C \to \infty$ theory. This interpretation is also in accordance with the zero width of all states, including the excited ones. It is interesting to note that there exists a genuine supersymmetric approach to the meson-baryon relation relying on the $N_C \to \infty$ limit. Armoni and Patella [19] consider $\mathcal{N} = 1$ supersymmetric $SU(N_C)$; in their approach, the meson is formed by a bosonic string from a quark to an antiquark, whereas the baryon is formed by a fermionic string between two quarks. In the large N_C limit the string tension for both objects become equal: "supersymmetric relics" [20] from the supersymmetric theory lead to equal string tension for mesons and baryons in $SU(N_C)$.

We emphasize that the supersymmetric relations between the observed baryons and mesons, which we derive here, are not a consequence of supersymmetric QCD with scalar quarks and gluinos. Since no supersymmetric partners of the fundamental QCD fields have been observed, such a theory is evidently broken below the TeV scale. The relations derived here are relations between the wave functions of hadrons, not field operators. In fact, the relations obtained in the framework of supersymmetric quantum mechanics reflect properties of the confining mechanism in an effective semiclassical theory. One thus expects deviations from experiment which are of the same order as in light-front holographic QCD.

This article is organized as follows: After briefly reviewing some important results of light-front holographic QCD in Sec. II, we discuss in Sec. III the construction of the bound-state Hamiltonian within the superconformal algebra and the breaking of dilation invariance following [14]. The search for the supersymmetric partners of the baryon trajectories is discussed in Sec. IV. A summary of the main results and our conclusions are presented in Sec. V. Some useful formulae for the derivations presented in this article are given in the appendices.

II. LIGHT-FRONT HOLOGRAPHIC QCD

We first briefly review some principal results of light-front holographic QCD [21]. There an integer-spin field in AdS_5, with a free hadronic field at the four-dimensional border $\zeta = 0$, is split into a component $\Phi_J(\zeta)$, describing the behavior in the bulk, and a plane wave with an integer J-spinor describing the Minkowski space-time behavior:

$$\Phi_{\nu_1 \cdots \nu_J}(P, \zeta) = \Phi_J(\zeta) e^{iP\cdot x} \epsilon_{\nu_1 \cdots \nu_J}(P). \tag{1}$$

The four-momentum squared is the mass squared of the hadron represented by the free field, $P^2 = M^2$.

A Schrödinger-like wave equation [2, 7] follows from the AdS action for arbitrary integer spin-J modified by a dilaton term $e^{\varphi(\zeta)}$:

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L^2 - 1}{4\zeta^2} + U(\zeta, J)\right) \phi_J(\zeta) = 0, \tag{2}$$

where we have factored out the scale $(1/\zeta)^{J-3/2}$ and dilaton factors from the AdS field Φ_J by writing $\Phi_J(\zeta) = (R/\zeta)^{J-3/2}\, e^{-\varphi(\zeta)/2}\, \phi_J(\zeta)$. Equation (2) has exactly the form of a LF wave equation for massless quarks with a LF effective potential U and LF angular momentum L. The latter is related to the total spin J and the product of the AdS mass μ with the AdS radius R by

$$(\mu R)^2 = L^2 - (J - 2)^2. \tag{3}$$

The potential U is related to the dilaton profile by [6, 7]

$$U(\zeta, J) = \frac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2\zeta}\,\varphi'(\zeta). \tag{4}$$

The holographic variable ζ is identified with the LF invariant transverse separation: $\zeta^2 = b_\perp^2 u(1-u)$ [1, 2], where $b_\perp$ is the transverse separation of the constituents and u is the longitudinal light-front momentum fraction.

In the case of the quadratic dilaton profile $\varphi(\zeta) = \lambda_M \zeta^2$, the LF effective potential is $U(\zeta, J) = \lambda_M^2 \zeta^2 + 2\lambda_M(J-1)$, and the holographic bound-state wave equation (2) can be written as

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M\,(J-1) + \frac{4\nu^2-1}{4\zeta^2}\right)\phi_J = M^2\,\phi_J, \tag{5}$$

for a meson with total spin J. Near $\zeta = 0$ the regular solution behaves as $\phi_J(\zeta) \sim \zeta^{\nu+\frac{1}{2}}$, corresponding to twist $2+\nu$. In LF holographic QCD one thus has $\nu = L_M$, where L_M is the LF angular momentum of the meson, $L_M = |L_M^z|_{\max}$. The eigenvalues of (5) predict the meson spectrum

$$M^2_{n,L,J} = 4\left(n + \frac{J+L_M}{2}\right)\,\lambda_M, \tag{6}$$

for $\lambda_M > 0$, where n indicates the radial excitation quantum number: the number of nodes in the wave function.

Similarly, the AdS field for arbitrary half-integer spin-J can be factorized into a bulk wave function $\Psi_J^\pm(\zeta)$ and a plane wave with a Rarita-Schwinger or Dirac spinor with momentum P and mass M, representing a freely propagating baryon at the AdS border:

$$\Psi^\pm_{\nu_1\cdots\nu_{J-1/2}}(P,\zeta) = \Psi_J^\pm(\zeta)\,e^{iP\cdot x}u^\pm_{\nu_1\cdots\nu_{J-1/2}}(P), \tag{7}$$

where the chiral spinors $u^\pm_{\nu_1\cdots\nu_{J-1/2}} = \frac{1}{2}(1\pm\gamma_5)u_{\nu_1\cdots\nu_{J-1/2}}$ satisfy the equations

$$\gamma\cdot P\;u^\pm_{\nu_1\cdots\nu_{J-1/2}}(P) = M u^\mp_{\nu_1\cdots\nu_{J-1/2}}(P)\,; \qquad \gamma_{\nu_1}\,u^\pm_{\nu_1\cdots\nu_{J-1/2}}(P) = 0. \tag{8}$$

The spinors $u^\pm$ have positive and negative chirality, respectively.

The bound-state wave equations for the AdS bulk wave functions $\Psi_J^\pm$ can be derived from the action for arbitrary half-integer spin-J if one includes the effective interaction $V(\zeta) = \lambda_B\zeta$. The result is [7]

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,(\nu+\frac{1}{2}) + \lambda_B + \frac{4\nu^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\,\psi_J^+, \tag{9}$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,(\nu+\frac{1}{2}) - \lambda_B + \frac{4(\nu+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\,\psi_J^-, \tag{10}$$

where we have factored out the scale $(1/\zeta)^{J-5/2}$ by writing $\Psi_J^\pm(\zeta) = (R/\zeta)^{J-5/2}\,\psi_J^\pm(\zeta)$, and ν is related to the product of the AdS fermionic mass and the AdS radius R by

$$\nu = \mu R - \frac{1}{2}. \tag{11}$$

The baryon spectrum which follows from (9,10) is

$$M_{n,\nu}^2 = 4(n + \nu + 1)\,\lambda_B, \tag{12}$$

for $\lambda_B > 0$. The eigenvalues given by (12) do not depend explicitly on J, an important result also found in Ref. [22].

III. SUPERCONFORMAL ALGEBRA AND BREAKING OF DILATATION SYMMETRY

We will now show how the preceding results can be systematically derived using superconformal algebra, but with important new consequences. One starts with the simplest supersymmetric algebra of two fermionic operators, the supercharges Q and $Q^\dagger$, and a Hamiltonian H [23]

$$\{Q, Q^\dagger\} = 2H, \tag{13}$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \tag{14}$$

$$[Q, H] = [Q^\dagger, H] = 0. \tag{15}$$

A simple realization is:

$$Q = \psi^\dagger(-ip + W), \qquad Q^\dagger = \psi(ip + W), \tag{16}$$

where p is the canonical momentum operator; ψ and $\psi^\dagger$ are fermionic operators with anti-commutation relation

$$\{\psi, \psi^\dagger\} = 1, \tag{17}$$

and W is an arbitrary potential (the superpotential).

A realization using Pauli matrices $\vec{\sigma}$ is:

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \qquad \psi^\dagger = \frac{1}{2}(\sigma_1 + i\sigma_2), \tag{18}$$

leading to

$$B = \frac{1}{2}[\psi^\dagger, \psi] = \frac{1}{2}\sigma_3, \tag{19}$$

where B is the generator of $U(1)$ transformations $\psi \to e^{i\alpha}\psi$, $\psi^\dagger \to e^{-i\alpha}\psi^\dagger$ with eigenvalues $+\frac{1}{2}$ and $-\frac{1}{2}$.

In the Schrödinger picture the supercharges are realized as operators in $\mathcal{L}_2(R_1)$, with $p = -i\, d/dx$:

$$Q = \psi^\dagger \left(-\frac{d}{dx} + W(x) \right), \tag{20}$$

and

$$Q^\dagger = \psi \left(\frac{d}{dx} + W(x) \right), \tag{21}$$

leading to the supersymmetric Hamiltonian:

$$H = \frac{1}{2}\{Q, Q^\dagger\} = \frac{1}{2}\left(-\frac{d^2}{dx^2} + W^2(x) - 2W'(x)\, B \right). \tag{22}$$

The Hamiltonian operates on 2-spinors

$$|\phi\rangle = \begin{pmatrix} \phi_1 \\ \\ \phi_2 \end{pmatrix}, \tag{23}$$

of which one component can be attributed to fermion number 1 and the other 0. Imposing conformal symmetry leads to an unique choice of W [14, 24], namely

$$W(x) = \frac{f}{x}, \tag{24}$$

with a dimensionless constant f.

Introducing the spinor operators

$$S = \psi^\dagger\, x, \qquad S^\dagger = \psi\, x, \tag{25}$$

one can construct the larger algebra [25] (superconformal algebra), which contains the conformal algebra with the dilatation generator D and the special conformal transformation generator K. The extended algebraic structure is

$$\frac{1}{2}\{Q, Q^\dagger\} = H, \qquad \frac{1}{2}\{S, S^\dagger\} = K, \tag{26}$$

$$\{Q, S^\dagger\} = f - B + 2iD, \tag{27}$$

$$\{Q^\dagger, S\} = f - B - 2iD, \tag{28}$$

where the operators

$$H = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 + 2Bf}{x^2}\right), \tag{29}$$

$$K = \frac{1}{2}x^2, \tag{30}$$

$$D = \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right), \tag{31}$$

satisfy the conformal algebra

$$[H, D] = iH, \qquad [H, K] = 2iD, \qquad [K, D] = -iK. \tag{32}$$

The anticommutators of all the other generators vanish: $\{Q, Q\} = \{Q, S\} = \cdots = 0$.

Fubini and Rabinovici considered several ways to construct a new compact quantum-mechanical evolution operator inside the superconformal algebra. The most straightforward way is to directly follow the procedure of dAFF [8] and construct a linear combination of the (old) Hamiltonian and the generator of special conformal transformations [14, 24], which breaks supersymmetry explicitly. There is, however, the interesting possibility of constructing a new Hamiltonian using the superposition of generalized supercharges within the superconformal algebra [14] and thus preserving supersymmetry. This is the procedure we shall follow here. To this end, we slightly generalize the definitions of FR [14] and introduce a new supercharge R as a linear combination of the generators Q and S

$$R_\lambda = Q + \lambda\, S. \tag{33}$$

This leads, in analogy to the dAFF procedure in conformal quantum mechanics, to the introduction of a constant with nonzero dimensions; in fact, since Q has dimension $[x^{-1}]$, and S has dimension $[x^1]$, λ must therefore have dimension $[x^{-2}]$.

One can now construct a new evolution operator G inside the superconformal algebra in terms of the new supercharge R:

$$\{R_\lambda, R_\lambda^\dagger\} = G, \tag{34}$$

$$\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0, \tag{35}$$

$$[R_\lambda, G] = [R_\lambda^\dagger, G] = 0. \tag{36}$$

We find

$$G = 2H + 2\lambda^2 K + 2\lambda\,(f\mathbf{I} - B), \tag{37}$$

which is a compact operator for $\lambda \in \mathcal{R}$.

The supercharge operator $R_\lambda^\dagger$ transforms a state $|\phi\rangle$ into the state $R^\dagger|\phi\rangle$ with different fermion number (See Appendix B). By construction, the evolution operator G commutes with R_λ; it thus follows that the states $|\phi\rangle$ and $R^\dagger|\phi\rangle$ have identical eigenvalues. In fact, if $|\phi_E\rangle$ is an eigenstate of G with $E \neq 0$,

$$G\,|\phi_E\rangle = E\,|\phi_E\rangle, \tag{38}$$

then $G\,R_\lambda^\dagger\,|\phi_E\rangle = R_\lambda^\dagger\,G\,|\phi_E\rangle = E\,R_\lambda^\dagger|\phi_E\rangle$, and thus $R_\lambda^\dagger\,|\phi_E\rangle$ is also an eigenstate of G with the same eigenvalue.

The new Hamiltonian G is diagonal. In the Schrödinger representation:

$$G_{11} = -\frac{d^2}{dx^2} + \lambda^2\,x^2 + 2\lambda\,f - \lambda + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}, \tag{39}$$

$$G_{22} = -\frac{d^2}{dx^2} + \lambda^2\,x^2 + 2\lambda\,f + \lambda + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}, \tag{40}$$

with $G_{10} = G_{01} = 0$. For $f \geq \frac{1}{2}$ and $\lambda > 0$ the spectra of both operators are identical:

$$E_n = 4\left(n + f + \frac{1}{2}\right)\lambda. \tag{41}$$

Comparing (9, 10) with (39, 40) we recover the result of Ref. [13], namely that the modified Hamiltonian G of superconformal quantum mechanics is the same as the Hamiltonian derived in LF holographic QCD, provided we identify $\phi_2(x)$, the eigenfunction of G_{22}, with the positive chirality wave function $\psi_J^+(\zeta)$, identify $\phi_1(x)$, the eigenfunction of G_{11}, with $\psi_J^-(\zeta)$; and take $f - \frac{1}{2} = \nu = L_B$ and $\lambda = \lambda_B$. The consequences of this remarkable result have been discussed extensively in Ref. [13].

In Ref. [13] the $U(1)$ operator (19) $B = [\psi^\dagger, \psi]$ was identified in the light-front with the Dirac matrix γ_5 which acts on physical spinors. In that paper we have shown that the supercharges relate the chirality-plus component of a baryonic wave function with the chirality-minus component of the same baryonic state. In the usual applications of supersymmetry, however, the supercharges connect bosonic to fermionic states. We therefore shall explore in the next section the possibility to relate mesonic with baryonic wave functions by the supercharges within the superconformal algebra. In this case, the supercharges act on some internal space. The supercharges in [13] and those used in the following are therefore only formally related. The bosonic operators H, D and K, however, have in both cases the same physical meaning. In particular, we will show that the G_{11} and G_{22}, equations (39) and

TABLE I. Orbital quantum number assignment for the leading-twist parameter ν for baryon trajectories according to parity P and internal spin S.

	$S=\frac{1}{2}$	$S=\frac{3}{2}$
P = +	$\nu = L_B$	$\nu = L_B + \frac{1}{2}$
P = −	$\nu = L_B + \frac{1}{2}$	$\nu = L_B + 1$

(40), match our light-front holographic equations for both the pion and nucleon trajectories. The extension of this superconformal connection to the Δ-ρ families will also be discussed.

IV. BARYON-MESON SUPERSYMMETRY

A. The Superpartner of the Nucleon Trajectory

In the case of baryons, the assignment of the leading-twist parameter ν in Eqs. (9, 10), as given in Table I [13], successfully describes the structure of the light baryon orbital and radial excitations [26]. The assignment $\nu = L_B$ for the lowest trajectory, the nucleon trajectory, is straightforward and follows from the stability of the ground state – the proton – and the mapping to LF quantized QCD.

The bound-state equations for the nucleon trajectory are (cf. Eqs. (9, 10)):

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\,\psi_J^+, \tag{42}$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B\,L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\,\psi_J^-. \tag{43}$$

We will now search for the meson supersymmetric partners of the nucleon trajectory. We choose as starting point the leading-twist chirality component $\psi_J^+(\zeta)$ which satisfies (42). With the identifications $x=\zeta,\ f-\frac{1}{2} = L_B$ and $\lambda = \lambda_B$, the plus chirality component $\psi_J^+(\zeta)$ is also an eigenfunction of G_{22}, Eq. (40). This identification allow us to define an effective "baryon number" N_B as a convenient convention to label our "meson" and "baryon" states. In terms of the $U(1)$ operator $B = \frac{1}{2}[\psi^\dagger, \psi]$

$$N_B = \frac{1}{2} - B, \tag{44}$$

with eigen-equations

$$N_B|\phi\rangle_M = 0, \tag{45}$$

$$N_B|\phi\rangle_B = |\phi\rangle_B, \tag{46}$$

where $|\phi\rangle_B$ has only a lower component ($\phi_1 = 0$) and $|\phi\rangle_M$ only an upper component ($\phi_2 = 0$):

$$|\phi\rangle_B = \begin{pmatrix} 0 \\ \phi_2 \end{pmatrix}, \qquad |\phi\rangle_M = \begin{pmatrix} \phi_1 \\ 0 \end{pmatrix}. \tag{47}$$

Therefore, the supersymmetric partner of the baryonic Hamiltonian G_{22} (40), the Hamiltonian G_{11} (39), should describe a meson trajectory. Indeed, the Hamiltonian G_{11} with the above mentioned substitutions agrees with the bound-state equation (5) for mesons with $J = L_M$, provided we identify $f + \frac{1}{2} = L_M = L_B + 1$ and set $\lambda_M = \lambda_B$. The lowest state on the mesonic trajectory, with $J = L_M = 0$ – the pion – is massless in the chiral limit. It corresponds to a negative value of f, namely $f = -\frac{1}{2}$ and thus its baryonic partner would have $L_B = -1$, which is an unphysical state. As discussed in Appendix A, this remarkable result, also follows directly from the superconformal algebra. As shown there, the operator which transforms a mesonic state into its baryonic supersymmetric counterpart, annihilates the meson state if $f = -\frac{1}{2}$.

We have thus derived the astonishing result that the pion has no supersymmetric partner even though no explicit breaking of supersymmetry has been introduced. Since the supercharges R_λ, $R_\lambda^\dagger$, which connect mesonic and baryonic wave functions, commute with the Hamiltonian G (34 - 36), it follows that if $|\phi\rangle_M$ is a mesonic state with eigenvalue E, $G\,|\phi\rangle_M = E\,|\phi\rangle_M$, then there exists also a baryonic state $R_\lambda^\dagger|\phi\rangle_M = |\phi\rangle_B$ with the same eigenvalue E. Indeed, as discussed in the proceeding section,

$$G\,|\phi\rangle_B = G\,R_\lambda^\dagger|\phi\rangle_M = R_\lambda^\dagger\,G\,|\phi\rangle_M = E\,|\phi\rangle_B. \tag{48}$$

However, for the specific eigenvalue $E = 0$ we can have the trivial solution

$$|\phi(E = 0)\rangle_B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{49}$$

This remarkable feature underlines the special role played by the pion in light-front holographic QCD. As a unique state of zero energy, it plays the same role as the unique vacuum state in a supersymmetric quantum field theory [23, 27].

In is interesting to note that the case of negative f was not considered in [14], since the classical potential $\frac{f}{2x^2} + \lambda^2 x^2$ has no stable ground state for $f < 0$. Nevertheless, the lowest lying bound state of G_{11} with $f = -\frac{1}{2}$ has the normalizable wave function $x^{\frac{1}{2}} e^{-\lambda_M x^2/2}$. This situation is reminiscent of the light-front holographic correspondence: angular momentum $L = 0$ corresponds to a tachyonic AdS mass $\mu^2 < 0$ (See Eq. (3)), but nonetheless the Breitenlohner-Freedman stability bound [29] is still satisfied.

We thus obtain from superconformal quantum mechanics a very satisfactory result: both the nucleon and the $I = 1, S = 0$ mesons lie on linear trajectories with the same slope and the same radial and orbital excitation energies. The lowest lying state on the meson trajectory is the massless pion. In superconformal quantum mechanics it corresponds to the value $f = -1/2$, and therefore it has no supersymmetric partner.

In the framework of superconformal quantum mechanics all eigenstates with eigenvalues different from zero have supersymmetric partners. We emphasize that the pion with $f = -\frac{1}{2}$ and zero mass is unique: it is annihilated by the fermion-number changing supercharge $R_\lambda^\dagger$, and it therefore has no supersymmetric partner (See Fig. 1). This is in accordance with the spectroscopy derived from light-front holographic QCD, where baryon and meson partners have the masses $M_B^2 = 4\lambda_B(n + L_B + 1)$ and $M_M^2 = 4\lambda_M(n + L_M)$ respectively. This result follows from Eqs. (12) and (6) with $\nu = L_B$ and $J = L_M$, respectively. If one takes $\lambda_B = \lambda_M$ in LF holographic QCD, which is automatic in the superconformal theory, the spectral results are then identical for $L_M = L_B + 1$.

The predictions of supersymmetric quantum mechanics are based on the fact that the supercharge operator R_λ transforms baryon states with angular momentum L_B into their mesonic superpartners with angular momentum $L_M = L_B + 1$. The operator $R_\lambda^\dagger$ operates in the opposite direction. The pion has a very special role: its existence is predicted by the superconformal algebra, and according to the formalism, it is massless and has no supersymmetric partner. We have thus established a complete correspondence between the light-front holographic QCD results and supersymmetric quantum mechanics.

The superconformal predictions presented in Fig. 1 should be understood as a zeroth-order approximation. There are, however, several phenomenological corrections to this initial approximation. First, the slope of the π/b_1 trajectory is not exactly identical to the slope of the nucleon trajectory: for the mesons $\sqrt{\lambda_M} = 0.59$ GeV, whereas for the nucleons $\sqrt{\lambda_B} = 0.49$ GeV [21]. This makes the b_1 heavier than its supersymmetric partner, the nucleon.

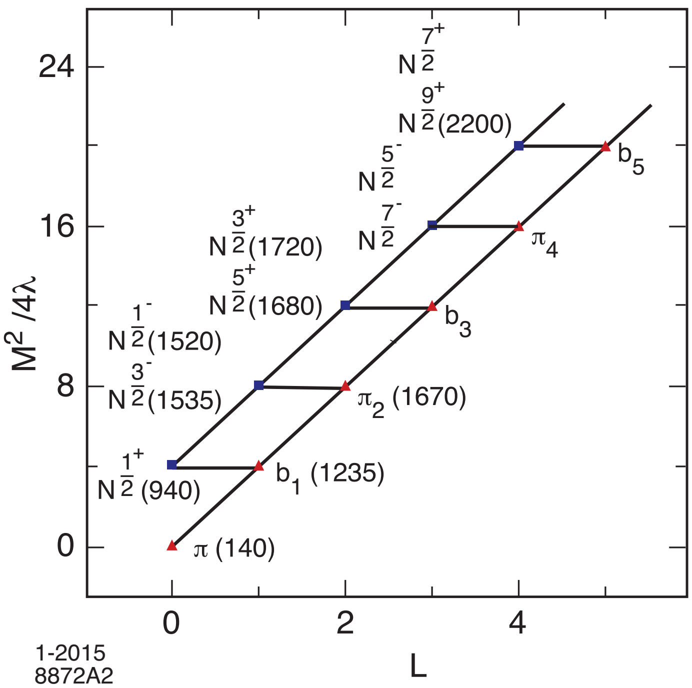

FIG. 1. Meson-nucleon superconformal connection. The predicted value of M^2 in units of 4λ for mesons with $S = 0$ (red triangles), and baryons with $S = \frac{1}{2}$ (blue squares) is plotted vs the orbital angular momentum L. The π-meson has no baryonic partner. The baryon quantum number assignment is taken from Ref. [13]. Nucleon trajectories for $J^z = L^z \pm S^z$ are degenerate.

In terms of LF holographic QCD this indicates that for this internal spin configuration, the confining force between the spectator and the cluster in the baryon is weaker than between the constituents of the meson; this makes the meson a more compact object since $\langle r^2 \rangle \sim 1/\lambda$. Second, the negative parity nucleon states are systematically higher than the nucleons with positive parity, a fact which in LF holographic QCD has been taken into account phenomenologically by the half-integer twist assignment $\nu = L + \frac{1}{2}$ given in Table I. It is expected that this effect could be explained by the different quark configurations and symmetries of the baryon wave function [30–32].

The nucleon-meson superpartner pairs are plotted in Fig. 2 with their measured masses. The observed difference in the squared masses of the supersymmetric partners indicates that the most important breaking of supersymmetry is due to the difference between λ_B and λ_M. Only confirmed PDG states have been included [33].

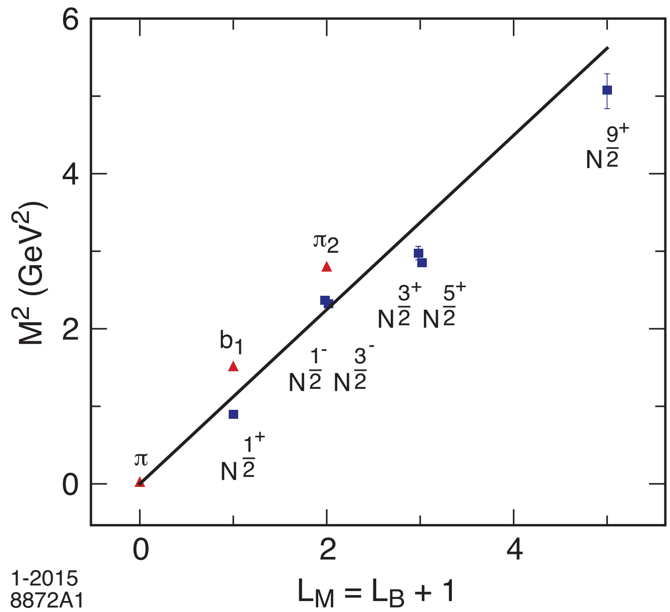

FIG. 2. Supersymmetric meson-nucleon partners: Mesons with $S = 0$ (red triangles) and baryons with $S = \frac{1}{2}$ (blue squares). The experimental values of M^2 are plotted vs $L_M = L_B + 1$. The solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV. The π has no baryonic partner.

B. The Mesonic Superpartners of the Delta Trajectory

The essential physics derived from the superconformal connection of nucleons and mesons follows from the action of the fermion-number changing supercharge operator R_λ. As we have discussed in the previous section, this operator transforms a baryon wave function with angular momentum L_B into a superpartner meson wave function with angular momentum $L_M = L_B + 1$ (See Appendix B), a state with the identical eigenvalue – the hadronic mass squared. We now check if this relation holds empirically for other baryon trajectories.

We first observe that baryons with positive parity and internal spin $S = \frac{3}{2}$, such as the $\Delta^{\frac{3}{2}^+}(1232)$, and baryons with with negative parity and internal spin $S = \frac{1}{2}$, such as the $\Delta^{\frac{1}{2}^-}(1620)$, lie on the same trajectory; this corresponds to the phenomenological assignment $\nu = L_B + \frac{1}{2}$, given in Table I. From (12) we obtain the spectrum [34]

$$M^{2(+)}_{n,L_B,S=\frac{3}{2}} = M^{2(-)}_{n,L_B,S=\frac{1}{2}} = 4\left(n + L_B + \frac{3}{2}\right)\lambda_B. \tag{50}$$

If we now apply the superconformal relation $L_M = L_B + 1$ and $\lambda_M = \lambda_B$ we predict a meson trajectory with eigenvalues

$$M^2_{n,L_M} = 4\left(n + L_M + \frac{1}{2}\right)\lambda_M, \tag{51}$$

which is, precisely, the expression for the spectrum of the ρ-meson (6) for $J = L_M + 1$. Again, one sees that the lowest-lying mesonic state, in this case the ρ meson, has no superpartner, since L_M would be negative.

Since the phenomenological value of λ for the Δ trajectory is close to that of the ρ trajectory, $\sqrt{\lambda_\Delta} = 0.51$ and $\sqrt{\lambda_\rho} = 0.54$ (See Ref. [21]), one can expect good agreement for the masses of the supersymmetric partners. This is indeed the case, as can be seen from Fig. 3, where we have included the confirmed Δ and $J = L + S$, $S = 1$, vector-meson states from Ref. [33].

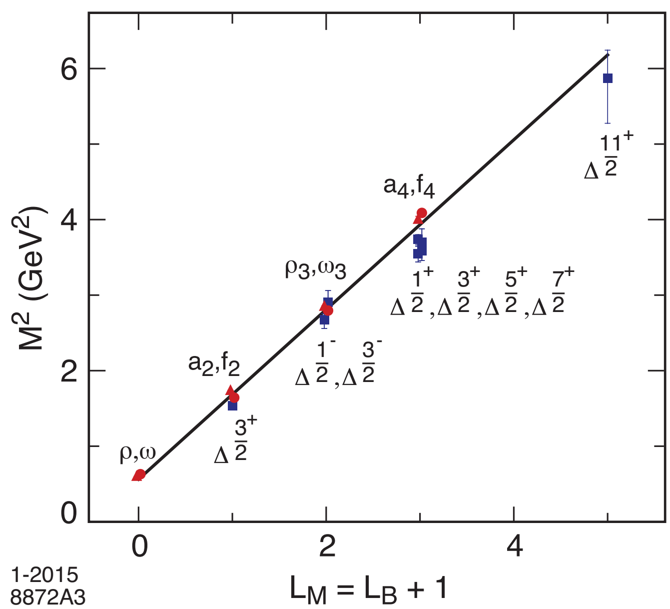

FIG. 3. Supersymmetric vector-meson and Δ partners: Mesons with $I = 1$ (red triangles) and $I = 0$ (red circles) and Δ states with $S = \frac{3}{2}$ and $S = \frac{1}{2}$ (blue squares) for plus and minus parity respectively. The experimental values of M^2 are plotted vs $L_M = L_B + 1$. The solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV. The ρ and ω have no baryonic partner, since it would imply a negative value of L_B.

Using the assignment $\nu = L_B + \frac{1}{2}$ from (Table I) and the comparison of Eqs. (9) with (40) (or (10) with (39)), we obtain the relation $f = \nu + \frac{1}{2} = L_B + 1 = L_M$ for the superconformal relation $L_M = L_B + 1$. Thus from (39) we obtain the LF-Hamiltonian for the superpartner vector meson trajectory

$$G_{11} = -\frac{d^2}{d\zeta^2} + \lambda_M^2\, \zeta^2 + 2\lambda_M(\, L_M - 1) + \frac{4(L_M + \frac{1}{2})^2 - 1}{4\zeta^2}, \tag{52}$$

with $\lambda = \lambda_M = \lambda_B$. This expression is to be compared with the light-from holographic Hamiltonian which follows from (5) for $J = L_M + 1$ and $\nu = L_M$:

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \lambda_M^2\, \zeta^2 + 2\lambda_M\, L_M + \frac{4L_M^2 - 1}{4\zeta^2}. \tag{53}$$

Thus, by extending the meson-baryon connection for baryons with $\nu = L_B + \frac{1}{2}$ we obtain an identical expression for the vector meson-spectrum, but with a different LF Hamiltonian. This somewhat less satisfactory feature of the Δ-ρ relations is reflected in the transformation under the supercharge $R_\lambda^\dagger$ (Appendix B). The ρ meson wave function ϕ_1, that is the eigenfunction of G_{11} with $f = 0$, is not annihilated by the action of $R_\lambda^\dagger$ (B31). Indeed the terms which determine the angular momentum, the singular terms in the two Hamiltonians G_{11} and G_{22}, Eqs. (39) and (40) respectively, are identical for $f = 0$. Thus in this case, the unphysical value of the angular momentum, $L_B = -1$, is the only reason to exclude the baryonic superpartner of the ρ. This is in contrast to the case of the pion, where the fermion-number changing operator $R_\lambda^\dagger$ actually annihilates the pion wave function (B29), since it is a zero mass eigenmode.

V. SUMMARY AND CONCLUSIONS

Conformal and superconformal quantum mechanics [8, 14], together with light-front holographic QCD [21], has revealed the importance of conformal symmetry and its breaking within the algebraic structure for understanding the confinement mechanism of QCD.

If one introduces the mass scale scale for hadrons using the method developed by de Alfaro *et al.* [8], one obtains a confining theory for mesons while retaining a conformally invariant action. If one applies the dAFF procedure to light-front Hamiltonian theory, the form of the LF potential is uniquely fixed to that of a harmonic oscillator in the invariant LF radial variable ζ [9]. It predicts color confinement and linear Regge meson trajectories with the same slope in the radial and orbital excitations n and L. If one compares the construction of the confining LF potential with the Hamiltonian obtained in light-front holographic QCD, then the dilaton factor in the modified AdS action is uniquely fixed [5, 6]. The appearance of the extra spin-dependent constant term in the LF potential is a consequence of the specific embedding of the LF wave equations in AdS for arbitrary integer-spin [7]. This extra term is essential for agreement with experiment, including the prediction of a massless pion in the chiral limit.

In the case of half-integer spin, the dilaton in the AdS action does not lead to confinement for baryons since such a term can be absorbed into the wave function. Confinement thus requires the addition of a Yukawa-like term in the half-integer spin Lagrangian. However, this apparent deficiency is cured [13] by the application of superconformal quantum mechanics.

Superconformal quantum mechanics can be constructed by restricting the superpotential in Witten's construction [23] to a conformally invariant expression [14, 24]. Remarkably, it is possible to introduce a mass scale into the quantum-mechanical evolution equations, without violating supersymmetry, by introducing a new supercharge which is a linear combination of generators of the super conformal algebra [14]. Furthermore, by connecting the resulting wave equations to the light-front holographic formalism, one fixes not only the confining term for baryons and mesons for all spins, but also the constant terms in the LF potential. The resulting spectra reproduces the principal observed features of mesonic and baryonic Regge trajectories: the resulting trajectories are linear, and the spacing of the radial excitations equals the spacing of the orbital ones. Furthermore, the baryon masses depend only on the LF angular momentum L, but not on the total spin J, as observed in experiment.

There are striking phenomenological similarities between the baryon and meson spectra which would not be expected from the underlying quark degrees of freedom, given that in QCD the valence state in the meson case consists of confined $q\bar{q}$ excitations, and baryons are normally considered qqq bound states. However, the observed Regge trajectories are linear in the squared mass for both cases, with equal spacings of the orbital and angular excitations – both features which are typical for the proto-string theory such as the Veneziano model [35]. These essential features also follow from the light-front clustering properties of the semiclassical approximation to strongly coupled QCD and its holographic embedding in AdS space. In this approximation a nucleon behaves as an active quark and a spectator cluster, which resembles the usual quark-diquark picture, and it is also described by a one-dimensional effective theory. Furthermore, the coefficients of the confining term for mesons and baryons agree within $\pm 10\,\%$, although they would seem to be completely unrelated.

These similarities suggest that supersymmetric relations are responsible for these remarkable features.

In Ref. [13] superconformal quantum mechanics was used to describe baryonic states. There, the supercharges were shown to relate the positive and negative chirality components of the baryon wave functions, consistent with parity conservation. In this paper we have shown that supercharges, constructed formally as in [13], can also be used to relate hadronic states with different fermion number. This leads to remarkable relations between the spectroscopy of baryons and mesons, thus extending the applicability of light-front superconformal quantum mechanics to hadronic physics.

An important feature of the Hamiltonian operators (39, 40), which act on the two components of a supermultiplet $|\phi\rangle$, is the difference in the singular term of the potential. For one component of the Hamiltonian, it is $\frac{1}{4x^2}\left((f+\frac{1}{2})^2-1\right)$; for the other component, it is $\frac{1}{4x^2}\left((f-\frac{1}{2})^2-1\right)$. This has the consequence that the power behavior of the wave function at the origin (twist) differs by one unit for the two components. In light-front holographic QCD this implies a difference of the LF angular momentum by one unit $L_M = L_B + 1$. Comparing the spectra of the nucleon and the π/b_1 trajectory one indeed observes this approximate degeneracy (See Fig. 2). The leading-twist wave function of the baryons is identified with the component ϕ_2 of the supermultiplet $|\phi\rangle$, and the wave function of the mesons is identified with the component ϕ_1. As a consequence, the shared symmetric features of mesons and baryons are in fact a consequence of the properties of the superconformal algebra.

The problem for supersymmetry posed by the pion, which is massless in the chiral limit, and therefore can have no baryonic superpartner, is solved in a simple way: The value of the dimensionless constant f of the conformal potential (24) has for the pion and its radial excitations the value $f = L_M - \frac{1}{2} = -\frac{1}{2}$. The supercharge $R_\lambda^\dagger$, (33), which transforms the meson into the baryonic partner, annihilates the pion state, and therefore there cannot be a baryonic partner. The case $f = -\frac{1}{2}$ was not considered by Fubini and Rabinovici [14], since the classical potential in this case has no lower limit. Nevertheless, the pion wave function is regular at the origin and normalizable.

We have previously demonstrated a correspondence between superconformal quantum mechanics and light-front holographic QCD [13]. In this approach, one must explicitly assume in LF holographic QCD the same value for the bosonic and baryonic gap scale λ. In contrast, in the superconformal effective theory described here, the equality of λ for mesons and baryons is a consequence of the approach.

We have also applied the same procedure to the ρ/a_2 and the Δ-trajectories. The wave functions of the ρ-trajectory are identified with the component ϕ_1, and the component ϕ_2 of the super multiplet is identified with the Δ-states. As for the case of the π-nucleon connection, the properties of the fermion-changing supercharge R_λ imply that the meson angular momentum L_M is one unit larger than the baryon angular momentum L_B, $L_M = L_B + 1$ consistent with the Hamiltonians (39, 40). One indeed obtains excellent agreement between the spectra of the mesonic and baryonic states (See Fig. 3). The values of $\sqrt{\lambda_M}$ and $\sqrt{\lambda_B}$ are nearly degenerate as predicted by superconformal quantum mechanics.

There is, however, a problem with the ρ/a_2-Δ connection in that half-integer twist is apparently required. For the Δ trajectory the observed spectrum corresponds to half-integer twist $2 + L_B + \frac{1}{2}$, which also implies half-integer twist for the mesons on the ρ/a_2 trajectory. Although the spectra of this half-integer twist obtained with the superconformal Hamiltonian operator (39) correspond fully to those obtained by LF holographic QCD (and experiment), the wave functions do not; they differ by a factor $x^{\frac{1}{2}}$. Related to this problem is the fact that the supercharge $R_\lambda^\dagger$ does not annihilate the ρ wave function, but it formally leads to a baryonic state with the same mass. However, this state is excluded as a physical state, since it would have the angular momentum $L_B = -1$.

It should be noted that the semiclassical equations of light-front holographic QCD and superconformal quantum mechanics are intended to be a zeroth order approximation to the complex problem of bound states in QCD. We also emphasize that the quantum-mechanical supersymmetric relations derived here are not a consequence of a supersymmetry of the underlying quark and gluon fields; they are instead a consequence of the superconformal-confining dynamics of the semi-classical theory and the clustering inherent in light-front holographic QCD. Breaking of conformal invariance by quark masses leads to a mass splitting, but the supersymmetric connection between mesons and baryons is not affected. This will be discussed elsewhere.

In this paper we have concentrated on the consequences of superconformal algebra for the spectral properties of meson and baryons. Since the meson and baryon wave functions are also related, there are also interesting dynamical consequences; *e.g.*, for elastic and transition form factors. The b_1 wave function is predicted to be identical to the non-leading-twist wave function of the nucleon, which in turn is related to the leading-twist wave function via a parity transform – see [13]; therefore, at low resolution the form factors of the nucleon and the b_1 are related. Another dynamical consequence of the model is that for high resolution, at large momentum transfer when the baryon cluster is resolved into its individual constituents, the twists of the superpartners are equal: the higher value of L of the meson, $L_M = L_B + 1$, is compensated by the additional constituent in the baryon.

Acknowledgments

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Appendix A: Other Possible Evolution Operators

Fubini and Rabinovici have discussed three different ways of constructing compact Hamiltonians from the superconformal algebra. Some care should be taken, however, in transferring their interpretation to our application. The emphasis in [8] and later in [14, 24] was on quantum mechanics as a one dimensional field theory and the investigation of the vacuum structure in this field theory. Therefore only the case with a stable classical potential, implying $f > 0$, was considered. In our search for semiclassical bound-state equations, however, the lowest state is a hadronic state. Furthermore, in the field theoretical investigations of FR the dimensionless constant f is an arbitrary positive parameter, each value of f representing a different field theory with a different vacuum. In our investigations, where the procedure of dAFF [8] and its extension by FR [14] has been embedded in LF holographic QCD the dimensionless constant f determines the angular momentum and we are confined to the series of discrete values representing the orbital excitations. Nevertheless, it is informative to discuss the three different ways to construct the compact Hamiltonian representing hadronic bound states also from our perspective. For generality purposes we use in the appendices for the dimensionful constants the symbols w and v, which can be positive or negative. In our applications to meson and baryon spectroscopy we are restricted $w > 0$.

The simplest way to construct a Hamiltonian with discrete spectrum in the frame of the superconformal algebra is to apply directly the method of dAFF [8]. This yields the

Hamiltonian [14, 24], again in the slightly generalized notation

$$G_0 = \{Q, Q^\dagger\} + w^2 K. \tag{A1}$$

Both supersymmetry and dilatation symmetry are broken here. The two components of the eigen-spinor of G_0 have different spectra

$$(G_0)_{11}\phi_1 = (4n + 2f + 3)|w|\phi_1, \tag{A2}$$

$$(G_0)_{22}\phi_2 = (4n + 2f + 1)|w|\phi_2, \tag{A3}$$

and thus supersymmetry is broken from the onset for all levels. This approach would yield a LF potential $U(\zeta) = w^2\zeta^2$, without any additional constants which occur in LF holographic QCD (See 5, 9, 10), and which are phenomenologically very important.

On the other hand, the approach where supersymmetry is conserved by constructing a new Hamiltonian from the spinor operator R_w, a superposition of the supercharges Q and S within the superalgebra [14],

$$R_w = Q + wS, \tag{A4}$$

conserves supersymmetry for $f > \frac{1}{2}$, since R_w commutes with the evolution operator

$$G(w) = \{R_w, R_w^\dagger\}. \tag{A5}$$

Therefore $R_w|\phi_w\rangle$ is an eigenstate of G_w with identical eigenvalue as the eigenstate $|\phi_w\rangle$.

The spectra of $G(w)$ for real values of f and w are:

$$E_1 = (4n + 2)|w| + 2\left|f + \frac{1}{2}\right| |w| + 2(f - \frac{1}{2})\, w, \tag{A6}$$

$$E_2 = (4n + 2)|w| + 2\left|f - \frac{1}{2}\right| |w| + 2(f + \frac{1}{2})\, w, \tag{A7}$$

where E_1 are eigenvalues of G_{11} representing mesonic states and E_2 the eigenvalues of G_{22} for baryons. For $w < 0$ and $f > -\frac{1}{2}$ the spectra are independent of f:

$$E_1 = 4(n + 1)|w|, \tag{A8}$$

$$E_2 = 4n|w|, \tag{A9}$$

and therefore cannot lead to angular excitations of the corresponding LF Hamiltonians. For $f = -\frac{1}{2}$ and $w > 0$, we have

$$E_1 = 4n\, w, \tag{A10}$$

$$E_2 = 4(n + 1)\, w. \tag{A11}$$

There exists no baryonic state

$$|\phi\rangle = \begin{pmatrix} 0 \\ \\ \phi_B \end{pmatrix}, \tag{A12}$$

with zero energy. The reason for this seeming contradiction with the above mentioned commutation relation, lies in the fact that the operator $R_w^\dagger$ annihilates the mesonic state (See (B29)).

Appendix B: Transformation Operators and Quantum-Mechanical Evolution

The generalized hypercharge R has the commutation relations

$$[G(w), R_v] = -2(w - v)\, R_{-w}, \tag{B1}$$

$$[G(w), R_v^\dagger] = 2(w - v)\, R_{-w}^\dagger, \tag{B2}$$

with the new Hamiltonian $G(w) = \{R_w, R_w^\dagger\}$.

For $v = w$ the commutator vanishes, therefore if $|\phi\rangle$ is an eigenstate of G, also $R_w\,|\phi\rangle$ is an eigenstate with the same eigenvalue. Therefore the spinor supercharge R transforms the baryonic superpartner with angular momentum L_B, into the mesonic one with angular momentum $L_M = L_B + 1$. The operator $R_w^\dagger$ acts in the opposite direction.

For $v = -w$, however, we have the typical commutation behavior of a raising and lowering operator respectively:

$$[G(w), R_{-w}] = -4w\, R_{-w}, \tag{B3}$$

$$[G(w), R_{-w}^\dagger] = 4w\, R_{-w}^\dagger. \tag{B4}$$

That is, if $|\phi\rangle$ is eigenfunction of G with eigenvalue E, then $R_{-w}\,|\phi\rangle$ is eigenstate with the energy $E + 4w$. This means that a baryonic state with angular momentum L_B and radial excitation n is transformed into a mesonic state with angular momentum $L_M = L_B + 1$ and radial excitation $n + 1$, which has the same energy as the baryonic state with angular momentum L_B and radial excitation $n + 1$.

There is also a bosonic raising operator, that is, a raising operator which does not change fermion number. It is composed of the bosonic operators of the superconformal algebra. Generalizing again slightly the operators introduced by FR in Ref. [14]

$$L_v = H + v^2\, K + 2i\, v\, D, \tag{B5}$$

one obtains from the algebra (26) the commutation relations

$$[G(w), L_w] = 4w\, L_w, \tag{B6}$$

$$[G(w), L_{-w}] = -4w\, L_w. \tag{B7}$$

These relations imply that also L_w is a raising operator, which transforms a baryon with L_B, n into a baryon with L_B, $n+1$ and the same with the mesons. Since it is composed of operators of the conformal group, it can also be applied to the lowest mesonic state, although there is no supersymmetric partner.

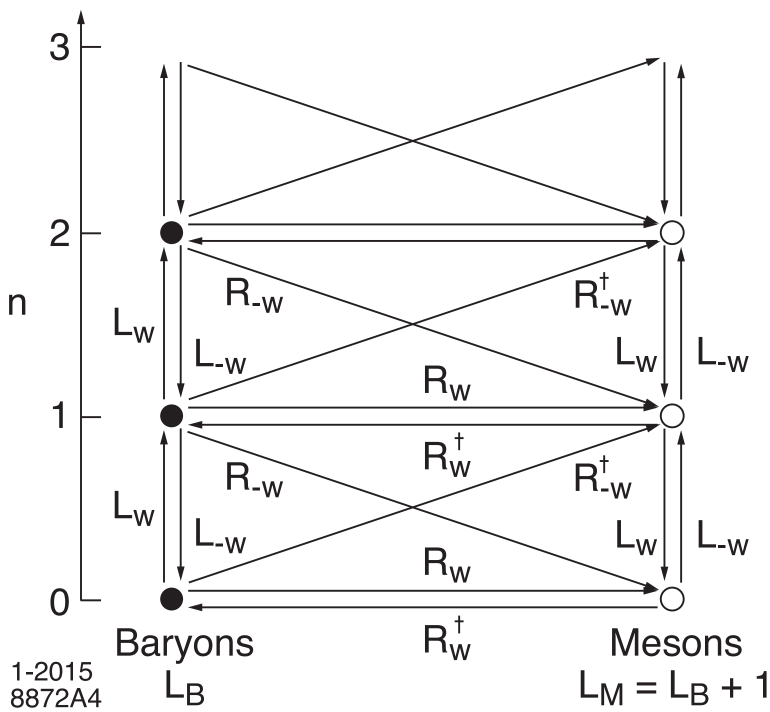

FIG. 4. Radial excitations and transformations by elements of the superconformal algebra for a baryon-meson system with a given $f - \frac{1}{2} = L_B \geq 0$.

Since the hypercharges R_w change the angular momentum by one unit, it is tempting to look for an operator which also leads to angular excitations. Such an operator which increases the angular momentum by one unit is easily constructed and has the form:

$$\Lambda_w = \{Q, \psi\} + w\{S^\dagger, S\} + \frac{1}{\zeta}\psi^\dagger\psi. \tag{B8}$$

If $|\phi\rangle_L$ is an eigenstate to the Hamiltonian operator G_L constructed with $f = L + \frac{1}{2}$, then $|\phi\rangle_{L+1} = \Lambda_w\, |\phi\rangle_L$ is eigenstate to G_{L+1}, constructed with $f = L + 1 + \frac{1}{2}$. This operator

Λ is, however, not an element of the superconformal algebra. The action of the different operators in the baryon-meson system is illustrated in Fig. 4.

1. Quantum-Mechanical Evolution

In this paper, as in [13], we have concentrated on algebraic aspects and its consequences for the spectra. We now briefly discuss the quantum-mechanical time evolution. The Hamiltonian of unbroken superconformal quantum mechanics, H Eq. (22), is the translation operator for the time variable t

$$i\,\frac{d}{dt}|\phi\rangle = H\,|\phi\rangle. \tag{B9}$$

The quantum-mechanical evolution of the operator (37)

$$G = 2H + 2w^2\,K + 2w\,(f\mathbf{I} - B), \tag{B10}$$

follows from the action of the generators H and K on the state $|\phi\rangle$. We have (See Appendix C in Ref. [21]),

$$e^{-i\,H\,\epsilon}|\phi(t)\rangle = |\phi(t)\rangle + \frac{d}{dt}|\phi(t)\rangle\epsilon + O(\epsilon^2), \tag{B11}$$

$$e^{-i\,K\,\epsilon}|\phi(t)\rangle = |\phi(t)\rangle + \frac{d}{dt}|\phi(t)\rangle\epsilon\,t^2 + O(\epsilon^2). \tag{B12}$$

There follows

$$G\,|\phi(\tau)\rangle = \left(i\frac{d}{d\tau} + 2w\,(f\mathbf{I} - B)\right)|\phi(\tau)\rangle, \tag{B13}$$

where the new evolution parameter τ is related to t in (B9) by

$$d\tau = \frac{dt}{2(1 + w^2 t^2)}, \tag{B14}$$

as in dAFF [8]. From the eigenvalue equation $G\,|\phi_E\rangle = E\,|\phi_E\rangle$ follows the stationary state solution

$$|\phi_E(\tau)\rangle = |\phi_E(0)\rangle e^{-i\big((E\mathbf{I} - 2w\,(f\mathbf{I} - B)\big)\tau}. \tag{B15}$$

2. Operators in Matrix Form

It is sometimes convenient to work with a special matrix representation of the superconformal algebra. For convenience we give here an explicit realization in the Schrödinger picture. Define

$$q = -\frac{d}{dx} + \frac{f}{x}, \qquad q^\dagger = \frac{d}{dx} + \frac{f}{x}. \tag{B16}$$

Then we can write the spinor operators Q and S as

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \tag{B17}$$

and

$$S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}, \qquad S^\dagger = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}. \tag{B18}$$

The Hamiltonian $H = \frac{1}{2}\{Q, Q^\dagger\}$ in matrix form is

$$2H = \begin{pmatrix} q\,q^\dagger & 0 \\ 0 & q^\dagger\,q \end{pmatrix} = \begin{pmatrix} -\frac{d^2}{dx^2} + \frac{f(f+1)}{x^2} & 0 \\ 0 & -\frac{d^2}{dx^2} + \frac{f(f-1)}{x^2} \end{pmatrix}. \tag{B19}$$

The Hamiltonian $G = \{R_w, R_w^\dagger\}$ is

$$G = \begin{pmatrix} -\frac{d^2}{dx^2} + w^2\,x^2 + 2w\,f - w + \frac{4(f+\frac{1}{2})^2-1}{4x^2} & 0 \\ 0 & -\frac{d^2}{dx^2} + w^2\,x^2 + 2w\,f + w + \frac{4(f-\frac{1}{2})^2-1}{4x^2} \end{pmatrix}, \tag{B20}$$

where

$$R_w = \begin{pmatrix} 0 & -\frac{d}{dx} + \frac{f}{x} + w\,x \\ 0 & 0 \end{pmatrix}, \tag{B21}$$

and

$$R_w^\dagger = \begin{pmatrix} 0 & 0 \\ \frac{d}{dx} + \frac{f}{x} + w\,x & 0 \end{pmatrix}. \tag{B22}$$

The operator L_w (B5) is

$$L_w = H + \frac{1}{2}\left(w^2\,x^2 - \frac{d}{dx}\,x - x\,\frac{d}{dx}\right)\mathbf{I}, \tag{B23}$$

and its adjoint

$$L_w^\dagger = H + \frac{1}{2}\left(w^2\,x^2 + \frac{d}{dx}\,x + x\,\frac{d}{dx}\right)\mathbf{I}. \tag{B24}$$

Finally, the orbital raising operator (B8) is

$$\Lambda_w = \begin{pmatrix} -\frac{d}{dx} + \frac{f+1}{x} + w\,x^2 & 0 \\ 0 & -\frac{d}{dx} + \frac{f}{x} + w\,x^2 \end{pmatrix}. \tag{B25}$$

In this matrix form the upper component of the state $|\phi\rangle$ is the meson, the lower one the baryon

$$|\phi\rangle = \begin{pmatrix} \phi_M \\ \\ \phi_B \end{pmatrix}. \tag{B26}$$

Thus the effective baryon number operator $N_B = \frac{1}{2}(1 - [\psi^\dagger, \psi])$ is in matrix form:

$$N_B = \begin{pmatrix} 0 & 0 \\ \\ 0 & 1 \end{pmatrix}. \tag{B27}$$

It is easy to check that the state containing the pion, that is the eigenstate of (39) with $f = -\frac{1}{2}$, namely

$$\phi_\pi = \frac{1}{N}\sqrt{x}\, e^{-w\,x^2/2}, \tag{B28}$$

has no supersymmetric partner, since

$$R_w^\dagger\,|\phi\rangle = \begin{pmatrix} 0 \\ \\ (q^\dagger + w\,x)\phi_\pi \end{pmatrix} = \begin{pmatrix} 0 \\ \\ 0 \end{pmatrix}. \tag{B29}$$

Likewise, one checks that the state containing the ρ-meson, where $f = 0$, with the wave function

$$\phi_\rho = \frac{1}{N} x\, e^{-w\,x^2/2}, \tag{B30}$$

has formally a superpartner, but with negative angular momentum $L_B = -1$. Indeed

$$R_w^\dagger\,|\phi\rangle = \begin{pmatrix} 0 \\ \\ (q^\dagger + w\,x)\phi_\rho \end{pmatrix} = \begin{pmatrix} 0 \\ \\ \phi_\rho \end{pmatrix}. \tag{B31}$$

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