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Geoffrey T. Bodwin, Hee Sok Chung, U-Rae Kim, and Jungil Lee

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Quark fragmentation into spin-triplet S-wave quarkonium

Geoffrey T. Bodwin and Hee Sok Chung

High Energy Physics Division, Argonne National Laboratory,

9700 South Cass Avenue, Argonne, Illinois 60439, USA

U-Rae Kim and Jungil Lee

Department of Physics, Korea University, Seoul 136-713, Korea

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Abstract

We compute fragmentation functions for a quark to fragment to a quarkonium through an S-wave spin-triplet heavy quark-antiquark pair. We consider both color-singlet and color-octet heavy quark-antiquark ($Q\bar{Q}$) pairs. We give results for the case in which the fragmenting quark and the quark that is a constituent of the quarkonium have different flavors and for the case in which these quarks have the same flavors. Our results for the sum over all spin polarizations of the $Q\bar{Q}$ pairs confirm previous results. Our results for longitudinally polarized $Q\bar{Q}$ pairs agree with previous calculations for the same flavor cases and correct an error in a previous calculation for the different-flavor case.

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I. INTRODUCTION

Quarkonium production at large transverse quarkonium momentum p_T proceeds at leading power (LP) in p_T through processes in which a high-energy collision produces a single parton, which subsequently fragments into a quarkonium [1]. Fragmentation functions for a parton to fragment into a quarkonium play a central role in calculations of such processes. In this paper, we compute the fragmentation functions for a quark q to fragment into a quarkonium through heavy-quark-antiquark ($Q\bar{Q}$) pair channels in which the $Q\bar{Q}$ pair is in a spin-triplet S-wave state and a color-singlet or a color-octet state. We calculate fragmentation functions for the case in which the flavors of q and Q are the same, as well as for the case in which the flavors are different. We carry out these calculations at the leading nontrivial order in the strong coupling α_s and at order v^0, where v is the relative velocity of the Q and the $\bar{Q}$ in the quarkonium rest frame.

Previous calculations have given the fragmentation functions for a quark to fragment into an S-wave, spin-triplet $Q\bar{Q}$ pair for the case in which a sum over the $Q\bar{Q}$ spin polarizations has been taken. The case in which the initial quark and final quark have different flavors and are in a color-octet state is discussed in Refs. [2, 3]. The cases in which the initial quark and final quark have the same flavor and are in a color-octet or a color-singlet state is discussed in Ref. [3]. Our calculation confirms all of these results. We also verify a previous calculation of the fragmentation function for a quark to fragment into an S-wave, spin-triplet, color-singlet $Q\bar{Q}$ pair in which a sum over the $Q\bar{Q}$ spin polarizations is taken [4].

We have extended all of these spin-summed calculations to the cases in which the $Q\bar{Q}$ pair is in a longitudinally polarized state. After our calculation was completed, we learned that these longitudinal-polarization fragmentation functions had been calculated in Ref. [5]. Our calculation agrees with the results in Ref. [5] for the color-octet and color-singlet same-flavor cases and corrects an error in Ref. [5] for the color-octet different-flavor case.

The remainder of this paper is organized as follows. In Sec. II, we introduce our notation and the kinematics that we use in the calculation and present projectors for the $Q\bar{Q}$ spin and color. In Sec. III, we present the Collins-Soper fragmentation function for an initial quark [1] and give the Feynman rules for its computation. Sections IV and V contain, respectively, the calculations of the color-octet fragmentation functions for the case in which the initial and final quarks have different flavors and the case in which the initial and final quarks have the same flavor. In Sec. VI, we present the fragmentation functions for the color-singlet case. Section VII contains a summary and discussion of our results.

II. NOTATION, KINEMATICS, AND PROJECTORS

In this paper, we use the following light-cone coordinates for a four-vector V in the $d = 4 - 2\epsilon$ space-time dimensions:

$$V = (V^+, V^-, \boldsymbol{V}_\perp), \tag{1a}$$

$$V^+ = (V^0 + V^{d-1})/\sqrt{2}, \tag{1b}$$

$$V^- = (V^0 - V^{d-1})/\sqrt{2}, \tag{1c}$$

where we call V^{d-1} the longitudinal component of the $(d-1)$-dimensional spatial vector $\boldsymbol{V}$, and $\boldsymbol{V}_\perp$ is the $(d-2)$-dimensional component of $\boldsymbol{V}$ that is transverse to V^{d-1}. In this coordinate system, the scalar product of two four-vectors V and W is given by $V \cdot W = V^+W^- + V^-W^+ - \boldsymbol{V}_\perp \cdot \boldsymbol{W}_\perp$.

At the leading nontrivial order in α_s, a quark fragments into the $Q\bar{Q}$ pair that form the quarkonium plus an additional final-state quark. We denote the momentum and mass of the fragmenting quark by k and m_q, respectively. We denote the momentum of the final-state quark by k_1, and we denote the momentum of the $Q\bar{Q}$ pair by P. We work at order v^0, and so we take the Q and the $\bar{Q}$ to have identical momenta $p = P/2$, with $p^2 = m_Q^2$, where m_Q is the mass of the Q or $\bar{Q}$. The mass of the $Q\bar{Q}$ state is then given by $M = 2m_Q$.

We work in the frame in which the transverse momentum of the $Q\bar{Q}$ pair vanishes. In this frame, the momenta are

$$k = \left(k^+, k^- = \frac{k^2 + (P_\perp/z)^2}{2k^+}, -\frac{\boldsymbol{P}_\perp}{z}\right), \tag{2a}$$

$$P = \left(zk^+, \frac{M^2}{2zk^+}, \boldsymbol{0}_\perp\right), \tag{2b}$$

$$k_1 = \left(z_1 k^+, \frac{m_q^2 + k_{1\perp}^2}{2z_1 k^+}, \boldsymbol{k}_{1\perp} = -\frac{\boldsymbol{P}_\perp}{z}\right), \tag{2c}$$

where z and $z_1 \equiv 1 - z$ are the longitudinal momentum fractions of the $Q\bar{Q}$ pair and the final-state quark, respectively:

$$z = \frac{P^+}{k^+}, \tag{3a}$$

$$z_1 = \frac{k_1^+}{k^+}. \tag{3b}$$

We note that

$$k_1 \cdot P = P^+ k_1^- + P^- k_1^+ = \frac{z^2(m_q^2 + k_{1\perp}^2) + z_1^2 M^2}{2zz_1}. \tag{4}$$

We wish to project the $Q\bar{Q}$ pair onto spin-triplet states. The required spin-triplet projectors in order v^0 are [6–11]

$$\Pi_3(p,p,\lambda) = -\frac{1}{2\sqrt{2}m_Q}\not{\epsilon}^*(\lambda)(\not{p} + m_Q), \tag{5a}$$

$$\bar{\Pi}_3(p,p,\lambda) = \gamma^0 \Pi_3^\dagger(p,p,\lambda)\gamma^0 = \frac{1}{2\sqrt{2}m_Q}\not{\epsilon}(\lambda)(\not{p} - m_Q), \tag{5b}$$

where $\epsilon(\lambda)$ is the polarization vector for the spin state λ. These projectors correspond to nonrelativistic normalization of the heavy-quark spinors.

The absolute squares of $\epsilon(\lambda)$ for various polarization states can be written in covariant forms. The result for the sum over all λ is

$$I_{\mu\nu} \equiv \sum_{\lambda=0,\pm1} \epsilon_\mu^*(\lambda)\epsilon_\nu(\lambda) = -g_{\mu\nu} + \frac{P_\mu P_\nu}{P^2}. \tag{6a}$$

The result for the sum over transverse polarizations is [12]

$$I_{\mu\nu}^T \equiv \sum_{\lambda=\pm1} \epsilon_\mu^*(\lambda)\epsilon_\nu(\lambda) = -g_{\mu\nu} + \frac{P_\mu n_\nu + P_\nu n_\mu}{n \cdot P} - \frac{P^2}{(n\cdot P)^2} n_\mu n_\nu, \tag{6b}$$

where

$$n = (0, 1, \mathbf{0}_\perp). \tag{6c}$$

Then, for the longitudinal polarization, we have

$$I_{\mu\nu}^L \equiv \epsilon_\mu^*(0)\epsilon_\nu(0) = I_{\mu\nu} - I_{\mu\nu}^T. \tag{6d}$$

The color-singlet and color-octet projection operators for the $Q\bar{Q}$ pair are

$$\Lambda_1 = \frac{\mathbf{1}}{\sqrt{N_c}}, \tag{7a}$$

$$\Lambda_8^a = \sqrt{2}\, T^a, \tag{7b}$$

where $\mathbf{1}$ is a unit SU(N_c)-color matrix, T^a is a generator of the fundamental representation of SU(N_c), $a \in \{1,\, 2,\, \cdots,\, N_c^2 - 1\}$, and $N_c = 3$.

III. COLLINS-SOPER DEFINITION OF FRAGMENTATION FUNCTION

Collins and Soper have given the following gauge-invariant definition of the quark fragmentation function in d dimensions [1]:

$$D_{q\to H}(z) = \frac{z^{d-3}}{N_c \times 4 \times 2\pi} \int_{-\infty}^{+\infty} dx^- e^{-iP^+x^-/z} \mathrm{tr}\left[\not{n}\langle 0|\psi(0)\mathcal{E}^\dagger(0)\mathcal{P}_{H(P,\lambda)}\mathcal{E}(x^-)\bar{\psi}(x)|0\rangle\right], \quad (8)$$

where $\psi(x)$ is the field of the initial quark and $\mathcal{E}(x^-)$ is the gauge link (eikonal line)

$$\mathcal{E}(x^-) = \mathcal{P}\exp\left[+ig_s\int_{x^-}^{\infty} dz^- \, A^+(0^+, z^-, \mathbf{0}_\perp)\right]. \quad (9)$$

Here, $\mathcal{P}$ indicates path ordering, and $g_s = \sqrt{4\pi\alpha_s}$ is the QCD coupling constant. The spinor field ψ is an SU(N_c)-color column vector in the fundamental representation, and the gluon field $A^\mu = A^\mu_a T^a$ is an SU(N_c)-color matrix in the fundamental representation. The trace is over the color and Dirac indices. The factors N_c and 4 in the denominator of Eq. (8) arise from the average over the color and Dirac indices of the initial-state quark, respectively. $\mathcal{P}_{H(P,\lambda)}$ is a projector onto states that include a hadron H with momentum P and polarization λ:

$$\mathcal{P}_{H(P,\lambda)} = \sum_X |H(P,\lambda)+X\rangle\langle H(P,\lambda)+X|, \quad (10)$$

where the summation is over all possible degrees of freedom.

The Feynman rules for the fragmentation function are the standard ones for QCD, with the following exceptions. First, there is an overall factor

$$C_{\mathrm{qfrag}} = \frac{z^{1-2\epsilon}}{8\pi N_c}, \quad (11)$$

which arises from the definition of the fragmentation function. Second, there are additional Feynman rules for the eikonal lines. We state the rules for the part of the Feynman diagram that lies to the left of the final-state cut. The rules for the part of the diagram that lies to the right of the final-state cut can be obtained by complex conjugation. Each eikonal-line propagator that carries momentum ℓ, flowing from the cut side to the operator side, contributes a factor $i\delta_{ij}/(\ell\cdot n + i\varepsilon)$, where i and j are color indices. Each eikonal-line–gluon vertex contributes a factor $ig_s n_\mu T^a_{ij}$, where μ is the four-vector index of the gluon. The final-state cut in an eikonal line carrying momentum ℓ contributes a factor $2\pi\delta(\ell\cdot n)$.

In general, the final-state phase space for a fragmentation function for n unobserved particles in the final state is given by

$$d\Phi_n = \frac{4\pi M}{S}\delta\left(k^+ - P^+ - \sum_{i=1}^{n} k_i^+\right)\left(\frac{\mu^2}{4\pi}e^{\gamma_{\rm E}}\right)^{n\epsilon}\prod_{i=1}^{n}\theta(k_i^+)\frac{dk_i^+ d^{2-2\epsilon}\boldsymbol{k}_{i\perp}}{2k_i^+(2\pi)^{3-2\epsilon}}, \tag{12}$$

where S is the statistical factor for identical final-state particles, M and P are the mass and momentum of the observed particle, respectively, and k_i is the momentum of the ith unobserved particle. We have included a factor $2M$ in the phase space in order to compensate for the fact that we use nonrelativistic normalization for the heavy-quark spinors. We associate the standard modified-minimal-subtraction ($\overline{\rm MS}$) scale factor $[\mu^2 e^{\gamma_{\rm E}}/(4\pi)]^\epsilon$ with each dimensionally regulated integration in $d = 4 - 2\epsilon$ space-time dimensions. Here, μ is the dimensional-regularization scale and $\gamma_{\rm E}$ is the Euler-Mascheroni constant.

For our specific kinematics, with one unobserved particle in the final state, the phase-space reduces to

$$\begin{aligned} d\Phi &= 4\pi M\delta(k^+ - P^+ - k_1^+)\left(\frac{\mu^2}{4\pi}e^{\gamma_{\rm E}}\right)^{\epsilon}\theta(k^+)\frac{dk_1^+}{4\pi k_1^+}\frac{d^{d-2}\boldsymbol{k}_{1\perp}}{(2\pi)^{d-2}} \\ &= \frac{4\pi M}{k^+}\delta(1 - z - z_1)\left(\frac{\mu^2}{4\pi}e^{\gamma_{\rm E}}\right)^{\epsilon}\theta(z_1)\frac{dz_1}{4\pi z_1}\frac{d^{2-2\epsilon}\boldsymbol{k}_{1\perp}}{(2\pi)^{2-2\epsilon}}. \end{aligned} \tag{13}$$

IV. COLOR-OCTET FRAGMENTATION: DIFFERENT-FLAVOR CASE

In this section, we compute the fragmentation function for quark fragmentation into a color-octet $Q\bar{Q}$ pair for the case in which the initial quark q and the quark Q that is a constituent of the quarkonium have different flavors. The Feynman diagrams for this calculation are shown in Fig. 1. In this calculation, and throughout the remainder of this paper, we work in the Feynman gauge.

In each of the contributions from the diagrams in Fig. 1, there is a common factor that arises from the annihilation of a virtual gluon into a color-octet spin-triplet S-wave pair $Q\bar{Q}(^3S_1^{[8]})$. The contribution to this factor from the left side of the final-state cut can be written as

$$J_\mu^{ab}(\lambda) = \frac{-ig_{\mu\nu}}{P^2 + i\varepsilon}{\rm tr}[(-ig_s\gamma^\nu T^a)\Pi_3(p,p,\lambda)\Lambda_8^b]. \tag{14}$$

A straightforward calculation gives

$$J_\mu^{ab}(\lambda) = \frac{g_s}{M^2 + i\varepsilon}\delta^{ab}\epsilon_\mu^*(\lambda). \tag{15}$$

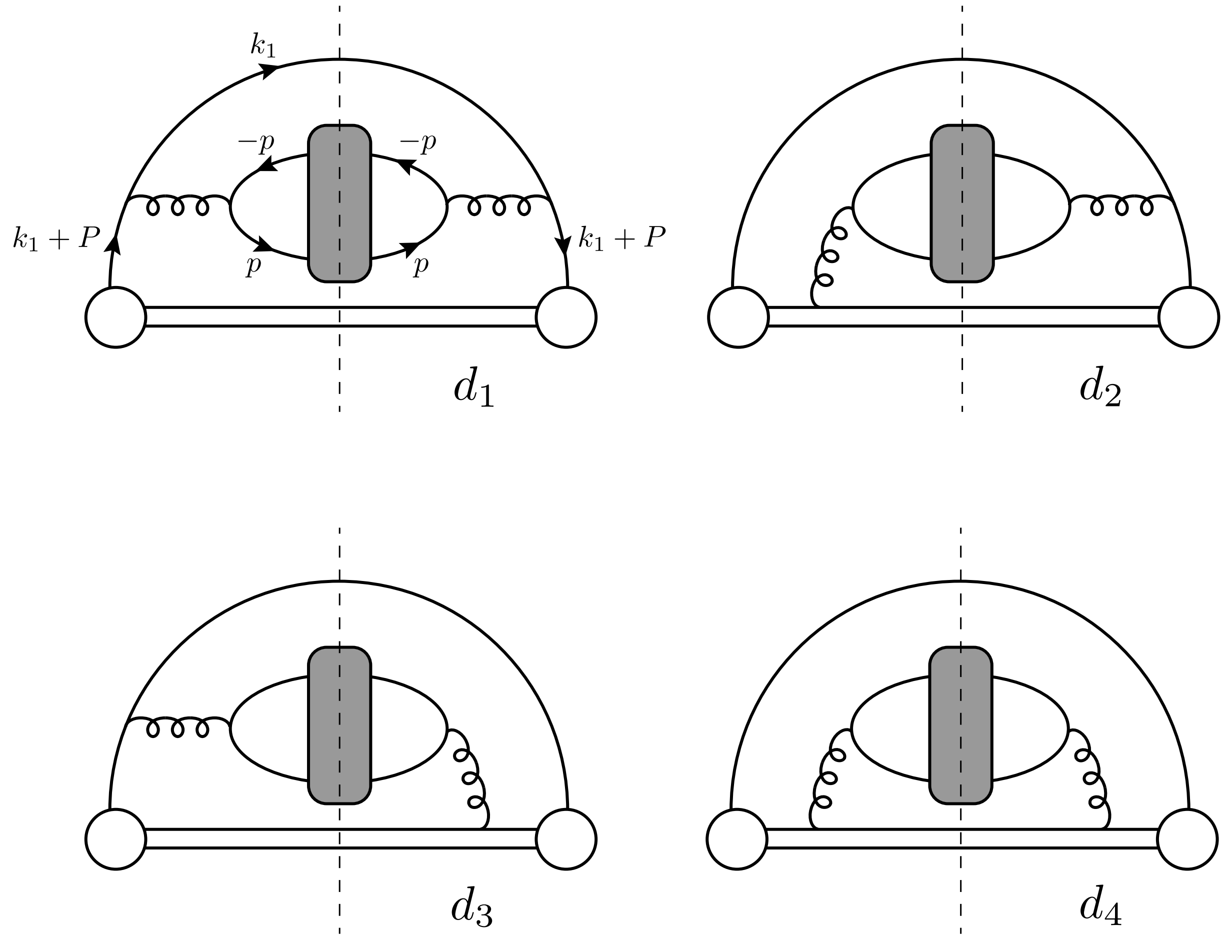

FIG. 1: Feynman diagrams for quark fragmentation into a color-octet $Q\bar{Q}$ pair for the case in which the initial quark q and the quark Q that is a constituent of the quarkonium have different flavors. The diagram labels d_i correspond to the quantities that appear in Eq. (17).

Multiplying by the complex conjugate and summing over the final-state color index, we obtain

$$X^{ab}_{\mu\nu}(\lambda) = J^{ac}_{\mu}(\lambda) J^{bc*}_{\nu}(\lambda) = \frac{g_s^2}{M^4} \delta^{ab} \epsilon^*_{\mu}(\lambda) \epsilon_{\nu}(\lambda). \tag{16}$$

Then, the diagrams of Fig. 1 give the following contributions to the fragmentation function:

$$
\begin{aligned}
d_1(z) &= C_{\mathrm{qfrag}}\mathrm{tr}\Bigg[(\not{k}_1+m_q)(-ig_s\gamma^\mu T^a)\frac{i}{\not{k}_1+\not{P}-m_q+i\varepsilon}\not{n} \\
&\qquad\times\frac{-i}{\not{k}_1+\not{P}-m_q-i\varepsilon}(+ig_s\gamma^\nu T^b)\Bigg]X^{ab}_{\mu\nu}d\Phi, &\text{(17a)}\\
d_2(z) &= C_{\mathrm{qfrag}}\mathrm{tr}\Bigg[(\not{k}_1+m_q)\frac{i}{(k-k_1)\cdot n+i\varepsilon}(ig_s n^\mu T^a)\not{n} \\
&\qquad\times\frac{-i}{\not{k}_1+\not{P}-m_q-i\varepsilon}(+ig_s\gamma^\nu T^b)\Bigg]X^{ab}_{\mu\nu}d\Phi, &\text{(17b)}\\
d_3(z) &= C_{\mathrm{qfrag}}\mathrm{tr}\Bigg[(\not{k}_1+m_q)(-ig_s\gamma^\mu T^a)\frac{i}{\not{k}_1+\not{P}-m_q+i\varepsilon}\not{n} \\
&\qquad\times(-ig_s n^\nu T^b)\frac{-i}{(k-k_1)\cdot n-i\varepsilon}\Bigg]X^{ab}_{\mu\nu}d\Phi, &\text{(17c)}\\
d_4(z) &= C_{\mathrm{qfrag}}\mathrm{tr}\Bigg[(\not{k}_1+m_q)\frac{i}{(k-k_1)\cdot n+i\varepsilon}(+ig_s n^\mu T^a)\not{n} \\
&\qquad\times(-ig_s n^\nu T^b)\frac{-i}{(k-k_1)\cdot n-i\varepsilon}\Bigg]X^{ab}_{\mu\nu}d\Phi. &\text{(17d)}
\end{aligned}
$$

In each contribution in Eq. (17), the overall color factor, including the color factor in $X^{ab}_{\mu\nu}$, is

$$\frac{1}{2}\delta^{ab}\mathrm{tr}(T^aT^b) = \frac{C_F N_c}{2}, \tag{18}$$

where $C_F = (N_c^2-1)/(2N_c)$. The dependence on $\boldsymbol{k}_{1\perp}$ in these expressions comes from the quark-propagator denominators, which contribute factors $(\boldsymbol{k}_{1\perp}^2 + m_q^2 + \frac{1-z}{z^2}M^2)^{-1}$. Thus, the integrals of these expressions over $\boldsymbol{k}_{1\perp}$ can be written in terms of the scalar integrals $J_n(m_q^2 + \frac{1-z}{z^2}M^2)$, where

$$
\begin{aligned}
J_n(s) &\equiv \left(\frac{\mu^2}{4\pi}e^{\gamma_{\mathrm{E}}}\right)^\epsilon \int \frac{d^{2-2\epsilon}\boldsymbol{k}_{1\perp}}{(2\pi)^{2-2\epsilon}}\frac{1}{(\boldsymbol{k}_{1\perp}^2+s)^n} \\
&= \frac{(\mu^2 e^{\gamma_{\mathrm{E}}})^\epsilon}{4\pi}\frac{\Gamma(n-1+\epsilon)}{\Gamma(n)}s^{1-n-\epsilon}. &\text{(19)}
\end{aligned}
$$

The only integral that diverges as $\epsilon \to 0$ is $J_1(s)$:

$$J_1(s) = \frac{(\mu^2 e^{\gamma_{\mathrm{E}}}/s)^\epsilon}{4\pi}\frac{\Gamma(1+\epsilon)}{\epsilon}. \tag{20}$$

For $n \geq 2$, $J_n(s)$, we can write

$$J_n(s) = \frac{s^{1-n}}{4\pi(n-1)} + O(\epsilon) \quad (n \geq 2). \tag{21}$$

Summing over the four contributions in Eq. (17), using the expressions for the absolute squares of the polarizations in Eq. (6), multiplying by the factor in Eq. (11), and carrying out the phase-space integration in Eq. (13), we obtain the following results for the fragmentation functions:

$$\sum_\lambda D_{q\to Q\bar{Q}({}^3S_1^{[8]})(\lambda)}(z) = \frac{g_s^4 C_F}{\pi M^3 z^{1+2\epsilon}}\{[1+(1-z)^2-\epsilon z^2]J_1(m_q^2+\tfrac{1-z}{z^2}M^2)$$
$$-[(1-\epsilon)M^2+2m_q^2](1-z)J_2(m_q^2+\tfrac{1-z}{z^2}M^2)\}, \quad \text{(22a)}$$
$$D_{q\to Q\bar{Q}({}^3S_1^{[8]})(\lambda=0)}(z) = \frac{2g_s^4 C_F}{\pi M}\frac{(1-z)^2}{z^{3+2\epsilon}}J_2(m_q^2+\tfrac{1-z}{z^2}M^2). \quad \text{(22b)}$$

Here, we retain the full ϵ dependence, as it may be useful for calculations of fragmentation functions at higher orders in α_s.

The expression for $\sum_\lambda D_{q\to Q\bar{Q}({}^3S_1^{[8]})(\lambda)}$ contains a pole in ϵ. We renormalize this expression using the $\overline{\rm MS}$ procedure [1]:

$$D^{\overline{\rm MS}}_{q\to A}(z,\mu) = D_{q\to A}(z,\mu) - \frac{1}{\epsilon}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y}\, P_{gq}(z/y)\, D_{g\to A}(y), \quad \text{(23)}$$

where the $D_{q\to A}(z)$ and the $D_{g\to A}(z)$ are the bare quark and gluon fragmentation functions and $P_{gq}(z)$ is the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) splitting function, which is given at lowest order in α_s by

$$P_{gq}(z) = C_F\frac{1+(1-z)^2}{z}. \quad \text{(24)}$$

The bare gluon fragmentation functions for the unpolarized and longitudinally polarized states are given at leading order in α_s by [13]

$$\sum_\lambda D_{g\to Q\bar{Q}({}^3S_1^{[8]})(\lambda)}(z) = \frac{\pi\alpha_s}{m_Q^3}\delta(1-z), \quad \text{(25a)}$$
$$D_{g\to Q\bar{Q}({}^3S_1^{[8]})(\lambda=0)}(z) = 0. \quad \text{(25b)}$$

At this order in α_s, the bare gluon fragmentation functions do not depend on ϵ.[1] Carrying out the renormalization and dropping contributions of order ϵ and higher, we obtain

$$\sum_\lambda D^{\overline{\rm MS}}_{q\to Q\bar{Q}({}^3S_1^{[8]})(\lambda)}(z,\mu) = \frac{\alpha_s^2 C_F}{2m_Q^3}\left\{\frac{z^2-2z+2}{z}\left[\log\frac{\mu^2}{4m_Q^2}-\log(1-z+rz^2)\right]\right.$$
$$\left. -z-\frac{z(1-z)(1+2r)}{1-z+rz^2}\right\}\frac{\langle O^{Q\bar{Q}({}^3S_1^{[8]})}({}^3S_1^{[8]})\rangle}{3(N_c^2-1)}, \quad \text{(26a)}$$
$$D_{q\to Q\bar{Q}({}^3S_1^{[8]})(\lambda=0)}(z) = \frac{\alpha_s^2 C_F}{2m_Q^3}\frac{2(1-z)}{z}\frac{1-z}{1-z+rz^2}\frac{\langle O^{Q\bar{Q}({}^3S_1^{[8]})}({}^3S_1^{[8]})\rangle}{3(N_c^2-1)}, \quad \text{(26b)}$$

[1] The ϵ-dependence in Eq. (25) is different from that in the corresponding expression in Ref. [13]. In Ref. [13], a factor $[\mu^2 e^{\gamma_{\rm E}}/(4\pi)]^\epsilon$ was associated with each factor g_s^2. In the present paper, we associate a factor $[\mu^2 e^{\gamma_{\rm E}}/(4\pi)]^\epsilon$ with each dimensionally regulated integral. This difference between these conventions does not affect the finite result.

where $r \equiv m_q^2/M^2 = m_q^2/(2m_Q)^2$. In Eq. (26), we have written the perturbative fragmentation function in the NRQCD-factorized form of a short-distance coefficient times an NRQCD long-distance matrix element (LDME) [14] by making use of the fact that, at order α_s^0,

$$\langle O^{Q\bar{Q}(^3S_1^{[8]})}(^3S_1^{[8]})\rangle = (N_c^2-1)(d-1).^2 \tag{27}$$

The NRQCD LDME $\langle O^{Q\bar{Q}(^3S_1^{[8]})}(^3S_1^{[8]})\rangle$ is defined by

$$\langle O^{Q\bar{Q}(^3S_1^{[8]})}(^3S_1^{[8]})\rangle = \langle 0|\chi^\dagger\sigma^i T^a\psi \sum_\lambda \mathcal{P}_{Q\bar{Q}(^3S_1^{[8]})(P,\lambda)}\psi^\dagger\sigma^i T^a\chi|0\rangle. \tag{28}$$

Here, ψ is the two-component (Pauli) spinor field operator that annihilates a heavy quark and $\chi^\dagger$ is the two-component (Pauli) spinor field operator that annihilates a heavy antiquark. The projection operator $\mathcal{P}_{Q\bar{Q}(^3S_1^{[8]})(P,\lambda)}$ is the free $Q\bar{Q}$ analogue of $\mathcal{P}_{H(P,\lambda)}$, except that, because we are considering an NRQCD LDME, the intermediate state can contain only light degrees of freedom in addition to the explicit $Q\bar{Q}$ pair.

We can now obtain the quarkonium fragmentation functions for the unpolarized and the longitudinally polarized states by replacing the free $Q\bar{Q}$ LDMEs in Eq. (26) with the quarkonium LDMEs:

$$\begin{aligned}\sum_\lambda D^{\overline{\rm MS}}_{q\to H(\lambda)}(z,\mu) = \frac{\alpha_s^2 C_F}{2m_Q^3}\Bigg\{&\frac{z^2-2z+2}{z}\left[\log\frac{\mu^2}{4m_Q^2} - \log(1-z+rz^2)\right]\\ &-z-\frac{z(1-z)(1+2r)}{1-z+rz^2}\Bigg\}\frac{\langle O^H(^3S_1^{[8]})\rangle}{3(N_c^2-1)},\end{aligned} \tag{29a}$$

$$D_{q\to H(\lambda=0)}(z) = \frac{\alpha_s^2 C_F}{2m_Q^3}\frac{2(1-z)}{z}\frac{1-z}{1-z+rz^2}\frac{\langle O^H(^3S_1^{[8]})\rangle}{3(N_c^2-1)}, \tag{29b}$$

where the NRQCD LDME $\langle O^H(^3S_1^{[8]})\rangle$ is defined by

$$\langle O^H(^3S_1^{[8]})\rangle = \langle 0|\chi^\dagger\sigma^i T^a\psi \sum_\lambda \mathcal{P}_{H(P,\lambda)}\psi^\dagger\sigma^i T^a\chi|0\rangle. \tag{30}$$

Identical expressions hold for the case of an initial antiquark.

The result for the polarization-summed fragmentation function in Eq. (29a) confirms the result in Eq. (4.2) of Ref. [2] and the result in Eq. (C37) of Ref. [3]. The result for the

[2] Our convention for the LDME is to sum over all spin states of the quarkonium or $Q\bar{Q}$ pair, even in the case of fragmentation into a single (longitudinally polarized) spin state. In that case, we use the fact that the LDMEs for different spin states are identical, up to corrections of relative order v^2. Note that our LDMEs for the longitudinally polarized case are larger than those in Ref. [5] by a factor three.

longitudinal-polarization fragmentation function in Eq. (29b) disagrees with the result in Eq. (C.10) of Ref. [5].[3] The author of Ref. [5] has confirmed that the result in Eq. (29b) is correct.

In the case of light initial quarks, it is useful to take the limit $m_q \to 0$, which gives

$$\sum_\lambda D^{\overline{\rm MS}}_{q\to H(\lambda)}(z,\mu) = \frac{\alpha_s^2 C_F}{2m_Q^3}\left[\frac{z^2-2z+2}{z}\log\frac{\mu^2}{4m_Q^2(1-z)} - 2z\right]\frac{\langle O^H(^3S_1^{[8]})\rangle}{3(N_c^2-1)}, \quad (31a)$$

$$D_{q\to H(\lambda=0)}(z) = \frac{\alpha_s^2 C_F}{2m_Q^3}\frac{2(1-z)}{z}\frac{\langle O^H(^3S_1^{[8]})\rangle}{3(N_c^2-1)}. \quad (31b)$$

We have compared our result in Eq. (31a) with the result in Eq. (20) of Ref. [15] for quark fragmentation into lepton pairs, taking into account differences in the color and phase-space factors, and have found that they are consistent with each other.

V. COLOR-OCTET FRAGMENTATION: SAME-FLAVOR CASE

Now let us consider the fragmentation function for the case in which the initial quark q and the quark Q that is a constituent of the quarkonium have the same flavor. In this case, there are contributions from the diagrams that are shown in Fig. 1. These are given by the expressions in Eq. (17), but with m_q set equal to m_Q. In addition, there are contributions from the diagrams that are shown in Fig. 2 and in Fig. 3. The diagrams in Fig. 2 differ from those in Fig. 1 in that the identical quarks have been interchanged in the amplitudes on both the left and right sides of the final-state cut. The diagrams in Fig. 3 differ from those in Fig. 1 in that the identical quarks have been interchanged in an amplitude on only one side of the cut.

[3] We find that a denominator factor in Eq. (C.10) of Ref. [5] should be $\eta z^2 - 4z + 4$, rather than $\eta^2 z^2 - 4z + 4$, where $\eta = m_q^2/m_Q^2 = 4r$. We also find that the result in Eq. (C.10) of Ref. [5] should be multiplied by an overall factor three. Here, we have taken into account the fact the LDME in Eq. (29b) is a factor three larger than the corresponding LDME in Ref. [5].

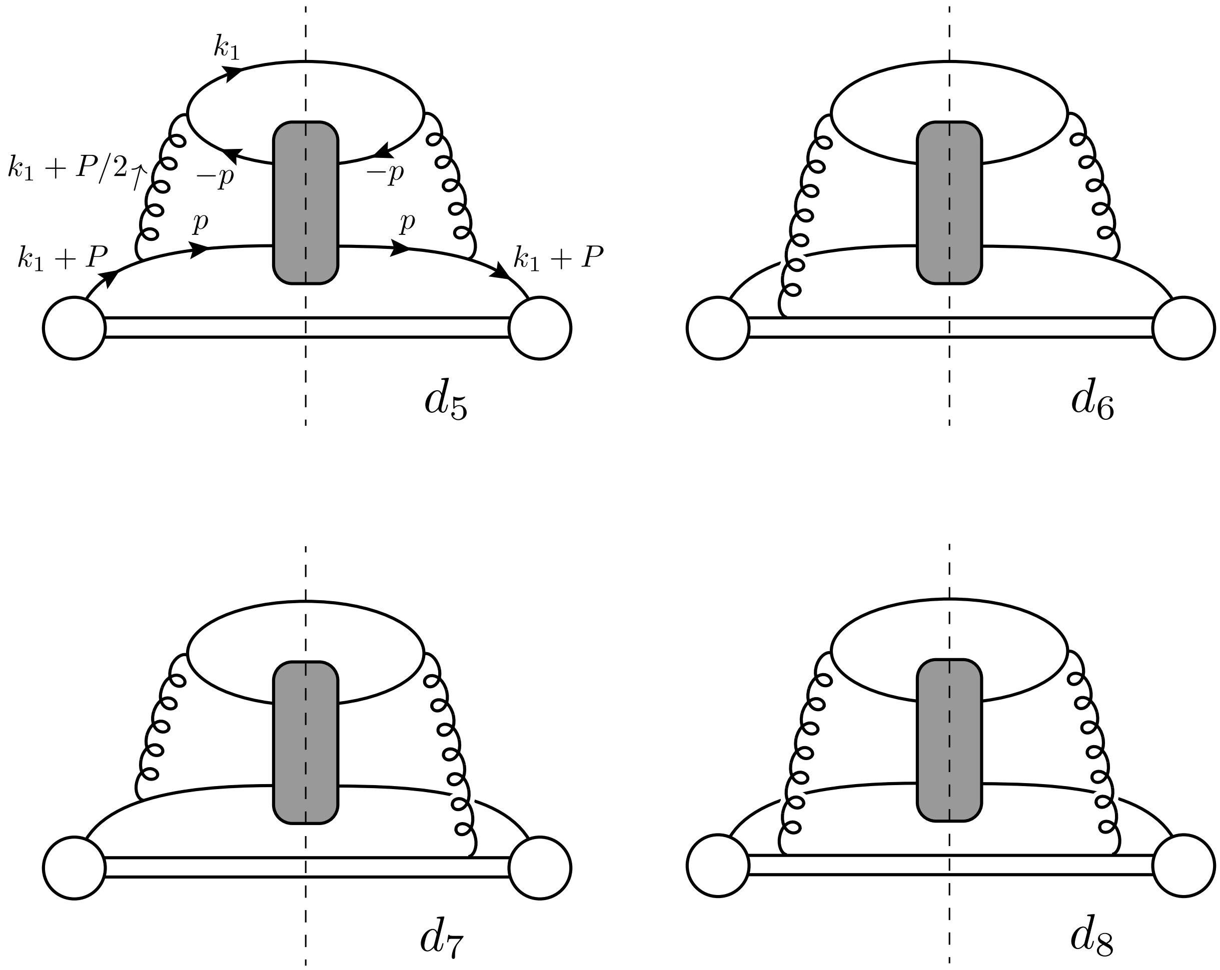

FIG. 2: Additional Feynman diagrams for quark fragmentation into a color-octet $Q\bar{Q}$ pair for the case in which the initial quark q and the quark Q that is a constituent of the quarkonium have the same flavor. These diagrams differ from those in Fig. 1 in that the identical quarks have been interchanged in the amplitudes on both the left and right sides of the final-state cut. The diagram labels d_i correspond to the quantities that appear in Eq. (32).

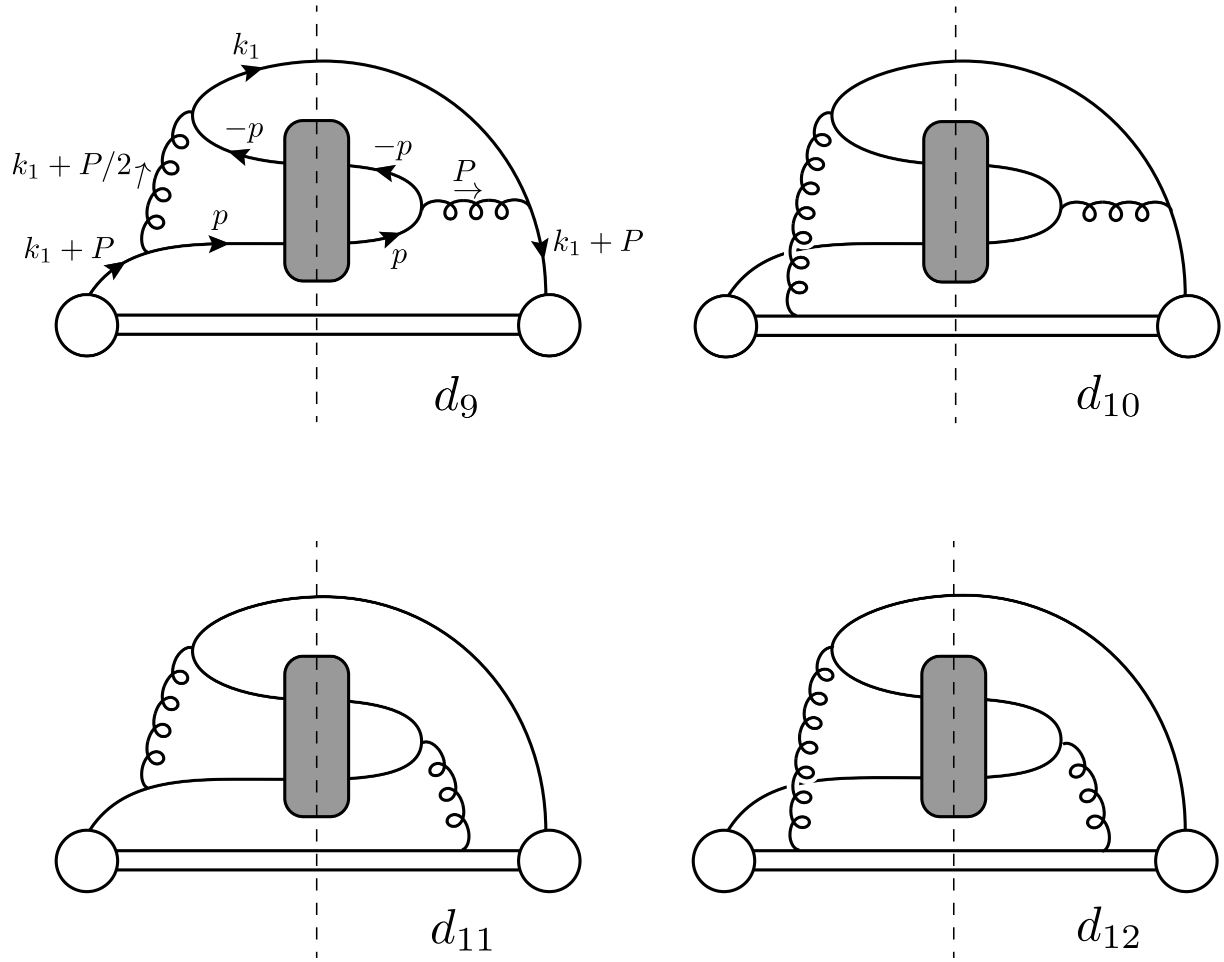

FIG. 3: Additional Feynman diagrams for quark fragmentation into a color-octet $Q\bar{Q}$ pair for the case in which the initial quark q and the quark Q that is a constituent of the quarkonium have the same flavor. These diagrams differ from those in Fig. 1 in that the identical quarks have been interchanged in an amplitude on only one side of the final-state cut. The diagram labels d_i correspond to the quantities that appear in Eq. (34).

The contributions of the diagrams in Fig. 2 are

$$
\begin{aligned}
d_5(z,\lambda) \;=\; & C_{\rm qfrag}{\rm tr}\Bigg[\Pi_3(p,p,\lambda)\Lambda_8^c(-ig_s\gamma^\mu T^a)\frac{i}{\not k_1+\not P-m_Q+i\varepsilon}\not n\frac{-i}{\not k_1+\not P-m_Q-i\varepsilon} \\
& \times(+ig_s\gamma^\nu T^b)\bar\Pi_3(p,p,\lambda)\Lambda_8^c(+ig_s\gamma_\nu T^b)(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Bigg] \\
& \times\frac{-i}{(k_1+P/2)^2+i\varepsilon}\frac{+i}{(k_1+P/2)^2-i\varepsilon}d\Phi, \qquad (32a) \\
d_6(z,\lambda) \;=\; & C_{\rm qfrag}{\rm tr}\Bigg[\Pi_3(p,p,\lambda)\Lambda_8^c\frac{i}{(k-P/2)\cdot n+i\varepsilon}(ig_sn^\mu T^a)\not n\frac{-i}{\not k_1+\not P-m_Q-i\varepsilon} \\
& \times(+ig_s\gamma^\nu T^b)\bar\Pi_3(p,p,\lambda)\Lambda_8^c(+ig_s\gamma_\nu T^b)(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Bigg] \\
& \times\frac{-i}{(k_1+P/2)^2+i\varepsilon}\frac{+i}{(k_1+P/2)^2-i\varepsilon}d\Phi, \qquad (32b) \\
d_7(z,\lambda) \;=\; & C_{\rm qfrag}{\rm tr}\Bigg[\Pi_3(p,p,\lambda)\Lambda_8^c(-ig_s\gamma^\mu T^a)\frac{i}{\not k_1+\not P-m_Q+i\varepsilon}\not n(-ig_sn^\nu T^b) \\
& \times\frac{-i}{(k-P/2)\cdot n-i\varepsilon}\bar\Pi_3(p,p,\lambda)\Lambda_8^c(+ig_s\gamma_\nu T^b)(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Bigg] \\
& \times\frac{-i}{(k_1+P/2)^2+i\varepsilon}\frac{+i}{(k_1+P/2)^2-i\varepsilon}d\Phi, \qquad (32c) \\
d_8(z,\lambda) \;=\; & C_{\rm qfrag}{\rm tr}\Bigg[\Pi_3(p,p,\lambda)\Lambda_8^c\frac{i}{(k-P/2)\cdot n+i\varepsilon}(ig_sn^\mu T^a)\not n(-ig_sn^\nu T^b) \\
& \times\frac{-i}{(k-P/2)\cdot n-i\varepsilon}\bar\Pi_3(p,p,\lambda)\Lambda_8^c(+ig_s\gamma_\nu T^b)(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Bigg] \\
& \times\frac{-i}{(k_1+P/2)^2+i\varepsilon}\frac{+i}{(k_1+P/2)^2-i\varepsilon}d\Phi. \qquad (32d)
\end{aligned}
$$

In each contribution in Eq. (32), the overall color factor is

$$
{\rm tr}\left(\Lambda_8^cT^aT^b\Lambda_8^cT^bT^a\right)=\frac{C_F}{2N_c}. \qquad (33)
$$

The contributions from the diagrams in Fig. 3 are

$$\begin{aligned}
d_9(z,\lambda) &= -C_{\rm qfrag}{\rm tr}\Big[\Pi_3(p,p,\lambda)\Lambda_8^c(-ig_s\gamma^\mu T^a)\frac{i}{\not k_1+\not P-m_Q+i\varepsilon}\not n \\
&\quad\times\frac{-i}{\not k_1+\not P-m_Q-i\varepsilon}(+ig_s\gamma^\nu T^b)(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Big]\times J_\nu^{bc*} \\
&\quad\times\frac{-i}{(k_1+P/2)^2+i\varepsilon}d\Phi+{\rm c.c.}, &(34a) \\
d_{10}(z,\lambda) &= -C_{\rm qfrag}{\rm tr}\Big[\Pi_3(p,p,\lambda)\Lambda_8^c\frac{i}{(k-P/2)\cdot n+i\varepsilon}(ig_s n^\mu T^a)\not n \\
&\quad\times\frac{-i}{\not k_1+\not P-m_Q-i\varepsilon}(+ig_s\gamma^\nu T^b)(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Big]\times J_\nu^{bc*} \\
&\quad\times\frac{-i}{(k_1+P/2)^2+i\varepsilon}d\Phi+{\rm c.c.}, &(34b) \\
d_{11}(z,\lambda) &= -C_{\rm qfrag}{\rm tr}\Big[\Pi_3(p,p,\lambda)\Lambda_8^c(-ig_s\gamma^\mu T^a)\frac{i}{\not k_1+\not P-m_Q+i\varepsilon}\not n \\
&\quad\times(-ig_s n^\nu T^b)\frac{-i}{(k-k_1)\cdot n-i\varepsilon}(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Big]\times J_\nu^{bc*} \\
&\quad\times\frac{-i}{(k_1+P/2)^2+i\varepsilon}d\Phi+{\rm c.c.}, &(34c) \\
d_{12}(z,\lambda) &= -C_{\rm qfrag}{\rm tr}\Big[\Pi_3(p,p,\lambda)\Lambda_8^c\frac{i}{(k-P/2)\cdot n+i\varepsilon}(ig_s n^\mu T^a)\not n \\
&\quad\times(-ig_s n^\nu T^b)\frac{-i}{(k-k_1)\cdot n-i\varepsilon}(\not k_1+m_Q)(-ig_s\gamma_\mu T^a)\Big]\times J_\nu^{bc*} \\
&\quad\times\frac{-i}{(k_1+P/2)^2+i\varepsilon}d\Phi+{\rm c.c.}, &(34d)
\end{aligned}$$

where c.c. stands for complex conjugate. In each contribution in Eq. (34), the overall color factor, including the color factor from J_ν^{bc*}, is

$$\frac{\delta^{bc}}{\sqrt{2}}{\rm tr}\left(\Lambda_8^c T^a T^b T^a\right) = -\frac{C_F}{2}. \qquad (35)$$

We note that the contributions in Eq. (34) contain a phase -1 relative to the contributions in Eqs. (17) and (32) that arises because of the interchange of the identical quarks in an amplitude on only one side of the final-state cut. We also note that the quantities $\sum_{i=1}^4 d_i(z,\lambda)$ [Eq. (17)], $\sum_{i=5}^8 d_i(z,\lambda)$ [Eq. (32)], and $\sum_{i=9}^{12} d_i(z,\lambda)$ [Eq. (34)] are separately gauge invariant.

The dependence on $\boldsymbol{k}_{1\perp}$ in the expressions in Eqs. (32) and (34) comes from the quark- and gluon-propagator denominators, which contribute factors $[\boldsymbol{k}_{1\perp}^2+\left(\frac{2-z}{2z}M\right)^2]^{-1}$. Thus, the integrals of these expressions over $\boldsymbol{k}_{1\perp}$ can be expressed in terms of the scalar integrals

$J_n[\left(\frac{2-z}{2z}M\right)^2]$ [Eq. (19)]. Summing over the contributions in Eqs. (32) and (34), using the expressions for the absolute squares of the polarizations in Eq. (6), multiplying by the factor in Eq. (11), and carrying out the phase-space integration[4] in Eq. (13), we obtain the following contributions to the fragmentation functions:

$$\sum_{\lambda=0,\pm1}\sum_{i=5}^{12} d_i(z,\lambda) \;=\; \frac{g_s^4 C_F(1-z)}{2\pi N_c^2 z^{3+2\epsilon}(2-z)^2 M}\left[a_2^{[8]}J_2 + M^2 a_3^{[8]}J_3 + M^4 a_4^{[8]}J_4\right], \tag{36a}$$

$$\sum_{i=5}^{12} d_i(z,\lambda=0) \;=\; \frac{g_s^4 C_F(1-z)^2}{2\pi N_c^2 z^{5+2\epsilon}(2-z)^2 M}\left[l_2^{[8]}J_2 + M^2 l_3^{[8]}J_3 + M^4 l_4^{[8]}J_4\right], \tag{36b}$$

where the dimensionless coefficients $a_n^{[8]}$ and $l_n^{[8]}$ are given in terms of z and ϵ by

$$\begin{aligned} a_2^{[8]} \;=&\; z^2(z-1)[-(9z^2+4z+4)+2\epsilon(3z^2+4)+\epsilon^2(5z^2-4z-12)+2\epsilon^3(4-4z+z^2)] \\ &-2N_c z(z-2)[-2(5z^2-5z+2)+\epsilon(-z^3+8z^2-6z+4)+\epsilon^2(z^3-2z^2)], \end{aligned} \tag{37a}$$

$$\begin{aligned} a_3^{[8]} \;=&\; -z(z-1)(z-2)[-2(z^2+6z-4)+\epsilon z(z+6)+2\epsilon^2 z(z-2)] \\ &-2N_c z(z-1)(z-2)^2(3-2\epsilon), \end{aligned} \tag{37b}$$

$$a_4^{[8]} \;=\; -(3-2\epsilon)(z-1)^2(z-2)^2, \tag{37c}$$

$$l_2^{[8]} \;=\; z^4[z+2+\epsilon(z-2)]^2 - 4N_c z^3(z-2)[z+2+\epsilon(z-2)], \tag{37d}$$

$$\begin{aligned} l_3^{[8]} \;=&\; -z^2(z-2)[4(z-1)(z+2)+\epsilon(z-2)(z^2+4z-8)] \\ &+2N_c z(z-2)^2[4(z-1)+\epsilon(z-2)^2], \end{aligned} \tag{37e}$$

$$l_4^{[8]} \;=\; 2[2(z-1)+\epsilon(z-2)^2](z-1)(z-2)^2. \tag{37f}$$

Here, the terms that are proportional to N_c come from the contributions in Eq. (34). Such terms do not appear in $a_4^{[8]}$ and $l_4^{[8]}$. Again, we retain the full ϵ dependence, as it may be useful for calculations of fragmentation functions at higher orders in α_s.

Taking the limit $\epsilon \to 0$, writing the result in the NRQCD-factorized form, and replacing

[4] We note that, although the final state contains two identical quarks, the statistical factor S in the phase space is unity. This follows from the fact that there is no integration over the momentum of the $Q\bar{Q}$ pair or from the fact that the two final-state particles, namely, the $Q\bar{Q}$ pair and the single Q, are distinct.

free $Q\bar{Q}$ LDMEs with quarkonium LDMEs, we obtain

$$\sum_{\lambda=0,\pm1} D^{(5\text{–}12)}_{Q\to H(\lambda)} = \frac{\alpha_s^2 C_F(1-z)}{N_c^2(2-z)^6 m_Q^3}\big[z(1-z)(5z^4-32z^3+72z^2-32z+16)$$
$$+8N_c(2-z)^2(z^3-6z^2+6z-2)\big]\times\frac{\langle O^H({}^3S_1^{[8]})\rangle}{3(N_c^2-1)}, \tag{38a}$$

$$D^{(5\text{–}12)}_{Q\to H(\lambda=0)} = \frac{\alpha_s^2 C_F z(1-z)^2}{3N_c^2(2-z)^6 m_Q^3}\big[3z^4-24z^3+64z^2-32z+16$$
$$+12N_c(2-z)^2(4-z)\big]\frac{\langle O^H({}^3S_1^{[8]})\rangle}{3(N_c^2-1)}. \tag{38b}$$

One obtains the complete fragmentation functions for the case in which the initial quark q and the quark Q that is a constituent of the quarkonium have the same flavor by adding the contributions in Eq. (39a) to the contributions in Eq. (29) with $r=m_q^2/(2m_Q)^2$ set equal to 1/4. The results are

$$\sum_\lambda D^{\overline{\text{MS}}}_{Q\to H(\lambda)}(z,\mu) = \frac{\alpha_s^2 C_F}{2zN_c^2(2-z)^6 m_Q^3}\Bigg[N_c^2(z^2-2z+2)(2-z)^6\log\frac{\mu^2}{(2-z)^2m_Q^2}$$
$$-N_c^2z^2(2-z)^4(z^2-10z+10)$$
$$+16N_cz(2-z)^2(1-z)(z^3-6z^2+6z-2)$$
$$+2z^2(1-z)^2(5z^4-32z^3+72z^2-32z+16)\Bigg]\frac{\langle O^H({}^3S_1^{[8]})\rangle}{3(N_c^2-1)}, \tag{39a}$$

$$D_{Q\to H(\lambda=0)}(z) = \frac{\alpha_s^2 C_F(1-z)^2}{3N_c^2z(2-z)^6m_Q^3}\Bigg[12N_c^2(2-z)^4+12N_cz^2(2-z)^2(4-z)$$
$$+z^2(3z^4-24z^3+64z^2-32z+16)\Bigg]\frac{\langle O^H({}^3S_1^{[8]})\rangle}{3(N_c^2-1)}. \tag{39b}$$

Identical expressions hold for the case of an initial antiquark.

Our polarization-summed result in Eq. (39a) differs from the result in Eq. (11) of Ref. [16], which was duplicated in Eq. (A3) of Ref. [17]. In Ref. [16], the color-octet fragmentation function was obtained by multiplying the color-singlet fragmentation function by a color factor. Consequently, the contributions of the diagrams of Figs. 1 and 3 were omitted. Our result in Eq. (39a) agrees with that in Eq. (C29) of Ref. [3].

Our longitudinal-polarization result in Eq. (39b) agrees with the result in Eq. (C.16) of Ref. [5], once one takes into account the fact that the LDME in Eq. (39b) is a factor three larger than the corresponding LDME in Ref. [5].

VI. COLOR-SINGLET FRAGMENTATION

In this section, we compute the fragmentation function for a quark to fragment through a spin-triplet color-singlet S-wave pair $Q\bar{Q}({}^3S_1^{[1]})$. This process proceeds at leading order in α_s only if the fragmenting quark has the same flavor as the quark in the $Q\bar{Q}$ pair. The Feynman diagrams that contribute to this process at leading order in α_s are those in Fig. 2. The corresponding contributions to the fragmentation function for a quark to fragment into a $Q\bar{Q}({}^3S_1^{[1]})$ pair are identical to the contributions d_5–d_8 in Eq. (32), except that the color projectors Λ_8 are replaced with the color projector Λ_1. In each contribution, the overall color factor is now

$$\mathrm{tr}(\Lambda_1 T^a T^b \Lambda_1 T^b T^b) = C_F^2, \tag{40}$$

instead of $C_F/(2N_c)$ [Eq. (33)].

The dependence on $\boldsymbol{k}_{1\perp}$ in the expressions in Eq. (32) comes from the quark- and gluon-propagator denominators, which contribute factors $[\boldsymbol{k}_{1\perp}^2 + \left(\frac{2-z}{2z}M\right)^2]^{-1}$. Thus, the integrals of these expressions over $\boldsymbol{k}_{1\perp}$ can be expressed in terms of the scalar integrals $J_n[\left(\frac{2-z}{2z}M\right)^2]$ [Eq. (19)]. Summing over the contributions in Eq. (32), but taking the color factor in Eq. (40), using the expressions for the absolute squares of the polarizations in Eq. (6), multiplying by the factor in Eq. (11), and carrying out the phase-space integration, we obtain the following contributions to the fragmentation functions

$$\begin{aligned}
\sum_{\lambda=0,\pm1} D^{(5-8)}_{Q\to Q\bar{Q}({}^3S_1^{[1]})(\lambda)} &= \frac{g_s^4 C_F^2 (1-z)^2}{3\pi z^{3+2\epsilon}(2-z)^2 M} \Bigg\{ a_2^{[1]} J_2\left[\left(\tfrac{2-z}{2z}M\right)^2\right] + M^2 a_3^{[1]} J_3\left[\left(\tfrac{2-z}{2z}M\right)^2\right] \\
&\quad + M^4 a_4^{[1]} J_4\left[\left(\tfrac{2-z}{2z}M\right)^2\right] \Bigg\}, \qquad (41\text{a}) \\
D^{(5-8)}_{Q\to Q\bar{Q}({}^3S_1^{[1]})(\lambda=0)} &= \frac{g_s^4 C_F^2 (1-z)^2}{3\pi z^{5+2\epsilon}(2-z)^2 M} \Bigg\{ l_2^{[1]} J_2\left[\left(\tfrac{2-z}{2z}M\right)^2\right] + M^2 l_3^{[1]} J_3\left[\left(\tfrac{2-z}{2z}M\right)^2\right] \\
&\quad + M^4 l_4^{[1]} J_4\left[\left(\tfrac{2-z}{2z}M\right)^2\right] \Bigg\}, \qquad (41\text{b})
\end{aligned}$$

where the dimensionless coefficients $a_n^{[1]}$ and $l_n^{[1]}$ are given in terms of z and ϵ by

$$a_2^{[1]} = z^2[(9z^2+4z+4) - \epsilon(6z^2+8) - \epsilon^2(5z^2-4z-12) - 2\epsilon^3(z^2-4z+4)], \tag{42a}$$

$$a_3^{[1]} = z(z-2)[-2(z^2+6z-4) + \epsilon z(2\epsilon z - 4\epsilon + z + 6)], \tag{42b}$$

$$a_4^{[1]} = (3-2\epsilon)(z-1)(z-2)^2, \tag{42c}$$

$$l_2^{[1]} = [z+2+\epsilon(z-2)]^2 z^4, \tag{42d}$$

$$l_3^{[1]} = -(z-2)z^2[4(z+2)(z-1) + \epsilon(z^2+4z-8)(z-2)], \tag{42e}$$

$$l_4^{[1]} = 2(z-1)(z-2)^2[2(z-1) + \epsilon(z-2)^2]. \tag{42f}$$

Taking the limit $\epsilon \to 0$, writing the result in the NRQCD-factorized form, and replacing free $Q\bar{Q}$ LDMEs with quarkonium LDMEs, we obtain

$$\sum_{\lambda=0,\pm1} D_{Q\to H(\lambda)} = \frac{\alpha_s^2 C_F^2 z(1-z)^2(5z^4-32z^3+72z^2-32z+16)}{9N_c(2-z)^6 m_Q^3}\langle O^H(^3S_1^{[1]})\rangle, \tag{43a}$$

$$D_{Q\to H(\lambda=0)} = \frac{\alpha_s^2 C_F^2 z(1-z)^2(3z^4-24z^3+64z^2-32z+16)}{27N_c(2-z)^6 m_Q^3}\langle O^H(^3S_1^{[1]})\rangle, \tag{43b}$$

where the NRQCD LDME is defined by

$$\langle O^H(^3S_1^{[1]})\rangle = \langle 0|\chi^\dagger\sigma^i\psi \sum_\lambda \mathcal{P}_{H(P,\lambda)}\psi^\dagger\sigma^i\chi|0\rangle, \tag{44}$$

and, in writing the result in the NRQCD-factorized form, we have used the fact that

$$\langle O^{Q\bar{Q}(^3S_1^{[1]})}(^3S_1^{[1]})\rangle = 2N_c(d-1). \tag{45}$$

The result in Eq. (43a) agrees with the result in Eq. (14) of Ref. [4] and with the result in Eq. (C24) of Ref. [3], once one takes into account the fact that the LDME in Eq. (43a) is a factor $2N_c$ larger than the corresponding LDME in Ref. [3]. Our result in Eq. (43b) agrees with the result in Eq. (C.11) of Ref. [5], once one takes into account the fact that the LDME in Eq. (43b) is a factor $6N_c$ larger than the LDME in Ref. [5].

VII. SUMMARY AND DISCUSSION

In this paper we computed fragmentation functions for a quark q to fragment into a heavy quarkonium through a heavy $Q\bar{Q}$ channel in which the $Q\bar{Q}$ pair is in a spin-triplet, S-wave state. Our computations are at the leading nontrivial order in α_s and at leading order in the heavy-quark velocity v. We have considered the following cases: (i) the q and Q have different flavors, in which case the $Q\bar{Q}$ pair must be in a color-octet state; (ii) the q and Q have the same flavor and the $Q\bar{Q}$ pair is in a color-octet state; (iii) the q and Q have the same flavor and the $Q\bar{Q}$ pair is in a color-singlet state. In each case, we have computed both the fragmentation function summed over all spin polarizations of the $Q\bar{Q}$ pair and the fragmentation function for longitudinal polarization of the $Q\bar{Q}$ pair. We have also presented expressions in $d = 4 - 2\epsilon$ dimensions, which may be useful in carrying out calculations of fragmentation functions at higher orders in α_s.

Our results for case (i) are given for finite m_q in Eq. (29) and for $m_q = 0$ in Eq. (31). The result in Eq. (29a) for the sum over $Q\bar{Q}$ polarizations agrees with previous calculations in Refs. [2, 3], and the result in Eq. (31a) for the sum over $Q\bar{Q}$ polarizations agrees with a calculation of quark fragmentation into lepton pairs in Ref. [15], once differences in the phase-space and color factors have been taken into account. The result in Eq. (29b) corrects the result in Ref. [5]. This correction has been confirmed by the author of Ref. [5].

Our result for case (ii) for the fragmentation function summed over $Q\bar{Q}$ polarizations is given by in Eq. (38a) and agrees with the result in Ref. [3]. Our result for case (ii) for the longitudinal-polarization fragmentation function is given in Eq. (39b) and agrees with the result in Ref. [5].

Our result for case (iii) for the fragmentation function summed over $Q\bar{Q}$ polarizations is given in Eq. (43a) and agrees with the results in Refs. [3, 4]. Our result for case (iii) for the longitudinal-polarization fragmentation function is given in Eq. (43b) and agrees with the result in Ref. [5].

The new results for longitudinally polarized fragmentation functions that we have obtained in this paper will make it possible to compute quark-initiated leading-power fragmentation contributions to the production of polarized S-wave spin-triplet quarkonia. While gluon-initiated fragmentation dominates quark-initiated fragmentation in quarkonium hadroproduction, quark-initiated fragmentation may be important for other quarkonium production processes, such as production in e^+e^- annihilation.

As we have mentioned, our calculations are at the leading nontrivial order in α_s and v. A complete calculation at order α_s^5 of the LP contributions to quarkonium production in the color-octet channels requires corrections to the fragmentation functions for the $^1S_0$ and ^3P_J channels at next-to-leading order (NLO) in α_s and for the $^3S_1$ channel through next-to-next-to-leading order (NNLO) in α_s. Corrections of NLO in α_s to the fragmentation function for a gluon to fragment into a quarkonium through the ${}^3S_1$ color-octet channel have already been computed [3, 18] and give a contribution that is numerically large in comparison with the leading-order (LO) contribution [19]. Corrections of higher order in v are also known to be large, relative to the LO contribution, for the fragmentation functions for gluons to fragment into quarkonia through the ${}^3S_1$ color-singlet and color-octet channels [13, 20]. These large corrections of higher order in α_s and v suggest that it may be important to compute higher-order corrections for additional quarkonium production channels and for quark-initiated, as well as gluon-initiated, fragmentation processes.

Note added: After the first version of this paper was submitted to the arXiv, Ref. [21] appeared. That paper confirms our results.

Acknowledgments

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