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Soft Factor Subtraction and Transverse Momentum Dependent Parton Distributions on Lattice

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Abstract

We study the transverse momentum dependent (TMD) parton distributions in the newly proposed quasi-parton distribution function framework in Euclidean space. In this framework, the parton distributions can be extracted from lattice observables in a systematic expansion of $1/P_z$ where P_z is the hadron momentum. A soft factor subtraction is found to be essential to make the TMDs calculable on lattice. We show that the quasi-TMDs with the associated soft factor subtraction can be applied in hard QCD scattering processes such as Drell-Yan lepton pair production in hadronic collisions. This allows future lattice calculations to provide information on the non-perturbative inputs and energy evolutions for the TMDs. Extension to the generalized parton distributions and quantum phase space Wigner distributions will lead to a complete nucleon tomography on lattice.

1. Introduction. Transverse momentum dependent (TMD) parton distributions have attracted great attentions in hadron physics research [1]. They provide a unique aspect of the partonic structure of nucleon, by extending the conventional description of Feynman parton distribution to the transverse dimension. Experimental investigations of the TMDs in the last few years have stimulated great theoretical developments, such as the QCD factorization, the energy (scale) evolution, and the universality of the TMDs [2–6]. These developments have laid solid foundation in phenomenological applications in hard QCD processes and hence allow us to extract the TMDs from the experiments.

In addition to the experimental accesses, the lattice QCD shall also be able to compute the TMDs. Early attempts to formulate the TMDs on lattice have been performed in Refs. [7] following the definitions of Collins-Soper [4]. These results have generated great interests in the hadron physics community. In order to obtain the TMDs applicable in phenomenology, we have to include the soft factor subtraction (see discussions below), which could not be included in the set-up of Refs. [7]. In this paper, we investigate the TMD formalism based on the recent proposal [8] to calculate the parton distributions (PDFs) in Euclidean space directly, in the large momentum effective theory approach (LaMET) [9]. This effective theory allows parton physics to be calculated in lattice QCD at large hadron momentum P_z in a systematic approximation. In particular, the PDF observables are evaluated as the matrix elements of the space-like correlators between the nucleon states which has a finite momentum P_z. Final results are obtained by a systematic expansion of $1/P_z$. The PDFs extracted in this way are also referred as qusi-PDFs [8].

To compute the parton distributions on lattice is conceptually important in the applications of QCD in hadron physics [7–12]. Progresses have been made concerning the technique issues associated with the applications of the quasi-PDFs, mainly on the integrated parton distributions [10–12]. In the current paper, we will apply the idea of Ref. [8] to the TMD case and, in particular, build the QCD factorization description of the hard scattering process such as Drell-Yan lepton pair production in *pp* collisions. The goal is to identify the TMD operators which can be computed on lattice and applied in QCD hard process in a consistent and rigorous fashion. A key point of the effective theory approach is that the theoretical uncertainties are under control [8].

The lattice calculations can also help the phenomenological applications of the TMDs. In particular, if we want to make predictions for future experiments, not only the TMDs at lower scale but also the relevant energy evolutions become important [13, 14]. In previous phenomenological studies, various assumptions are made [14–18] and they differ from each other. If we can compute the TMDs on lattice, it will provide important guidelines for the phenomenological studies.

We will carry out an explicit one-loop perturbative calculation for the TMDs, and demonstrate the QCD factorization in terms of the quasi-TMDs for the Drell-Yan process. By doing so, we will find that a soft factor subtraction in the TMD definition is essential to fulfill the factorization argument. The soft factor is constructed in such a way that it can be computed on lattice. This provides a solid foundation for future lattice applications for the TMDs and many other distributions, such as the generalized parton distributions and quantum phase space Wigner distributions.

Soft factor is an important aspect of the TMD factorization for hard QCD processes, which is also related to the regulation for the light-cone singularity in the TMD parton distributions. In the literature, there have been several proposals, and each of them introduces a way to construct the soft factor in the final factorization formula [2–5]. Following the quasi-PDF framework, we will derive a unique soft factor subtraction. Most importantly, both TMDs and the soft factor can be computed on lattice.

The rest of this paper is organized as follows. In Sec.2, we introduce the definition of the TMDs in Euclidean space, and will show the soft factor subtraction is necessary. In Sec. 3, we apply the TMDs to the Drell-Yan process and show that the QCD factorization at one-loop order can be achieved, where the soft factor plays an essential role. We briefly discuss the Collins-Soper evolution of the TMDs in Sec. 4 and conclude our paper in Sec. 5.

2. TMD Definition and Soft Factor Subtraction. For convenience, we consider the proton moving in $+\hat{z}$ direction with momentum,

$$P = \Lambda p + \frac{M^2}{2\Lambda} n \ , \tag{1}$$

where $\Lambda = P^+$ is a large momentum scale and M is the proton mass, and we have introduced two light-like vectors $p = (0^-, 1^+, 0_\perp) = \bar{n}$ and $n = (1^-, 0^+, 0_\perp)$: $p^2 = n^2 = 0$ and $p \cdot n = 1$. We further introduce a space-like vector $n_z = \frac{1}{\sqrt{2}}(n - p)$, such that $n_z^2 = -1$. P_z is related to the projection of P along with n_z, $n_z \cdot P = -P_z$. In the limit of $P_z \gg M$ or massless case, we have $\Lambda = \sqrt{2} P_z$.

In applying the TMD parton distributions and the associated QCD factorization, we keep the leading power contribution in the limit of $P_z \gg k_\perp$ where $k_\perp$ is the transverse momentum. We neglect all higher power corrections of $k_\perp/P_z$. This power counting analysis is consistent with the large momentum effective theory arguments to compute the parton distribution on lattice [9]. In this framework, the TMD quark distribution is written as,

$$q(x_z, k_\perp) = \frac{1}{2} \int \frac{d^3 z}{(2\pi)^3} e^{ik\cdot z} \langle PS | \overline{\psi}(0) \mathcal{L}^\dagger_{n_z(0,-\infty)} \gamma^z \mathcal{L}_{n_z(z,-\infty)} \psi(z) | PS \rangle \ , \tag{2}$$

where $x_z = k_z/P_z$. In the above definition, $\mathcal{L}_{n_z(y,-\infty)} = \mathcal{P}exp\left\{-ig\int_0^{-\infty} d\lambda n_z \cdot A(\lambda n_z + y)\right\}$ represents the gauge link along the $\hat{z}$ direction [1]. It has been known that the TMDs are process-dependent, and we have chosen the gauge link path to $-\infty$ which indicates that the above definitions are for the Drell-Yan process. In the TMD factorization, the cross section and the parton distributions are conveniently written in the $b_\perp$-space, which is the Fourier transformation respect to the transverse momentum: $q(x_z, b_\perp) = \int \frac{d^2k_\perp}{(2\pi)^2} e^{-ik_\perp \cdot b_\perp} q(x_z, k_\perp)$.

The TMD quark distribution defined as Eq. (2) contains the soft gluon radiation, which has to be subtracted in the final factorization formula. Similar to the idea proposed by Collins in Ref. [2], we introduce the following subtraction,

$$q^{sub.}(x, b_\perp) = q^{unsub.}(x, b_\perp) \sqrt{\frac{S^{n_x,n_y}(b_\perp)}{S^{n_x,n_z}(b_\perp) S^{n_z,n_y}(b_\perp)}} \ , \tag{3}$$

where $q^{unsub.}(x_z, b_\perp)$ is the Fourier-transformed un-subtracted PDF in Eq. (2) and S is defined as

$$S^{\bar{v},v}(b_\perp) = \langle 0 | \mathcal{L}^\dagger_{\bar{v}(-\infty,0)}(b_\perp) \mathcal{L}^\dagger_{v(0,-\infty)}(b_\perp) \mathcal{L}_{v(0,-\infty)}(0) \mathcal{L}_{\bar{v}(-\infty,0)}(0) | 0 \rangle \ , \tag{4}$$

[1] We focus our discussions in covariant gauge. In a singular gauge, such as the axial gauge $n_z \cdot A = 0$, we have to include an extra gauge link in the spatial infinity [19].

with $\mathcal{L}_v$ the gauge link to infinity along the direction v. In the above subtraction, we have chosen two transverse Wilson lines: $n_x^2 = n_y^2 = -1$ and $n_x \cdot n_z = n_y \cdot n_z = n_x \cdot n_y = 0$, to construct the associated soft factor. From the factorization point of view, light-like vectors for $n_{x,y}$ could be used as well. However, such soft factor can not be calculated on lattice.

An important consequence of subtracting the soft factor from the TMDs defined in Eq. (2) is that the gauge link self-interaction diagrams (such as Figs. 1(c) and 2(c) shown in the following section) are canceled out by the similar contribution from the soft factor of the last term in Eq. (3). These diagrams, in general, can introduce a pinch singularity [5] in the TMD calculations, and the subtraction is essential to fulfill the factorization for the associated hard processes [2].

After the subtraction, the TMD quark distribution of Eq. (3) is well defined and calculable on lattice. This kind of subtraction method in lattice QCD has been applied earlier in the literature, see, for example, Ref. [20]. This technique will have profound implications in the quasi-PDFs framework. In the following calculations, we will first focus on the applications in the TMD factorization.

3. One-loop Calculations and Factorization in Drell-Yan Process. It is instructive to have one-loop calculations and investigate the associated factorizations in terms of the new TMDs defined in the last section. For the one-loop calculations, we take an on-shell quark target. Clearly, the leading order quark distribution can be written as $q^{(0)}(x_z, k_\perp) = \delta(1-x_z)\delta^{(2)}(k_\perp)$, which leads to the expression in $b_\perp$-space: $q^{(0)}(x_z, b_\perp) = \delta(1-x_z)$. One-loop corrections contain real and virtual diagrams as shown in Figs. (1,2). First, the calculations for the virtual diagrams are similar to those in Ref. [3, 14]. We only need to change $\zeta^2 \to -\zeta^2$ where $\zeta^2 = (2n_z \cdot P)^2/(-n_z^2)$,

$$q^{(1)}(x_z, b_\perp)|_{vir} = \frac{\alpha_s}{2\pi}\delta(1-x_z)\left[-\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} + \frac{1}{\epsilon}\ln\frac{\zeta^2}{\mu^2} + \ln\frac{\zeta^2}{\mu^2} - \frac{1}{2}\left(\ln\frac{\zeta^2}{\mu^2}\right)^2 + \frac{1}{12}\pi^2 - 2\right] \quad (5)$$

For the real diagrams, different from the conventional TMDs, the quasi-TMDs will have contributions from $x_z > 1$ region, similar to that calculated for the integrated parton distributions [10, 11]. However, these contributions are power suppressed in the limit of $k_\perp \ll P_z$. For example, Fig. 1(a) contributes,

$$q^{(1)}(x_z, k_\perp)|_{\text{Fig. 1(a)}} = \frac{\alpha_s}{4\pi^2} C_F \frac{1-\epsilon}{k_\perp^2} \frac{(1-x_z)\left(\sqrt{k_\perp^2 + P_z^2(1-x_z)^2} + P_z(1-x_z)\right)}{\sqrt{k_\perp^2 + P_z^2(1-x_z)^2}} \ . \quad (6)$$

Clearly, the contribution in the region of $x_z > 1$ is power suppressed. Therefore, in the limit of $P_z \gg k_\perp$, it reduces to

$$\frac{\alpha_s}{2\pi^2} C_F \frac{1-\epsilon}{k_\perp^2}(1-x_z) \ , \quad (7)$$

for $0 < x_z < 1$, where $2\epsilon = 4 - D$ with D the dimension. In our calculations, we take dimension regulation to regulate singularities in both ultra-violet and infra-red regions. Fig. 1(b) contributes,

$$q^{(1)}(x_z, k_\perp)|_{\text{Fig. 1(b)}} = \frac{\alpha_s}{4\pi^2} C_F \frac{1}{k_\perp^2} \frac{x_z}{1-x_z} \frac{\left(\sqrt{k_\perp^2 + P_z^2(1-x_z)^2} + P_z(1-x_z)\right)}{\sqrt{k_\perp^2 + P_z^2(1-x_z)^2}} \ . \quad (8)$$

[2] Similar idea can work out for the case of the TMD factorization studied in Ref. [5] extending the original Collins-Soper 81 definition of the TMDs in axial gauge [4] to a covariant gauge.

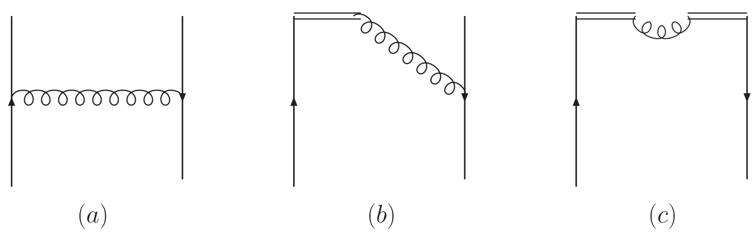

FIG. 1: Real diagrams contributions to the TMD quark distributions at one-loop order.

Again, the contribution in the region of $x_z > 1$ is also power suppressed. However, there is a singularity at $x_z = 1$. In order to evaluate the leading power contribution, we introduce a plus distribution and take the limit of $k_\perp^2 \ll P_z^2$,

$$\frac{\alpha_s}{2\pi^2} C_F \frac{1}{k_\perp^2} \left(\frac{2x_z}{(1-x_z)_+} + \delta(1-x_z) \ln \frac{\zeta^2}{k_\perp^2} \right) . \tag{9}$$

To derive the above result, we have taken into account the fact that there are contributions below and above $x_z = 1$ in Eq. (8), and a principal value prescription has been applied to evaluate the second term in Eq. (9). After this procedure, the leading power contributions are again limited to the region of $0 < x_z \leq 1$.

As we discussed in the previous section, Figs. 1(c) and 2(c) will be cancelled out by similar diagrams from the soft factor subtraction, and they will not contribute to the subtracted TMDs. Adding the contributions from Figs. 1(a) and (b) together, we obtain the real contributions at one-loop order,

$$q^{sub.(1)}(x_z, k_\perp)|_{real} = \frac{\alpha_s}{2\pi^2} C_F \frac{1}{k_\perp^2} \left(\frac{1-\epsilon}{k_\perp^2}(1-x_z) + \frac{2x_z}{(1-x_z)_+} + \delta(1-x_z) \ln \frac{\zeta^2}{k_\perp^2} \right) . \tag{10}$$

Fourier transforming into $b_\perp$-space, we will obtain

$$\begin{aligned} q^{sub.(1)}(x_z, b_\perp)|_{real} &= \frac{\alpha_s}{2\pi} C_F \left\{ \left(-\frac{1}{\epsilon} + \ln \frac{c_0^2}{b^2\mu^2} \right) \left[\frac{1+x_z^2}{(1-x_z)_+} \right] + (1-x_z) \right. \\ &\left. + \delta(1-x_z) \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon} \ln \frac{\zeta^2}{\mu^2} + \frac{1}{2} \left(\ln \frac{\zeta^2}{\mu^2} \right)^2 - \frac{1}{2} \left(\ln \frac{\zeta^2 b_\perp^2}{c_0^2} \right)^2 - \frac{\pi^2}{12} \right] \right\} \end{aligned} \tag{11}$$

where $c_0 = 2e^{-\gamma_E}$. By summing the virtual and real contributions, we find that the total TMD quark distribution at one-loop order, will be

$$\begin{aligned} q^{sub.}(x_z, b_\perp; \zeta) &= \frac{\alpha_s}{2\pi} C_F \left\{ \left(-\frac{1}{\epsilon} + \ln \frac{c_0^2}{b_\perp^2 \bar{\mu}^2} \right) \mathcal{P}_{q\to q}(x_z) + (1-x_z) \right. \\ &\left. + \delta(1-x_z) \left[\frac{3}{2} \ln \frac{b_\perp^2 \mu^2}{c_0^2} + \ln \frac{\zeta^2}{\mu^2} - \frac{1}{2} \left(\ln \frac{\zeta^2 b_\perp^2}{c_0^2} \right)^2 - 2 \right] \right\} , \end{aligned} \tag{12}$$

in the $b_\perp$-space, where $\mathcal{P}_{q\to q}(x) = \left(\frac{1+x^2}{1-x} \right)_+$ is the leading order splitting kernel for the quark. It is interesting to note that the above result is very similar to that obtained in the Ji-Ma-Yuan 2004 scheme [3], except two major differences. First, in Ji-Ma-Yuan 2004 scheme, the

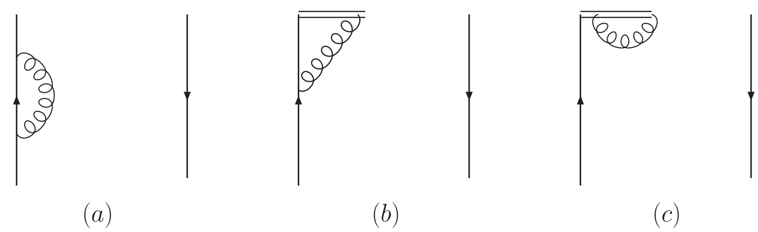

FIG. 2: Virtual diagrams contributions to the TMD quark distributions at one-loop order.

soft factor subtraction was formulated on the cross section level, such that the TMD quark distribution will have soft factor contribution. This can be seen, for example, by comparing the above equation to Eq. (20) of Ref. [14]. With soft factor subtraction at the TMD level, the expression for the quark distribution is much simpler in current scheme. Second, in Ji-Ma-Yuan 2004, ζ^2 is defined through a time-like gauge link, whereas here we have used a space-like gauge link. The difference is seen from the replacement $\zeta^2 \leftrightarrow -\zeta^2$.

To apply the above TMD quark distribution in hard QCD process as Drell-Yan lepton pair production in pp collisions, we need to calculate the TMD antiquark distribution as well. Similar to ζ introduced above, for the antiquark distribution we introduce the energy parameter $\bar{\zeta}^2 = (2n_z \cdot \bar{P})^2/(-n_z^2)$ where $\bar{P}$ is the momentum for the hadron moving in the $-\hat{z}$ direction. The differential cross section depending on the transverse momentum of the lepton pair can be written in the following factorization form,

$$W(Q, b_\perp) = q^{sub.}(x_z, b_\perp; \zeta)\overline{q}^{sub.}(\bar{x}_z, b_\perp; \bar{\zeta})H(Q, \mu) \ , \tag{13}$$

in the $b_\perp$-space. The Fourier transform of the above $W(Q, b_\perp)$ will lead to the transverse momentum distribution of the differential cross section. From the factorization, we obtain the hard factor for the Drell-Yan process as

$$H(Q) = \frac{\alpha_s}{2\pi}C_F\left[\ln\frac{Q^2}{\mu^2} + \pi^2 - 4\right] \ , \tag{14}$$

where we have chosen $\zeta^2 = \bar{\zeta}^2 = Q^2$ for simplicity [3].

4. TMD evolution. Similar to the previous formalisms for the TMDs, the TMDs in the quasi-parton distribution framework in Euclidean space also depend on the energy of the hadron. This can be seen from the one-loop calculations in the last section, in particular, from a double logarithms term $\ln^2(\zeta^2 b_\perp^2)$ as shown in Eq. (12). The Collins-Soper evolution can be derived as differential equation respect to ζ^2 for the TMDs. Because ζ^2 is defined through the gauge link vector n_z and the hadron momentum P, we can evaluate the Collins-Soper evolution by varying either n_z or P. In Collins-Soper 81 [4] and Ji-Ma-Yuan 2004 [3], it is done through n_z. By following a similar method, we expect to be able to derive an evolution equation of the form,

$$\frac{\partial}{\partial \ln \zeta} q(x_z, b_\perp, \zeta) = (K(b_\perp, \mu) + G(\zeta, \mu)) \times q(x_z, b_\perp, \zeta) \ , \tag{15}$$

[3] After resummation of large logarithms by solving the Collins-Soper evolution equations, the dependence on ζ and $\bar{\zeta}$ will cancel out and lead to the unique final results for the differential cross sections depending on the transverse momentum.

where K and G are the soft and hard parts, respectively. In particular, the derivative respect to ζ can be evaluated as derivative respect to n_z,

$$\zeta\frac{\partial}{\partial\zeta} = \delta v^\alpha \frac{\partial}{\partial n_z^\alpha} \ , \tag{16}$$

where δv is another dimensionless vector: $\delta v^- = n_z^-$, $\delta v^+ = -n_z^+$, and $\delta v_\perp = 0$. So that, we have $\delta v^2 = -n_z^2 > 0$ and $\delta v \cdot n_z = 0$. The relevant Feynman rules can be derived in the TMD limit: $P_z \gg k_\perp$. These derivations will be very much the same as those in Refs. [3, 4]. With the above evolution equation, we can resum the large logarithms from high order corrections, following the Collins-Soper-Sterman resummation formalism.

On the other hand, if we can compute the TMDs from lattice QCD, we can not only directly calculate the differential cross sections for the SIDIS and Drell-Yan processes from the TMDs, but also extract the evolution information. In practice, we need to perform the lattice calculations for several different values of P_z, and we can calculate numerically the dependence of the TMDs on the energy of the hadrons. This is of importance for phenomenological applications, for example, to investigate the energy dependence of the Sivers asymmetries, which is one of the top questions in hadronic spin physics.

5. Conclusion and Discussions. In summary, we have shown that the proposed framework of Ref. [8] for parton distributions can be applied to the transverse momentum dependent parton distributions, where the soft factor subtraction plays a very important role. We have calculated the TMDs at one-loop order, and demonstrated the associated factorization for the Drell-Yan lepton pair production.

We would like to emphasize that the soft factor subtraction is crucial to achieve the factorization. More importantly, this soft factor can be calculated from lattice. Future lattice QCD calculations of the TMDs can serve as important inputs for hard processes, and can also be used to study the parton distribution in three-dimension fashion. Extending to the quantum phase space Wigner distributions is straightforward, which, in return, will provide computational access to the nucleon tomography in parton picture.

In the above calculations, we have shown the perturbative calculations at one-loop order. From the generic factorization argument, we expect the soft factor subtraction will be also important to understand the quasi-PDFs at two-loop order and beyond. We will carry out the detailed analysis in a future publication.

In addition, the soft factor subtraction deals with the self-interaction diagrams from the gauge links in the quasi-PDF definition as those in Fig. 1(c) and Fig. 2(c). The existence of these diagrams come from the fact that the gauge links are along the non-light-like directions. The contributions of these diagrams lead to subtle ultra-violet behaviors in the perturbative calculations at one-loop order, which have to be carefully handled in the matching between the quasi-PDFs and the conventional ones [10–12]. Since the subtraction method introduced above is very general, and should apply to various parton distributions. This will help the convergence of the matching calculations in these papers.

Needless to say that the LaMET approach to calculating the various PDFs, including the TMDs discussed in this paper, has just begun to attract the lattice practitioners' attention. Calculations requires large momentum configurations for the nucleon, which leads to very small lattice spacing along z-direction particularly challenging for a meaningful simulation. Any progress in the LaMET will crucially depend on how we implement this requirement in lattice computations. Early attempts in Ref. [12] in LaMET shall encourage further efforts along this direction. It has also been discussed in Ref. [9] that small-x parton distributions are more challenging to calculate. This argument applies to the TMDs at small-x as well.

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