Top quark forward-backward asymmetry at approximate N^{3}LO

Nikolaos Kidonakis

The top quark forward-backward asymmetry at approximate N^{3}LO

Nikolaos Kidonakis

Department of Physics, Kennesaw State University,
Kennesaw, GA 30144, USA

Abstract

I calculate the top quark forward-backward asymmetry at the Tevatron in both the laboratory frame and the $t\bar{t}$ rest frame. I show that soft-gluon corrections are the dominant contribution to the asymmetry and closely approximate exact results through next-to-next-to-leading order (NNLO). I present a calculation of the asymmetry including approximate next-to-next-to-next-to-leading-order (N^{3}LO) soft-gluon contributions from next-to-next-to-leading-logarithm (NNLL) resummation as well as electroweak corrections. Thus approximate N^{3}LO (aN^{3}LO) results are obtained, which significantly enhance and improve previous NNLO results. The theoretical aN^{3}LO result for the top quark forward-backward asymmetry at the Tevatron in the laboratory frame is $(6.8\pm0.3)\%$, and in the $t\bar{t}$ rest frame it is $(10.0 \pm 0.6)\%$ which is in excellent agreement with recent Tevatron data.

1   Introduction

Top quark production has been studied extensively at the Tevatron and the LHC. The soft-gluon corrections are now known approximately to next-to-next-to-next-to-leading-order (N^{3}LO) [1, 2], based on the next-to-next-to-leading-logarithm (NNLL) two-loop soft-gluon resummation formalism of [3], which allows the calculation of approximate N^{3}LO (aN^{3}LO) results for the total cross section [1] and the transverse momentum and rapidity distributions [2]. The agreement between theory and experimental results from the LHC and the Tevatron is excellent for all available energies for both the total and differential cross sections.

A quantity of particular interest is the top quark forward-backward asymmetry, $A_{\rm FB}$, which is closely related to the rapidity distribution of the top quark. The definition of the asymmetry is

$$A_{\rm FB} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)} \equiv \frac{\Delta\sigma}{\sigma} \,, \tag{1.1}$$

where σ denotes cross sections and y_t is the top-quark rapidity in a given Lorentz frame, which may be the $p\bar{p}$ laboratory frame or the $t\bar{t}$ rest frame (in the latter frame the asymmetry is equivalently defined in terms of $\Delta y = y_t - y_{\bar{t}}$, with $y_{\bar{t}}$ the antitop rapidity; see Ref. [4] for more details on various definitions and further references). At a $p\bar{p}$ collider such as the Tevatron, charge conjugation invariance in QCD implies that $\sigma(y_{\bar{t}} > 0) = \sigma(y_t < 0)$, so the forward-backward asymmetry is equivalent to the charge asymmetry. The $t\bar{t}$ rest frame asymmetry is larger than the laboratory frame asymmetry by about fifty percent due to kinematics.

Early measurements of $A_{\rm FB}$ were performed by CDF [5] and D0 [6] and returned values substantially larger than standard model predictions. Recent measurements of $A_{\rm FB}$ in the $t\bar{t}$

rest frame with more data and precision have been reported in [7, 8]. Thus, it is important to perform the most accurate calculation for the asymmetry in order to have confidence in the theoretical prediction while looking for hints of new physics.

The QCD corrections to the asymmetry were calculated with next-to-leading-logarithm (NLL) resummation of soft-gluon contributions in the moment-space formalism in [9]. This was later extended to NNLL accuracy in [10] where approximate next-to-next-to-leading-order (NNLO) results were obtained. Results at NNLL were also derived using Soft-Collinear Effective Theory (SCET) in [11]. An approach using the principle of maximum conformality was used in [12] to calculate the asymmetry. Recently the complete NNLO QCD contribution to the asymmetry has appeared in [13].

A variety of resummation formalisms and methods have been proposed in the literature for top quark production over the past twenty years (see the discussions in [3, 14, 15, 16] and references therein). These differences include whether resummation is performed for the total cross section or for double-differential cross sections (i.e. absolute vs partonic threshold); the inclusion and motivation for various subleading terms and whether or not they are warranted; the use of damping factors away from threshold; whether resummation is performed in moment-space perturbative QCD vs SCET resummation; the use of prescriptions (and their validity) in resummed results vs performing prescription-independent fixed-order expansions at NNLO or NNNLO; the choice of kinematics, single-particle-inclusive (1PI) vs pair-invariant-mass (PIM), for double-differential resummations, and various choices for writing the analytical terms via partonic threshold relations of kinematical variables; etc. More details and references may be found in [3, 14, 15, 16]. In most recent formalisms (see Refs. [3, 10, 15, 16, 17, 18]) the soft-gluon corrections were found to be large and dominant.

Most resummation formalisms for top production use methods applicable only to calculations of total cross sections. Only two formalisms have been developed at NNLL for double-differential cross sections and applied to the asymmetry, one using moment-space QCD resummation [10] and the other using SCET [11]. The differences between the moment-space QCD and the SCET resummation approaches in top production were discussed in the review paper [15], where results for the asymmetry in different formalisms were shown. In the formalism of [10] the approximate NNLO contributions to the asymmetry were found to be significant while in the approach of [11] they were rather small. This discrepancy was not unexpected since the overall contributions from soft-gluon resummation to the total cross section and differential distributions were also quite different in the two approaches. There are several differences between the two resummations, as described in [15], but the numerically prominent one is the inclusion of certain terms in [11] that arise in the SCET approximation and are not strictly of soft gluon origin. Those SCET terms severely depress the results in [11] relative to [10]. As we will show in Section 2, the result with large approximate NNLO corrections in [10] precisely predicted the exact NNLO asymmetry; the results in [11] are much smaller, with tiny corrections beyond NLO.

The fact that the resummation formalism used in [3, 10] predicts the exact NNLO cross section better than all other approaches, to better than one per mil accuracy for the central value at the Tevatron, as shown in [16], had already strongly indicated that the results for the asymmetry in [10] would be very close to the exact NNLO result before the latter was known. This is indeed the case as we discuss in the next section. We note that there are strong

theoretical reasons for the choices we have made in our approach. Formally, the resummation uses renormalization group methods to fully determine terms at a given logarithmic accuracy (NNLL in this case). Numerically, subleading terms have to be treated carefully to approximate and predict the corrections correctly. At next-to-leading order (NLO) the total cross sections as well as the transverse momentum and rapidity distributions at all Tevatron and LHC energies were reproduced nearly exactly with our method. It was also shown that with our method the differences between approximate NNLO results in 1PI and PIM kinematics is minimized, which indicated that any missing terms were numerically small [19]. This fact together with the correct description at NLO was a very strong indicator that our NNLO predictions would be successful and, not surprisingly, this has indeed been the case, to very high accuracy, which validates our method and the theoretical arguments in support of it.

Parenthetically we note that it has been known for a while that the NLO expansion of the results from soft-gluon resummation for the asymmetry reproduces the complete NLO result rather well (this was noted already in [9] where NLL resummation was used). The extension to NNLL in [10] has a similar effect in the NNLO expansion, in predicting the exact NNLO results of [13] very well.

The authors of [13] also found a significant NNLO contribution, as expected from [10]. However, the authors of [13] stated erroneously that the effects of soft-gluon contributions are negligible. As discussed above, and as will be shown in more detail in the next section, this is factually incorrect. The soft-gluon contributions have been known to be large and dominant in the total and differential top production cross sections for at least twenty years (for references see the review in [15]), and in the asymmetry in particular at least since 2011 [10]. In fact the NNLO enhancements to the asymmetry found in [13] are very close to the predictions in [10]. The accuracy, precision, reliability, and robustness of the resummation formalism that we use has been amply demonstrated and discussed in [3, 16] for total cross sections and differential distributions.

We also note that while our resummation formalism uses the soft-gluon eikonal approximation for partonic threshold, the integrations are over all allowable gluon energies. Furthermore, partonic threshold is more general than absolute production threshold, i.e. the top quark is not necessarily produced at rest but may have arbitrary transverse momentum and rapidity as allowed by the kinematics. As a consequence, the effects of these approximate contributions to the top-quark transverse momentum (p_T) distribution increase with p_T at large p_T since that is a region closer to partonic threshold (see e.g. Fig. 3 in the first paper of Ref. [16]). Thus, we find the arguments made in [13] regarding the contributions of soft gluons to the transverse momentum distribution of the top-antitop pair to be invalid or irrelevant in our approach.

In this paper we will improve on the NNLO results for the asymmetry by further calculating N^{3}LO soft-gluon corrections, and including electroweak corrections [4, 20, 21, 22], thus producing an aN^{3}LO estimate. We present the results for the aN^{3}LO asymmetry in both the laboratory frame and the $t\bar{t}$ rest frame.

2 The top-quark forward-backward asymmetry at the Tevatron

We write the perturbative QCD expansion for the top-antitop pair cross section as

$$\sigma = \sum_{n=0}^{\infty} \alpha_s^{2+n} \, \sigma^{(n)} \tag{2.1}$$

with α_s the strong coupling. At leading order the cross section is of order α_s^2, the NLO corrections $\sigma^{(1)}$ are of order α_s^3, etc. We include the complete $\sigma^{(1)}$ and $\sigma^{(2)}$ corrections and the approximate $\sigma^{(3)}$ corrections from NNLL resummation.

At each order in α_s, the cross section includes plus-distributions of the form $[\ln^k(s_4/m_t^2)/s_4]_+$ where $s_4 = s+t+u-2m_t^2$, with s, t, u the usual kinematical variables and m_t the top-quark mass. The variable s_4 measures distance from partonic threshold and, for the n-th order corrections, the power of the logarithm, k, in the plus distributions can range from the leading value of $2n-1$ down to the lowest value of 0. Thus, at NLO the leading value of k is 1, at NNLO it is 3, and at N³LO it is 5. With NNLL resummation one can derive the coefficients of all powers of the logarithms at NLO and NNLO, but not at N³LO. We use the inverse transform of the moment-space logarithms of the Mellin moment to the specified order, including constant ζ_i terms (see also discussion in [15]). As discussed in [1], at N³LO the coefficients of the powers of the logarithms with $k = 5, 4, 3, 2$ can be fully determined, but those with $k = 1, 0$ are only partially known and include factorization and renormalization scale terms as well as terms arising from the inversion from moment space back to momentum space (see Ref. [23] for explicit expressions). The inversion terms, involving ζ_i constants, are known to dominate the subleading powers of the logarithms, and the impact of the remaining subleading terms is much smaller than the scale variation [1].

The top quark forward-backward asymmetry is then written as

$$A_{\mathrm{FB}} = \frac{\Delta\sigma^{\mathrm{EW}} + \alpha_s^3 \, \Delta\sigma^{(1)} + \alpha_s^4 \, \Delta\sigma^{(2)} + \alpha_s^5 \, \Delta\sigma^{(3)} + \cdots}{\alpha_s^2 \, \sigma^{(0)} + \alpha_s^3 \, \sigma^{(1)} + \alpha_s^4 \, \sigma^{(2)} + \alpha_s^5 \, \sigma^{(3)} + \cdots} \tag{2.2}$$

where again $\Delta\sigma^{(n)}$ denotes the QCD asymmetric terms as in Eq. (1.1), i.e. the difference of nth-order QCD corrections integrated over positive and negative rapidities, and $\Delta\sigma^{\mathrm{EW}}$ denotes the asymmetric term from electroweak corrections.

One may further reexpand the above ratio in the definition of A_{FB} in powers of α_s, thus obtaining the expression

$$
\begin{aligned}
A_{\mathrm{FB}} &= \frac{\Delta\sigma^{\mathrm{EW}}}{\alpha_s^2 \, \sigma^{(0)}} + \alpha_s \frac{\Delta\sigma^{(1)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{\mathrm{EW}} \sigma^{(1)}}{\alpha_s \, (\sigma^{(0)})^2} + \alpha_s^2 \left[\frac{\Delta\sigma^{(2)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{(1)} \, \sigma^{(1)}}{(\sigma^{(0)})^2} \right] \\
&\quad + \frac{\Delta\sigma^{\mathrm{EW}}}{(\sigma^{(0)})^3} \left[(\sigma^{(1)})^2 - \sigma^{(0)} \, \sigma^{(2)} \right] + \alpha_s^3 \left[\frac{\Delta\sigma^{(3)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{(2)} \, \sigma^{(1)}}{(\sigma^{(0)})^2} + \frac{\Delta\sigma^{(1)} (\sigma^{(1)})^2}{(\sigma^{(0)})^3} - \frac{\Delta\sigma^{(1)} \, \sigma^{(2)}}{(\sigma^{(0)})^2} \right] \\
&\quad + \cdots
\end{aligned} \tag{2.3}
$$

Explicit formulas and numerical values for the quantities $\Delta\sigma^{(i)}$ and $\sigma^{(i)}$ may be found in Refs. [1, 4, 13, 23]. In the following we will provide results for both definitions, Eq. (2.2) and Eq. (2.3), in both the laboratory and $t\bar{t}$ rest frames.

In the Standard Model the forward-backward asymmetry is due mainly to QCD effects. At leading-order in perturbative QCD, the production channels $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ are symmetric in rapidity, thus A_{FB} vanishes at that order. Furthermore, the gg channel remains symmetric at all orders in perturbative QCD. However an asymmetry arises in the $q\bar{q}$ channel (as well as a much smaller contribution from the the qg channel) starting at NLO. Therefore by applying soft-gluon resummation we expect the gg channel to remain symmetric, but we have contributions to the asymmetry from higher orders in the $q\bar{q}$ channel.

2.1 $p\bar{p}$ laboratory frame

We first provide results for the asymmetry in the laboratory frame. We use MSTW2008 [24] parton distribution functions and a top quark mass $m_t = 173.3$ GeV in the calculations. The central values refer to the choice of scale $\mu = m_t$, and theoretical errors are estimated with scale variation by a factor of two.

Using the NNLO approximate rapidity distributions it was found in [10] that the approximate NNLO top quark forward-backward asymmetry in the laboratory frame at the Tevatron (from QCD effects only) is 5.2%, using Eq. (2.2). This is a large increase of thirty percent over the NLO asymmetry (and as we will see later, we find essentially the same percent increase in the $t\bar{t}$ rest frame). We also note that this NNLO / NLO ratio is very similar to the increase that was found in [13], as we also discuss in the next subsection. We confirm the results of [10] in this paper and we add the aN³LO QCD corrections as well as electroweak corrections. Furthermore, we calculate the corresponding numbers using Eq. (2.3).

At aN³LO we find that the asymmetry from QCD alone is $(5.6^{+0.3}_{-0.4})\%$ using Eq. (2.2), and $(6.0 \pm 0.1)\%$ using Eq. (2.3). The uncertainties indicated in these numbers are estimated by varying the scale μ between $m_t/2$ and $2m_t$. If we include the electroweak corrections as well as the aN³LO QCD corrections we find for the asymmetry the results $(6.4^{+0.5}_{-0.6})\%$ using Eq. (2.2), and $(6.8 \pm 0.3)\%$ using Eq. (2.3). The aN³LO results with and without electroweak corrections are tabulated in Table 1.

We also note that the differences between using Eq. (2.2) and Eq. (2.3) decrease as higher orders are included in the calculation of A_{FB}.

Top-quark asymmetry at the Tevatron		
aN³LO A_{FB} %	$p\bar{p}$ frame	$t\bar{t}$ frame
QCD only Eq. (2.2)	$5.6^{+0.3}_{-0.4}$	$8.1^{+0.4}_{-0.6}$
QCD only Eq. (2.3)	6.0 ± 0.1	8.7 ± 0.2
QCD+EW Eq. (2.2)	$6.4^{+0.5}_{-0.6}$	$9.4^{+0.7}_{-0.9}$
QCD+EW Eq. (2.3)	6.8 ± 0.3	10.0 ± 0.6

Table 1: The top-quark forward-backward asymmetry in $p\bar{p}$ collisions at the Tevatron in aN³LO QCD with and without electroweak (EW) corrections, using $\sqrt{S} = 1.96$ TeV and $\mu = m_t = 173.3$ GeV. Results are shown in the $p\bar{p}$ (laboratory) frame and the $t\bar{t}$ rest frame.

2.2 $t\bar{t}$ rest frame

In the $t\bar{t}$ rest frame the overall asymmetry is around fifty percent larger than in the laboratory frame, due to kinematics, but the overall ratios of contribution levels at different orders in QCD are similar. As in the laboratory frame, in the $t\bar{t}$ rest frame the increase from NNLO contributions from QCD is around thirty percent on top of the NLO asymmetry, using Eq. (2.2). More precisely, the ratio of the aNNLO to the NLO asymmetry is 1.29, which is very close to the value of 1.27 for the NNLO over NLO ratio that was found in [13]. Thus, approximate NNLO results for the asymmetry are nearly identical to exact NNLO results. Our aNNLO result is $(7.6^{+0.4}_{-0.9})\%$ which is only one percent higher than the exact NNLO result in [13] and has very similar scale uncertainty. The NNLO result and the increase over NLO is practically the same whether one uses the exact NNLO corrections [13] or the NNLO soft-gluon contributions, as before. As we will show in the figures in the next subsection, this agreement holds not just for the total asymmetry but also in separate rapidity bins.

There is also good agreement between exact and approximate results if one uses Eq. (2.3) instead. We find an aNNLO/NLO ratio of 1.14 vs the NNLO/NLO ratio of 1.13 in [13]. Specifically, our value for the QCD aNNLO asymmetry is $(8.4 \pm 0.2)\%$, which again is only one percent higher than the exact NNLO value in [13]. All these results again confirm the fact that the overwhelming QCD contribution to the asymmetry can be derived from the soft-gluon approximation of Ref. [3], which again is fully consistent with similar findings for the total cross section and the transverse momentum and rapidity distributions of the top quark [1, 2, 3, 10, 16].

At aN^{3}LO we find that the asymmetry from QCD alone is $(8.1^{+0.4}_{-0.6})\%$ using Eq. (2.2), and $(8.7 \pm 0.2)\%$ using Eq. (2.3). The uncertainties indicated in these numbers are estimated by varying the scale μ between $m_t/2$ and $2m_t$, as before. If we include the electroweak corrections as well as the aN^{3}LO QCD corrections, we find for the asymmetry the results $(9.4^{+0.7}_{-0.9})\%$ using Eq. (2.2), and $(10.0 \pm 0.6)\%$ using Eq. (2.3). The aN^{3}LO results with and without electroweak corrections are again tabulated in Table 1.

We note that the N^{3}LO soft-gluon corrections provide an 8% increase over the NNLO QCD result for the asymmetry using Eq. (2.2). The increase over NNLO becomes 5% if instead Eq. (2.3) is used. Thus, the increase from the aN^{3}LO corrections is very substantial; in addition, the scale dependence is reduced at aN^{3}LO relative to NNLO.

The best estimate for A_{FB} in the $t\bar{t}$ rest frame is $(10.0 \pm 0.6)\%$ which is in excellent agreement with the D0 result of $(10.6 \pm 3.0)\%$ [8] and the CDF result of $(16.4 \pm 4.7)\%$ [7].

2.3 Differential A_{FB}

In this subsection we present the differential forward-backward asymmetry defined by

$$A_{\mathrm{FB}}^{\mathrm{bin}} = \frac{\sigma_{\mathrm{bin}}^{+}(\Delta y) - \sigma_{\mathrm{bin}}^{-}(\Delta y)}{\sigma_{\mathrm{bin}}^{+}(\Delta y) + \sigma_{\mathrm{bin}}^{-}(\Delta y)} \qquad (2.4)$$

with $\Delta y = y_t - y_{\bar{t}}$ and where the bin determines the range of Δy. In the figures below we present results for four $|\Delta y|$ bins of width 0.5 over the range from 0 to 2, and we compare our aNNLO and aN^{3}LO QCD results with the NNLO QCD results in [13] using no expansion in
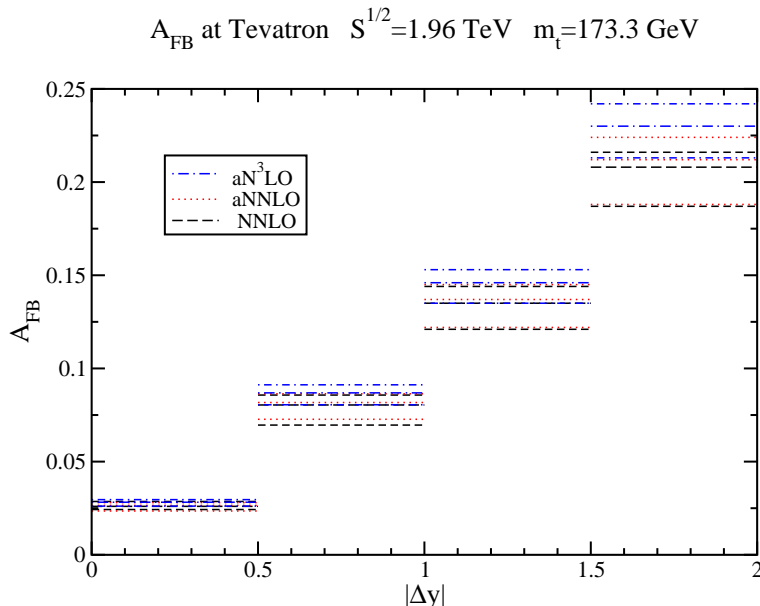
6

Figure 1: The top-quark forward-backward asymmetry at NNLO, aNNLO, and aN^{3}LO QCD (without EW corrections) as a function of $|\Delta y|$ in $t\bar{t}$ production at the Tevatron with $\sqrt{S} = 1.96$ TeV. The central lines at each order are with $\mu = m_t$, and the other lines display the upper and lower values from scale variation over $m_t/2 \leq \mu \leq 2m_t$.

α_S (i.e. using the analog of Eq. (2.2) without electroweak corrections). We also compare our aN^{3}LO QCD results with data from CDF [7] and D0 [8].

In Fig. 1 we show our aNNLO and aN^{3}LO QCD results in the various $|\Delta y|$ bins and compare them with the NNLO results from [13]. It is clear that the aNNLO results are very close to the exact NNLO results in both central value and scale uncertainty in all bins. This observation is of course fully consistent with the results for the total asymmetry. We also note that the aN^{3}LO results enhance the overall values and decrease the theoretical uncertainty in all $|\Delta y|$ bins.

In Fig. 2 we show the comparison of our aN^{3}LO QCD results with CDF [7] and D0 [8] data in various $|\Delta y|$ bins. The aN^{3}LO results describe the data better than at NNLO. Our results are fully consistent with D0 data in all bins; they are also in agreement with the CDF data in the first two bins and are within two standard deviations or better with CDF data in the highest two bins.

3   Conclusions

The aN^{3}LO top-quark forward-backward asymmetry at the Tevatron has been calculated. It has been shown that the asymmetry is dominated by soft-gluon corrections. In fact, at both NLO and NNLO the soft-gluon contribution closely approximates the complete QCD result. Results for A_{FB} have been presented in both the laboratory frame and the $t\bar{t}$ rest frame, with
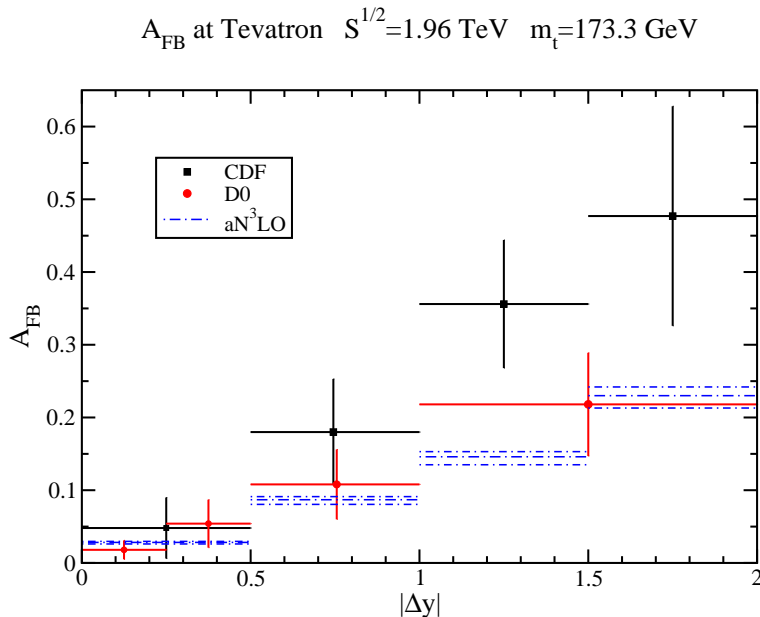
Figure 2: The top-quark forward-backward asymmetry at aN³LO QCD (without EW corrections) as a function of $|\Delta y|$ and comparison with CDF [7] and D0 [8] data in $t\bar{t}$ production at the Tevatron with $\sqrt{S} = 1.96$ TeV. The central aN³LO lines are with $\mu = m_t$, and the other lines display the upper and lower values from scale variation over $m_t/2 \leq \mu \leq 2m_t$.

and without including electroweak corrections, and using two different methods, expansion or no expansion in the strong coupling. The aN³LO soft-gluon corrections from NNLL resummation substantially enhance previous NNLO results and they reduce the theoretical uncertainty. The best estimate for the asymmetry at aN³LO in the $t\bar{t}$ rest frame is $(10.0 \pm 0.6)\%$ which is in excellent agreement with experimental results using the full Run II data at the Tevatron [7, 8]. This result for A_{FB} complements the recent aN³LO calculations [1, 2] for top-antitop pair production which represent the current state-of-the-art in higher-order calculations for this process.

Acknowledgments

References

[1] N. Kidonakis, Phys. Rev. D **90**, 014006 (2014) [arXiv:1405.7046 [hep-ph]].

[2] N. Kidonakis, Phys. Rev. D **91**, 031501(R) (2015) [arXiv:1411.2633 [hep-ph]].

[3] N. Kidonakis, Phys. Rev. D **82**, 114030 (2010) [arXiv:1009.4935 [hep-ph]].

[4] W. Bernreuther and Z.-G. Si, Phys. Rev. D **86**, 034026 (2012) [arXiv:1205.6580 [hep-ph]].

[5] CDF Collaboration, Phys. Rev. Lett. **101**, 202001 (2008) [arXiv:0806.2472 [hep-ex]]; Phys. Rev. D **83**, 112003 (2011) [arXiv:1101.0034 [hep-ex]].

[6] D0 Collaboration, Phys. Rev. Lett. **100**, 142002 (2008) [arXiv:0712.0851 [hep-ex]]; Phys. Rev. D **84**, 112005 (2011) [arXiv:1107.4995 [hep-ex]].

[7] CDF Collaboration, Phys. Rev. D **87**, 092002 (2013) [arXiv:1211.1003 [hep-ex]].

[8] D0 Collaboration, Phys. Rev. D **90**, 072011 (2014) [arXiv:1405.0421 [hep-ex]].

[9] L.G. Almeida, G. Sterman, and W. Vogelsang, Phys. Rev. D **78**, 014008 (2008) [arXiv:0805.1885 [hep-ph]].

[10] N. Kidonakis, Phys. Rev. D **84**, 011504(R) (2011) [arXiv:1105.5167 [hep-ph]].

[11] V. Ahrens, A. Ferroglia, M. Neubert, B.D. Pecjak, and L.L. Yang, Phys. Rev. D **84**, 074004 (2011) [arXiv:1106.6051 [hep-ph]].

[12] S.J. Brodsky and X.-G. Wu, Phys. Rev. D **85**, 114040 (2012) [arXiv:1205.1232 [hep-ph]].

[13] M. Czakon, P. Fiedler, and A. Mitov, arXiv:1411.3007 [hep-ph].

[14] N. Kidonakis, Phys. Rev. D **64**, 014009 (2001) [hep-ph/0010002].

[15] N. Kidonakis and B.D. Pecjak, Eur. Phys. J. C **72**, 2084 (2012) [arXiv:1108.6063 [hep-ph]].

[16] N. Kidonakis, Phys. Part. Nucl. **45**, 714 (2014) [arXiv:1210.7813 [hep-ph]]; in Snowmass 2013 Proceedings, arXiv:1304.7775 [hep-ph]; in Proceedings of HQ2013 Summer School, DESY-PROC-2013-03, p. 139, arXiv:1311.0283 [hep-ph].

[17] M. Aliev, H. Lacker, U. Langenfeld, S. Moch, P. Uwer, and M. Wiedemann, Comput. Phys. Commun. **182**, 1034 (2011) [arXiv:1007.1327 [hep-ph]].

[18] M. Beneke, P. Falgari, S. Klein, and C. Schwinn, Nucl. Phys. B **855**, 695 (2012) [arXiv:1109.1536 [hep-ph]].

[19] N. Kidonakis and R. Vogt, Phys. Rev. D **68**, 114014 (2003) [hep-ph/0308222].

[20] W. Bernreuther and Z.-G. Si, Nucl. Phys. B **837**, 90 (2010) [arXiv:1003.3926 [hep-ph]].

[21] W. Hollik and D. Pagani, Phys. Rev. D **84**, 093003 (2011) [arXiv:1107.2606 [hep-ph]].

[22] A.V. Manohar and M. Trott, Phys. Lett. B **711**, 313 (2012) [arXiv:1201.3926 [hep-ph]].

[23] N. Kidonakis, Phys. Rev. D **73**, 034001 (2006) [hep-ph/0509079].

[24] A.D. Martin, W.J. Stirling, R.S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009) [arXiv:0901.0002 [hep-ph]].