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Nucleon Tensor Charge from Collins Azimuthal Asymmetry Measurements

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We investigate the nucleon tensor charge from current experiments by a combined analysis of the Collins asymmetries in two hadron production in e^+e^- annihilations and semi-inclusive hadron production in deep inelastic scattering processes. The transverse momentum dependent evolution is taken into account, for the first time, in the global fit of the Collins fragmentation functions and the quark transversity distributions at the approximate next-to-leading logarithmic order. We obtain the nucleon tensor charge contribution from up and down quarks as: $\delta u = +0.30^{+0.12}_{-0.08}$ and $\delta d = -0.20^{+0.28}_{-0.11}$ at 90% of confidence level for momentum fraction $0.0065 \le x_B \le 0.35$ and $Q^2 = 10$ GeV^2.

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Introduction. — Nucleon tensor charge is one of the fundamental properties of the proton and its determination is among the main goals of existing and future experimental facilities [1–6]. It also plays an important role in constraining the nuclear physics aspects for probing new physics beyond the standard model, and has been an active subject from lattice QCD calculations [7, 8]. In terms of the partonic structure of the nucleon, the tensor charge is constructed from the quark transversity distribution, one of the three leading-twist quark distributions. However, the experimental exploration of the quark transversity distribution in high energy scattering is difficult because of its odd-chirality [2].

An important channel is to measure the Collins azimuthal asymmetries in semi-inclusive hadron production in deep inelastic scattering (SIDIS), where the transversity distribution is coupled to the chiral-odd Collins fragmentation function (FF) [9], as well as back-to-back two hadron production in e^+e^- annihilations where two Collins FFs are coupled to each other [10]. There have been great experimental efforts from both DIS and e^+e^- facilities to explore the Collins asymmetries, including HERMES [11, 12], COMPASS [13] and JLab [14] in DIS experiments, and BELLE [15, 16] and BABAR [17] at e^+e^- colliders of B-factories. Due to the universality of the Collins fragmentation functions [18], we will be able to combine the analysis of these two processes to constrain the quark transversity distributions. Earlier results of the phenomenological studies in Refs. [19–21] have demonstrated the powerful reach of the Collins asymmetry measurements in accessing the quark transversity distributions and eventually the nucleon tensor charge. In this paper, we go beyond the leading order (LO) framework of Refs. [19–21], and take into account the important higher order corrections, including, in particular, the large logarithms [22, 23].

Theoretically, the large logarithms in the above hard processes are controlled by the relevant QCD evolution, i.e., the transverse momentum dependent (TMD) evolution [22, 23]. It was pointed out in Ref. [24] that the TMD evolution plays an important role in evaluating the Collins asymmetries. Because of the large energy difference between the existing DIS and e^+e^- experiments [11–17], the QCD evolution effects have to be carefully examined when one extracts the quark transversity distributions. In this paper, for the first time, we demonstrate that the TMD evolution can describe the experimental data and constrain the nucleon tensor charge with improved theoretical accuracy. To achieve that, we include the most recent developments from both theory and phenomenology sides [25–34] and apply the TMD evolution at the next-to-leading-logarithmic (NLL) order within the Collins-Soper-Sterman (CSS) [22, 23] formalism. We show that our results improve the theoretical description of the experimental data in various aspects, especially, in formulating the transverse momentum dependence of the asymmetries in e^+e^- annihilations [17].

The quark transversity distribution has also been an important subject to explore other transverse spin related phenomena, such as the di-hadron fragmentation processes [35, 36], and inclusive hadron production at large transverse momentum in single transversely polarized pp collisions [37–39]. Our results will provide an important cross check and a step further toward a global analysis of all these spin asymmetries associated with the quark transversity distributions.

Collins Asymmetries in SIDIS and e^+e^- annihilation. — In SIDIS, a lepton scatters off the nucleon target N, and produces an identified hadron h in the final state, $lN \to lhX$. The Collins effect leads to a transverse spin asymmetry: $\sigma(S_\perp) \sim F_{UU}(1 + A_{UT}^{\sin(\phi_h+\phi_s)} \sin(\phi_h + \phi_s))$, where ϕ_s and ϕ_h are the azimuthal angles of the nucleon's transverse polarization vector $\vec{S}_\perp$ and the trans-

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verse momentum vector $\vec{P}_{h\perp}$ of the final-state hadron, respectively. The asymmetry $A_{UT}^{\sin(\phi_h+\phi_s)}$ can be calculated as

$$A_{UT}^{\sin(\phi_h+\phi_s)}(x_B, y, z_h, P_{h\perp}) = \frac{2(1-y)}{1+(1-y)^2}\frac{F_{UT}}{F_{UU}} \ , \quad (1)$$

with usual SIDIS kinematic variables x_B, y, z_h, and $Q^2 \simeq x_B y S$, and S is the lepton-nucleon center of mass energy. The structure functions F_{UU} (F_{UT}) depend on the kinematic variables and can be factorized into the TMD quark distribution (transversity) and fragmentation (Collins) functions in the low transverse momentum region. Applying the TMD evolution, we can write down F_{UU}, F_{UT} as [22–24, 28, 40]

$$\begin{aligned} F_{UU} &= \frac{1}{z_h^2}\int \frac{db\, b}{2\pi} J_0\!\left(\frac{P_{h\perp} b}{z_h}\right) e^{-S_{\rm PT}(Q,b_*)-S_{\rm NP}^{\rm (SIDIS)}(Q,b)} \\ &\quad \times\ C_{q\leftarrow i}\otimes f_1^i(x_B,\mu_b)\ \hat{C}_{j\leftarrow q}^{\rm (SIDIS)}\otimes \hat{D}_{h/j}(z_h,\mu_b), \quad (2)\\ F_{UT} &= -\frac{1}{2z_h^3}\int \frac{db\, b^2}{2\pi} J_1\!\left(\frac{P_{h\perp} b}{z_h}\right) e^{-S_{\rm PT}(Q,b_*)-S_{\rm NP\,coll}^{\rm (SIDIS)}(Q,b)} \\ &\quad \times\ \delta C_{q\leftarrow i}\otimes h_1^i(x_B,\mu_b)\ \delta\hat{C}_{j\leftarrow q}^{\rm (SIDIS)}\otimes \hat{H}_{h/j}^{(3)}(z_h,\mu_b), (3) \end{aligned}$$

where b is Fourier conjugate variable to the measured final hadron momentum $P_{h\perp}$, J_1 is the Bessel function, $\mu_b = c_0/b_*$ with $c_0 \simeq 1.12$, and the symbol $\otimes$ represents the usual convolution in momentum fractions. Summation over quark flavors q weighted with quark charge $\sum_q e_q^2$ and summation over $i, j = q, \bar{q}, g$ is implicit in all formulas for structure functions. C, $\hat{C}$ and δC, $\delta\hat{C}$ are coefficient functions for unpolarised distribution, fragmentation function, and transversity and Collins FF that can be calculated perturbatively.

The b_*-prescription ($b \to b_* \equiv b/\sqrt{1+b^2/b_{max}^2}$ with b_{max} =1.5 GeV^{-1} in our calculations) was applied to introduce the non-perturbative form factors $S_{\rm NP}^{\rm (SIDIS)}$ and $S_{\rm NP\,coll}^{\rm (SIDIS)}$ that contain information on initial conditions of evolution. The Collins fragmentation function [9] enters as the transverse momentum moment [26], $\hat{H}_{h/q}^{(3)}(z_h) = \int d^2 p_\perp \frac{|p_\perp^2|}{M_h} H_{1\,h/q}^\perp(z_h, p_\perp)$, where $H_{1\,h/q}^\perp(z_h, p_\perp)$ is the quark Collins function defined in [26], and differs by a factor of $(-1/z_h)$ from the so-called "Trento convention" [41],

$$H_{1\,h/j}^\perp(z_h, p_\perp) = -\frac{1}{z_h} H_{1\,h/j}^\perp(z_h, p_\perp)|_{\rm Trento}, \quad (4)$$

with $p_\perp$ the transverse component of the hadron with respect to the fragmenting quark momentum.

Three important ingredients have to be included to achieve the NLL formalism for the above structure functions and asymmetries. First, the perturbative Sudakov form factor [42],

$$S_{\rm PT}(Q, b_*) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[A \ln\frac{Q^2}{\mu^2} + B\right] \ , \quad (5)$$

with perturbative coefficients $A^{(1,2)} \sim \alpha_s^{(1,2)}$ and $B^{(1)} \sim \alpha_s^1$ [42, 43]. Second, the scale evolutions of the quark transversity distribution and the Collins fragmentation functions up to the scale of μ_b. The evolution for the quark transversity is known

$$\frac{\partial}{\partial \ln\mu^2} h_1^q(x,\mu) = \frac{\alpha_s}{2\pi} P_{q\leftarrow q}^{h_1} \otimes h_1^q(x,\mu) \ , \quad (6)$$

with the splitting kernel $P_{q\leftarrow q}^{h_1}$ given in [44]. The evolution equation for $\hat{H}_{h/q}^{(3)}$ is more complicated [26, 27, 45]. However, if we keep only the homogenous term, it reduces to a simpler form as

$$\frac{\partial}{\partial \ln\mu^2} \hat{H}_{h/q}^{(3)}(z_h,\mu) = \frac{\alpha_s}{2\pi} P_{q\leftarrow q}^{\rm coll} \otimes \hat{H}_{h/q}^{(3)}(z_h,\mu) \ , \quad (7)$$

and it is interesting to find out that the splitting kernel $P_{q\leftarrow q}^{\rm coll}$ for the homogenous term is the same [27] as that for the quark transversity distribution. As a first study, we will use this approximation and call resulting resummation NLL$'$.

Third, the C-coefficients are calculated at one-loop order ($C^{(1)}$) [42, 43], for which we have [26, 30, 33, 40]: $\delta C_{q\leftarrow q}^{(1)}(x,\mu_b) = \frac{\alpha_s}{\pi}\left(-2C_F\delta(1-x)\right)$ and $\delta\hat{C}_{q\leftarrow q}^{\rm (SIDIS)(1)}(z,\mu_b) = \frac{\alpha_s}{\pi}\left(P_{q\leftarrow q}^{\rm coll}(z)\ln z - 2C_F\delta(1-z)\right)$. Again, we only keep the homogenous term in the latter coefficient. In the CSS formalism, there is a freedom to include part of C-coefficient contributions into a hard factor [25, 46], and the difference is in higher order NNLL. This difference is negligible in our numeric calculations.

In the two hadron productions in e^+e^- annihilations, $e^+ + e^- \to h_1 + h_2 + X$, a quark-antiquark pair is produced and fragments into hadrons, where two of them are observed in the final state in opposite hemispheres. The center of mass energy $S = Q^2 = (P_{e^+} + P_{e^-})^2$, and the final state two hadrons have momenta P_{h1} and P_{h2}, respectively. The Collins effect leads to an azimuthal angular $\cos(2\phi_0)$ asymmetries between the two hadrons [10], and can be quantified as

$$R^{h_1h_2} \equiv 1 + \cos(2\phi_0)\frac{\sin^2\theta}{1+\cos^2\theta}\frac{Z_{\rm coll}^{h_1h_2}}{Z_{uu}^{h_1h_2}} \ , \quad (8)$$

where θ is the polar angle between the hadron h_2 and the beam direction of e^+e^-, and ϕ_0 is defined as the azimuthal angle of hadron h_1 relative to that of hadron h_2. To cancel possible acceptance effects as well as radiative effects, experiments measure the so-called double ratio asymmetries A_0 and A_{12} , which are related to the ratios of $R^{h_1h_2}$ from different hadron pair combinations, for details, see [15–17]. In the current study, we focus on the so-called A_0 [15–17] asymmetry. With TMD evolution included, the final results for Z functions are given by

[24, 40],

$$Z_{uu}^{h_1h_2} = \frac{1}{z_{h1}^2}\int \frac{db\, b}{(2\pi)} J_0\!\left(\frac{P_{h\perp}b}{z_{h1}}\right)\, e^{-S_{\rm PT}(Q,b_*)-S_{\rm NP}^{(e^+e^-)}(Q,b)} \times \hat{C}_{i\leftarrow q}^{(e^+e^-)} \otimes D_{h_1/i}(z_{h1},\mu_b) \times \hat{C}_{j\leftarrow \bar{q}}^{(e^+e^-)} \otimes D_{h_2/j}(z_{h2},\mu_b)\ , \tag{9}$$

$$Z_{\rm coll}^{h_1h_2} = \frac{1}{z_{h1}^2}\frac{1}{4z_{h1}z_{h2}}\int \frac{db\, b^3}{(2\pi)} J_2\!\left(\frac{P_{h\perp}b}{z_{h1}}\right)\, e^{-S_{\rm PT}(Q,b_*)} \times e^{-S_{\rm NP\ coll}^{(e^+e^-)}(Q,b)}\ \delta\hat{C}_{i\leftarrow q}^{(e^+e^-)} \otimes \hat{H}_{h_1/i}^{(3)}(z_{h1},\mu_b) \times \delta\hat{C}_{j\leftarrow \bar{q}}^{(e^+e^-)} \otimes \hat{H}_{h_2/j}^{(3)}(z_{h2},\mu_b)\ , \tag{10}$$

where $z_{hi} = 2|P_{hi}|/Q$, $P_{h\perp}$ is the transverse momentum of hadron h_1, and the coefficient for the Collins function at one-loop order is given by $\delta\hat{C}_{q\leftarrow q}^{(e^+e^-)(1)}(z,\mu_b) = \frac{\alpha_s}{\pi}\left(P_{q\leftarrow q}^{\rm coll}(z)\ln z + \frac{C_F}{4}\left(\pi^2-8\right)\delta(1-z)\right)$, while the coefficient $\hat{C}_{j\leftarrow q}^{(e^+e^-)(1)}(z,\mu_b)$ are derived in [40, 47]. The TMD factorization for the so-called A_{12} asymmetry can not be straightforwardly formulated [40] because of additional requirement of jet axis involved in experiments.

Global analysis with TMD evolution. — To perform the global analysis of the experimental data, we should parametrize the non-perturbative form factors. For the spin-averaged cross sections, we follow the parameterizations in Ref. [34],

$$S_{\rm NP}^{(\rm SIDIS)} = g_2\ln(b/b_*)\ln(Q/Q_0) + \left(g_q + g_h/z_h^2\right) b^2\ , \tag{11}$$

$$S_{\rm NP}^{(\rm e^+e^-)} = g_2\ln(b/b_*)\ln(Q/Q_0) + g_h b^2\left(1/z_{h1}^2 + 1/z_{h2}^2\right)\ , \tag{12}$$

where the initial scale is chosen to be $Q_0^2 = 2.4$ GeV^2, and other parameters are determined from the analysis of unpolarized SIDIS and Drell-Yan processes in Ref. [34]: $g_q = 2g_1 = 0.424$, $g_2 = 0.84$, $g_h = 0.042$ (GeV^2). The presence of α_s^1 contributions to C-coefficients require normalization factors in the fit of Ref. [34], however they affect both polarised and unpolarised parts equally thus there is no need in any additional normalization factor in the asymmetry. The parameterization of $S_{\rm NP}^{(e^+e^-)}$ follows the universality arguments of the TMDs. For the Collins asymmetries, we need to take into account different initial conditions for transversity and Collins FF. We introduce a new parameter g_c to take into account different b-shape of the Collins fragmentation function and write, using universality of the Collins function between these two processes,

$$S_{\rm NP\ coll}^{(\rm SIDIS)} = S_{\rm NP}^{(\rm SIDIS)} - g_c b^2/z_h^2\ , \tag{13}$$

$$S_{\rm NP\ coll}^{(\rm e^+e^-)} = S_{\rm NP}^{(\rm e^+e^-)} - g_c b^2\left(1/z_{h1}^2 + 1/z_{h2}^2\right)\ . \tag{14}$$

In the global fit, we parameterize the quark transversity distributions at the initial scale Q_0 to satisfy the Soffer bound [48, 49] as,

$$h_1^q(x,Q_0) = N_q^h x^{a_q}(1-x)^{b_q}\frac{(a_q+b_q)^{a_q+b_q}}{a_q^{a_q}b_q^{b_q}} \times \frac{1}{2}\left(f_1(x,Q_0) + g_1(x,Q_0)\right)\ , \tag{15}$$

with $\left|N_q^h\right| \le 1$ for up and down quarks $q = u, d$, respectively, where f_1 are the unpolarized CT10 NLO quark distributions [50] and g_1 are the DSSV helicity NLO distributions [51]. In the current study, we assume all the sea quark transversity distributions are negligible.

Similarly, we parameterize the moments for the Collins fragmentation functions in terms of the unpolarized fragmentation functions,

$$\hat{H}_{fav}^{(3)}(z,Q_0) = N_u^c z^{\alpha_u}(1-z)^{\beta_u} D_{\pi^+/u}(z,Q_0)\ , \tag{16}$$

$$\hat{H}_{unf}^{(3)}(z,Q_0) = N_d^c z^{\alpha_d}(1-z)^{\beta_d} D_{\pi^+/d}(z,Q_0)\ , \tag{17}$$

for the favored and unfavored Collins fragmentation functions, respectively. The rest can be obtained by applying the isospin relations. We also neglect possible difference of favoured/unfavoured fragmentation functions of $\bar{u}, \bar{d}$ and u, d. In our fit, we include the strange quark Collins FF, which is parameterized similar to unfavoured function in Eq. (17) with unpolarized strange FF. We also utilize the newest NLO extraction of fragmentation functions [52]. The new DSS FF set is capable of describing pion multiplicities measured by COMPASS and HERMES collaborations.

$N_u^h = 0.85 \pm 0.09$	$a_u = 0.69 \pm 0.04$	$b_u = 0.05 \pm 0.04$
$N_d^h = -1.0 \pm 0.13$	$a_d = 1.79 \pm 0.32$	$b_d = 7.00 \pm 2.65$
$N_u^c = -0.262 \pm 0.025$	$\alpha_u = 1.69 \pm 0.01$	$\beta_u = 0.00 \pm 0.54$
$N_d^c = 0.195 \pm 0.007$	$\alpha_d = 0.32 \pm 0.04$	$\beta_d = 0.00 \pm 0.79$
$g_c = 0.0236 \pm 0.0007$	(GeV^2)	
$\chi^2_{min} = 218.407$	$\chi^2_{min}/n.d.o.f = 0.88$	

TABLE I. Fitted parameters of the transversity quark distributions for u and d and Collins fragmentation functions. The fit is performed by using MINUIT minimization package. Quoted errors correspond to MINUIT estimate.

In total we have 13 parameters in our global fit: N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, α_u, α_d, β_u, β_d, g_c. In the fit, we include all existing SIDIS data ($n_{SIDIS} = 140$ points), all points in x_B, z_h, and $P_{h\perp}$ where formalism is valid (we limit $P_{h\perp} < 0.8$ GeV) for $\pi^\pm$ pion production from HERMES [11, 12], COMPASS [13] and JLab HALL A [14]. For the Collins asymmetries in e^+e^- annihilation experiments we have $n_{e^+e^-} = 122$ data points, measurements as function of z_{h1}, z_{h2}, and $P_{h\perp}$ (we limit $P_{h\perp}/z_{h1} < 3.5$ GeV) from BELLE [16] and BABAR [17] collaborations. We use MINUIT minimization package to perform the fit. The resulting parameters are presented in Table I. The total $\chi^2 = 218.407$, $n_{d.o.f.} = 249$, and

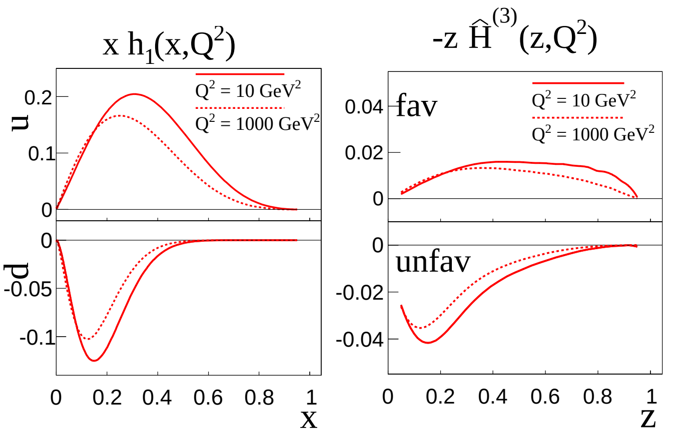

FIG. 1. Extracted transversity distribution and Collins fragmentation function at two different scales $Q^2 = 10$ (solid lines) and $Q^2 = 1000$ (dashed lines) GeV^2.

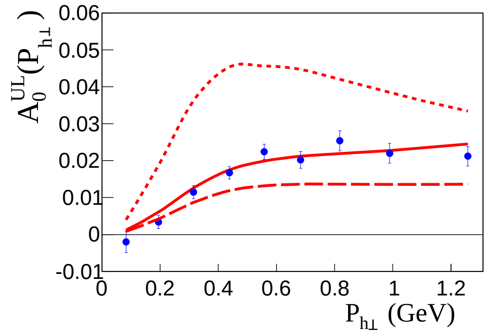

FIG. 2. Collins asymmetries measured by BABAR [17] collaboration as a function of $P_{h\perp}$ in production of unlike sign "U" over like sign "L" pion pairs at $Q^2 = 110$ GeV^2. The solid line corresponds to the full NLL$'$ calculation, the dashed line to the LL calculation, and the dotted to the calculation without TMD evolution. Calculations are performed with parameters from Table I.

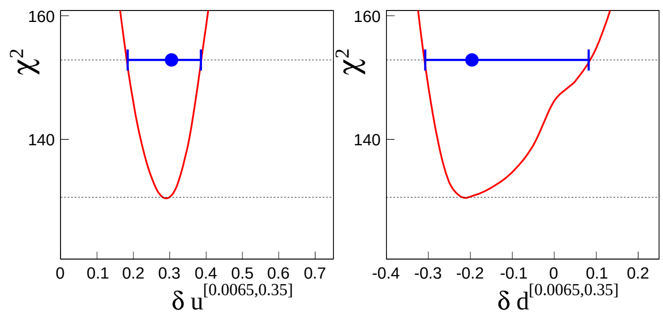

FIG. 3. χ^2 profiles for up and down quark contributions to the tensor charge. The errors of points correspond to 90% C.L. interval.

$\chi^2/n_{d.o.f} = 0.88$. The fit is equally good for SIDIS and e^+e^- data $\chi^2_{SIDIS}/n_{SIDIS} = 0.93$, $\chi^2_{e^+e^-}/n_{e^+e^-} = 0.72$. The goodness of resulting fit is 90% [40, 53] and inclusion of more parameters does not improve it. We estimate flavor dependence of functions by allowing a flavor dependent functional form. Note that our resulting d quark transversity is very close to its bound, the same feature was also found in Refs. [35, 36]. We plot the extracted transversity and Collins fragmentation function in Fig. 1 at two different scales $Q^2 = 10$ and 1000 GeV^2. Only relative sign of transversity can be determined and we present here a solution with positive u quark transversity as in Refs. [19–21, 35, 36]. Favorite and unfavorite Collins FFs are of opposite signs as suggested by sum rules [54, 55].

We also show an example of description of experimental data, namely $P_{h\perp}$ dependence of asymmetry in e^+e^- from BABAR [17] collaboration in Fig. 2. One can see that NLL$'$ accuracy adequately describes the data. In this plot we also show theoretical computations without TMD evolution (dotted line), LL accuracy (dashed line), and the complete NLL$'$ accuracy (solid line). The difference between these computations diminishes when we include higher orders, it means that the theoretical uncertainty improves. We conjecture that the difference between NLL$'$ and NNLL will be smaller than difference between NLL$'$ and LL and thus be comparable to experimental errors. One can also observe that asymmetry at $Q^2 = 110$ GeV^2 is suppressed by factor $2-3$ with respect to tree-level calculations due to the Sudakov form factor.

Finally, we present an estimate at 90% confidence level (C.L.) interval for the nucleon tensor charge contributions using the strategy outlined in Refs. [56, 57]. Transversity enters directly in SIDIS asymmetry and we find that the main constraints come from SIDIS data only, its correlations with errors of Collins FF turn out to be numerically negligible. Since the experimental data has only probed the limited region $0.0065 < x_B < 0.35$, we define the following partial contribution to the tensor charge

$$\delta q^{[x_{\min},x_{\max}]}\left(Q^2\right) \equiv \int_{x_{\min}}^{x_{\max}} dx\, h_1^q(x,Q^2)\ . \qquad (18)$$

In Fig. 3, we plot the χ^2 Monte Carlo scanning of SIDIS data for the contribution to the tensor charge from such a region, and find

$$\delta u^{[0.0065,0.35]} = +0.30^{+0.12}_{-0.08}\ , \qquad (19)$$

$$\delta d^{[0.0065,0.35]} = -0.20^{+0.28}_{-0.11}\ , \qquad (20)$$

at 90% C.L. at $Q^2 = 10$ GeV^2. We notice that this result is comparable with previous TMD extractions without evolution [19–21] and di-hadron method [35, 36].

Existing experimental data covers a limited kinematic region, thus a simple extension of our fitted parametrization to the whole range of $0 < x_B < 1$ will significantly underestimate the uncertainties, in particular, in the dominant large-x_B regime. It is extremely important to extend the experimental study of the quark transversity distribution to both large and small x_B to constrain the total tensor charge contributions. This requires future experiments to provide measurements at the Jefferson Lab 12 GeV upgrade [4] and the planned Electron Ion Collider [5, 6].

Conclusions and outlook. — We have performed a global analysis of the Collins azimuthal asymmetries in e^+e^- annihilation and SIDIS processes, by taking into account the appropriate TMD evolution effects at the NLL$'$ order and constrained the nucleon tensor charge contributions from the valence up and down quarks in the kinematics covered by the existing experiments . The resulting transversity and Collins fragmentation functions will be made available upon request in a form of a computer library. Future developments will include analysis of other spin asymmetries including those from *pp* scattering. We emphasize the importance of future experiments to further constrain the total tensor charge contribution of the nucleon.

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