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Testing the Copernican Principle with Hubble Parameter

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Using the longitudinal expression of Hubble expansion rate for the general Lemaître-Tolman-Bondi (LTB) metric as a function of cosmic time, we examine the scale on which the Copernican Principle holds in the context of a void model. By way of performing parameter estimation on the CGBH void model, we show that the Hubble parameter data favors a void with characteristic radius of $2 \sim 3$ Gpc. This brings the void model closer, but not yet enough, to harmony with observational indications given by the background kinetic Sunyaev-Zel'dovich effect and the normalization of near-infrared galaxy luminosity function. However, the test of such void models may ultimately lie in the future detection of the discrepancy between longitudinal and transverse expansion rates, a touchstone of inhomogeneous models. With the proliferation of observational Hubble parameter data and future large-scale structure observation, a definitive test could be performed on the question of cosmic homogeneity. Particularly, the spherical LTB void models have been ruled out, but more general non-spherical inhomogeneities still need to be tested by observation. In this paper, we utilise a spherical void model to provide guidelines into how observational tests may be done with more general models in the future.

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I. INTRODUCTION

The Copernican Principle (CP) is the hypothesis that we do not occupy a privileged position in the Universe. It leads to the Friedmann-Robertson-Walker (FRW) metric as the metric of the homogeneous and isotropic background spacetime [1]. However, one may not expect the CP to hold on all scales of cosmological interest, for both theory and observation shows that large-scale structure can emerge even if a homogeneous and isotropic initial background is assumed. Recently, the observed near-infrared luminosity function from a complete sample of galaxies indicates that the data cannot rule out the possibility of our vicinity being described by a void model [2]. In addition, the void model may also serve as a possible explanation to the emergence of accelerated expansion of the Universe without employing an exotic component dubbed ‘dark energy’. To further investigate the ramifications of such a non-CP scenario and ascertain the possible existence of a local void, we consider a variety of other cosmological tests, as laid out in this paper. We mainly make use of the observational Hubble parameter data (OHD) which is independent of CMB and galaxy distribution measurement, and their observational properties have not been elucidated well enough in the inhomogeneous void model.

Although the spherical symmetric LTB void models have been ruled out, this does not imply that inhomogeneity, as a whole, has “died” as an alternative to the concordance model. Apparently, LTB voids are simple and mathematically tractable, but its inherent spherical inhomogeneity is very special and certainly non-generic. It is then impossible to know *a priori* if the more generic forms and profiles of inhomogeneity will fare like LTB voids and end up failing to fit the joint current tests such as SN, the kSZ effect or BAO or OHD and so on. Thus, we in this paper still use spherical LTB models to provide a pathfinder of how future non-spherical models could be tested in the future.

II. LTB DYNAMICS AND THE VOID MODEL

The Lemaître-Tolman-Bondi (LTB) line element reads

$$ds^2 = -dt^2 + \frac{A'(r,t)^2}{1 - k(r)} dr^2 + A^2(r,t) d\Omega^2, \quad (1)$$

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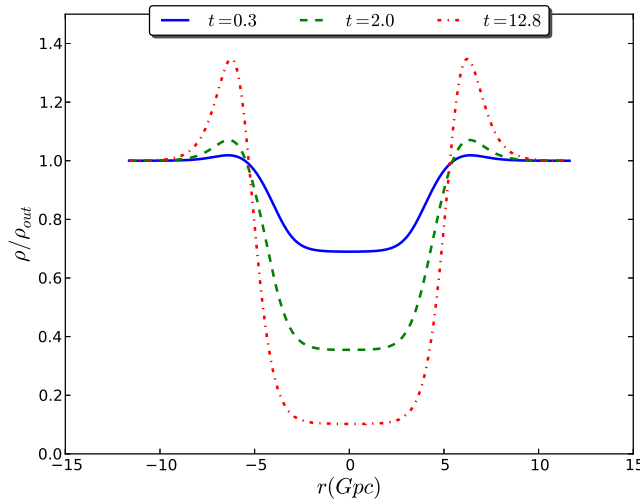


FIG. 1: Density profiles at different cosmic times. Time is in unit of Gyrs, and the densities are normalized to the background density.

where $'$ denotes $\partial/\partial r$, and $k(r)$ is associated with the spatial curvature. The Friedmann-Robertson-Walker (FRW) metric can be recovered by imposing $A(r, t) = a(t)r$ and $k(r) = kr^2$. Besides the most popular inhomogeneous exact solution of LTB model in cosmology, another interesting family of that are those found by Szekeres[3], which are much less idealised than spherical LTB models. Generally, these models have no symmetries (i.e. no killing-vectors [4]) and are constructed by six arbitrary metric functions: one freedom being represented to rescale the radial coordinate and remaining five degrees of freedom to model inhomogeneity. In fact, all of LTB quantities given in coordinate independent manner can be readily generalised to that in Szekeres models[5, 6]. The Gpc-size spherical symmetric LTB void modes are able to fit CMB data without dark energy under assumption that our cosmic observing position is very close to the void centre[7, 8]. This certainly leads to an unacceptable fine tuning and is a direct effect of spherical symmetry, and can also be corrected by considering non-spherical models. As shown in [7], even the still idealised deviation from spherical symmetry furnished by a quasi-spherical Szekeres model allows for a significant improvement on this fine tuning of the centre position that has always plagued LTB models.

From the LTB metric one can go on writing down and solving the dynamical equations for LTB void models. One notices along the way that the spherical symmetric configuration gives rise to two expansion rates

$$H_{\perp} \equiv \frac{\dot{A}}{A}, \quad H_{\parallel} \equiv \frac{\dot{A}'}{A'} \quad (2)$$

After choosing a gauge $A_0(r) = r$, and a homogeneous ‘bang time’, one needs only the boundary conditions to finally obtain the evolution history(see Ref. [9, 10] for more detailed treatments). Expressed as two functions, Ω_m and $H_{\perp 0}$, these boundary conditions define an LTB void model. Throughout this work, we employ the Constrained GBH (CGBH) model [11], in which

$$\Omega_m(r) = 1 + (\Omega_0 - 1) \left(\frac{1 - \tanh[(r - r_0)/2\Delta r]}{1 + \tanh(r_0/2\Delta r)} \right), \quad (3)$$

where Ω_0 describes the density at the symmetric center, r_0 is the characteristic size of the void, and Δr describes the steepness of the void near the edge.

To illustrate how the universe and its evolution in CGBH model look like, we choose $\Omega_0 = 0.05$, $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $r_0 = 6 \text{ Gpc}$, $\Delta r = 0.1 r_0$, and plot in Fig. 1 and Fig. 2 the density profile on different cosmic time (in *Gyr*) slices and on the light cone, respectively, and in Fig. 3 and Fig. 4 the profiles of the two expansion parameters on time slices and the light cone, respectively.

It is seen from Fig. 1 that the void gets deeper and its shell gets denser as the universe evolves. On the other hand, in a homogeneous universe one always has $H_{\parallel} = H_{\perp}$. Observation of a difference, like that shown in Fig. 4, within the redshift ranges of about 1-6, would imply spatial inhomogeneity.

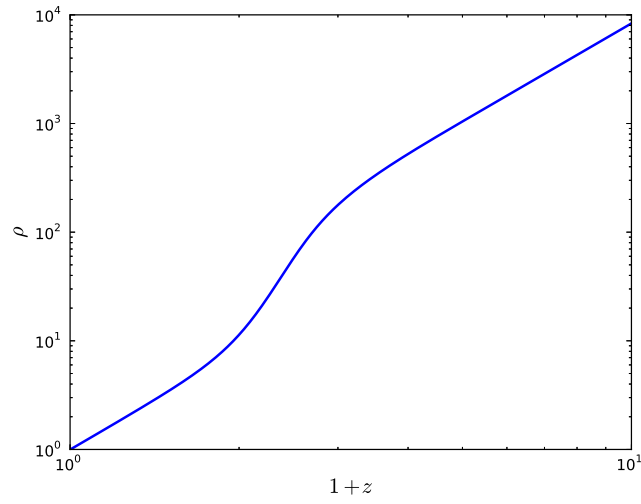


FIG. 2: Densities on the past light cone, normalized to the value at the void center.

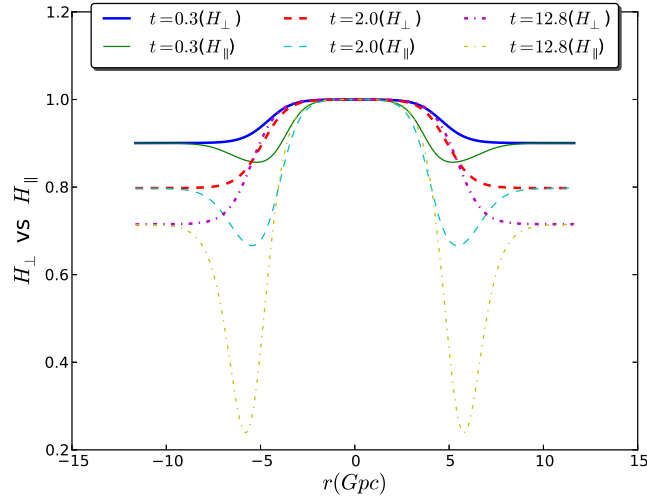


FIG. 3: Comparison of the H_{\perp} and H_{\parallel} profiles at different cosmic times. Time is in unit of Gyr, and the expansion parameters are normalized to the value at the void center.

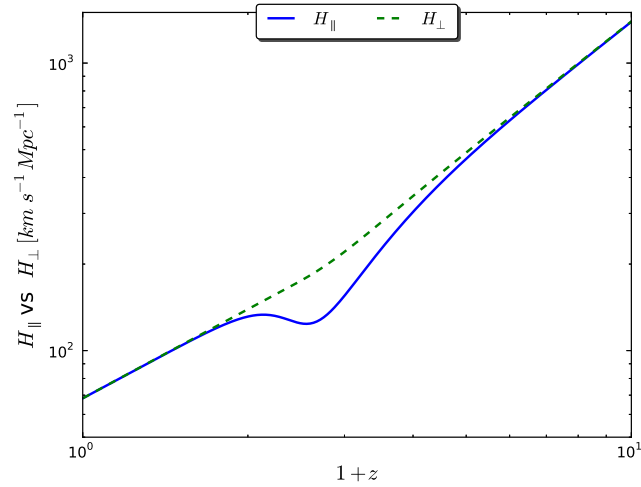


FIG. 4: Comparison of the H_{\perp} and H_{\parallel} at different redshifts (on the past light cone).

III. OBSERVATIONAL HUBBLE PARAMETER DATA

There are four main methods to measure the Hubble parameter $H(z)$: by measuring the differential age of passively evolving galaxies (differential age method) [12–17], the baryon acoustic oscillation (BAO) along the line-of-sight direction from the spectroscopic galaxy sample [18], the dipole of the luminosity distance d_L of gravitational wave sources (luminosity dipole method of standard sirens) [19, 20], and by measuring the Sandage-Loeb signal of the Lyman- α forest of QSOs (Sandage-Loeb signal or redshift drift method) [21]. The radial BAO size method depends on the detailed evolution of perturbations not well understood in the LTB cosmology, although progresses have been made [22, 23]. The luminosity dipole method of standard sirens by now has produced no observational data yet. Therefore, the OHD used in this work refer exclusively to that by the differential age method.

The Hubble parameter for FRW models with scale factor a reads

$$H \equiv \frac{\dot{a}}{a} = -\frac{1}{1+z} \frac{dz}{dt_{ct}}, \quad (4)$$

where dt_{ct} is the variation of the cosmic time due to a small change in the redshift dz . For any galaxy one has $T_{CA}(z) = T_F + T_{GA}(z)$, which simply states that the cosmic age T_{CA} at redshift z equates the summation of the formation time of the galaxy, T_F , and the age of this galaxy, T_{GA} that can be determined spectroscopically. If we could find a group of galaxies that share a uniform formation time, i. e. $T_F = \text{const.}$, we would then get a handle of dt_{ct} by simply measuring the age difference of those galaxies: $dt_{ct}(z) = dT_{CA} = dT_F + dT_{GA}(z) = dT_{GA}(z)$. The passively evolving galaxies can be identified by figuring out at every redshift the oldest galaxies, which together define the ‘red envelop’. One assumes in this process the oldest galaxies formed at the same time (standard cosmic chronometers), which is a natural assumption in an FRW universe (one may call it the galaxy-formation version of the cosmic Copernican Principle).

Of the two expansion rates defined in Eq. (2), the longitudinal expansion rate H_{\parallel} turns out to have the same form as Eq. (4) $H_{\parallel} = -[1/(1+z)](dz/dt_{ct})$, and hence corresponds to the observed $H(z)$ [10]. For a general LTB model, the bang time function $t_B(r)$ should not be zero, hence the age of the Universe is: $T = t - t_B(r)$ so we have $dT/dz = (dt/dz) - (dt_B/dr)(dr/dz)$. But as emphasized in our previous paper [10], the gradients (dt_B/dr) in the bang time, t_B , correspond to a currently non-vanishing decaying mode (also see: [22, 24]. This would imply a very inhomogeneous early universe, and hence violate inflation. Further, it is true that the gradient dt_B/dr is associated with density decaying modes of linear dust perturbations [24] that can be generalised to fully non-linear LTB models [22], though this has been updated by recent study of dust density modes in LTB models [25]. More importantly for the current work, non-zero gradient will lead to great inhomogeneities in the galaxy formation time and make the OHD data set invalid. Therefore, $t_B = \text{constant}$ must be set, i.e., $dt_B/dr = 0$, and we set it to be zero in this paper. Of course, a mathematically zero of big bang time gradient can cripples the dynamical freedom of the models and is not strictly necessary to prevent the violation of inflation. Near homogeneous conditions prevailing in the last scattering surface $z \sim 1100$ can be realised by LTB models in which the density decaying mode is not zero but becomes subdominant at such redshift, which is argued in [26–28] and also show that the extreme inhomogeneity and violation of inflation is no longer valid for times close to the big bang, well before the last scattering surface. We also notice that empiric calculations are not affected if the models allow for a small gradient of the big bang time function (i.e. a small position dependent variation in cosmic ages of different observers). Thus, it evidently does not need the LTB models (or any other late universe inhomogeneous model) to be valid all the way back to the big bang.

The problem of using OHD in LTB models is that the basic assumption that the oldest galaxies share a same formation time might not hold any more, as discussed recently in Ref. [10], because the background in LTB models has considerable inhomogeneities. However, we argue that OHD is still valid in our context (see *Discussion*). In the following we will use the latest 23 data entries as listed in Refs. [15, 17], where the data sample is larger than that of 11 OHD used in our previous paper [10].

IV. CONSTRAINTS ON THE VOID MODEL

We perform the constraints on the void model by both OHD and the background inhomogeneity-induced kSZ (BlkSZ) effect [29, 30]. The direction dependent, therefore observable, temperature shift reads

$$\Delta T_{\text{BI}} = T_{\text{CMB}} \times \int_0^{z_e} \delta_e(\hat{n}, z) \frac{\vec{v}_H(\hat{n}, z) \cdot \hat{n}}{c} d\tau_e, \quad (5)$$

where $T_{\text{CMB}} = 2.73$ K, $z_e = 100$ (the result essentially does not change as long as $z(r_0) \ll z_e$), and

$$v_H(z) \approx [H_{\parallel}(r(z), t(z)) - H_{\parallel}(r(z_e), t(z))] A(r(z), t(z)) \quad (6)$$

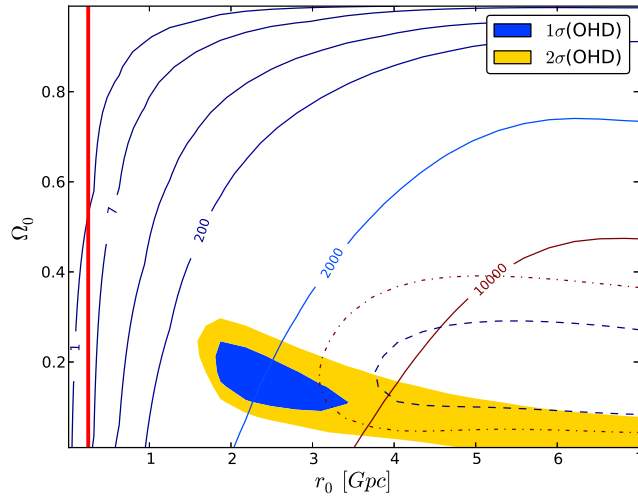


FIG. 5: Background inhomogeneity-induced kSZ effect ΔT_{BI}^2 in μK^2 . The line marked with 7 corresponds to the 95% observational upper limit of the South Pole Telescope [32, 33] which is significant to the constraint of void models [29]. Also plotted are the 1σ and 2σ confidence regions from the OHD as well as those from the supernovae Union2 dataset. The vertical (red) bar marks the characteristic void radius $r = 250$ Mpc in the small-void models of Refs. [7, 34] that are comparable with recent observational data of near-infrared galaxy luminosity function [2].

Here, V_H in void model is generally contributed from both Doppler and Sachs-Wolfe anisotropies induced by the void, and it is qualitatively dependent on the size r_0 of the void considered ([29], and therein). Similarly, the β function is employed instead of V_H in works [27, 31]. However, for small voids considered in this paper, such as Gpc-size void with size of less than a few Gpc, Sachs-Wolfe anisotropies could be neglected, and only Doppler contribution is left as Eq.(6) above. So only for cases of larger void cases, the general expression of V_H contributed from both Doppler and Sochs-Wolfe anisotropies should be adopted. For a more general non-spherical pattern of inhomogeneity such as Szekeres models, the full integral β function should be contributed from both Doppler and Sochs-Wolfe anisotropies when used to examine kSZ effect.

We calculate BkSZ power spectrum ΔT_{BI}^2 at $l = 3000$ and its constraints on the (r_0, Ω_0) parameter plane, with $\Delta r/r_0 = 0.21$, $H_0 = 74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ fixed at their respective best-fit values, which are in turn obtained from the Hubble parameter dataset. The resulting contours, as well as confidence regions from the OHD and the supernovae Union2 dataset, are plotted in Fig. 5. The OHD data favor a smaller (and more tightly constrained) void than what the supernovae Union2 data do. Indeed, there is a clear discrepancy between those two datasets, as found in Ref. [10]. This is a sign of inadequacy for this specific LTB model. Also, as pointed out in Ref. [29], one can see from Fig. 5 that the Gpc-sized voids, as those favored by the supernovae data, are incompatible with the BkSZ measurement, hence are largely excluded. Now we can tell from the figure that the OHD dataset give slightly weaker, though basically the same, conclusion. Furthermore, the observed normalization of the near-IR galaxy luminosity function indicates that a void, if exists, amounts to a few hundred Mpc [2]. This could in principle be consistent with the BkSZ measurement.

V. FUTURE BAO CONSTRAINT

As is shown in Fig. 4 above, unlike the homogeneous cosmological models, H_{\parallel} can differ from H_{\perp} in LTB models at redshift z roughly ranging from 1 to 6. Therefore one straightforward way is to define the ratio between these two expansion rates $\mathcal{E} = H_{\parallel}/H_{\perp}$, which always equals to 1 for the homogeneous cosmological models, but deviates from 1 in LTB models. Specifically, we can further write \mathcal{E} as

$$\mathcal{E} = H_{\parallel}/H_{\perp} = 1 + \frac{A}{A'} \frac{H'_{\perp}}{H_{\perp}}, \quad (7)$$

since $H_{\parallel} = \frac{\dot{A}'}{A'} = H_{\perp} + \frac{A}{A'} H'_{\perp}$.

Therefore, the violation of $\mathcal{E} = 1$ could also be an indicator of LTB-type models. To adopt this criteria however, one needs independent measurements of H_{\parallel} and H_{\perp} at the same redshift or just the variation of H_{\perp} on the light cone. The BAO feature imprinted in the non-relativistic matter such as galaxies distribution yields a further geometric test

of homogeneity, and future large-volume BAO surveys will also allow us detect the BAO scale in both radial and transverse directions, although the transverse H_{\perp} could not be measured in current measurement accuracy of BAO. Thus, we expect future BAO measurement would supply the information of the criteria \mathcal{E} by radial Hubble parameter H_{\parallel} and transverse Hubble parameter H_{\perp} , and improve greatly the testing of violation of homogeneity.

VI. DISCUSSION

In our calculation of OHD constraints on the void model, we assume that OHD could be used in LTB models. Actually, the same formation time of the oldest galaxies is a basic assumption in obtaining OHD. However, in LTB models where the universe has a considerable background inhomogeneity, this assumption becomes unreasonable and some arguments are also recently given in Ref. [35] where galaxy ages are used. First, despite the overall uniform-formation-age assumption, the validity of an $H(z)$ data point requires the same formation time only inside the redshift bin where OHD is locally defined and obtained (Eq. [4]), even if the global density – hence the formation time of the oldest galaxies – at different redshifts varies much. Secondly, a standard viewpoint (referred to as the onion approximation) is to treat the LTB void universe as a group of thin shells structured together, and inside each of these spherical shells the matter is homogeneously distributed [36]. For the OHD used in this paper, the size of each redshift bin is between 0.1 and 0.15, where the first limit is so chosen that the age evolution between the two bins is larger than the error in the age determination [13]. As the precision of the age determination improves, we expect an even smaller bin size. To be sure about the validity of OHD used in LTB models, one needs the exact knowledge about the thickness of the shell given the size of a redshift bin, as well as the steepness of the density profile at the time the oldest galaxies formed. We ever discussed this issue in Ref. [10].

On the other hand, future observation is expected to yield ~ 2000 measurements for passively evolving galaxies in the redshift range $0 < z < 1.5$ in the future [13]. It has been estimated that about 1000 OHD entries at a 15% accuracy level will be determined with 10% error of the galaxies ages. In Ref. [37] the power of OHD in the context of Λ CDM model has been assessed. With the increase of high quantity OHD, the power of OHD constraining void model should also be greatly improved for constraining the void models [10].

Although the large void model appear to be ruled out by some cosmological observations, future OHD measurement in both radial and transverse directions, as an alternative and complementary cosmological test, could give a tight constraint on LTB model with a small void. If the transverse BAO information can be realized from future large-scale structure observations, we should be able to arrive at a definite test of spatial homogeneity of the Universe. In this context, the role played by the transverse BAO is complementary to the radial BAO discussed in Ref. [38].

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Appendix A: H_{\parallel} and H_{\perp} in a coordinate independent manner

The “longitudinal” and “transverse” Hubble parameter in Eq.(2) has only a covariant meaning for spherical symmetry: they are the components, tangent and orthogonal to the orbits of $SO(3)$, of the expansion tensor $H_b^a = H h_b^a + \sigma_b^a$, where $H = (1/3)\nabla_a u^a$ is the Hubble expansion scalar and σ_b^a is the shear tensor. Since $H_r^r = H_{\parallel}$ and $H_{\theta}^{\theta} = H_{\phi}^{\phi} = H_{\perp}$, and we can always choose an orthonormal tetrad with u^a as the timelike tetrad vector, one vector n^a orthogonal to the orbits and two vectors $m_{(1)}^a, m_{(2)}^a$ tangent to them, then $H_{\parallel} = H_{ab}n^a n^b$ and $H_{\perp} = H_{ab}m_{(1)}^a m_{(1)}^b = H_{ab}m_{(2)}^a m_{(2)}^b$, which renders the parameter \mathcal{E} introduced in Eq.(7) as the ratio

$$\mathcal{E} = \frac{H_{ab}n^a n^b}{H_{ab}m_{(1)}^a m_{(1)}^b} = \frac{H_{ab}n^a n^b}{H_{ab}m_{(2)}^a m_{(2)}^b}$$

For non-spherical models, these quantities can always be computed in terms of an orthonormal tetrad, but their interpretation as “longitudinal” and “transverse” becomes coordinate dependent. Another possible comparison that

provides a measure of local inhomogeneity is given by the ratios σ_1/H and σ_2/H , where σ_1 and σ_2 are the eigenvalues of the shear tensor (being trace-free and it admits two eigenvalues in general). For LTB models, $\sigma_1 = \sigma_2 = \sigma = -(1/3)(H_{||} - H_{\perp})$ is the unique eigenvalue: $\sigma_b^a = \sigma e_b^a$ with $e_b^a = h_b^a - 3n^a n_b$ and $H = 2H_{||} + H_{\perp}$. For Szekeres models, there is also a single shear eigenvalue, so by replacing \mathcal{E} with σ/H as a measure of inhomogeneity one can readily generalise the interpretation of observational tests from LTB to Szekeres models. In fact, all of LTB quantities given in coordinate independent manner can be readily generalised to that in Szekeres models[5, 6].

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