Quintessence's last stand?

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Current cosmological data puts increasing pressure on models of dark energy in the freezing class, e.g. early dark energy or those with equation of state w substantially different from -1. We investigate to what extent data will distinguish the thawing class of quintessence from a cosmological constant. Since thawing dark energy deviates from $w=-1$ only at late times, we find that deviations $1+w\lesssim 0.1$ are difficult to see even with next generation measurements; however, modest redshift drift data can improve the sensitivity by a factor of two. Furthermore, technical naturalness prefers specific thawing models.

I. INTRODUCTION

Cosmic acceleration indicates crucial new physics exists outside the standard model of particle physics and cosmology. Cosmological data imply the expansion history of the universe acts like the matter component is supplemented with a fluid of strongly negative effective pressure: dark energy with an effective equation of state, or pressure to energy density, ratio $w\approx -1$. One possibility is the cosmological constant, with $w=-1$ for all times, or redshifts z, and no spatial perturbations. For data sensitive to only the background cosmic expansion, or Hubble parameter $H(z)$, the effects are fully described by $w(z)$, even if there is no physical dark energy fluid but rather a modification of the form of the action.

Cosmic microwave background (CMB) measurements impose two stringent constraints on the dark energy. From the distance to last scattering, the energy density weighted time average of $w(z)$ must be close to -1 [1]. From the structure of the CMB temperature power spectrum, the dark energy density around recombination ($z\approx 10^3$) is less than about 1% of the critical energy density [1–3]. Late time observations of the cosmic expansion, such as supernovae or baryon acoustic oscillation distances, also indicate that $w\approx -1$ at recent times [4, 5]. These results disfavor the first quintessence models, tracker models with insensitivity to initial conditions, but moreover most of the freezing region of dark energy dynamics (Ref. [6] showed that the effective dark energy field either began dominated by Hubble friction – thawing fields – or by the steepness of the potential – freezing fields, unless fine tuned). Also see [7] for how current observations restrict scans through spaces of potentials.

The remaining dark energy behavior favored by data is the thawing class. While thawing fields do not have the trackers' insensitivity to initial conditions, some of them do have the attractive quality of technical naturalness – protection against quantum radiative corrections. Since thawing models only deviate from $w\approx -1$ at late times, they are consistent with the data but are also harder to distinguish from a static cosmological constant, despite having very different physics implications. Given the current state of data, and the advancing plans for next generation experiments, we assess whether continued consistency of the data with a cosmological constant Λ will enable all quintessence models to be disfavored, or at least relegated to a region fine tuned to be close to Λ.

Beyond quintessence, cosmic acceleration models with deviations in growth of structure relative to their expansion behavior, such as from modified gravity theories, or clustered or coupled dark energy, can also be constrained by growth measurements. However within quintessence – canonical, minimally coupled dark energy – no new signatures arise within growth. Thus the question of whether we will be able to distinguish thawing dark energy, and hence most of the remaining viable phase space of quintessence, from Λ will focus on the expansion history.

In Sec. II we discuss methods of treating the thawing class of dark energy as a whole, drawing a clear distinction with inappropriate slow roll assumptions. We examine the leverage of future data in separating thawing quintessence from a cosmological constant in Sec. III, highlighting the role of potential redshift drift measurements. We explore some particle physics implications of fields nearly indistinguishable from Λ, and the virtues of technical naturalness, in Sec. IV and conclude in Sec. V.

II. THAWING QUINTESSENCE

The class of thawing quintessence includes common potentials such as monomials $V(\phi)\sim\phi^n$ and pseudo-Nambu Goldstone boson (PNGB, or axion) fields with $V(\phi)\sim[1+\cos(\phi/f)]$, where f is the symmetry breaking energy scale. Two members of this class are of particular interest since they have shift symmetry protecting against high energy radiative corrections. One is the PNGB case, long studied in terms of both natural inflation [8] and dark energy [9]. The other is the linear potential [10, 11]. Recently the linear potential has garnered renewed interest in connection with both inflation [12] and modified gravity [13].

We are interested in constraining the thawing class as a whole, rather than individual models. To do this efficiently it is useful to parametrize its behavior. The standard w_0–w_a form for the dark energy equation of

state, $w(a) = w_0 + w_a(1-a)$, where $a = 1/(1+z)$ is the scale factor, works for a broad variety of dark energy models, beyond just the thawing class. Indeed it was derived from exact solutions of the field dynamics [14] (and is completely unrelated to a linear or Taylor expansion). The w_0–w_a form matches exact solutions of the observables – distances and Hubble expansion H – to better than 0.1% for a broad range of currently allowed models [15]. However, because it involves two parameters, constraints from even next generation data have difficulty distinguishing thawing models with $w_0 \lesssim -0.9$ from a cosmological constant at 95% confidence level, due to covariances between parameters.

Since we focus our interest on the thawing class, we could use more specialized parametrizations, such as [16–21]. We emphasize though that from an observational perspective the w_0–w_a parametrization with its 0.1% matching is wholly sufficient. To obtain tighter model constraints, we would need a one parameter form. We cannot rely on the usual inflationary slow roll field approximation since even thawing dark energy does not slow roll, in the sense that all terms in the Klein-Gordon equation for its dynamics are comparable [22]. For example, during the vast majority of e-folds when the field is frozen (e.g. during the matter dominated era), the terms are in the ratio of $1:2:-3$. Note also that a small field assumption is somewhat uncertain: while many thawing fields roll a distance $\Delta\phi \approx 0.24\,M_{\rm Pl}$ to get to $w_0 = -0.9$ today, they start at several $M_{\rm Pl}$ from their minima.

We can take two approaches to adopting a one parameter thawing equation of state. Ref. [15] showed that not only was w_0–w_a an excellent approximation to exact solutions of the field evolution, and to observational quantities, but it could also be derived as a calibration relation for classes of dark energy dynamics. By appropriate choice of w_0 and w_a, the spread of evolution in the w–w' plane calibrated into narrow tracks for different dark energy physics. In particular, a broad range of thawing models could all be tightly fit by a constrained w_a form:

$$w_a \approx -1.58\,(1+w_0)\ . \tag{1}$$

This combines all the virtues of the general w_0–w_a form (e.g. 0.1% distance reconstruction) with the leverage of a one parameter fit.

While this fits the expansion observables (distances and Hubble parameter) superbly, an alternate approach is to follow the thawing physics more closely. For example, thawing models have $w = -1$ at high redshift. This is not a problem for the form Eq. (1) since thawing dark energy density fades quickly into the past and so using $w(z \gg 1) = w_0 + w_a \neq -1$ has negligible effect. (Recall that even accounting for this the distance to high redshifts is accurately matched to better than 0.1%.) If we felt more comfortable approximating $w(z)$, despite it not being an observable, then the algebraic thawing form of Ref. [17] works quite well:

$$1 + w(a) = (1+w_0)\,a^p \left(\frac{1+b}{1+ba^{-3}}\right)^{1-p/3}\ , \tag{2}$$

where the constant $b = 0.5$. This is derived from the physics of how the field evolves upon leaving the matter dominated era and automatically has the correct high redshift behavior.

We can turn it into a one parameter form by fixing $p = 1$, so the final algebraic form we will use is

$$1 + w(a) = (1+w_0)\,a^3 \left(\frac{3}{1+2a^3}\right)^{2/3}\ . \tag{3}$$

Note that of course it can fit a cosmological constant as well, with $w_0 = -1$. Both the (general) algebraic thawed and the general w_0–w_a forms have the advantage that the Hubble parameter $H(z)$ is analytic.

Figure 1 demonstrates the fit of the algebraic form to the exact solutions for thawing $w(z)$, with $w_0 = -0.9$. (Note that thawing models with greater present deviation from -1 are already in some tension with current data.) The agreement in $w(z)$ is better than 0.1% (and even better for the observables like distances and H, as we show below) for the favored models of the linear potential and PNGB, and better than 0.2% and 0.3% for the quadratic and quartic potentials.

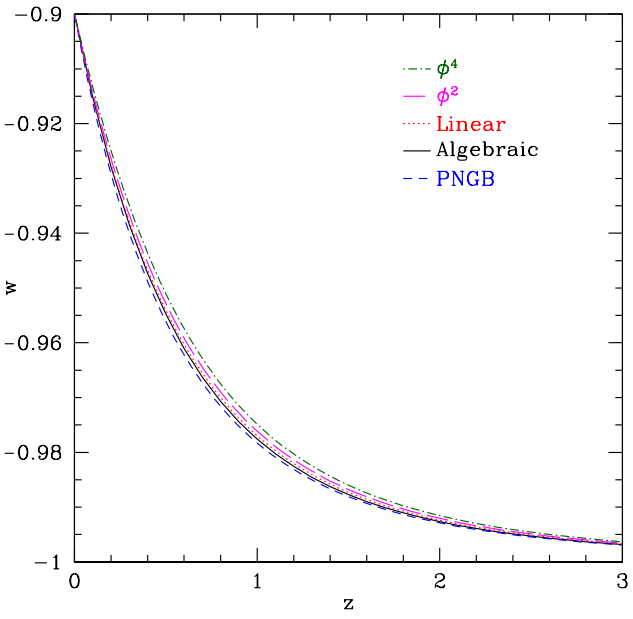

FIG. 1. The one parameter algebraic thawing form (solid black curve, Eq. 3) accurately fits $w(z)$ at all redshifts, for the thawing models of linear, quadratic, quartic, and PNGB potentials.

While monomial potentials indeed have only one parameter to describe the equation of state, PNGB models

have two: the steepness of the potential, given by the symmetry breaking scale f, and the initial field position. For steeper potentials, the algebraic form fit can degrade to $\approx 1\%$ in w (alternately we could adjust the values of b or p in the original algebraic form of Eq. 2, but there will still be a range of PNGB models that cannot be captured to much better than 1% in $w(z)$ by only one parameter).

Despite the success, it is important to note we are not trying to fit $w(z)$ per se (as it is not an observable), but rather to obtain a realistic enough representation that our constraints on distinguishing thawing models from Λ in the next section are accurate. Table I demonstrates the success of both the algebraic thawer (Eq. 3) and constrained w_0–w_a form (Eq. 1) in matching the observables. (Changing the value of, e.g., the matter density would slightly degrade the accuracy of fitting $w(z)$, but improve the accuracy of fitting d or H since there is more freedom allowed for the fit.) The accuracy is more than sufficient for next generation observations seeking to distinguish thawing dark energy from a cosmological constant Λ.

Model	$d(z)$	$H(z)$	$d_{\rm lss}$
Linear potential	0.06%	0.01%	10^{-5}
ϕ^2	0.08%	0.02%	10^{-5}
ϕ^4	0.16%	0.03%	10^{-5}
PNGB ($f/M_{\rm Pl}=2$)	0.03%	0.01%	10^{-5}
Constrained w_a	0.09%	0.01%	4×10^{-5}
Λ	3.5%	0.7%	0.5%

TABLE I. Maximum deviations in the observables – the comoving distance $d(z)$, Hubble parameter $H(z)$, and distance to CMB last scattering $d_{\rm lss}$ – over all redshifts, relative to the algebraic thawing form, are given for the exact solutions of various thawing dark energy models with $w_0=-0.9$. The matter density and Hubble parameter are fixed ($\Omega_m=0.3$, $h=0.7$).

III. CONSTRAINING THAWERS

As Table I shows, the maximum difference between thawing dark energy deviating from the cosmological constant at late times to $w_0=-0.9$ is 3.5% in distance (at $z=0.5$). However, this was with fixing all other cosmological parameters so in reality covariances make the distinction more challenging. Let us see if our two one-parameter thawing forms give data the leverage to distinguish such dynamical dark energy from a cosmological constant.

To test these models with expansion history measurements we consider supernova distances and the CMB distance to last scattering. (We could use baryon acoustic oscillation distances, but supernovae are somewhat more sensitive to the equation of state, especially at the needed low redshift.) For supernovae (SN), we consider a future sample of 150 SN at $z<0.1$, 900 between $z=0.1$–1, and 42 between $z=1$–1.7, with a magnitude systematic of $0.02(1+z)$. Given the systematic control out to $z=1$, this roughly represents a sample from ground based imaging surveys such as the Dark Energy Survey or Large Synoptic Survey Telescope (LSST). We also analyze an extended sample inspired by the DESIRE proposal [23] for the Euclid satellite, with 7200 SN between $z=0.1$–1 and 1000 at $z=1$–1.6, with the same systematics control out to $z=1.6$. These are simply rough estimates of potential data. For the CMB we use Planck quality distance to last scattering measurement at the 0.2% level and a prior on the physical matter density $\Omega_m h^2$ of 0.9%.

The parameters for constraining the expansion history include the matter density Ω_m, dynamical equation of state parameter w_0, supernova absolute magnitude $\mathcal{M}$, and Hubble constant h. Spatial flatness is assumed. Using the future data, the constraint on the equation of state parameter is $\sigma(w_0)\approx 0.06$. If the thawing dark energy has reached $w_0=-0.9$ today (and a stronger deviation is already disfavored by present data), then this yields at most a $\approx 1.7\sigma$ distinction from Λ. A stronger discriminant would be useful.

Interestingly, the extended distance sample does not improve substantially the constraints here. While the baseline sample gives $\sigma(w_0)=0.063$ and 0.062 for the algebraic thawer and calibrated w_a forms, the extended sample gives 0.056 and 0.055. The reason for this insensitivity is that thawers tend to deviate appreciably from Λ only at low redshift; for example thawers as represented by the algebraic form have $w(z=1)=-0.98$ while eventually reaching $w(z=0)=-0.9$. While supernova distances are indeed sensitive to the equation of state value at low redshift, they are not sensitive enough.

The question then is what observational probe can more keenly test the low redshift equation of state. Recently, cosmic redshift drift has been recognized as highly sensitive at low redshift [24]. This involves observing the cosmic evolution of a source redshift with observer time, hence measuring the change in the expansion factor over time. Such observations would constrain the quantity

$$\dot z = (1+z)\,H_0 - H(z)\ . \tag{4}$$

For the (general) algebraic thawer the Hubble parameter can be written analytically as

$$\left(\frac{H(z)}{H_0}\right)^2 = \Omega_m a^{-3} + (1-\Omega_m)\, e^{\frac{3(1+w_0)}{\alpha p}\left[1-(\alpha a^3+\beta)^{p/3}\right]} \tag{5}$$

where $\alpha=1/(1+b)$, $\beta=b/(1+b)$. However, redshift drift requires highly accurate spectroscopy on thirty meter class telescopes (see, e.g., [25]; such spectroscopy is also driven by exoplanet research) and is currently wholly unproven, so we conjecture a single, modest 5% measurement at redshift $z=0.5$. This one data point improves the distinction between thawers and Λ by a factor of two, reducing $\sigma(w_0)$ to 0.030, marginalized over the other cosmological parameters, and hence a 3.3σ distinction from a cosmological constant.

Of course as $w_0\to-1$ the dark energy would be increasingly difficult to distinguish from Λ. Section IV discusses particle physics implications of such near-Λ fields.

IV. FINE TUNED FIELDS

Thawing fields have a limit in which they are identical to a cosmological constant, i.e. when $w_0 = -1$, and so the full class can never be ruled out if the data are consistent with Λ. We examine whether fields in such a limit are particularly fine tuned or disfavored in particle physics terms.

First consider quadratic and quartic potentials as thawers. Like all thawers, the fields do not roll very far during their evolution up to the present, as shown in Fig. 2. For $1+w_0 \ll 1$, the field excursion $\Delta\phi/M_{\rm Pl} \approx 0.76\,(1+w_0)^{1/2}$. However, to attain even only $w_0 = -0.9$ today, the fields must start $2.6\,M_{\rm Pl}$ and $5.1\,M_{\rm Pl}$ respectively from the minima. The potentials must be sufficiently steep locally to be able to reach $\Omega_{de,0} = 0.7$ today once they are released from Hubble friction to roll. Values of w_0 closer to -1 exacerbate the situation, with the fields needing to start at $7.9\,M_{\rm Pl}$ and $15.7\,M_{\rm Pl}$ respectively to reach, say, $w_0 = -0.99$ and hence be easily confused with Λ. Especially since such potentials have no protection against radiative corrections, we might be concerned about such superPlanckian values (despite the field rolling over less than a Planck mass). If we restrict the field initial position to lie within one Planck mass of the minimum, these potentials cannot deliver dark energy density $\Omega_{de} > 0.45$ and 0.17 respectively.

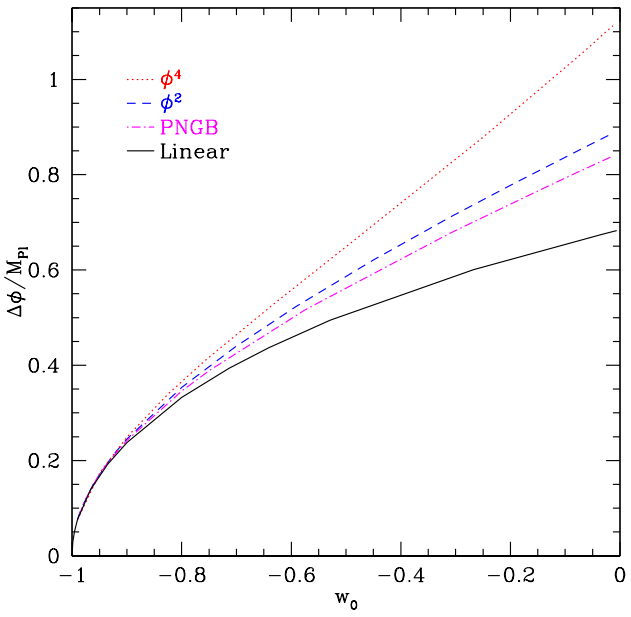

FIG. 2. The distance the field rolls up to the present is plotted vs the present equation of state for several thawer models. The field only traverses a short distance for, say, $w_0 = -0.9$ but must start several $M_{\rm Pl}$ from the potential minimum.

The linear potential, however, is robust. For $V(\phi) = V_0(1+\alpha\phi)$, while the slope of the potential $\alpha V_0 \approx 10^{-120}$ as for any quintessence, the parameter α itself is of order unity. For $w_0 = -0.9$ we have $\alpha = 0.72$, and behavior close to Λ can be obtained without fine tuning α, e.g. $w_0 = -0.99$ comes from $\alpha = 0.25$. Furthermore the linear potential is substantially protected against quantum corrections. In the future it leads to a cosmic doomsday [11], actually essential for the recent solution of the cosmological constant problem known as the sequester [13]. Short of doomsday, the linear potential can be extremely difficult to distinguish from Λ, however, and so upcoming observations will not be able to rule out fully the thawing class.

For the PNGB model, a shift symmetry protects the potential form against quantum corrections. Nevertheless, one might be concerned about the physics validity if the symmetry energy scale $f > M_{\rm Pl}$. As f decreases, and the potential steepens, it becomes harder to attain $\Omega_{de,0} = 0.7$. The field must be fine tuned to an initial position ϕ_i closer and closer to the potential maximum, out of all the range $[0, \pi f]$. For fixed ϕ_i/f, $\Omega_{de,\rm max} \propto f^2$, while for fixed $\Omega_{de,0}$, $(\phi_i/f)_{\rm max} \sim e^{-1/f}$ [15]. Thus small f is problematic. To obtain $w_0 < -0.99$ with $f = 0.5\,M_{\rm Pl}$ requires $\phi_i/f < 0.18$, i.e. only allowing 6% of the full range, while for $f = 0.2\,M_{\rm Pl}$ a 0.5% fine tuning is required. Still, a reasonable region near $f/M_{\rm Pl} \approx 1$ is allowed and can give an expansion history effectively indistinguishable from Λ.

Thus we see that nothing prevents thawing fields, and in particular the two technically natural and hence robust models of the linear potential and PNGB, from approaching Λ beyond the ability of upcoming experiments to distinguish.

V. CONCLUSIONS

Great strides have been made in constraining dark energy physics with ever improving observations. Freezing dark energy, including early dark energy, is substantially limited by the data as an explanation for cosmic acceleration. Probes of the expansion history will continue to become more incisive, and probes of the growth of large scale structure will impose tighter constraints on more elaborate theories involving dark energy clustering, coupling, and modified gravity. However even if the data continue to be consistent with a cosmological constant, quintessence remains a viable option in its thawing class.

The standard dark energy equation of state parametrization $w(a) = w_0 + w_a(1-a)$ is an excellent global approximation over a wide range of freezer, thawed, or modified gravity models, to the 0.1% level in the observables. A constrained, one parameter form with $w_a = -1.58(1+w_0)$ can be used to model specifically the thawing class. Another option is to use the algebraic thawer form of Eq. (3) that follows the thawing physics to match $w(z)$ as well as the observables. We use these not as fitting forms per se, but to represent the thawing class as a whole for comparison to a cosmological constant, to evaluate the discriminatory

leverage of forthcoming data.

We find that next generation distance data can reveal modest, but consistent distinctions of thawers from Λ if $w_0 \gtrsim -0.9$. The leverage in distinguishing the physics doubles with inclusion of a single, 5% measurement by the prospective probe of cosmic redshift drift. This gives further motivation for its exploration and development. In particular, since the signature of the thawing deviation increases at very low redshift – e.g. the deviation is five times greater at $z = 0$ than at $z = 1$ – then the recently identified power of redshift drift at low redshift [24] is especially valuable.

While quintessence in the form of thawing fields cannot be ruled out even if the data become increasingly consistent with a cosmological constant, despite the very different physics, such behavior does point to preferred models. Both the linear potential and PNGB are technically natural, easing high energy quantum effects by their shift symmetry. Indeed the linear potential has recently been found essential in one method for solving the original cosmological constant problem [13]. If future data show some definite signature in expansion or growth away from Λ, we will be able to focus our efforts on specific physical properties. Even if Λ continues to be a good fit, we will be drawn either to explaining its magnitude or to finding the informative high energy physics origins of these robust thawing quintessence potentials.

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