

This is the accepted manuscript made available via CHORUS. The article has been published as:

SU(5) grand unification in pure gravity mediation

Jason L. Evans, Natsumi Nagata, and Keith A. Olive

Phys. Rev. D **91**, 055027 — Published 25 March 2015

DOI: [10.1103/PhysRevD.91.055027](https://doi.org/10.1103/PhysRevD.91.055027)

SU(5) Grand Unification in Pure Gravity Mediation

Jason L. Evans^a, Natsumi Nagata^{a,b}, and Keith A. Olive^a

^a*William I. Fine Theoretical Physics Institute, School of Physics and Astronomy,
University of Minnesota, Minneapolis, MN 55455, USA*

^b*Kavli IPMU (WPI), TODIAS, University of Tokyo, Kashiwa 277-8583, Japan*

Abstract

We discuss the proton lifetime in pure gravity mediation models with non-universal Higgs soft masses. Pure gravity mediation offers a simple framework for studying SU(5) grand unified theories with a split supersymmetry like spectra. We find that for much of the parameter space gauge coupling unification is quite good leading to rather long lifetimes for the proton. However, for $m_{3/2} \sim 60$ TeV and $\tan\beta \sim 4$, for which gauge coupling unification is also good, the proton lifetime is short enough that it could be in reach of future experiments.

1 Introduction

After the initial run of the LHC, the constraints on new physics are rather severe [1]. Although models can still be made to realize weak scale mass spectra, sfermion masses of generic models like the constrained minimal supersymmetric standard model (CMSSM) [2] are now required to be larger than about a TeV. As a result, the naturalness of supersymmetry has been called in to question. However, it was perhaps naive to expect nature to fall into our strict definition of naturalness with less than 10% fine tuning. Supersymmetry, with sfermion masses larger than a TeV, still solves the larger hierarchy problem associated with grand unification and/or the Plank scale. Furthermore, if the sfermion masses are set by the gravitino mass, $m_{3/2}$, and are larger than about 10 TeV, the gravitino lifetime is short enough that it decays before BBN [3]. Moreover, as the mass scale of the sfermions is pushed beyond the weak scale, the constraints on SUSY models from flavor and CP violation in the sfermion sector are greatly relaxed [4]. These advantages, plus the fact that sfermion masses this large are consistent with a larger Higgs mass like the 126 GeV Higgs boson seen at the LHC [5] suggest we relax our strict definition of naturalness.

Large sfermion masses like those found in split supersymmetry [6] are realized in models such as pure gravity mediation (PGM) [7, 8], which can be parameterized by a single parameter [9] $m_{3/2}$. This minimal model of pure gravity mediation is similar in many ways to minimal supergravity (mSUGRA). Universal masses equal to $m_{3/2}$ are imposed at the (grand unified) GUT scale based on the assumption that the Kähler manifold is flat for all matter fields. Unlike the CMSSM, gauginos do not get a tree-level mass. This is because the supersymmetry breaking field is not a singlet and so is excluded from coupling to the gauge kinetic function to leading order. Thus, the leading order contribution to the mass of the gauginos comes from anomaly mediation [10] and are loop suppressed relative to the sfermion masses. The B -term, which contributes to electroweak symmetry breaking, is identical to that in mSUGRA, $B = A - m_{3/2}$. However, since the A -terms of PGM are effectively zero, $B = -m_{3/2}$ and B is fixed for a given value of $m_{3/2}$. This makes radiative electroweak symmetry breaking (EWSB) difficult. However, by adding a Giudice-Masiero term [11], B is no longer fixed by $m_{3/2}$ alone, but also depends on the coupling of the Giudice-Masiero term. This additional freedom in B makes radiative EWSB possible, but only for small values of $\tan \beta$. Once the Higgs mass constraint is taken into consideration, these models have a single free parameter which is some combination of $m_{3/2}$ and $\tan \beta$ [9]. However, because $\tan \beta$ is restricted to be less than about 3, $m_{3/2}$ tends to be rather large. The constraints on $\tan \beta$ can be removed, if the Higgs soft masses at the GUT scale are taken to be non-universal [12]. In this case, $m_{3/2}$ can be taken to be smaller for larger values of $\tan \beta$.

Another important motivation for SUSY is grand unification [13]. In the SM, the gauge couplings approach each other as they are run up to the high scale [14]. However, the quality of the coupling unification is less than convincing. If the SM is supersymmetrized, on the other hand, the unification of the gauge couplings becomes quite good [15]. Furthermore, grand unification in the SM would generate enormous quadratic di-

vergences for the Higgs boson. However, these quadratic divergences are significantly reduced for supersymmetric grand unified theories, even if the sfermions are larger than a TeV. Clearly, grand unification is another motivation for PGM.

The signatures of these simple PGM type models are limited. One possible signature at the LHC for small $m_{3/2}$ is the wino [16]. For larger $m_{3/2}$, on the other hand, the wino cannot be seen at the LHC but could be a viable thermal relic dark matter candidate [17]. If this is indeed the case, it could be seen by indirect detection experiments in the near future [18]. However, this scenario is already under tension from existing indirect detection experiments [19]. The direct detection of wino dark matter is challenging as its scattering cross section with a nucleon is as small as 10^{-47} cm^2 [20]. A Higgsino signature at the LHC is another possible observable which arises from tuning μ to be small [21]. However, this is also difficult to see. This scenario could also have Higgsino-like dark matter which could possibly be seen in future indirect detection experiments [22]. The scattering cross section of the Higgsino with a nucleon is dependent on the size of the wino component of the LSP, and may be probed in future experiments [23].

In this work, we will examine another possible signature of these models. Since the colored triplet Higgs gives threshold corrections to the gauge couplings when integrated out, the quality of the coupling unification determines the mass of the colored triplet Higgs [24, 25, 26] and so affect the lifetime of the proton. When the colored Higgs is integrated out, it also generates a dimension 5 operator proportional to down type Yukawa couplings which lead to proton decay [27]. Since this dimension 5 operator is proportional to the down type Yukawa couplings, it will be enhanced for large $\tan\beta$. Proton decay from this dimension 5 operator arises from a loop diagram with a Higgsino mass insertion [28]. Proton decay of this type can then be suppressed for small μ . When unification is not ideal and $\tan\beta$ is large, a larger Higgsino mass can increase the rate of proton decay from this dimension 5 operator. Parameters of this size are viable in PGM models. Since proton decay of this type is also suppressed by $m_{3/2}$, the more interesting parameters space will be for smaller $m_{3/2}$ and larger $\tan\beta$. Therefore, we will need to consider non-universal Higgs soft masses. We will find that if $m_{3/2}$ is small and $\tan\beta$ is larger, which is also consistent with the Higgs mass measurement, the proton lifetime may be in reach of future experiments. However, for much of the parameter space the lifetime tends to be well beyond the reach of future experiments. We will also look at the quality of the gauge coupling unification determined by the deviation of the colored triplet Higgs mass, M_{H_C} , from the GUT scale as well as the deviation of $(M_X^2 M_\Sigma)^{1/3}$, where X represents the GUT scale SU(5) gauge bosons that become massive and Σ is the **24** which breaks SU(5) at the GUT scale.

2 Minimal SUSY SU(5) GUT

In this section, we will outline the SU(5) SUSY GUT theory [29, 30] we will consider. Additional details on these models can be found in appendix A. The superpotential for

this minimal SU(5) SUSY GUT is given by

$$W = W_{\text{Higgs}} + W_{\text{Yukawa}} , \quad (1)$$

where

$$W_{\text{Higgs}} = \frac{1}{3}\lambda_{\Sigma}\text{Tr}\Sigma^3 + \frac{1}{2}m_{\Sigma}\text{Tr}\Sigma^2 + \lambda_H\bar{H}\Sigma H + m_H\bar{H}H , \quad (2)$$

$$W_{\text{Yukawa}} = \frac{1}{4}h^{ij}\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\Psi_i^{\hat{a}\hat{b}}\Psi_j^{\hat{c}\hat{d}}H^{\hat{e}} - \sqrt{2}f^{ij}\Psi_i^{\hat{a}\hat{b}}\Phi_{j\hat{a}}\bar{H}_{\hat{b}} , \quad (3)$$

and $\hat{a}, \hat{b}, \dots = 1-5$ represent the SU(5) indices and $\epsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}$ is the totally antisymmetric tensor with $\epsilon_{12345} = 1$. Φ_i and Ψ_i are the chiral superfields in the $\mathbf{\bar{5}}$ and $\mathbf{10}$ representations, respectively, with i denoting the generation index. H and \bar{H} are the $\mathbf{5}$ and $\mathbf{\bar{5}}$ containing the MSSM doublets. In these expressions, we have assumed R -parity conservation which forbids terms like $\Psi\Phi\Phi$ and $H\Phi$. The adjoint Higgs field, Σ , gets a vacuum expectation value (VEV) in the direction

$$\langle \Sigma \rangle = V \cdot \text{diag}(2, 2, 2, -3, -3) , \quad (4)$$

breaking the SU(5) gauge group to the SM gauge groups $\text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y$. Because SUSY remains unbroken for SU(5) breaking, we have $V = m_{\Sigma}/\lambda_{\Sigma}$. For this set up, the masses of Σ_3 , Σ_8 , Σ_{24} , and H_C are given as

$$M_{\Sigma} \equiv M_{\Sigma_8} = M_{\Sigma_3} = \frac{5}{2}\lambda_{\Sigma}V , \quad M_{\Sigma_{24}} = \frac{1}{2}\lambda_{\Sigma}V , \quad M_{H_C} = 5\lambda_H V , \quad (5)$$

while the μ term for the MSSM Higgs fields is

$$\mu_0 = m_H - 3\lambda_H V . \quad (6)$$

As is usually done, we tune the parameter m_H to realize $\mu_0 \ll m_H$ which is typically referred to as the doublet-triplet splitting.¹ In addition, the gauge interactions of the adjoint Higgs field yields an X -boson mass of $M_X = 5\sqrt{2}g_5V$ where g_5 is the unified gauge coupling constant. The components $\Sigma_{(3^*,2)}$ and $\Sigma_{(3,2)}$ become the longitudinal component of the X -bosons, and thus do not appear as physical states.

The Yukawa couplings h^{ij} and f^{ij} in Eq. (3) have redundant degrees of freedom, most of which are eliminated by the field re-definition of Ψ and Φ . Since h^{ij} is a symmetric matrix, h^{ij} and f^{ij} have six and nine complex degrees of freedom, respectively. The field redefinition of the SM fields form the $\text{U}(3) \otimes \text{U}(3)$ transformation group, and thus the physical degrees of freedom turn out to be $(12 + 18) - 9 \times 2 = 12$. Among these degrees of freedom, six of them are the quark mass eigenvalues and four are for the CKM matrix

¹This is another fine-tuning besides that for the Higgs mass. Note that the fine-tuning for the Higgs mass becomes worse as the μ parameter is taken to be larger, while the doublet-triplet fine-tuning becomes less severe. This tension may explain why μ is much larger than the electroweak scale [31, 32].

elements and we are left with two phases [33]. In this paper, we take the same basis used in Ref. [25] such that

$$h^{ij} = e^{i\varphi_i} \delta_{ij} f_{u_i}(Q_G) , \quad (7)$$

$$f^{ij} = V_{ij}^* f_{d_j}(Q_G) , \quad (8)$$

where $f_{u_i}(Q_G)$ and $f_{d_j}(Q_G)$ are the up-type and down-type Yukawa couplings, respectively, at a scale Q_G around the GUT scale, and V_{ij} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The phase factors φ_i satisfy the condition $\sum_i \varphi_i = 0$, and thus only two of them are independent. In this basis, the MSSM superfields are embedded into the SU(5) matter multiplets as

$$\Psi_i \ni \{Q_i, e^{-i\varphi_i} \bar{U}_i, V_{ij} \bar{E}_j\} , \quad \Phi_i \ni \{\bar{D}_i, L_i\} . \quad (9)$$

Then, Eq. (3) leads to

$$\begin{aligned} W_{\text{Yukawa}} = & f_{u_i}(Q_i^a \cdot H_2) \bar{U}_{ia} - V_{ij}^* f_{d_j}(Q_i^a \cdot H_1) \bar{D}_{ja} - f_{d_i} \bar{E}_i (L_i \cdot H_1) \\ & - \frac{1}{2} e^{i\varphi_i} \epsilon_{abc} f_{u_i}(Q_i^a \cdot Q_i^b) H_C^c + V_{ij}^* f_{d_j}(Q_i^a \cdot L_j) \bar{H}_{Ca} \\ & + f_{u_i} V_{ij} \bar{U}_{ia} \bar{E}_j H_C^a - V_{ij}^* f_{d_j} e^{-i\varphi_i} \epsilon^{abc} \bar{U}_{ia} \bar{D}_{jb} \bar{H}_{Cc} . \end{aligned} \quad (10)$$

The new phase factors appear only in the couplings of the color-triplet Higgs multiplets.

3 Mass Spectrum and Coupling Unification

To compute the proton decay rate, we need to evaluate the masses of the GUT scale particles which induce the baryon-number violating interactions. In this section, we estimate these masses using the method discussed in Refs. [24, 25, 26]. The mass of the heavy particles are determined by first RG running the couplings to the scale where they approximately unify. Then, because the thresholds at the GUT scale depend on these superheavy particles, their masses can be determined by assuming the deviation in gauge coupling unification is solely due to these thresholds. Note, we will use the $\overline{\text{DR}}$ scheme [34] in the following calculation. At the scale Q_G near the GUT scale, the one-loop matching conditions for the gauge coupling constants are as follows [35, 36]:

$$\begin{aligned} \frac{1}{g_1^2(Q_G)} &= \frac{1}{g_G^2(Q_G)} + \frac{1}{8\pi^2} \left[\frac{2}{5} \ln \frac{Q_G}{M_{H_C}} - 10 \ln \frac{Q_G}{M_X} \right] , \\ \frac{1}{g_2^2(Q_G)} &= \frac{1}{g_G^2(Q_G)} + \frac{1}{8\pi^2} \left[2 \ln \frac{Q_G}{M_\Sigma} - 6 \ln \frac{Q_G}{M_X} \right] , \\ \frac{1}{g_3^2(Q_G)} &= \frac{1}{g_G^2(Q_G)} + \frac{1}{8\pi^2} \left[\ln \frac{Q_G}{M_{H_C}} + 3 \ln \frac{Q_G}{M_\Sigma} - 4 \ln \frac{Q_G}{M_X} \right] , \end{aligned} \quad (11)$$

where g_G is the unified gauge coupling constant. Note that the conditions do not include constant (scale independent) terms since we use the $\overline{\text{DR}}$ scheme for renormalization.

Assuming the above equations contain the major thresholds for the gauge couplings, they can be used to solve for the masses

$$\begin{aligned}\frac{3}{g_2^2(Q_G)} - \frac{2}{g_3^2(Q_G)} - \frac{1}{g_1^2(Q_G)} &= -\frac{3}{10\pi^2} \ln\left(\frac{Q_G}{M_{H_C}}\right), \\ \frac{5}{g_1^2(Q_G)} - \frac{3}{g_2^2(Q_G)} - \frac{2}{g_3^2(Q_G)} &= -\frac{9}{2\pi^2} \ln\left(\frac{Q_G}{M_G}\right),\end{aligned}\tag{12}$$

with $M_G \equiv (M_X^2 M_\Sigma)^{\frac{1}{3}}$. The above expressions allow us to find the masses of the heavy particles in the combination², M_{H_C} and $M_X^2 M_\Sigma$. The value of M_{H_C} and M_G found from these relationships will be used below to find the lifetime of the proton.

4 Proton Decay

In the minimal SUSY GUT, proton decay is induced by the exchange of the color-triplet Higgs boson, and the dominant decay mode is, generally, $p \rightarrow K^+ \bar{\nu}$ [27]. We will only give details of the contributions from the colored Higgs boson since it will often be the dominant source of proton decay in PGM. At the GUT scale, the triplet Higgs boson is integrated out. The most important interaction for our considerations is the color-triplet Higgs exchange which we match at the scale Q_G on to the dimension-five effective Lagrangian

$$\mathcal{L}_5^{\text{eff}} = C_{5L}^{ijkl} \mathcal{O}_{ijkl}^{5L} + C_{5R}^{ijkl} \mathcal{O}_{ijkl}^{5R} + \text{h.c.}, \tag{13}$$

where the effective operators \mathcal{O}_{ijkl}^{5L} and \mathcal{O}_{ijkl}^{5R} are defined by

$$\begin{aligned}\mathcal{O}_{ijkl}^{5L} &\equiv \int d^2\theta \frac{1}{2} \epsilon_{abc} (Q_i^a \cdot Q_j^b) (Q_k^c \cdot L_l), \\ \mathcal{O}_{ijkl}^{5R} &\equiv \int d^2\theta \epsilon^{abc} \bar{U}_{ia} \bar{E}_j \bar{U}_{kb} \bar{D}_{lc},\end{aligned}\tag{14}$$

and the Wilson coefficients C_{5L}^{ijkl} and C_{5R}^{ijkl} are given by

$$\begin{aligned}C_{5L}^{ijkl}(Q_G) &= \frac{1}{M_{H_C}} f_{u_i} e^{i\varphi_i} \delta^{ij} V_{kl}^* f_{d_l}, \\ C_{5R}^{ijkl}(Q_G) &= \frac{1}{M_{H_C}} f_{u_i} V_{ij} V_{kl}^* f_{d_l} e^{-i\varphi_k}.\end{aligned}\tag{15}$$

Note, the color indices must be completely antisymmetric for these interactions and as a result, only operators with at least two generations will be allowed. For this reason, the dominant decay modes contain a strange quark in their final state, *i.e.*, $p \rightarrow K^+ \bar{\nu}$.

As can be seen in Eq. (10), at the GUT scale the lepton and down type quark Yukawa couplings should be equal. However, in running up from the weak scale, we find them to

²The third conditions is used to determine $g_G^2(Q_G)$.

be quite different especially those for the first two generations. The difference is, however, easily compensated by effects above the GUT scale; for instance, the higher-dimensional operators induced at the Planck scale contribute to the Yukawa couplings, which may account for this difference [37, 38, 39]. Because it is not known which of these values is close to the correct value for the Yukawa coupling at the GUT scale, in the discussion below, we use both the down quark and lepton type Yukawa couplings to calculate the proton lifetime. This will allow us to quantify our uncertainty in the lifetime of the proton.

The relevant operators in Eq. (15) can be further reduced by keeping only those with the largest Yukawa couplings. We find that only the operators \mathcal{O}_{3312}^{5R} and \mathcal{O}_{3311}^{5R} yield a sizable contribution to proton decay, even though the contribution is suppressed by a flavor changing element of the CKM matrix. This contribution turns out to be dominant because of the large third generation Yukawa couplings involved [28]. The relevant Wilson coefficients are then

$$\begin{aligned} C_{5R}^{3311}(Q_G) &= \frac{1}{M_{H_C}} f_t f_d(Q_G) e^{-i\varphi_1} V_{tb} V_{ud}^* , \\ C_{5R}^{3311}(Q_G) &= \frac{1}{M_{H_C}} f_t f_s(Q_G) e^{-i\varphi_1} V_{tb} V_{us}^* . \end{aligned} \quad (16)$$

Notice that the coefficients include a common phase factor $e^{-i\varphi_1}$, which is therefore not important for proton decay.

The Wilson coefficients in Eq. (16) are then evolved down to the SUSY scale. At the SUSY scale, the sfermions of these dimension-five operators are integrated out via the one-loop diagram found in Fig. 7 of appendix B. The process proceeds via the exchange of either a charged wino or a Higgsino³. In PGM, we generally have $|\mu| \gg |M_2|$ and so the contribution from Higgsino exchange dominates [41]⁴. For these reasons, we focus on the charged Higgsino exchange process in what follows.

The loop diagram in Fig. 7 is then matched onto the baryon-number violating four-fermion operators [42, 43, 44]

$$\mathcal{L}_6^{\text{eff}} = C_i \epsilon_{abc} (u_{R1}^a d_{Ri}^b) (Q_{L3}^c \cdot L_{L3}) , \quad (17)$$

with

$$C_i(Q_S) = \frac{f_t f_\tau}{(4\pi)^2} C_{5R}^{*331i}(Q_S) F(\mu, m_{\tilde{t}_R}^2, m_{\tau_R}^2) , \quad (18)$$

where $i = 1, 2$, and Q_S is the SUSY breaking scale taken to be around $m_{3/2}$. The loop function F is found in appendix B. The above expression shows that the proton decay rate depends on the SUSY spectra through the loop function. We will see this dependence in

³This is the dominant contribution to proton decay, unless there is flavor violation in the sfermion sector. In this paper, we assume there is no flavor violation in the sfermion sector. The flavor violating case is discussed in Ref. [40].

⁴ Higgsino exchange dominates in this limit because the gauginos and Higgsinos in the one-loop diagrams are required to flip their chirality, and thus their contribution to proton decay is proportional to their masses, as can be seen from the expression for the function F given in Eq. (41).

Sec. 6 for the PGM scenario. Note that the loop function is suppressed by the sfermion masses. Thus, we expect that for large $m_{3/2}$ the proton lifetime is long enough [41, 45] to evade the current bound, $\tau(p \rightarrow K^+ \bar{\nu}) > 5.9 \times 10^{33}$ years [46]. This can be compared to the weak-scale SUSY scenarios; in these cases, the proton decay rate is in general predicted to be so large that the minimal SUSY GUT is excluded [47] and thus some additional conspiracy is required to realize a SUSY GUT.

We now run the Wilson coefficients down to the hadronic scale, $Q_{\text{had}} = 2$ GeV. The Lagrangian at this scale takes the form⁵

$$\mathcal{L}(p \rightarrow K^+ \bar{\nu}_\tau) = C_{usd}[\epsilon_{abc}(u_R^a s_R^b)(d_L^c \nu_\tau)] + C_{uds}[\epsilon_{abc}(u_R^a d_R^b)(s_L^c \nu_\tau)] . \quad (19)$$

Using these Wilson coefficients, we then evaluate the partial decay width of the $p \rightarrow K^+ \bar{\nu}$ and find

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = \frac{m_p}{32\pi} \left(1 - \frac{m_K^2}{m_p^2}\right)^2 |\mathcal{A}(p \rightarrow K^+ \bar{\nu})|^2 , \quad (20)$$

where m_p and m_K are the masses of proton and kaon, respectively, and

$$\mathcal{A}(p \rightarrow K^+ \bar{\nu}) = C_{usd}(Q_{\text{had}}) \langle K^+ | (us)_R d_L | p \rangle + C_{uds}(Q_{\text{had}}) \langle K^+ | (ud)_R s_L | p \rangle . \quad (21)$$

The hadron matrix elements in the above equation have been recently computed in Ref. [48] using a lattice simulation of QCD,

$$\begin{aligned} \langle K^+ | (us)_R d_L | p \rangle &= -0.054(11)(9) \text{ GeV}^2 , \\ \langle K^+ | (ud)_R s_L | p \rangle &= -0.093(24)(18) \text{ GeV}^2 , \end{aligned} \quad (22)$$

where the first and second parentheses represent statistical and systematic errors, respectively. The matrix elements are computed at the scale $Q_{\text{had}} = 2$ GeV.

Before concluding this section, we comment on other possible contributions to proton decay. Firstly, the dimension-five baryon-number violating operators in Eq. (14) can also be generated at the Planck scale, M_P . If the coefficients of the operators are $\mathcal{O}(1/M_P)$, that is, there is no suppression from Yukawa couplings, then they will give the dominant contribution to proton decay and result in a lifetime which is too short [49]. It is expected, however, that there is some underlying mechanism such as a flavor symmetry which is responsible for the structure of the Yukawa couplings. This symmetry could give additional suppression to these Planck-scale operators. In this paper, we assume that the contribution of these operators are less significant compared with the color Higgs contribution, and neglect them in the following analysis.

Secondly, the exchange of the X bosons will also induce proton decay. This decay mode is via a dimension-six GUT scale effective operator and is thus usually sub-dominant compared to the contribution of the dimension-five operator discussed above. An approximate expression for the lifetime of the proton from the dimension six operator is

$$\tau(p \rightarrow e^+ \pi^0) \simeq 3 \times 10^{35} \times \left(\frac{M_X}{1.0 \times 10^{16} \text{ GeV}} \right)^4 . \quad (23)$$

⁵For more details of how we arrived at this expression see appendix B.

There is a slight dependence on the masses of SUSY particles we have neglected. As can be seen from this expression, the proton decay width from the dimension six operator will in general give lifetimes too long to be detected, at least much longer than the present bound: $\tau(p \rightarrow e^+ \pi^0) > 1.4 \times 10^{34}$ years [50, 51].

5 Pure Gravity Mediation

As discussed above, the lifetime of the proton depends on the SUSY parameters. Motivated by the 126 GeV Higgs boson [5] and other cosmological considerations [52], we will analyze the proton lifetime for PGM models. The scalar potential of PGM takes the same form as that of mSUGRA

$$V = \left| \frac{\partial W}{\partial \phi^i} \right|^2 + (A_0 W^{(3)} + B_0 W^{(2)} + \text{h.c.}) + m_{3/2}^2 \phi^i \phi_i^*, \quad (24)$$

which is determined by the flat Kähler manifold⁶ and the superpotential W is given in Eq. (1). $W^{(2)}$ and $W^{(3)}$ are the bi- and tri-linear parts of the superpotential. For PGM, the SUSY breaking field is a non-singlet and strongly stabilized [54] which suppresses the gaugino masses and A -terms respectively. The gaugino masses are regenerated by anomalies and take the form⁷

$$M_1 = \frac{33}{5} \frac{g_1^2}{16\pi^2} m_{3/2}, \quad (25)$$

$$M_2 = \frac{g_2^2}{16\pi^2} m_{3/2}, \quad (26)$$

$$M_3 = -3 \frac{g_3^2}{16\pi^2} m_{3/2}. \quad (27)$$

In order to account for radiative EWSB, mSUGRA is further modified by including a Giudice-Masiero term for the Higgs fields in the Kähler manifold [11]. This modifies the Higgs boson parameters to

$$\mu = \mu_0 + c_H m_{3/2}, \quad (28)$$

$$B\mu = \mu_0 (A_0 - m_{3/2}) + 2c_H m_{3/2}^2, \quad (29)$$

where μ_0 is the superpotential Higgs bilinear term found in $W^{(2)}$. This allows us to vary both μ and $B\mu$ independently in order to satisfy the EWSB conditions. This leaves $m_{3/2}$ and c_H as free parameters. In this case, $\tan \beta$ is an output of the EWSB conditions, but in practice one can trade c_H for $\tan \beta$ and use $m_{3/2}$ and $\tan \beta$ as free inputs. Since this simplest of PGM models tends to require small $\tan \beta$ and larger $m_{3/2}$, we will allow the

⁶If the Kähler manifold for the first two generations is no-scale like, these models can explain $g - 2$ experiments [53]. However, in this case the proton decay calculation is more complicated because of an additional wino contribution but should give a similar order of magnitude for the proton lifetime.

⁷The A -terms are also regenerated by anomalies. However, they are too small to be of importance.

Higgs soft masses to be free parameters. This will allow $\tan\beta$ to be larger and so allow for $m_{3/2}$ to be smaller [12]. As was seen in the previous sections, both larger $\tan\beta$ and smaller $m_{3/2}$ will lead to shorter lifetimes of the proton. We will not discuss the origin of these non-universal Higgs soft masses here. However, discussion about this can be found in Ref. [12]. Lastly, we note that the non-universal Higgs soft masses, m_1 and m_2 , can also be parameterized in terms of the low scale values of μ and m_A which are otherwise also outputs of the EWSB conditions. We will take advantage of this in the results below in order to zoom in on some features of the proton lifetime.

6 Results

We are now in a position to discuss the proton lifetime and mass scales associated with gauge coupling unification in a variety of models which have varying degrees of non-universality in the Higgs sector. We begin by displaying in Fig. 1 the $m_1 = m_2$ vs. $\tan\beta$ plane for fixed gravitino mass. This is a one-parameter extension of the minimal (two-parameter) PGM model and resembles NUHM1 models [55]. In the left panel we have fixed $m_{3/2} = 60$ TeV. For this value of the gravitino mass, the Higgs mass lies between 124 and 128 GeV⁸ for $\tan\beta$ roughly between 4–9 as shown by the red dot-dashed curves. The thin blue lines show the values of the LSP (wino) mass⁹ and are solid for $\mu > 0$ and dashed for $\mu < 0$. The anomaly mediated contribution to m_χ for $m_{3/2} = 60$ TeV is 170 GeV. At low $\tan\beta$, threshold corrections from the heavy Higgs bosons and the Higgsinos increase the mass for $\mu < 0$ and decrease the mass for $\mu > 0$. At large $\tan\beta$, the wino mass, for both positive and negative μ , tends to its anomaly mediated value. The curves end at high and low values of the Higgs soft masses due to the absence of radiative electroweak symmetry breaking. For large and negative values of $m_1^2 = m_2^2$ (the sign on the axis refers to the sign of the mass squared), the Higgs pseudoscalar mass squared is negative, and for large positive $m_1^2 = m_2^2$, the electroweak conditions yield $|\mu|^2 < 0$.

The thicker black curves in Fig. 1 show the values of the proton lifetime. As discussed earlier, as there is some uncertainty as to how we match the Yukawa couplings at the GUT scale, we have results based on quark Yukawa couplings (shown by the solid curves) and results based on lepton Yukawa couplings (shown by the dashed curves). As one can see from the figure, the calculated proton lifetime is sensitive to $\tan\beta$ yet relatively insensitive to the value of $m_{1,2}$ for fixed gravitino mass. In general, the proton lifetime is lower at high $\tan\beta$ due to the increase in the down-like Yukawa couplings when $\tan\beta$ is increased, whereas the Higgs mass increases with $\tan\beta$. For these relatively low values of the gravitino mass used in the left panel, the proton lifetimes based on quark Yukawas drop below 5×10^{34} years only when $\tan\beta > 7$ where $m_h > 127$ GeV. The lifetime increases rapidly at lower $\tan\beta$ and exceeds 5×10^{35} years when $\tan\beta < 4$ where $m_h < 124$ GeV.

⁸We refer to this extended range of Higgs masses to account for the uncertainty in the calculation of the Higgs mass. Note also the Higgs masses calculated here differ slightly from those calculated in [12] as here we are not imposing strict gauge coupling unification at the GUT scale.

⁹The present lower limit on the wino mass from the LHC experiment is about 270 GeV [56].

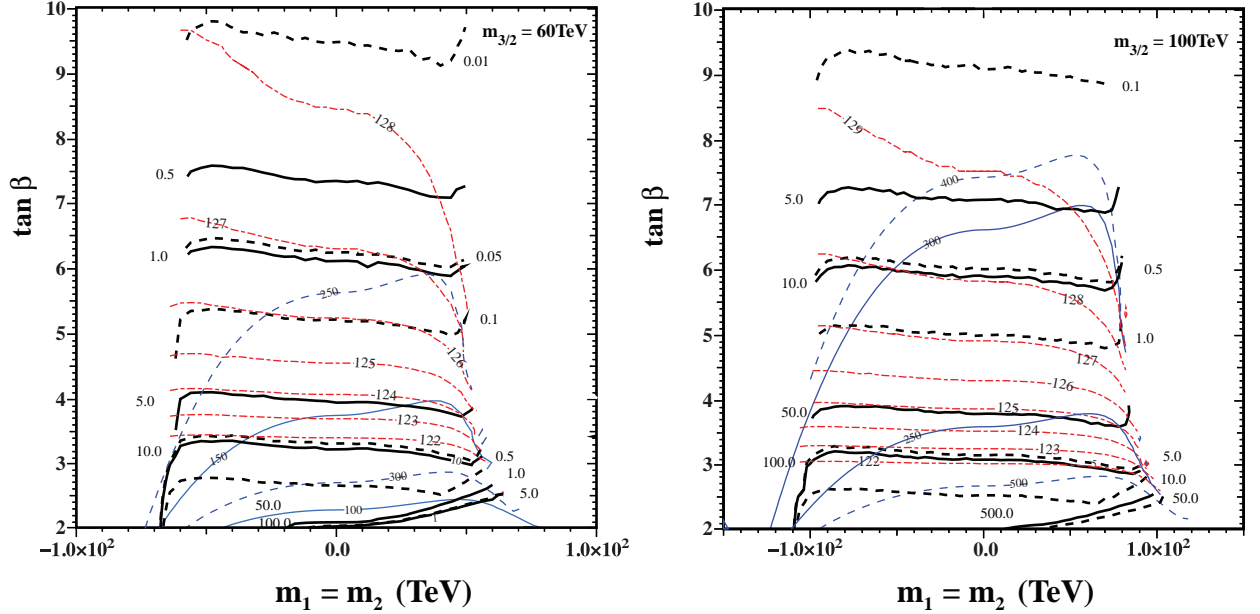


Figure 1: The $\tan\beta$ - $m_{1,2}$ plane for a) $m_{3/2} = 60$ TeV and b) $m_{3/2} = 100$ TeV. The Higgs mass is shown by the nearly horizontal thin red contours in 1 GeV intervals. The wino/chargino mass is shown by the thin solid ($\mu > 0$) and dashed ($\mu < 0$) contours. The thick black contours show the value of the proton lifetime based on the quark Yukawa couplings (solid) and lepton Yukawa couplings (dashed) in units of 10^{35} years. Lifetime contours for the solid curves are labeled to the left of the contours whereas dashed contours are labelled to the right.

However, the wino mass requires $\mu < 0$ and $\tan\beta \gtrsim 6$. Recall that these lifetimes are computed from Eq. (20) and when the lifetime exceeds 3×10^{35} years, the dominant contribution to the decay rate comes from the dimension six operator given in Eq. (23). Proton lifetimes based on lepton Yukawas are significantly smaller (by a factor of roughly 20), so that $\tau_p^l < 5 \times 10^{33}$ years when $\tan\beta \gtrsim 6$ and is still smaller than 2×10^{34} years when $\tan\beta > 4$.

In the right panel of Fig. 1, we have taken $m_{3/2} = 100$ TeV and as expected the Higgs mass for a given value of $\tan\beta$ is higher. The range 124 – 128 GeV now requires $\tan\beta \simeq 3.5 - 6$. The uncorrected wino mass is now about 290 GeV and in the figure we see lower (higher) wino masses when $\mu > (<)$ 0. The proton lifetimes are now significantly higher. At $\tan\beta = 6$, the quark based value of τ_p determined by the dimension five operator is now 10^{36} years and increases as $\tan\beta$ is lower. The lepton based lifetimes remain a factor of about 20 lower and may still be as low as 5×10^{34} years at $\tan\beta = 6$.

To see more clearly the dependence of the proton lifetime on the PGM parameters, we show in left panel of Fig. 2 the behavior of the proton lifetime as a function of $\tan\beta$ for fixed $m_{3/2} = 60$ and 200 TeV with $m_1 = m_2 = 0$. The lifetime falls off monotonically with $\tan\beta$. The ratio between the quark and lepton evaluation of τ_p is seen to be nearly

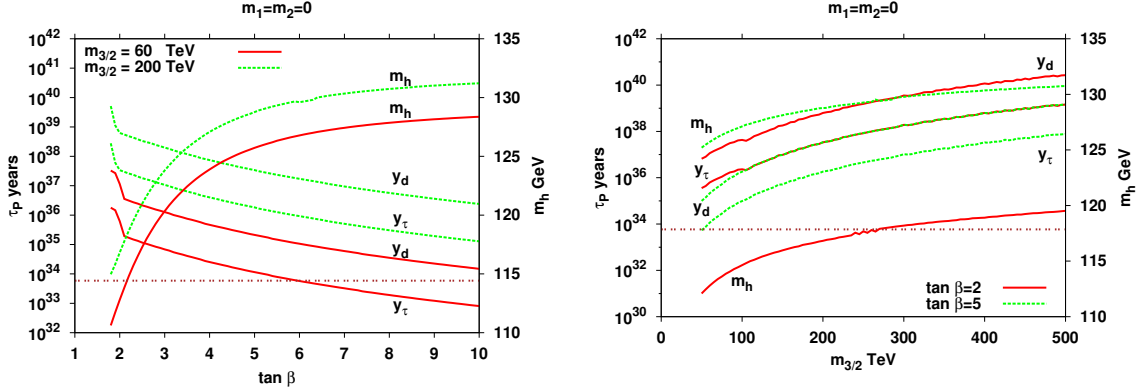


Figure 2: The dependence of the proton lifetime on $\tan \beta$ (left) for fixed $m_{3/2} = 60$ and 200 TeV, and on $m_{3/2}$ (right) for fixed $\tan \beta = 2$ and 5. Both of the Higgs soft masses have been fixed $m_1 = m_2 = 0$. The dependence of the Higgs mass is also shown with its value given on the right side of each panel. The horizontal line indicates the present experimental bound [46].

constant as $\tan \beta$ is varied with a ratio of about 20. We also see the substantial increase in τ_p when $m_{3/2}$ is increased to 200 TeV. The experimental limit on the proton lifetime, $\tau(p \rightarrow K^+ \bar{\nu}) > 5.9 \times 10^{33}$ years [46], is shown by the horizontal line. In the right panel, we show this increase in τ_p with $m_{3/2}$ for fixed values of $\tan \beta = 2$ and 5. In both panels, we also show the Higgs mass as a function of $\tan \beta$ and $m_{3/2}$ with its value displayed on the right edge of each panel. Restricting the Higgs mass to the range 124–128 GeV allows one to focus on the relevant ranges of either $\tan \beta$ or $m_{3/2}$ and hence on the predicted proton lifetime.

In contrast to the proton lifetime, the relevant GUT mass scales, M_{H_C} and M_G , are relatively insensitive to the PGM parameter choices as seen in Fig. 3. As one can see in the left panel, there is very little dependence on $\tan \beta$. The mass parameter $M_G \equiv (M_X^2 M_\Sigma)^{1/3}$ is always close to 10^{16} GeV independent of $m_{3/2}$ (as also seen in the right panel). While the color triplet mass is insensitive to $\tan \beta$, it does have a mild dependence on the gravitino mass and ranges from a few $\times 10^{16}$ – few $\times 10^{17}$ GeV. Notice that in the weak-scale SUSY scenario the mass of the color-triplet Higgs multiplet is predicted to be around 10^{15} GeV [47]. A heavier color triplet mass makes the proton lifetime long enough to evade the current experimental bound. Furthermore, in some of the parameter space of PGM, the GUT scale parameters M_{H_C} and M_G are both of $\mathcal{O}(10^{16})$. In these cases, the threshold corrections at the GUT scale become very small, which implies the unification of the gauge couplings is quite good. In fact, for $m_{3/2} \sim 60$ TeV and $\tan \beta \sim 5$, we get good gauge coupling unification and a proton lifetime which could be in reach of future experiments.

In Fig. 4, we offer two additional planes which show the dependence of the proton lifetime on other PGM parameters. In the left panel, we plot the lifetime contours in the $m_1 = m_2$, $m_{3/2}$ plane. This is again a NUHM1-like model and we have fixed $\tan \beta = 5$. As in Fig. 1, the red-dashed curves show the Higgs mass contours which vary from about

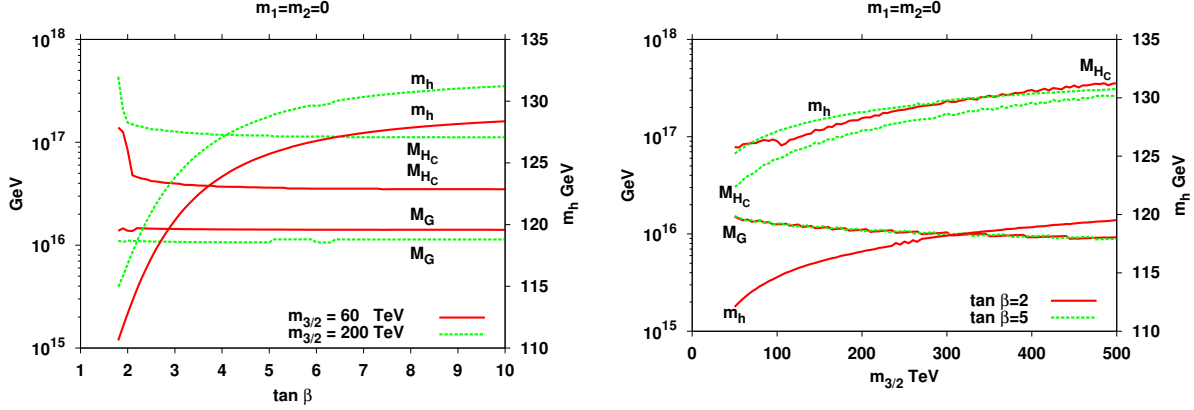


Figure 3: The dependence of the GUT scale mass parameters (M_{H_C} and M_G) on $\tan \beta$ (left) for fixed $m_{3/2} = 60$ and 200 TeV, and on $m_{3/2}$ (right) for fixed $\tan \beta = 2$ and 5 . Both of the Higgs soft masses have been fixed $m_1 = m_2 = 0$. The dependence of the Higgs mass is also shown with its value given on the right side of each panel.

124–128 GeV for the plane shown. As before, the curves extend across a limited range in $m_1 = m_2$ where the EWSB conditions can be satisfied. At large positive m_1^2 , μ^2 goes to 0 (where the curve is cutoff). At very small μ , the Higgs masses increases rapidly causing the sudden downturn in the mass contours. As expected, we see the wino mass varies considerably as $m_{3/2}$ is varied. For the range in $m_{3/2}$ shown, the proton lifetime varies from as low as 10^{33} years using the lepton Yukawas and low $m_{3/2}$ to as high as 10^{37} years using quark Yukawas and $m_{3/2} \approx 150$ TeV.

In the right panel of Fig. 4, we show a two-parameter extension of the two-parameter PGM similar to the NUHM2 [57]. Results are displayed in the μ, m_A plane for fixed $\tan \beta = 5$ and $m_{3/2} = 60$ TeV. In this case, the EWSB conditions, are used to solve for the two Higgs soft masses which now differ. As the Higgs mass is largely independent of m_A , the Higgs mass contours are nearly vertical. At the center of the plot, as $|\mu|$ gets to be very small, m_h gets large and exceeds 130 GeV. At large $|\mu|$, m_h is always larger than 125 GeV in the ranges shown. The threshold corrections to the wino mass are sensitive to μ and m_A and that accounts for the variation of m_χ as these parameters are varied.

The proton lifetime varies between 10^{34} and 10^{36} years but shows significantly more variability. This is due to the competing effects of changing μ . The proton lifetime depends both on the color-triplet Higgs mass and on μ itself.¹⁰ As μ is lowered, the color-triplet Higgs mass decreases which tends to decrease the proton lifetime. But as $|\mu|$ is further decreased, the proton lifetime dependence on μ overcomes its dependence on M_{H_C} and the lifetime increases very rapidly at small $|\mu|$ seen by the sharp downturn in the contours at near $\mu = 0$. These effects can be better understood by examining Figs. 5 and 6 which show the behavior of the proton lifetime and GUT-scale masses, including

¹⁰The proton decay rate directly depends on μ through the loop function F in Eq. (18). When $|\mu| \ll m_{3/2}$, $F \propto \mu/m_{3/2}^2$, while if $\mu \gg m_{3/2}$, $F \propto \log(\mu^2/m_{3/2}^2)/\mu$, as can be seen from Eq. (41).

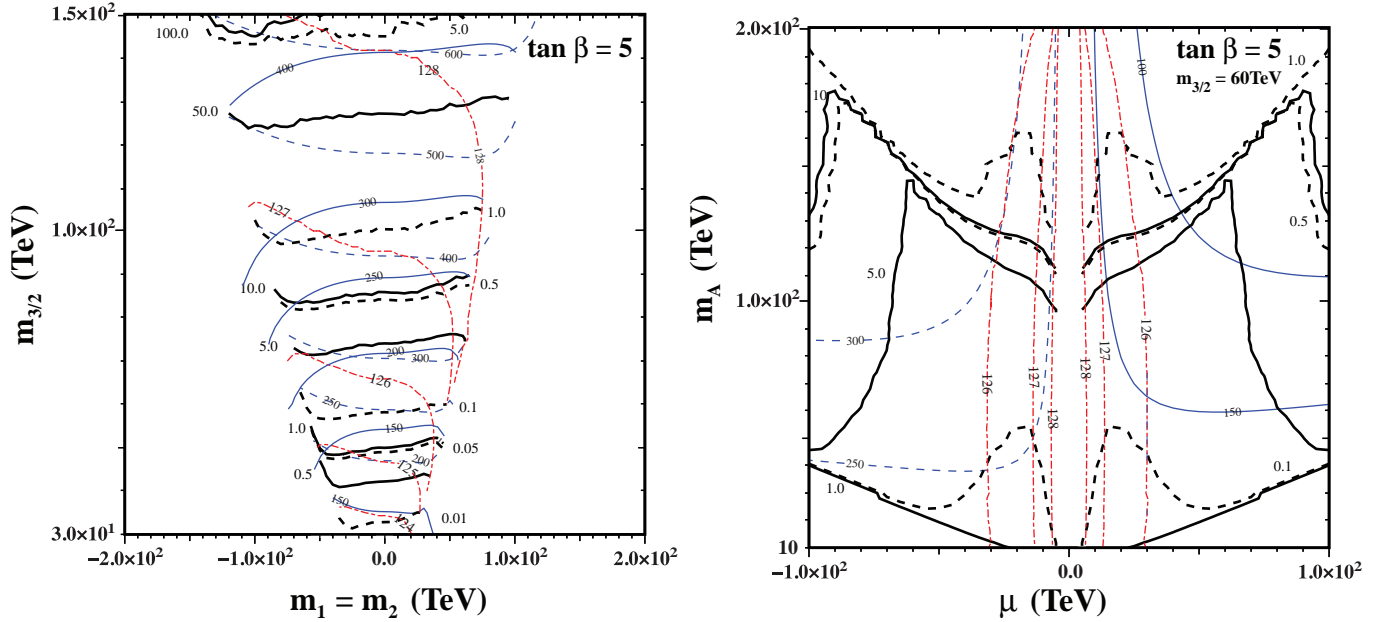


Figure 4: a) The $m_{3/2}$ - $m_{1,2}$ plane for $\tan \beta = 5$ and b) the μ - m_A plane for $\tan \beta = 5$ and $m_{3/2} = 60$ TeV. In both panels, the Higgs mass is shown by thin red dot-dashed contours in 1 GeV intervals. The wino/chargino mass is shown by the thin solid ($\mu > 0$) and dashed ($\mu < 0$) contours. The thick black contours show the value of the proton lifetime based on the quark Yukawa couplings (solid) and lepton Yukawa couplings (dashed) in units of 10^{35} years. Lifetime contours for the solid curves are labeled to the left of the contours whereas dashed contours are labelled to the right.

the heavy Higgs mass, as a function of μ for fixed $\tan \beta$ and $m_{3/2}$. Here we see the first gradual and then rapid decrease in the color-triplet mass as $|\mu|$ is lowered from large values toward $\mu = 0$. There is no substantial difference in this behavior between the two values of $\tan \beta$ shown. Once again, we see that M_G depends very little on our parameter choices and is always near 10^{16} GeV.

Finally, in Fig. 6, we see the sharp increase in the proton lifetime as $|\mu|$ gets small¹¹. Here we see also that the Higgs mass rises sharply as μ tends to zero. It is important to recall that the lifetime plotted corresponds only to that given by the dimension-5 operator given in Eq. (20) and would not exceed 3×10^{35} years when the dimension six operator is included. The latter is fairly insensitive to parameter choices.

7 Conclusion and Discussion

As we await new results for physics beyond the standard model from the LHC, we have been forced to consider supersymmetric models with sfermion masses larger than what

¹¹Our calculations are only valid for $|\mu|$ much greater than the wino mass.

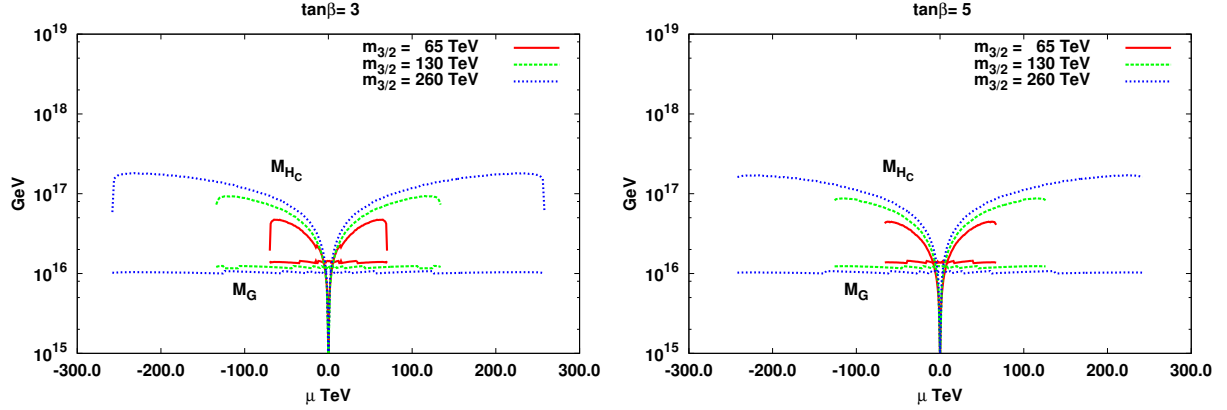


Figure 5: The dependence of the GUT-scale masses (m_{H_C} and M_G) on μ for fixed $\tan\beta = 3$ (left) and $\tan\beta = 5$ (right) and fixed $m_{3/2} = 65, 130$, and 260 TeV. Both of the Higgs soft masses have been fixed $m_1 = m_2 = 0$. The extent of the curves is determined by the validity of radiative EWSB.

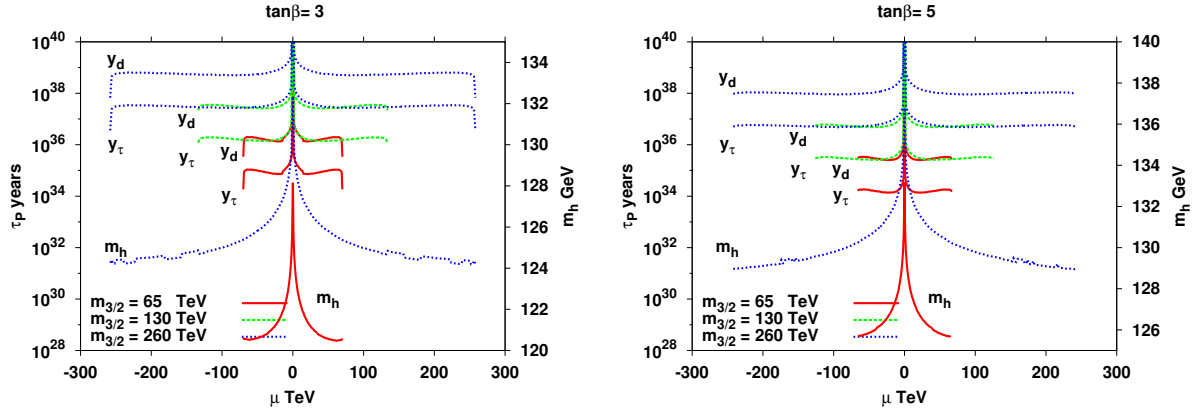


Figure 6: The dependence of the proton lifetime on μ for fixed $\tan\beta = 3$ (left) and $\tan\beta = 5$ (right) and fixed $m_{3/2} = 65, 130$, and 260 TeV. Both of the Higgs soft masses have been fixed $m_1 = m_2 = 0$. The extent of the curves is determined by the validity of radiative EWSB. The dependence of the Higgs mass is also shown with its value given on the right side of each panel.

was previously considered ‘natural’. While a great deal of attention had been focused on relatively simple models such as the CMSSM or mSUGRA (with 4 and 3 parameters respectively) or the NUHM1,2 with 5,6 parameters, pure gravity mediation models can be described with as few as 2 parameters at the cost of a mass spectrum which approaches the PeV scale. As we hope the actual theory of nature is in the realm of experimental science, it is imperative to find means to test these models. Here we have examined one additional possibility for testing these models despite their generally heavy mass spectra.

PGM theories, with all their economy, are still able to resolve many of the questions

their lower energy cousins (such as the CMSSM) were motivated from. These include the ability to achieve gauge coupling unification at the GUT scale, radiative breaking of the electroweak symmetry, the stability of the Higgs potential, and they also provide a suitable candidate for dark matter. The latter is definitely more difficult in PGM models, as the wino is usually the lightest supersymmetric particle and as such would require a wino mass near 3 TeV to supply the correct relic density. This pushes the gravitino mass up to several hundred TeV. Alternatives within PGM are possible if μ is relatively small and the Higgsino is the lightest supersymmetric particle [58] or if the theory contains additional vector-like states and bino-gluino co-annihilation controls the relic bino density [59], or even axion dark matter [58, 60]. In contrast to their lower energy counterparts, PGM models have a relatively easy time obtaining a Higgs mass in agreement with the experimental measurement [5].

Thus experimental verification of PGM models remains challenging. While there is the chance that the lightest supersymmetric particle is within reach of the LHC, the bulk of the PGM spectrum is not. Here we have calculated the proton lifetime in PGM models. We have found that typically the lifetime is long and in many cases significantly above the current experimental bounds. However in cases where $m_{3/2}$ is relatively small and $\tan \beta$ is relatively high, the proton lifetime is low and may be at the level of current experimental searches. While proton decay itself, can not point directly to PGM supersymmetry, it may provide one more handle on an ever increasingly elusive theory beyond the standard model.

Acknowledgments

The work of J.E. and K.A.O. was supported in part by DOE grant DE-SC0011842 at the University of Minnesota. The work of N.N. is supported by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists.

Appendix

A Minimal SU(5) Notation and Conventions

Here, we review the minimal SUSY SU(5) GUT [29, 30] and clarify our notation and conventions. In these models, the MSSM matter fields are embedded into a $\mathbf{\bar{5}}$ and $\mathbf{10}$ representations of the SU(5) gauge group for each generation. Let Φ_i and Ψ_i be the chiral superfields in the $\mathbf{\bar{5}}$ and $\mathbf{10}$ representations, respectively, with i denoting the generation

index. These fields decompose into the MSSM superfields as

$$\Phi_i = \begin{pmatrix} \bar{D}_{i1} \\ \bar{D}_{i2} \\ \bar{D}_{i3} \\ E_i \\ -N_i \end{pmatrix}, \quad \Psi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{U}_{i3} & -\bar{U}_{i2} & U_i^1 & D_i^1 \\ -\bar{U}_{i3} & 0 & \bar{U}_{i1} & U_i^2 & D_i^2 \\ \bar{U}_{i2} & -\bar{U}_{i1} & 0 & U_i^3 & D_i^3 \\ -U_i^1 & -U_i^2 & -U_i^3 & 0 & \bar{E}_i \\ -D_i^1 & -D_i^2 & -D_i^3 & -\bar{E}_i & 0 \end{pmatrix}, \quad (30)$$

with

$$L_i = \begin{pmatrix} N_i \\ E_i \end{pmatrix}, \quad Q_i^a = \begin{pmatrix} U_i^a \\ D_i^a \end{pmatrix}, \quad (31)$$

where $a = 1, 2, 3$ denotes the color index. The MSSM Higgs superfields, on the other hand, are embedded into a **5** and $\bar{\mathbf{5}}$:

$$H = \begin{pmatrix} H_C^1 \\ H_C^2 \\ H_C^3 \\ H_2^+ \\ H_2^0 \end{pmatrix}, \quad \bar{H} = \begin{pmatrix} \bar{H}_{C1} \\ \bar{H}_{C2} \\ \bar{H}_{C3} \\ H_1^- \\ -H_1^0 \end{pmatrix}, \quad (32)$$

where the last two components are the MSSM Higgs superfields,

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}. \quad (33)$$

The other piece of the **5** and $\bar{\mathbf{5}}$ Higgs bosons are labeled by H_C^a and \bar{H}_{Ca} and will be referred to as the color-triplet Higgs bosons.

The gauge boson of SU(5) is a **24**. In supersymmetry this corresponds to a real vector superfield, \mathcal{V}^A , where $A = 1, \dots, 24$ represents the gauge index. \mathcal{V}^A can be decomposed into the SM gauge fields, plus the additional massive gauge bosons of SU(5) breaking, as follows

$$\mathcal{V} \equiv \mathcal{V}^A T^A = \frac{1}{\sqrt{2}} \begin{pmatrix} G - \frac{2}{\sqrt{30}}B & X^{\dagger 1} & Y^{\dagger 1} \\ X_1 & X^{\dagger 2} & Y^{\dagger 2} \\ X_2 & X^{\dagger 3} & Y^{\dagger 3} \\ Y_1 & Y_2 & Y_3 \\ \frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B & W^+ & -\frac{1}{\sqrt{2}}W^3 + \frac{3}{\sqrt{30}}B \end{pmatrix}, \quad (34)$$

where T^A is the generator of the fundamental representation of the SU(5), and G , B , and W denote the MSSM gauge vector superfields with there associated generators. The massive gauge bosons associated with the breaking of SU(5) typically referred to as X_a and Y_a will be called just the X -bosons with definition

$$(X)_a^\alpha = \begin{pmatrix} X_a^1 \\ X_a^2 \end{pmatrix} \equiv \begin{pmatrix} X_a \\ Y_a \end{pmatrix}. \quad (35)$$

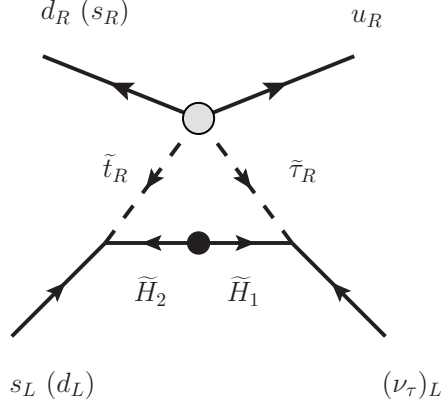


Figure 7: *One-loop Higgsino-exchanging diagram which gives rise to the dominant contribution to the baryon-number violating four-Fermi operators. Gray dot indicates the dimension-five effective interaction, while black dot represents the Higgsino mass term.*

Here α, β, \dots denote the $SU(2)_L$ indices.

The simplest means of breaking $SU(5)$ to the SM gauge symmetries $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is via an adjoint **24** discussed in the text. The **24** decomposes as follows

$$\Sigma \equiv \Sigma^A T^A \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^*,2)} & \Sigma_3 \end{pmatrix} + \frac{1}{2\sqrt{15}} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \Sigma_{24} . \quad (36)$$

Without losing any generality and for simplicity, we assume all $SU(5)$ breaking occurs along the Σ_{24} direction which is separated in the above equation.

B Proton Decay

In this appendix, we give additional details of our calculation of the proton lifetime. The important Wilson coefficients arising from integrating out the colored Higgs triplet are

$$\begin{aligned} C_{5R}^{3311}(Q_G) &= \frac{1}{M_{H_C}} f_t f_d(Q_G) e^{-i\varphi_1} V_{tb} V_{ud}^* , \\ C_{5R}^{3311}(Q_G) &= \frac{1}{M_{H_C}} f_t f_s(Q_G) e^{-i\varphi_1} V_{tb} V_{us}^* . \end{aligned} \quad (37)$$

These coefficients are then evolved down to the SUSY scale using

$$\frac{d}{d \ln Q} C_{5R}^{331l} = \frac{1}{16\pi^2} \left[-\frac{12}{5} g_1^2 - 8g_3^2 + 2f_t^2 + 2f_\tau^2 \right] C_{5R}^{331l} , \quad (38)$$

where $l = 1, 2$ and Q is the renormalization scale.

At the SUSY scale Q_S , the sfermions are integrated out via the diagram in Fig. 7 to give

$$\mathcal{L}_6^{\text{eff}} = C_i \epsilon_{abc} (u_{R1}^a d_{Ri}^b) (Q_{L3}^c \cdot L_{L3}) , \quad (39)$$

with

$$C_i(Q_S) = \frac{f_t f_\tau}{(4\pi)^2} C_{5R}^{*331i}(Q_S) F(\mu, m_{\tilde{t}_R}^2, m_{\tau_R}^2) , \quad (40)$$

where $i = 1, 2$ and

$$F(M, m_1^2, m_2^2) \equiv \frac{M}{m_1^2 - m_2^2} \left[\frac{m_1^2}{m_1^2 - M^2} \ln \left(\frac{m_1^2}{M^2} \right) - \frac{m_2^2}{m_2^2 - M^2} \ln \left(\frac{m_2^2}{M^2} \right) \right] . \quad (41)$$

These Wilson coefficients C_i , which are initially defined at the SUSY scale, are then run down from the weak scale using [44]

$$\frac{d}{d \ln Q} C_i = \left[\frac{\alpha_1}{4\pi} \left(-\frac{11}{10} \right) + \frac{\alpha_2}{4\pi} \left(-\frac{9}{2} \right) + \frac{\alpha_3}{4\pi} (-4) \right] C_i . \quad (42)$$

At the weak scale the Lagrangian takes the form

$$\mathcal{L}(p \rightarrow K^+ \bar{\nu}_\tau) = C_{usd} [\epsilon_{abc} (u_R^a s_R^b) (d_L^c \nu_\tau)] + C_{uds} [\epsilon_{abc} (u_R^a d_R^b) (s_L^c \nu_\tau)] , \quad (43)$$

with

$$\begin{aligned} C_{usd} &= -V_{td} C_2(m_Z) , \\ C_{uds} &= -V_{ts} C_1(m_Z) . \end{aligned} \quad (44)$$

The new Wilson coefficients $C_{usd,uds}$ are then further run down to the hadronic scale $Q_{\text{had}} = 2 \text{ GeV}$. Below the electroweak scale, the RGEs of the Wilson coefficients are given by

$$\frac{d}{d \ln Q} C_{usd,uds} = - \left[4 \frac{\alpha_s}{4\pi} + \left(\frac{4}{3} + \frac{4}{9} N_f \right) \frac{\alpha_s^2}{(4\pi)^2} \right] C_{usd,uds} , \quad (45)$$

at the two-loop level [61]. The solution for this equation is

$$A_L \equiv \frac{C(Q_{\text{had}})}{C(m_Z)} = \left[\frac{\alpha_s(Q_{\text{had}})}{\alpha_s(m_b)} \right]^{\frac{6}{25}} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_Z)} \right]^{\frac{6}{23}} \left[\frac{\alpha_s(Q_{\text{had}}) + \frac{50\pi}{77}}{\alpha_s(m_b) + \frac{50\pi}{77}} \right]^{-\frac{173}{825}} \left[\frac{\alpha_s(m_b) + \frac{23\pi}{29}}{\alpha_s(m_Z) + \frac{23\pi}{29}} \right]^{-\frac{430}{2001}} , \quad (46)$$

This long-range renormalization factor is computed to be $A_L = 1.247$ and appears as a multiplicative factor to the Wilson coefficients defined at the weak scale. The Wilson coefficients at the hadronic scale are then

$$C_{usd,uds}(Q_{\text{had}}) = C_{usd,uds}(m_Z) A_L . \quad (47)$$

The partial decay width for $p \rightarrow K^+ \bar{\nu}$ is then found to be

$$\Gamma(p \rightarrow K^+ \bar{\nu}) = \frac{m_p}{32\pi} \left(1 - \frac{m_K^2}{m_p^2} \right)^2 |\mathcal{A}(p \rightarrow K^+ \bar{\nu})|^2 , \quad (48)$$

where m_p and m_K are the proton and kaon masses, respectively, and

$$\mathcal{A}(p \rightarrow K^+ \bar{\nu}) = C_{usd}(Q_{\text{had}}) \langle K^+ | (us)_R d_L | p \rangle + C_{uds}(Q_{\text{had}}) \langle K^+ | (ud)_R s_L | p \rangle . \quad (49)$$

References

- [1] G. Aad *et al.* [ATLAS Collaboration], arXiv:1405.7875 [hep-ex]. S. Chatrchyan *et al.* [CMS Collaboration], JHEP **1406** (2014) 055 [arXiv:1402.4770 [hep-ex]].
- [2] M. Drees and M. M. Nojiri, Phys. Rev. D **47** (1993) 376 [arXiv:hep-ph/9207234]; G. L. Kane, C. F. Kolda, L. Roszkowski and J. D. Wells, Phys. Rev. D **49** (1994) 6173 [arXiv:hep-ph/9312272]; H. Baer and M. Brhlik, Phys. Rev. D **53** (1996) 597 [arXiv:hep-ph/9508321]; Phys. Rev. D **57** (1998) 567 [arXiv:hep-ph/9706509]; J. R. Ellis, T. Falk, K. A. Olive and M. Schmitt, Phys. Lett. B **388** (1996) 97 [arXiv:hep-ph/9607292]; Phys. Lett. B **413** (1997) 355 [arXiv:hep-ph/9705444]; J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Schmitt, Phys. Rev. D **58** (1998) 095002 [arXiv:hep-ph/9801445]; V. D. Barger and C. Kao, Phys. Rev. D **57** (1998) 3131 [arXiv:hep-ph/9704403]; J. R. Ellis, T. Falk, G. Ganis and K. A. Olive, Phys. Rev. D **62** (2000) 075010 [arXiv:hep-ph/0004169]; H. Baer, M. Brhlik, M. A. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, Phys. Rev. D **63** (2001) 015007 [arXiv:hep-ph/0005027]; J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, Phys. Lett. B **510** (2001) 236 [arXiv:hep-ph/0102098]; V. D. Barger and C. Kao, Phys. Lett. B **518** (2001) 117 [arXiv:hep-ph/0106189]; L. Roszkowski, R. Ruiz de Austri and T. Nihei, JHEP **0108** (2001) 024 [arXiv:hep-ph/0106334]; A. Djouadi, M. Drees and J. L. Kneur, JHEP **0108** (2001) 055 [arXiv:hep-ph/0107316]; U. Chattopadhyay, A. Corsetti and P. Nath, Phys. Rev. D **66** (2002) 035003 [arXiv:hep-ph/0201001]; J. R. Ellis, K. A. Olive and Y. Santoso, New Jour. Phys. **4** (2002) 32 [arXiv:hep-ph/0202110]; H. Baer, C. Balazs, A. Belyaev, J. K. Mizukoshi, X. Tata and Y. Wang, JHEP **0207** (2002) 050 [arXiv:hep-ph/0205325]; R. Arnowitt and B. Dutta, arXiv:hep-ph/0211417.
- [3] S. Weinberg, Phys. Rev. Lett. **48**, 1303 (1982); J. Ellis, D. V. Nanopoulos, and M. Quiros, Phys. Lett. B **174**, 176 (1986); T. Moroi, M. Yamaguchi and T. Yanagida Phys. Lett. B **342**, 105 (1995) [hep-ph/9409367]; M. Kawasaki, K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D **78**, 065011 (2008) [arXiv:0804.3745 [hep-ph]]. R. H. Cyburt, J. Ellis, B. D. Fields, F. Luo, K. A. Olive and V. C. Spanos, JCAP **0910**, 021 (2009) [arXiv:0907.5003 [astro-ph.CO]]; R. H. Cyburt, J. Ellis, B. D. Fields, F. Luo, K. A. Olive and V. C. Spanos, JCAP **1305**, 014 (2013) [arXiv:1303.0574 [astro-ph.CO]].
- [4] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477**, 321 (1996) [hep-ph/9604387]; T. Moroi and M. Nagai, Phys. Lett. B **723**, 107 (2013) [arXiv:1303.0668 [hep-ph]]; D. McKeen, M. Pospelov and A. Ritz, Phys. Rev. D **87**, 113002 (2013) [arXiv:1303.1172 [hep-ph]]; W. Altmannshofer, R. Harnik and J. Zupan, JHEP **1311**, 202 (2013) [arXiv:1308.3653 [hep-ph]]; K. Fuyuto, J. Hisano, N. Nagata and K. Tsumura, JHEP **1312**, 010 (2013) [arXiv:1308.6493 [hep-ph]]; M. Baumgart, D. Stolarski and T. Zorawski, Phys. Rev. D **90**, 055001 (2014) [arXiv:1403.6118 [hep-ph]].

- [5] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].
- [6] J. D. Wells, hep-ph/0306127; N. Arkani-Hamed and S. Dimopoulos, JHEP **0506**, 073 (2005) [arXiv:hep-th/0405159]; G. F. Giudice and A. Romanino, Nucl. Phys. B **699**, 65 (2004) [Erratum-ibid. B **706**, 65 (2005)] [arXiv:hep-ph/0406088]; N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, Nucl. Phys. B **709**, 3 (2005) [arXiv:hep-ph/0409232]; J. D. Wells, Phys. Rev. D **71**, 015013 (2005) [arXiv:hep-ph/0411041].
- [7] M. Ibe, T. Moroi and T. T. Yanagida, Phys. Lett. B **644**, 355 (2007) [hep-ph/0610277]; M. Ibe and T. T. Yanagida, Phys. Lett. B **709**, 374 (2012) [arXiv:1112.2462 [hep-ph]]; M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D **85**, 095011 (2012) [arXiv:1202.2253 [hep-ph]].
- [8] L. J. Hall and Y. Nomura, JHEP **1201**, 082 (2012) [arXiv:1111.4519 [hep-ph]]; N. Arkani-Hamed, A. Gupta, D. E. Kaplan, N. Weiner and T. Zorawski, arXiv:1212.6971 [hep-ph]; A. Arvanitaki, N. Craig, S. Dimopoulos and G. Villadoro, JHEP **1302**, 126 (2013) [arXiv:1210.0555 [hep-ph]]; L. J. Hall, Y. Nomura and S. Shirai, JHEP **1301**, 036 (2013) [arXiv:1210.2395 [hep-ph]].
- [9] J. L. Evans, M. Ibe, K. A. Olive and T. T. Yanagida, Eur. Phys. J. C **73**, 2468 (2013) [arXiv:1302.5346 [hep-ph]].
- [10] M. Dine and D. MacIntire, Phys. Rev. D **46**, 2594 (1992) [hep-ph/9205227]; L. Randall and R. Sundrum, Nucl. Phys. B **557**, 79 (1999) [arXiv:hep-th/9810155]; G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998) [arXiv:hep-ph/9810442]; J. A. Bagger, T. Moroi and E. Poppitz, JHEP **0004**, 009 (2000) [arXiv:hep-th/9911029]; P. Binetruiy, M. K. Gaillard and B. D. Nelson, Nucl. Phys. B **604**, 32 (2001) [arXiv:hep-ph/0011081].
- [11] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988); K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. D **45**, 328 (1992); E. Dudas, Y. Mambrini, A. Mustafayev and K. A. Olive, Eur. Phys. J. C **72**, 2138 (2012) [arXiv:1205.5988 [hep-ph]].
- [12] J. L. Evans, K. A. Olive, M. Ibe and T. T. Yanagida, Eur. Phys. J. C **73**, 2611 (2013) [arXiv:1305.7461 [hep-ph]].
- [13] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
- [14] H. Georgi, H. R. Quinn and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
- [15] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); W. J. Marciano and G. Senjanovic, Phys. Rev. D **25**, 3092 (1982); M. B. Einhorn and

- D. R. T. Jones, Nucl. Phys. B **196**, 475 (1982); U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B **260**, 447 (1991); P. Langacker and M. x. Luo, Phys. Rev. D **44**, 817 (1991).
- [16] M. Cirelli, F. Sala and M. Taoso, JHEP **1410**, 033 (2014) [Erratum-ibid. **1501**, 041 (2015)] [arXiv:1407.7058 [hep-ph]].
 - [17] J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, Phys. Lett. B **646**, 34 (2007) [hep-ph/0610249].
 - [18] B. Bhattacharjee, M. Ibe, K. Ichikawa, S. Matsumoto and K. Nishiyama, JHEP **1407**, 080 (2014) [arXiv:1405.4914 [hep-ph]].
 - [19] T. Cohen, M. Lisanti, A. Pierce and T. R. Slatyer, JCAP **1310**, 061 (2013) [arXiv:1307.4082]; J. Fan and M. Reece, JHEP **1310**, 124 (2013) [arXiv:1307.4400 [hep-ph]]; M. Baumgart, I. Z. Rothstein and V. Vaidya, arXiv:1412.8698 [hep-ph].
 - [20] J. Hisano, K. Ishiwata and N. Nagata, Phys. Lett. B **690**, 311 (2010) [arXiv:1004.4090 [hep-ph]]; J. Hisano, K. Ishiwata and N. Nagata, Phys. Rev. D **82**, 115007 (2010) [arXiv:1007.2601 [hep-ph]]; J. Hisano, K. Ishiwata, N. Nagata and T. Takesako, JHEP **1107**, 005 (2011) [arXiv:1104.0228 [hep-ph]].
 - [21] N. Nagata and S. Shirai, JHEP **1501**, 029 (2015) [arXiv:1410.4549 [hep-ph]].
 - [22] H. Baer, V. Barger and D. Mickelson, Phys. Lett. B **726**, 330 (2013) [arXiv:1303.3816 [hep-ph]].
 - [23] J. Hisano, K. Ishiwata and N. Nagata, Phys. Rev. D **87**, 035020 (2013) [arXiv:1210.5985 [hep-ph]].
 - [24] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. **69**, 1014 (1992).
 - [25] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B **402**, 46 (1993) [hep-ph/9207279].
 - [26] J. Hisano, T. Kuwahara and N. Nagata, Phys. Lett. B **723**, 324 (2013) [arXiv:1304.0343 [hep-ph]].
 - [27] N. Sakai and T. Yanagida, Nucl. Phys. B **197**, 533 (1982); S. Weinberg, Phys. Rev. D **26**, 287 (1982).
 - [28] V. Lucas and S. Raby, Phys. Rev. D **55**, 6986 (1997) [hep-ph/9610293]; K. S. Babu and M. J. Strassler, hep-ph/9808447; T. Goto and T. Nihei, Phys. Rev. D **59**, 115009 (1999) [hep-ph/9808255].
 - [29] S. Dimopoulos and H. Georgi, Nucl. Phys. B **193**, 150 (1981).
 - [30] N. Sakai, Z. Phys. C **11**, 153 (1981).

- [31] Y. Nomura and S. Shirai, Phys. Rev. Lett. **113**, 111801 (2014) [arXiv:1407.3785 [hep-ph]].
- [32] T. T. Yanagida, in private communication.
- [33] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Phys. Lett. B **88**, 320 (1979).
- [34] W. Siegel, Phys. Lett. B **84**, 193 (1979).
- [35] S. Weinberg, Phys. Lett. B **91**, 51 (1980).
- [36] L. J. Hall, Nucl. Phys. B **178**, 75 (1981).
- [37] P. Nath, Phys. Rev. Lett. **76**, 2218 (1996) [hep-ph/9512415].
- [38] P. Nath, Phys. Lett. B **381**, 147 (1996) [hep-ph/9602337].
- [39] B. Bajc, P. Fileviez Perez and G. Senjanovic, hep-ph/0210374.
- [40] N. Nagata and S. Shirai, JHEP **1403**, 049 (2014) [arXiv:1312.7854 [hep-ph]].
- [41] J. Hisano, D. Kobayashi, T. Kuwahara and N. Nagata, JHEP **1307**, 038 (2013) [arXiv:1304.3651 [hep-ph]].
- [42] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
- [43] F. Wilczek and A. Zee, Phys. Rev. Lett. **43**, 1571 (1979).
- [44] L. F. Abbott and M. B. Wise, Phys. Rev. D **22**, 2208 (1980).
- [45] M. Liu and P. Nath, Phys. Rev. D **87**, 095012 (2013) [arXiv:1303.7472 [hep-ph]].
- [46] K. Abe *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D **90**, 072005 (2014) [arXiv:1408.1195 [hep-ex]].
- [47] H. Murayama and A. Pierce, Phys. Rev. D **65**, 055009 (2002) [hep-ph/0108104].
- [48] Y. Aoki, E. Shintani and A. Soni, Phys. Rev. D **89**, 014505 (2014) [arXiv:1304.7424 [hep-lat]].
- [49] M. Dine, P. Draper and W. Shepherd, JHEP **1402**, 027 (2014) [arXiv:1308.0274 [hep-ph]].
- [50] M. Shiozawa, talk presented at TAUP 2013, September 8–13, Asilomar, CA, USA.
- [51] K. S. Babu, E. Kearns, U. Al-Binni, S. Banerjee, D. V. Baxter, Z. Berezhiani, M. Bergevin and S. Bhattacharya *et al.*, arXiv:1311.5285 [hep-ph].
- [52] J. L. Evans, M. A. G. Garcia and K. A. Olive, JCAP **1403**, 022 (2014) [arXiv:1311.0052 [hep-ph]].

- [53] J. L. Evans, M. Ibe, K. A. Olive and T. T. Yanagida, *Eur. Phys. J. C* **74**, no. 2, 2775 (2014) [arXiv:1312.1984 [hep-ph]].
- [54] E. Dudas, C. Papineau and S. Pokorski, *JHEP* **0702**, 028 (2007) [hep-th/0610297]. H. Abe, T. Higaki, T. Kobayashi and Y. Omura, *Phys. Rev. D* **75**, 025019 (2007) [hep-th/0611024]; E. Dudas, A. Linde, Y. Mambrini, A. Mustafayev and K. A. Olive, *Eur. Phys. J. C* **73**, 2268 (2013) [arXiv:1209.0499 [hep-ph]].
- [55] H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, *Phys. Rev. D* **71**, 095008 (2005) [arXiv:hep-ph/0412059]; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, *JHEP* **0507** (2005) 065, hep-ph/0504001; J. R. Ellis, K. A. Olive and P. Sandick, *Phys. Rev. D* **78**, 075012 (2008) [arXiv:0805.2343 [hep-ph]].
- [56] G. Aad *et al.* [ATLAS Collaboration], *Phys. Rev. D* **88**, 112006 (2013) [arXiv:1310.3675 [hep-ex]].
- [57] J. Ellis, K. Olive and Y. Santoso, *Phys. Lett. B* **539**, 107 (2002) [arXiv:hep-ph/0204192]; J. R. Ellis, T. Falk, K. A. Olive and Y. Santoso, *Nucl. Phys. B* **652**, 259 (2003) [arXiv:hep-ph/0210205].
- [58] J. L. Evans, M. Ibe, K. A. Olive and T. T. Yanagida, arXiv:1412.3403 [hep-ph].
- [59] K. Harigaya, M. Ibe and T. T. Yanagida, *JHEP* **1312**, 016 (2013) [arXiv:1310.0643 [hep-ph]]; K. Harigaya, K. Kaneta and S. Matsumoto, *Phys. Rev. D* **89**, 115021 (2014) [arXiv:1403.0715 [hep-ph]]; J. L. Evans and K. A. Olive, *Phys. Rev. D* **90**, 115020 (2014) [arXiv:1408.5102 [hep-ph]].
- [60] J. L. Evans, M. Ibe, K. A. Olive and T. T. Yanagida, *Eur. Phys. J. C* **74**, 2931 (2014) [arXiv:1402.5989 [hep-ph]].
- [61] T. Nihei and J. Arafune, *Prog. Theor. Phys.* **93**, 665 (1995) [hep-ph/9412325].