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Making Supersymmetric Connected $\mathcal{N}=(0,2)$ Sigma Models

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Abstract

We construct "connected" (0,2) sigma models starting from n copies of (2,2) $\mathrm{CP}(N-1)$ models. General aspects of models of this type (known as $T+O$ deformations) had been previously studied in the context of heterotic string theories. Our construction presents a natural generalization of the non-minimally deformed (2,2) model with an extra (0,2) fermion superfield on tangent bundle $\mathrm{T}\big[\mathrm{CP}(N-1)\times\mathrm{C}^1\big]$. We had thoroughly analyzed the latter model previously, found the exact β function and a spontaneous breaking of supersymmetry. In contrast, in certain connected sigma models the spontaneous breaking of supersymmetry disappears. We study the connected sigma models in the large-N limit finding supersymmetric vacua and determining the particle spectrum. While the Witten index vanishes in all the models under consideration, in these special cases of connected models one can use a permutation symmetry to define a modification of the Witten index which does not vanish. This eliminates the spontaneous breaking of supersymmetry. We then examine the exact β functions of our connected (0,2) sigma models.

1 Introduction

Quiver gauge theories in four dimensions are useful in various applications. Most common in four dimensions are Yang-Mills theories of the type

$$\mathrm{SU}(N_1)\times\mathrm{SU}(N_2)\times\mathrm{SU}(N_3)\times \ldots$$

(the "nodes") with each factor group being cyclically connected to its neighbors by a set of bifundamental fermion fields transforming in the fundamental representation of a given SU(N) theory and in the antifundamental representation of its neighbor. These fermion fields can be represented in the quiver graph as arrows (see Fig. 1).

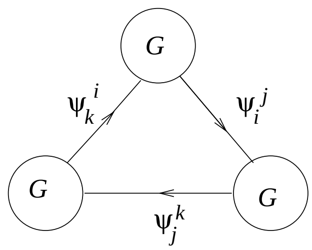

Figure 1: SU(N)×SU(N)×SU(N) Yang-Mills theory with bifundamental fermions.

Two-dimensional asymptotically free sigma models are long known to be excellent laboratories for modeling four-dimensional Yang-Mills theories.[1] The question we ask is whether one can construct an analog of quiver Yang-Mills in the context of two-dimensional sigma models. Moreover, we require a part of supersymmetry to be preserved in this construction.

In answering the above question we can use, for guidance, previous work carried out in the context of heterotic string theories in which models known as deformations $T+O$ with O being the trivial bundle where discussed [2,3]. In many instances models obtained are superconformal in the infrared (see e.g. [3]). Since we are interested in analogies with four-dimensional super-Yang Mills we would like to construct models with massive particle spectrum. To this end we turn our attention to a particular case of the $T+O$ deformations starting from a $\mathcal{N} = (2,2)$ theory associated to a product of n two-dimensional CP($N-1$) models. Dynamical connection between them is realized through one or several right-moving fermions from trivial tangent bundles. Somewhat related constructions were discussed in recent publications [4,5].

[1] It was forty years ago that A. Polyakov emphasized [1] that asymptotically free two-dimensional sigma models could present the best laboratory for the four-dimensional Yang-Mills theories. His prophecy came true.

Adding fermions in the bosonic sigma models (generally speaking, in a nonsupersymmetric manner) it is not difficult to "connect" them. For, instance, one of the options is to couple two CP($N-1$) models, both nonsupersymmetric, as follows:

$$\mathcal{L} = G\Big[\partial^\mu\phi^\dagger\,\partial_\mu\phi + i\bar\psi\nabla\!\!\!\!/\,\psi\Big] + \widetilde{G}\Big[\partial^\mu\widetilde{\phi}^\dagger\,\partial_\mu\widetilde{\phi} + i\bar{\widetilde{\psi}}\nabla\!\!\!\!/\,\widetilde{\psi}\Big] + \beta\Big(R\,\psi_L^\dagger\psi_R\Big)\Big(\widetilde{R}\widetilde{\psi}_R^\dagger\widetilde{\psi}_L\Big)\,, \qquad (1)$$

where the fields of the first CP($N-1$) are untilded, those of the second CP($N-1$) are tilded, β is a coupling constant, G and R stand for the CP($N-1$) metric and Ricci tensor, respectively. Moreover, ∇_μ is the target-space covariant derivative. The fermion fields ψ_L and $\widetilde{\psi}_R$ are chiral, left- and right-movers.

As was mentioned, we would like to find connected models with dynamical mass generation and a part of supersymmetry preserved. Nonminimal $\mathcal{N}=(0,2)$ models seem to be an ideal starting point. A nonminimal model to serve as our starting point appeared as a low-energy theory on the world sheet of a non-Abelian BPS-saturated flux tube supported in an $\mathcal{N}=1$ four-dimensional Yang-Mills theory [6,7]. Our tool is the large-N expansion, generalizing a number of results which had been obtained in the past in nonsupersymmetric and (2,2) supersymmetric $CP(N-1)$ models.

One starts from the bulk four-dimensional theory with $\mathcal{N}=2$ supersymmetry which supports 1/2-BPS strings. Then the low-energy theory on its world sheet has four supercharges and, thus, possesses $\mathcal{N}=(2,2)$ supersymmetry. The target space of the corresponding sigma model is CP($N-1$) [8]. More exactly, the full target space is CP($N-1$)$\times$C^1 where C^1 appears due to shifts of the string in transversal spatial directions. The bosonic fields living on C^1 as well as their fermionic partners on the tangent bundle TC^1 are free fields and for this reason usually are omitted from consideration.

If one slightly deforms the bulk theory, breaking $\mathcal{N}=2$ down to $\mathcal{N}=1$, four supercharges in the bulk survive. For relatively small deformations BPS saturation remains valid and so does the the target space of the two-dimensional sigma model. Now, the world-sheet model must have two, not four supercharges. However, Zumino's theorem [9] implies that given a Kähler target space any supersymmetric nonchiral model is automatically uplifted to $\mathcal{N}=(2,2)$, i.e. four supercharges.

Edalati and Tong [6] noted that in fact the above deformation of the bulk theory gives rise to interaction for right-moving fermions living on TC^1; in the bosonic background they start to mix with the right-moving fermions on TCP($N-1$). Thus, the bosonic target space stays the same CP($N-1$) (modulo free fields on C^1) while $\mathcal{N}=(2,2)$ is broken into $\mathcal{N}=(0,2)$ on the fermion tangent bundle T$\big[$CP(N–1)$\times$C$^1\big]$. They also conjectured a certain $\mathcal{N}=(0,2)$ model on the string world sheet with the field content of $\mathcal{N}=(2,2)$ CP($N-1$) sigma model plus a (0,2) spinor multiplet defined on C^1. This nonminimal theory (in a slightly different form) was explicitly derived by Shifman and Yung [7] from the analysis of the vortex solution. They also found a geometric formulation of this model, as well as its large-N solution [10]. This solution exhibits spontaneous breaking of supersymmetry, as it often happens in other $\mathcal{N} = (0, 2)$ models discussed in the literature.

Heterotic two-dimensional models (known as $\mathcal{N} = (0, 2)$ supersymmetric sigma models) have two chiral supercharges, say, Q_L and $Q_L^\dagger$, with the defining anticommutator

$$\{Q_L\,,\, Q_L^\dagger\} = 2(H - P)\,. \tag{2}$$

They were studied from the mathematical perspective [11–16] as well as from the standpoint of physical applications (see [17] and extensive references therein).

The $(0, 2)$ connected model we will construct has the bosonic target space

$$\mathrm{CP}(N_1-1)\times\mathrm{CP}(N_2-1)\times\,...\,\times\mathrm{CP}(N_n-1)\,. \tag{3}$$

As for the fermion fields they will live on the tangent bundles of the type

$$\mathrm{T}\Big[\mathrm{CP}(N_1-1)\times\mathrm{CP}(N_2-1)\times\,...\,\times\mathrm{CP}(N_n-1)\times C^1...\Big]\,. \tag{4}$$

In the simplest version to be considered in Sec. 3 there is a single connecting fermion ζ_R defined on the trivial tangent bundle TC^1. All fields from $\mathrm{CP}(N_p-1)$ interact with those from $\mathrm{CP}(N_q-1)$ (for all q, $p = 1, 2, ..., n$) through the coupling to the $(0, 2)$ Fermi multiplet consisting of ζ_R and an auxiliary field. The graph representation describing the case of $\mathrm{T}\big[\mathrm{CP}(N-1)\times\mathrm{CP}(N-1)\times\mathrm{CP}(N-1)\times C^1\big]$ is given in Fig. 2.

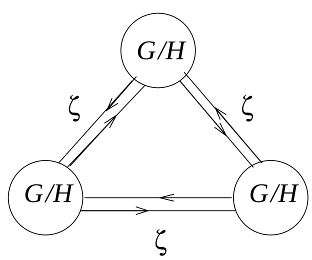

Figure 2: $\mathrm{T}\big[\mathrm{CP}(N-1)\times\,\mathrm{CP}(N-1)\times\,\mathrm{CP}(N-1)\times C^1\big]$ two-dimensional sigma model with one (0,2) fermion superfield. G and H are defined after Eq. (12).

In Sec. 4 we construct a (0,2) Lagrangian describing a connected sigma model with a cyclic graph of the type given in Fig. 1 with n nodes and n arrows $(\zeta_R)_{j,j+1}$.

Each arrow corresponds to its own (0,2) fermion superfield (Fig. 3), so that the target space structure is as follows:

$$\mathrm{T}\Big[\mathrm{CP}(N_1{-}1)\times C^1\times\mathrm{CP}(N_2{-}1)\times C^1\times\mathrm{CP}(N_3{-}1)\times C^1 ...\Big]\,. \qquad (5)$$

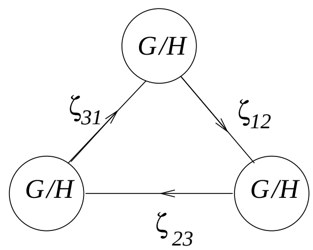

Figure 3: Graphic representation for the two-dimensional sigma model with the target space (5).

All models we consider have $\mathcal{N}=(0,2)$ supersymmetry at the Lagrangian level. In the leading order in $1/N$ we select the models (choosing $N_1,\ N_2,\ldots$ appropriately) where there is *no* spontaneous supersymmetry breaking. Once it happens in the leading $1/N$ order one can argue that then restoration of supersymmetry is an exact statement.

An important characteristic of the model associated with spontaneous breaking of supersymmetry is the Witten index of the model [18]. Spontaneous breaking can occur only when this index vanishes. In Sec. 5 we show that Witten's index vanishes for all the models we consider. However, one can introduce a modification of Witten index (a particular case of algebraic genera), following the same line of reasoning as in Sec. 6 of [19]. In massless $\mathcal{N}=1$ QED in four dimension $\mathrm{Tr}\,(-1)^F=0$, however $\mathrm{Tr}\,C(-1)^F=4$, where C stands for the C parity. In our case we can use flavor permutations to refine a modified index.

The paper is organized as follows. In Sec. 2 we briefly review the variant of the nonminimal heterotic modification of $\mathcal{N}=(2,2)$ theory to be used in our study. In Secs. 3 and 4 we fully specify the connected models with the target spaces (4) and (5), solve them at large N and determine the mass spectrum. Section 5 is devoted to the Witten index and its refinement. In Sec. 6 we obtain the exact β functions for both models. Finally, Sec. 7 summarizes our conclusions.

2 The nonminimal heterotic modification of $\mathcal{N}=(2,2)$ theories: generalities

The Lagrangian of generic $\mathcal{N}=(2,2)$ sigma model without torsion has the form

$$\mathcal{L} = \int d^4\theta \, K(\Phi, \Phi^\dagger)\,, \tag{6}$$

where the Kähler potential K depends on the chiral superfields Φ^i and antichiral $\Phi^{\dagger \bar{j}}$. Having in mind a manifold M which is a direct product of n manifolds M_F, $(F = 1, \ldots, n)$,

$$M = M_1 \times M_2 \times \cdots \times M_n\,, \tag{7}$$

we can rewrite $\mathcal{L}$ as a sum over manifolds,

$$\mathcal{L} = \sum_{F=1}^{n} \int d^4\theta \, K_F(\Phi_F, \Phi_F^\dagger)\,. \tag{8}$$

In terms of the (0,2) superfields the field Φ^i decomposes as

$$\begin{aligned} &\Phi^i(x_R + 2i\theta_R^\dagger\theta_R, x_L - 2i\theta_L^\dagger\theta_L, \theta_R, \theta_L) \\ &= A^i(x_R + 2i\theta_R^\dagger\theta_R, x_L - 2i\theta_L^\dagger\theta_L, \theta_R) + \sqrt{2}\,\theta_L B^i(x_R + 2i\theta_R^\dagger\theta_R, x_L, \theta_R), \end{aligned} \tag{9}$$

where $x_{R,L} = x^0 \pm x^1$ and the field A^i represents the chiral supermultiplets which on the mass shell consists of the scalar field and left-moving fermion,

$$A^i = \phi^i(x_R + 2i\theta^\dagger\theta,\, x_L) + \sqrt{2}\,\theta\, \psi_L^i(x_R + 2i\theta^\dagger\theta,\, x_L)\,, \tag{10}$$

while the field B^i describes the Fermi supermultiplet which on mass shell contains only a right-moving fermion (F^i is an auxiliary field),

$$B^i = \psi_R^i(x_R + 2i\theta^\dagger\theta,\, x_L) + \sqrt{2}\,\theta F_\psi^i(x_R + 2i\theta^\dagger\theta,\, x_L)\,. \tag{11}$$

Note a change in notation: in Eqs. (10) and (11) θ_R is substituted by θ because we do not use θ_L in the (0,2) superspace.

The (0,2) heterotic modification is due to adding one extra Fermi supermultiplet which we denote as $\mathcal{B}$,

$$\mathcal{B} = \zeta_R(x_R + 2i\theta^\dagger\theta,\, x_L) + \sqrt{2}\,\theta\, \mathcal{F}_\zeta(x_R + 2i\theta^\dagger\theta,\, x_L)\,, \tag{12}$$

which interacts with the fields A_F, B_F defined on each manifold M_F. Introduction of this right-moving fermion does not change geometry of the original bosonic manifold. Indeed, the manifold M_F we deal with is a symmetric space associated with G_F/H_F (in case of $\mathrm{CP}(N-1)$ we have $G = \mathrm{SU}(N)$ and $H = \mathrm{S}\left(\mathrm{U}(N-1)\times \mathrm{U}(1)\right)$). The additional field $\mathcal{B}$ is a singlet of the isometry group G_F, in contrast to A^i_F, B^i_F. Therefore, its interaction does not modify the isometry group.

The Lagrangian of the model is

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{F=1}^{n}\Big\{-\frac{1}{2}\,\mathrm{Re}\int d\theta\, G^F_{i\bar{j}}(A_F, A^\dagger_F)(\bar{D}A^{\dagger\bar{j}}_F)\left(i\partial_R A^i_F - 2\kappa_F\,\mathcal{B}B^i_F\right) \\
& +\; \frac{1}{2}\int d^2\theta\, Z_F\, G^F_{i\bar{j}}(A_F, A^\dagger_F)\, B^{\dagger\bar{j}}_F B^i_F\Big\} + \frac{1}{2}\int d^2\theta\, \mathcal{Z}\,\mathcal{B}^\dagger\mathcal{B}\,,
\end{aligned}
\tag{13}
$$

where $G^F_{i\bar{j}} = \partial^2 K^F/\partial A^i\partial A^{\dagger\,\bar{j}}$ is the Kähler metric of M_F, $\partial_{L,R} = \partial_{x^0} \pm \partial_{x^1}$ and $D = \partial_\theta - i\bar{\theta}\partial_L$, $\bar{D} = -\partial_{\bar{\theta}} + i\theta\partial_L$.

Moreover, the parameters κ_F are the deformation parameters, and Z_F, $\mathcal{Z}$ are wave function renormalization factors for B^i_F, $\mathcal{B}$ fields. When all $\kappa_F = 0$ the field $\mathcal{B}$ becomes a sterile field, and the (2, 2) supersymmetry in the nontrivial sector is restored. (The Z_F factors do not run at $\kappa = 0$ and can be taken to be 1.)

In components

$$
\begin{aligned}
\mathcal{L} = \sum_{F=1}^{n}\Big\{ & G^F_{i\bar{j}}\left[\partial_L\phi^{\dagger\bar{j}}_F\partial_R\phi^i_F + \psi^{\dagger\bar{j}}_{FL}\,i\nabla_R\,\psi^i_{FL} + Z_F\,\psi^{\dagger\bar{j}}_{FR}\,i\nabla_L\psi^i_{FR}\right] \\
& + Z_F R^F_{i\bar{j}k\bar{l}}\,\psi^{\dagger\bar{j}}_{FL}\psi^i_{FL}\,\psi^{\dagger\bar{l}}_{FR}\psi^k_{FR} + \left[\kappa_F\,\zeta_R\,G^F_{i\bar{j}}\big(i\,\partial_L\phi^{\dagger\bar{j}}_F\big)\psi^i_{FR} + \mathrm{H.c.}\right] \\
& + \frac{|\kappa_F|^2}{Z_F}\zeta^\dagger_R\,\zeta_R\big(G^F_{i\bar{j}}\,\psi^{\dagger\bar{j}}_{FL}\psi^i_{FL}\big)\Big\} - \frac{1}{\mathcal{Z}}\left|\sum_{F=1}^{n}\kappa_F G^F_{i\bar{j}}\psi^{\dagger\bar{j}}_{FL}\psi^i_{FR}\right|^2 + \mathcal{Z}\,\zeta^\dagger_R\,i\partial_L\,\zeta_R\,.
\end{aligned}
\tag{14}
$$

Here $\nabla_{L,R}$ are covariant derivatives,

$$
\nabla_{L,R}\,\psi^i_{R,L} = \partial_{L,R}\,\psi^i_{R,L} + \Gamma^i_{kl}\,\partial_{L,R}\,\phi^k\,\psi^l_{R,L}\,. \tag{15}
$$

3 The simplest connection of n CP($N-1$) models

The general idea is to choose the manifold $M_F = \mathrm{CP}(N-1)$ for all $F = 1, \ldots, n$ and couple all sectors through the field(s) $\mathcal{B}$ (see Fig. 2). This coupling can be realized in various forms. The simplest one is a universal coupling of a single $\mathcal{B}$ field to all CP($N-1$) sectors with one and the same coupling constant $\kappa_F=\kappa,\ F=1,\ldots,n$. The right-moving fermions then live on the tangent bundle of the form

$$\mathrm{T}\Big[\{\mathrm{CP}(N-1)\}^n\times C^1\Big]\,. \tag{16}$$

Let us first discuss this version and then move on to consider more elaborate models with the same underlying idea and a number of different $\mathcal{B}$ superfields (Fig. 3).

The geometric formulation of the models is given then by Eqs. (13) and (14) where the metric $G^F_{i\bar j}$, the heterotic coupling κ_F, and the wave function factors Z_F are the same for each $M_F=\mathrm{CP}(N-1)$. The field indices $i,\bar j$ run from 1 to $N-1$ and the explicit expressions for CP($N-1$) metric and related objects are of the form,

$$\begin{aligned}
&K=\frac{2}{g^2}\,\log\chi\,, && \chi=1+\sum_m^{N-1}\phi^{\dagger\,m}\phi^m\,, \\
&G_{i\bar j}=\frac{2}{g^2}\left(\frac{\delta_{i\bar j}}{\chi}-\frac{\phi^{\dagger\,i}\phi^{\bar j}}{\chi^2}\right), && G^{i\bar j}=\frac{g^2}{2}\,\chi\left(\delta^{i\bar j}+\phi^i\phi^{\dagger\,\bar j}\right), \\
&\Gamma^i_{kl}=-\frac{\delta^i_k\,\phi^{\dagger\,l}+\delta^i_l\,\phi^{\dagger\,k}}{\chi}\,, && \Gamma^{\bar i}_{\bar k\bar l}=-\frac{\delta^{\bar i}_{\bar k}\,\phi^{\bar l}+\delta^{\bar i}_{\bar l}\,\phi^{\bar l}}{\chi}\,, \\
&R_{i\bar jk\bar l}=-\frac{g^2}{2}\Big(G_{i\bar j}G_{k\bar l}+G_{k\bar j}G_{i\bar l}\Big), && R_{i\bar j}=-G^{k\bar j}R_{i\bar jk\bar l}=\frac{g^2N}{2}\,G_{i\bar j}\,.
\end{aligned} \tag{17}$$

The analogs of the gauge couplings $1/g^2$ are hidden in the metric tensors $G_{i\bar j}$, see Eq. (17). These couplings can be complexified by including θ terms,

$$\frac{1}{g^2}\Longrightarrow\frac{1}{g^2}+i\,\frac{\theta}{4\pi}\,. \tag{18}$$

Later we will use such complexification to our benefit.

The symmetry of the model is $\big[\mathrm{SU}(N)\big]^n$. With our choice of all parameters, the model acquires an additional flavor Z_n symmetry corresponding to interchanging different-F fields.[2] More exactly, we define the flavor symmetry as follows. Assume that the real parts of $1/g^2$ are the same for all n $CP(N-1)$ factors, while the θ terms take the values $0, 2\pi, 4\pi, ...2\pi(n-1)$. Since for all $\theta=2\pi\times$integer physics is the same, the permutation symmetry will be valid with the appropriate choice of vacua.

[2] This Z_n, which has no continuous analog, is not to be confused with the axial Z_N for each flavor which is a remnant of the continuous classical R-symmetry

3.1 Gauged formulation of modified $\big[\mathrm{CP}(N-1)\big]^n \times C^1$

The gauged formulation is defined by the groups G and H entering into the G/H symmetric space under consideration. In the CP$(N-1)$ case $G = \mathrm{SU}(N)$ and the gauged formulation has the form [6],

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{F=1}^{n} \Big\{ \mathcal{D}_\mu n^\dagger_{Fi} \mathcal{D}_\mu n^i_F - \big(2|\sigma_F|^2 + D_F\big) n^\dagger_{Fi} n^i_F + \xi^\dagger_{FLj}\, i\mathcal{D}_R\, \xi^j_{FL} + Z\, \xi^\dagger_{FRj}\, i\mathcal{D}_L\, \xi^j_{FR} \\
& + \Big[\sqrt{2Z}\, i\, \sigma_F \xi^\dagger_{FRj} \xi^j_{FL} + \sqrt{2}\, i\, n^\dagger_{Fj} \lambda_{FR} \xi^j_{FL} + \sqrt{2Z}\, i\, n^\dagger_{Fj} \lambda_{FL} \xi^j_{FR} + \text{H.c.}\Big] \\
& + \frac{2}{g^2}\, D_F - \frac{\theta}{2\pi}\, \epsilon^{\mu\nu} \partial_\mu A_{F\nu} + \Big[\frac{2\kappa}{g^2} \sqrt{2\mathcal{Z}}\, i\, \zeta_R\, \lambda^\dagger_{FL} \; + \text{H.c.}\Big] \Big\} - 8\, \frac{|\kappa|^2}{g^4} \Big| \sum_{F=1}^{n} \sigma_F \Big|^2 \\
& + \mathcal{Z}\, \zeta^\dagger_R\, i\partial_L\, \zeta_R \;. \qquad\qquad (19)
\end{aligned}
$$

Here n^i_F $(i = 1, \ldots, N)$ is the complex scalar field in the fundamental representation of SU(N), ξ^i_{FL}, ξ^i_{FR} are its fermion superpartners in unbroken $\mathcal{N} = (2, 2)$. The covariant derivatives, defined as $\mathcal{D}_\mu n^i_F = (\partial_\mu - iA_{F\mu}) n^i_F$, contain auxiliary Abelian gauge fields $A_{F\mu}$. The gauge field (2,2)-superpartners D_F, σ_F, and $\lambda_{FL}, \lambda_{FR}$ are other auxiliary fields which implement the constraints

$$
n^\dagger_{Fi} n^i_F = \frac{2}{g^2}\,, \qquad n^\dagger_{Fj} \xi^j_{FL} = 0\,, \qquad \sqrt{Z} n^\dagger_{Fj} \xi^j_{FR} = 2\sqrt{\mathcal{Z}}\, \frac{\kappa^*}{g^2}\, \zeta^\dagger_R\,. \qquad (20)
$$

3.2 Large-N solution

It is easy to solve the theory (19) in the 't Hooft limit, using the method of [20, 21]. In fact, at $N \to \infty$ only one-loop diagrams survive, as explained in detail in [20, 21] (and in [10] in application to the heterotic model (19) under consideration).

The running of the wavefunction factors Z and $\mathcal{Z}$ shows up only in the $1/N$ corrections (see Sec. 6 and [26] for further details of running), so in the leading large-N approximation we put $Z = \mathcal{Z} = 1$. Note also that in each CP$(N-1)$ sector the auxiliary fields A_μ, D, σ and $\lambda_{L,R}$ form a supermultiplet of $\mathcal{N} = (2, 2)$. The heterotic modification decomposes it into two (0,2) multiplets: a vector one, containing A_R, λ_R, $\lambda^\dagger_R$, D, and a chiral multiplet with σ and $\lambda^\dagger_L$ fields.

To determine the vacuum structure it is sufficient to set $A_\mu = 0$ and $\lambda_{L,R} = 0$, and treat D and σ as constant background fields, the critical values of which determine the vacuum energy density. The Lagrangian (19) is quadratic in both, the n fields and their fermion superpartners ξ. Therefore, they can be integrated out exactly. This yields

$$\prod_{F=1}^{n} \frac{\text{Det}\Big(-\partial_\alpha^2 - 2|\sigma_F|^2\Big)^N}{\text{Det}\Big(-\partial_\alpha^2 - D_F - 2|\sigma_F|^2\Big)^N}\,. \tag{21}$$

The denominator comes from the boson loop while the numerator from the fermion loop. Although σ is generically complex its phase can always be absorbed in the θ term in Eq. (19) by U(1) rotation of fermion fields. The one-loop graph contributions in (21) are simply calculable,

$$V_{\rm one-loop} = \frac{N}{4\pi}\sum_{F=1}^{n}\left[(D_F + 2|\sigma_F|^2)\Big(\log\frac{M_{\rm uv}^2}{D_F + 2|\sigma_F|^2} + 1\Big) - 2|\sigma_F|^2\Big(\log\frac{M_{\rm uv}^2}{2|\sigma_F|^2} + 1\Big)\right], \tag{22}$$

where $M_{\rm uv}$ is the ultraviolet cut-off. Then, the effective potential takes the form

$$\begin{aligned} V_{\rm eff} &= V_{\rm one-loop} - \frac{2}{g^2}\sum_{F=1}^{n} D_F + 8\,\frac{|\kappa|^2|}{g^4}\,\Big|\sum_{F=1}^{n}\sigma_F\Big|^2 \\ &= \frac{N}{4\pi}\left\{\sum_{F=1}^{n}\left[D_F\Big(\log\frac{\Lambda^2}{D_F + 2|\sigma_F|^2} + 1\Big) + 2|\sigma_F|^2\log\frac{2|\sigma_F|^2}{D_F + 2|\sigma_F|^2}\right]\right. \\ &\quad \left. + 2u\,\Big|\sum_{F=1}^{n}\sigma_F\Big|^2\right\}. \end{aligned} \tag{23}$$

Here we introduced the scaling parameter Λ,

$$\Lambda = M_{\rm uv}e^{-4\pi/Ng^2} \tag{24}$$

and the heterotic deformation parameter u,

$$u = \frac{16\pi|\kappa|^2}{Ng^4}\,. \tag{25}$$

The auxiliary field D_F can be excluded by the condition $\partial V_{\rm eff}/\partial D_F = 0\,$,

$$D_F = \Lambda^2 - 2|\sigma_F|^2\,, \tag{26}$$

and the effective potential for the σ_F fields becomes

$$V_{\rm eff}(\sigma) = \frac{N}{4\pi}\left\{\sum_{F=1}^{n}\left[\Lambda^2 - 2|\sigma_F|^2 + 2|\sigma_F|^2\log\frac{2|\sigma_F|^2}{\Lambda^2}\right] + 2u\,\Big|\sum_{F=1}^{n}\sigma_F\Big|^2\right\}. \tag{27}$$

3.3 Vacuum structure

Let us start with undeformed case when the heterotic parameter $u = 0$. Then we have just n disconnected copies of the (2,2) CP$(N-1)$ sigma models. Each of these copies has N supersymmetric vacua [20, 21],

$$\langle D_F\rangle = 0, \quad \langle\sigma_F\rangle_{k_F} = \frac{\Lambda}{\sqrt{2}}\exp\left(\frac{2\pi\, i\,\ell_F}{N}\right), \quad \ell_F = 0, 1, ..., (N-1)\,. \tag{28}$$

The value of $|\sigma_F|$ follows from minimization of $V_{\rm eff}$ in Eq. (27), the value of D_F is then given by Eq. (26). How the phase factor of σ_F appeared in Eq. (28)? The phase of the vacuum value of σ can always be absorbed in the θ term which, in turn, can be hidden in the definition (24) of Λ (we chose $\theta=0$ for simplicity). Given physical 2π periodicity in θ one arrives at the expression for σ_F presented in (28). The multi-valuedness of the vacuum expectation value of σ is the same as that in the condensate $\langle\xi_L^\dagger\xi_R\rangle$.

The next step to consider is the case of one CP$(N-1)$, $n = 1$, with nonvanishing heterotic parameter u. This was done in detail in Ref. [10]. The critical values are

$$\langle\sigma\rangle_k = \frac{\Lambda}{\sqrt{2}}\exp\left(-\frac{u}{2} + \frac{2\pi\, i\,\ell}{N}\right), \quad \ell = 0, 1, ..., (N-1)\,,$$
$$\langle D\rangle = \Lambda^2\left(1-e^{-u}\right), \quad \langle V_{\rm eff}\rangle = \frac{N}{4\pi}\langle D\rangle = \frac{N}{4\pi}\Lambda^2\left(1-e^{-u}\right)\,. \tag{29}$$

The fact that the vacuum energy density $\langle V_{\rm eff}\rangle \neq 0$ for $u \neq 0$ indicates that (0,2) supersymmetry is spontaneously broken. Of course, this implies the emergence of a massless Goldstino, its determination can be found in [10].

Now, let us turn to the quiver-like theories with $n > 1$ and show that for $n > 1$ supersymmetric vacua appear!

Unbroken supersymmetry implies that in the vacuum $\langle D_F\rangle = 0$ for all F; then Eq. (26) fixes $|\langle\sigma_F\rangle| = \Lambda/\sqrt{2}$ also for all F. Thus, in the supersymmetric vacua $\langle\sigma_F\rangle$ are the same as in the undeformed case and given by Eq. (28). The vacuum energy density (27) at this value of $|\langle\sigma_F\rangle|$ is given by $u\,|\sum\sigma_F|^2$ and vanishes when

$$\sum_{F=1}^{n}\exp\left(\frac{2\pi\, i\,\ell_F}{N}\right) = 0\,. \tag{30}$$

Take, for example, $n = 2$ where the total number of "prevacua" is N^2. The condition (30) is satisfied if $|\ell_2 - \ell_1| = N/2$. Of course, this is possible only for even N. We see the occurrence of N supersymmetric vacua, $\ell_1 = 0, \ldots, (N-1)$. In the remaining

$N(N-1)$ would-be vacua supersymmetry is spontaneously broken. These vacua have nonvanishing energy and are cosmologically unstable. In Sec. 5 we argue that the existence of supersymmetric vacua in our models extends beyond the leading $1/N$ approximation. This is an exact statement.

Absence of the spontaneous supersymmetry breaking in the heterotic (0,2) theories is not a new phenomenon. The (0,2) theories with supersymmetric vacua were recently discussed in Refs. [4,5]. Classes of theories such as the (0,2) Landau-Ginzburg models, as well as (0,2) GLSM constructions of heterotic string vacua – with supersymmetric vacua and superconformal regime in the IR – had been also considered in the past. Their dynamics is quite different from that we observe in our models.

The resurgence of the supersymmetric vacua due to (30) can be understood from different angles. To this end let us start discussing the mass spectrum of the models.

3.4 Mass spectrum

3.4.1 Undeformed theory

Let us start, again, with the undeformed case when the heterotic parameter $u=0$. Then the right-moving fermion ζ_R represent a sterile massless field, and we have $\mathcal{N}=(2,2)$ supersymmetry in the each $M_F=\mathrm{CP}(N{-}1)$ sector. The supersymmetry is unbroken and the mass spectrum at large N is well known [20,21]. The fundamentals of SU(N) – i.e. the fields n^i, ξ^i – get masses

$$m_n = m_\xi = \sqrt{2}\,|\langle\sigma\rangle| = \Lambda\,, \tag{31}$$

as it is visible from Eq. (19). It means that strong interaction in the infrared produces extra states as compared to the original Lagrangian of the sigma model. This leads to the linear representation of SU(N) and nonvanishing masses.

Besides, the kinetic terms for the gauge A_μ field and its (2,2) superpartners σ and λ as well as the Yukawa $\sigma\lambda\lambda$ coupling are dynamically generated at one loop in much the same way as in [20, 21],

$$\begin{aligned}\mathcal{L}^{\rm kin}_{\rm one-loop} = \frac{N}{4\pi\rho^2}\Big[&-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\,\partial_\mu\rho\,\partial^\mu\rho + \lambda_L^\dagger i\partial_R\lambda_L + \lambda_R^\dagger i\partial_L\lambda_R \\ &+2i\rho\big(e^{i\alpha}\lambda_R^\dagger\lambda_L - e^{-i\alpha}\lambda_L^\dagger\lambda_R\big)\Big] + \frac{N}{4\pi}\Big[\frac{1}{2}\,\partial_\mu\alpha\,\partial^\mu\alpha + 2\alpha\epsilon^{\mu\nu}\partial_\mu A_\nu\Big]\,. \end{aligned} \tag{32}$$

Here we represent the complex field σ in terms of the real fields ρ and α,

$$\sigma = \frac{1}{\sqrt{2}}\,\rho\, e^{i\alpha}\,, \tag{33}$$

i.e., the modulus and the phase of σ.

The Yukawa $\sigma\lambda\lambda$ coupling generates mass for the λ field, $m_\lambda = 2\langle\rho\rangle = 2\Lambda$. To read off this mass one should substitute fields ρ and α in Eq. (32) by their vacuum values,

$$\langle\rho\rangle = \Lambda\,, \qquad \langle\alpha\rangle = \frac{2\pi}{N}\,\ell\,, \tag{34}$$

and introduce canonically normalized fields

$$\tilde{\lambda}_L = \sqrt{\frac{4\pi}{N}}\,\frac{e^{i\langle\alpha\rangle/2}}{\langle\rho\rangle}\,\lambda_L\,, \qquad \tilde{\lambda}_R = \sqrt{\frac{4\pi}{N}}\,\frac{e^{-i\langle\alpha\rangle/2}}{\langle\rho\rangle}\,\lambda_R\,. \tag{35}$$

The same mass $m_\rho = m_\lambda$ follows for the ρ field from expansion of $V_{\rm eff}$ in $(\rho-\langle\rho\rangle)$ (see Eq. (27)) at $u=0$.

To show that the gauge field has the same mass note that equations of motion relate the deviation $\alpha-\langle\alpha\rangle$ to the gauge field,

$$\alpha - \langle\alpha\rangle = \frac{1}{2\rho^2}\,\epsilon^{\mu\nu}\partial_\mu A_\nu\,, \tag{36}$$

and lead to

$$m_{\rm ph} = m_\rho = m_\lambda = 2\langle\rho\rangle = 2\,\Lambda\,. \tag{37}$$

A crucial feature of the model is that the photon field A_μ, in addition to the kinetic term, acquires a nonvanishing mass: the presence of massless fermion fields in the model shifts the pole in the photon propagator away from zero. Thus, the gauge (2,2) multiplet A_μ, ρ, λ becomes propagating with the mass $m_{\rm ph}$.

Consequences of massless vs. massive photon in two dimensions are radically different. Massless gauge field in two-dimensions (bosonic CP(N–1)) imply confinement of charged particles, while the massive one (supersymmetric CP(N–1)) does not confine [20, 21]. In one-to-one correspondence with this is the existence of N degenerate vacua in the nonconfining case. In the confining case (i.e. massless gauge field, as in the bosonic CP($N-1$)) one of these vacua remains genuine while the remaining $N-1$ are uplifted and become quasistable states [22].

3.4.2 Deformed CP($N-1$)

Now let us consider one CP($N-1$), $n=1$, with nonvanishing heterotic parameter u. In this case supersymmetry is broken, but all N vacua (see the second line in (29)) remain degenerate. This degeneracy reflects spontaneous breaking of a Z_N symmetry present in the model. Hence, the n^i particles should remain unconfined.

Correspondingly, the photon becomes massive much in the same way as in the (2,2) model.

The masses of the fundamental fields n and ξ are different,

$$m_n = \Lambda, \quad m_\xi = \Lambda e^{-u/2} \tag{38}$$

Other particle masses change with u, see [10] for a detailed derivation. At small u they are close to 2Λ, with the massless Goldstino which predominantly coincides with ζ_R with a small admixture of λ_R. At large u the Goldstino becomes predominantly λ_R while ζ_R together with λ_L constitute two massive states with a large mass

$$m_{\lambda_L,\zeta_R} = \Lambda\sqrt{u}\,. \tag{39}$$

The gauge and ρ fields become light with masses

$$m_{\rm ph} = \sqrt{6}\Lambda e^{-u/2}\,, \qquad m_\rho = 2\sqrt{3}\Lambda e^{-u/2}\,. \tag{40}$$

3.4.3 Mass spectrum at $u \neq 0$

Let us find the mass spectrum of the quiver-like theory in more detail. For simplicity we consider the case $n = 2$ and even N. Vacuum values of the fields in the supersymmetric vacua in this case are

$$\langle\rho_1\rangle = \langle\rho_2\rangle = \Lambda\,, \quad \langle\alpha_1\rangle = \frac{2\pi}{N}\,k\,, \quad \langle\alpha_2\rangle = \langle\alpha_1\rangle + \pi\,. \tag{41}$$

Masses of the bosonic and fermionic fields n_F^i and ξ_F^i determined by the above VEVs are

$$m_n^F = m_\xi^F = \langle\rho_F\rangle = \Lambda\,, \quad (F = 1, 2)\,. \tag{42}$$

Note, that they are the same for all "flavors" F, do not depend on the deformation parameter u, and bosons and fermions remain degenerate. Note also that the D-term constraints are effectively lifted. For example, we have $2nN$ real degrees of freedom in the bosonic sector rather then $2n\,(N-1)$ seen quasiclassically.

Let us consider now the effective Lagrangian for fields of two gauge multiplets and the right-moving fermion ζ_R. Combining Eqs. (27) and (32) as well as the fermionic part of the heterotic deformation from Eq. (19) we have

$$\begin{aligned}\mathcal{L}_{\rm eff} = \zeta_R^\dagger i\partial_L \zeta_R + \frac{N}{4\pi}\sum_{F=1,2}\Bigg\{\frac{1}{\rho_F^2}\Big[-\frac{1}{4}F_{F\mu\nu}F_F^{\mu\nu} + \frac{1}{2}\,\partial_\mu\rho_F\partial^\mu\rho_F + \lambda_{FL}^\dagger i\partial_R\lambda_{FL} + \lambda_{FR}^\dagger i\partial_L\lambda_{FR}\\
+ 2i\rho_F\big(e^{i\alpha_F}\lambda_{FR}^\dagger\lambda_{FL} - e^{-i\alpha_F}\lambda_{FL}^\dagger\lambda_{FR}\big)\Big] + \Big[\frac{1}{2}\,\partial_\mu\alpha_F\,\partial^\mu\alpha_F + 2\alpha_F\epsilon^{\mu\nu}\partial_\mu A_{F\nu}\Big] \qquad (43)\\
-\Big[\Lambda^2 - \rho_F^2 + \rho_F^2\log\frac{\rho_F^2}{\Lambda^2}\Big] + \sqrt{\frac{8\pi u}{N}}\Big[i\,\zeta_R\,\lambda_{FL}^\dagger + \text{H.c.}\Big]\Bigg\} - u\,\frac{N}{4\pi}\Big|\sum_{F=1,2}\rho_F e^{i\alpha_F}\Big|^2 .\end{aligned}$$

Here we assumed that the parameter κ is real, its phase can be absorbed into a field redefinition of ζ_R.

Let us emphasize that this $\mathcal{L}_{\rm eff}$ is exact for constructing the large-N expansion for terms up to the second order in derivative. For fermions each fermionic field should be counted as a square root of derivative. The only term missing in Eq. (43) is of the fourth order in λ. It does not contribute in our leading-N calculation but will enter for $1/N$ corrections. (These terms can be found in [23].)

Now let us consider the effect of the heterotic modification for the mass spectrum. Consider first the bosonic masses. Expanding the modification term (the last one in Eq. (43)) near the vacuum values we get

$$\begin{aligned}&-u\,\frac{N}{4\pi}\Big|\sum_{F=1,2}\rho_F\,e^{i\alpha_F}\Big|^2 = -4u\,\frac{N}{4\pi}\left[\frac{1}{2}\Big(\frac{\tilde\rho_1-\tilde\rho_2}{\sqrt2}\Big)^2 + \frac{1}{2}\Big(\frac{\tilde\alpha_1-\tilde\alpha_2}{\sqrt2}\Big)^2\right], \qquad (44)\\
&\tilde\rho_F = \rho_F - \langle\rho_F\rangle\,,\quad \tilde\alpha_F = \alpha_F - \langle\alpha_F\rangle\,.\end{aligned}$$

It means that the mass of the $(\tilde\rho_1+\tilde\rho_2)/\sqrt2$ field as well as the mass of $(\tilde\alpha_1+\tilde\alpha_2)/\sqrt2$ and the corresponding gauge field combination is not modified by the heterotic coupling,

$$m\big[(\tilde\rho_1+\tilde\rho_2)/\sqrt2\big] = m_{\rm ph}\big[(A_1+A_2)/\sqrt2\,\big] = 2\Lambda\,, \qquad (45)$$

while for the orthogonal combinations we get

$$m\big[(\tilde\rho_1-\tilde\rho_2)/\sqrt2\big] = m_{\rm ph}\big[(A_1-A_2)/\sqrt2\,\big] = 2\Lambda\sqrt{1+u}\,. \qquad (46)$$

The fermionic part of the heterotic modification in Eq. (43) in terms of the canonical $\tilde\lambda$ fields introduced in Eq. (35) reduces to

$$2\sqrt{u}\,\Lambda\,e^{-i\langle\alpha_1\rangle/2}\,i\,\zeta_R\,\frac{\tilde\lambda_{1L}^\dagger + i\tilde\lambda_{2L}^\dagger}{\sqrt2} + \text{H.c.}\,. \qquad (47)$$

It implies that the mass of the orthogonal combination $(\tilde\lambda_1+i\tilde\lambda_2)/\sqrt{2}$ is not modified,

$$m\big[(\tilde\lambda_1+i\tilde\lambda_2)/\sqrt{2}\big]=2\Lambda\,, \tag{48}$$

The field $\tilde\lambda_{-R}=(\tilde\lambda_{1R}-i\tilde\lambda_{2R})/\sqrt{2}$ mixes with ζ_R, forming

$$\frac{\tilde\lambda_{-R}+\sqrt{u}\,e^{-i\langle\alpha_1\rangle/2}\,\zeta_R}{\sqrt{1+u}} \tag{49}$$

under diagonalization. In conjunction with the field $\tilde\lambda_{-L}$ it results in the mass

$$m\Big[\tilde\lambda_{-L},\big(\tilde\lambda_{-R}+\sqrt{u}\,e^{i\langle\alpha_1\rangle/2}\,\zeta_R\big)/\sqrt{1+u}\Big]=2\Lambda\sqrt{1+u}\,, \tag{50}$$

i.e. the same as in (46).

The combination

$$\frac{\zeta_R-\sqrt{u}\,e^{i\langle\alpha_1\rangle/2}\,\tilde\lambda_{+R}}{\sqrt{1+u}} \tag{51}$$

orthogonal to (49) represents a massless right-moving fermion. It is not a Goldstino fermion, however, since its residue to the supercurrent vanishes, together with the finishing of $\sum\sigma_F$.

Thus, our large-N study of the connected model mass spectrum demonstrates the following phenomenon. In addition to the extra massless fermion, we obtain two supermultiplets of $\mathcal{N}=(2,2)$ supersymmetry. The breaking of (2,2) supersymmetry in the mass spectrum down to $\mathcal{N}=(0,2)$ does not show up in the leading-N approximation at large N. The reason for this is visible in the above derivation: the effect of the heterotic modification, say, for fermions, appears just as an admixture of ζ to λ, which does not break the (2,2) supersymmetry. This feature is not maintained for higher than quadratic in fields terms in the effective action implying that breaking of (2,2) down to (0,2) supersymmetry shows up in the next order in $1/N$.

The breaking to (0,2) in the $1/N$ corrections also shows up in the running of the Z-factors in the model at hand. This running will be discussed in Sec. 6.

4 Pattern of quiver Yang-Mills: constructing a variety of connected sigma models

The simplest (0,2) model presented in Sec. 3 can be extended in many distinct ways similar to the pattern used in four-dimensional Yang-Mills (Fig. 1). For instance, the target space (16) can be expanded up to

$$\big[\mathrm{CP}(N-1)\big]^n\times\big[\mathrm{C}^1\big]^n \tag{52}$$

by replacing a single $\mathcal{B}$ superfield by an ensemble of n superfields

$$\mathcal{B}_{12},\, \mathcal{B}_{23},\, \ldots \mathcal{B}_{n-1,n},\, \mathcal{B}_{n,1}\,,$$

see Fig. 1. The Lagrangian in the geometric formulation takes the form

$$\begin{aligned}
\mathcal{L}_n \;=\; & \sum_{F=1}^{n}\Big\{-\frac{1}{4}\int d\theta\,\Big[G_{i\bar{j}}(A_F,A_F^\dagger)(\bar{D}A_F^{\dagger\bar{j}})\,i\partial_R A_F^i + \text{H.c.}\Big] \\
+\; & \frac{1}{2}\int d^2\theta\, G_{i\bar{j}}(A_F,A_F^\dagger)\,B_F^{\dagger\bar{j}}B_F^i\Big\} + \frac{1}{2}\sum_{F=1}^{n}\int d^2\theta\,\mathcal{B}^\dagger_{F,F+1}\mathcal{B}_{F,F+1} \\
-\; & \frac{\kappa}{2}\sum_{F=1}^{n}\int d\theta\,\Big\{G_{i\bar{j}}(A_F,A_F^\dagger)(\bar{D}A_F^{\dagger\bar{j}})B_F^i\,(\mathcal{B}_{F-1,F}+\mathcal{B}_{F,F+1}) + \text{H.c.}\Big\}\,.
\end{aligned} \tag{53}$$

In the gauged formulation we have

$$\begin{aligned}
\mathcal{L}_{n\ \text{gauged}} \;=\; & \sum_F\Big\{\big|\mathcal{D}_{F\mu}n_F^i\big|^2 - 2|\sigma_F|^2\,|n_F^i|^2 - D_F\,\big(|n_F^i|^2 - 2/g^2\big) \\
+\; & \big(\xi_F^\dagger\big)_{jR}\,i\mathcal{D}_{FL}\,(\xi_F)_R^j + \big(\xi_F^\dagger\big)_{jL}\,i\mathcal{D}_{FR}\,(\xi_F)_L^j \\
+\; & \Big[\sqrt{2}\sigma_F\big(\xi_F^\dagger\big)_{jR}\,(\xi_F)_L^j + \sqrt{2}\,n_{Fj}^\dagger\big(\lambda_{FR}\xi_{FL}^j + \lambda_{FL}\xi_{FR}^j\big) + \text{H.c.}\Big]\Big\} \\
+\; & \big(\zeta^\dagger_{F,F+1}\big)_R\,i\partial_L\,\zeta_R^{F,F+1} - \sum_F\left[\frac{4\kappa}{g^2}\,i\,\lambda^\dagger_{FL}\Big(\zeta_R^{F-1,F}+\zeta_R^{F,F+1}\Big) + \text{H.c.}\right] \\
-\; & \frac{8|\kappa|^2}{g^4}\sum_{F=1}^{n}\big|\sigma_{F-1}+\sigma_F\big|^2\,.
\end{aligned} \tag{54}$$

For even values of n this model also has supersymmetric vacua with vanishing energy, i.e. $\mathcal{N}=(0,2)$ is unbroken. To this end – keeping supersymmetry unbroken – one should choose the set of the vacuum values of σ_F to be sign-alternating, e.g. $(1/2)\,(\Lambda^2,\,-\Lambda^2,\,\Lambda^2,\,-\Lambda^2,...)$. Large-$N$ solution can be obtained along the same lines as in Sec. 3.

5 Witten's index and its generalization

An investigation of the Witten index in a general class of (0,2) models was carried out in [24]. In our case the Witten index vanishes for all connected sigma models but permutation symmetries of the models allow us to introduce a nonvanishing modified index.

The vanishing of the Witten index in the heterotically modified CP($N-1$) was clearly demonstrated by the large N solution [10], where spontaneous breaking of (0.2) supersymmetry is explicit. The connected extensions considered here preserve the feature of the vanishing Witten index.

To see that this is indeed the case let us consider the model on a finite-size circle, i.e., let us compactify the spacial dimension by imposing periodic boundary conditions both on bosons and fermions, which preserves supersymmetry. In the limit when all heterotic couplings κ_F in Eq. (13) are small we have the same bosonic vacua as in the unmodified (2,2) models. For example, in the case of $\prod \mathrm{CP}(N_i-1)$ the number of these bosonic vacua is $\prod N_i$. Besides, in the limit of $\kappa_F \to 0$, we have a free massless fermion field ζ_R which at the finite-size circle has two zero modes, one for ζ_R, another for $\zeta_R^\dagger$. Fermionic operators of creation and annihilation can be introduced in the standard way. The corresponding zero-energy fermion state can be either empty (bosonic vacuum) or once filled (fermionic counterpartner). Therefore, each bosonic and fermionic vacua always come together in the theories of Secs. 3 and 4.

The vanishing of the Witten index usually implies that in some higher approximation (e.g. nonperturbatively) supersymmetry will be spontaneously broken since there is no apparent robust protection against this breaking. Such a protection can exist, though, if there exists a nonvanishing extended "flavor" index, in the same vein as in [12]. In our model an extra flavor symmetry is the permutation symmetry of the CP($N-1$) factors from (4).

For simplicity let us consider the same case as in Sec. 3.3, i.e. $n=2$ and N even. Generalizations are straightforward. Let us choose $\theta_1 = 0$ and $\theta_2 = 2\pi\,\frac{N}{2}$ for the two CP($N-1$) factors. Classically we have a discrete symmetry

$$\zeta_R \to -\zeta_R\,, \quad \psi_R^f \to -\psi_R^f\,. \tag{55}$$

At the quantum level this symmetry is broken which is visible from the existence of fermion condensates $\langle \psi_{Lf}^\dagger \psi_R^f \rangle$. However, applying in addition the permutation of two CP($N-1$) factors:

$$\Phi^1 \leftrightarrow \Phi^2 \tag{56}$$

we get an invariance of the theory. The combination of (55) and (56) is a good symmetry which we will call *P-conjugation.* Now we introduce a modification of the Witten index $I_W = \mathrm{Tr}(-1)^F$ of the form

$$I_P = \mathrm{Tr}\big[P(-1)^F\big] \tag{57}$$

It is clear that that index does not vanish in contradistinction with I_W: an addition of the ζ_R fermion to the state now yields the positive sign because of P.

Another way to show the absence of the spontaneous supersymmetry breaking in the case is as follows. The order parameter for supersymmetry breaking is

$$\mathcal{F}_\zeta = \text{const} \cdot \kappa \left(\sum_{f=1,2} \psi^+_{L\,f}\, \psi^f_R\, G^f \right) \tag{58}$$

where G^f is the metric of the corresponding CP($N-1$) factor.

Now, let us apply the transformation (55). As was mentioned above this classical symmetry is broken at the quantum level due to the chiral anomaly, it changes the vacuum angles, namely,

$$\theta_1 \to 2\pi\,\frac{N}{2}\,, \qquad \theta_2 \to 2\pi\,\frac{N}{2} + 2\pi\,\frac{N}{2} = 2\pi N \;\text{ equiv }\, 0\,. \tag{59}$$

Under the rotation above the order parameter

$$\mathcal{F}_\zeta \to \exp(i\pi)\mathcal{F}_\zeta\,. \tag{60}$$

At the same time, interchanging the two CP($N-1$) factors we see that $\mathcal{F}_\zeta$ remains intact. Combining these two facts we conclude that $\mathcal{F}_\zeta = 0$. Note that $\mathcal{F}_\zeta$ presents also the coupling of the would-be-Goldstino to the supercurrent.

Another very simple example is $n = N$. In this case we must choose

$$\theta_1 = 0,\,,\;\; \theta_2 = 2\pi\,,\;\; \theta_3 = 4\pi,\,,\;\; ...,\theta_N = 2\pi(N-1)\,, \tag{61}$$

and the rotation

$$\psi^f_R \to e^{i2\pi/N}\,\psi^f_R \;\;\text{for all}\;\; f\,, \quad \zeta_R \to \zeta_R \exp(-i2\pi/N)\,. \tag{62}$$

The same argument as above implies $\mathcal{F}_\zeta = 0$.

6 Beta functions

The β functions of the basic heterotic model discussed in Sec. 2 were derived in [26, 27]. In the heterotic model one deals with two coupling constants, g^2 appearing in the metric, and the deformation parameter κ. The β function for g^2 is [26]

$$\beta_g = \mu\,\frac{dg^2}{d\mu} = -\frac{g^2}{4\pi}\,\frac{T_G\, g^2\,(1+\gamma_{\psi_R}/2) - h^2\,(\gamma_{\psi_R}+\gamma_\zeta)}{1-(h^2/4\pi)}\,, \tag{63}$$

where $\gamma_\zeta = -\mu\, d\log \mathcal{Z}/d\mu$ and $\gamma_{\psi_R} = -\mu\, d\log Z/d\mu$ are the anomalous dimensions of the corresponding fields, which to the leading order are proportional to the coupling

$$h^2 = \frac{|\kappa|^2}{Z\mathcal{Z}}\,. \tag{64}$$

Here Z and $\mathcal{Z}$ are field renormalization constants for ψ_R and ζ_R respectively (see Eqs. (13) and (14) for their definition). At one loop [27]

$$\gamma^{(1)}_{\psi_R} = \frac{h^2}{2\pi}\,, \qquad \gamma^{(1)}_{\zeta} = \frac{(N-1)\,h^2}{2\pi}\,. \tag{65}$$

Now, in the connected models the general relation (63) remains intact, while the expression for the anomalous dimension γ changes. In particular, for the model of Sec. 3

$$\gamma^{(1)}_{\psi_R} = \frac{h^2}{2\pi}\,, \qquad \gamma^{(1)}_{\zeta} = n\frac{(N-1)\,h^2}{2\pi}\,. \tag{66}$$

and $T_G = N$.

The β function for h^2 is also fixed by anomalous dimensions, see [26,27] for details. There is a fixed point for the ratio h^2/g^2,

$$\left.\frac{h^2}{g^2}\right|_c = \frac{1}{2}\cdot\frac{N}{n\,(N-1)+1}\,. \tag{67}$$

At large n it means that nh^2 and g^2 scale exactly in the same way (as it occurred at one loop).

7 Conclusions

In this paper we suggested a way to make connected two-dimensional $\mathcal{N}=(0,2)$ sigma models from $\mathcal{N}=(2,2)$ CP$(N-1)$ models. This method is easily extendible to any Grassmannian sigma model. To this end one introduces an extra fermion $\mathcal{N}=(0,2)$ superfield (or superfields) on C^1 coupled to all or some of the n copies of the $\mathcal{N}=(2,2)$ sigma model. The connected model emerging in this way can be solved in the large-N limit. Our solution demonstrates that $\mathcal{N}=(0,2)$ supersymmetry which is spontaneously broken without "connection" is restored in the connected version. This statement is unambiguously proved in the leading order in $1/N$.

Then in Sec. 5 we introduce a generalized Witten index which, being nonvanishing, provides us with the general proof of the exact statement: our connected models do have supersymmetric vacua.

We also find the excitation spectrum in the leading $1/N$ approximation and expressions for the beta functions in the quiver models.

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