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# Accidental symmetries and massless quarks in the economical 3-3-1 model

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## Abstract

In the framework of a 3-3-1 model with a minimal scalar sector, known as the economical 3-3-1 model, we study its capabilities of generating realistic quark masses. After a detailed study of the symmetries of the model, before and after the spontaneous symmetry breaking, we find a remaining axial symmetry that prevents some quarks to gain mass at all orders in perturbation theory. Since this accidental symmetry is anomalous, we also consider briefly the possibility to generate their masses for non-perturbative effects. However, we find that non-perturbative effects are not enough to generate the measured masses for that three massless quarks. Hence, these results imply that the economical 3-3-1 model is not a realistic description of the electroweak interaction.

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## I. INTRODUCTION

Up to now, from the experimental point of view, neutrino masses and their mixing, and dark matter are the only issues demanding for explanations beyond the standard model (SM). On the other hand, from the theoretical point of view, the quest for a more deeper understanding lead us to believe that a more fundamental model of the interactions is needed. That model should be able to answer simple but deep questions. Some of these questions are: why the number of families of quarks and leptons is three? Is there a more fundamental relation (symmetry) between quarks and leptons? Why does the observed pattern for the particle masses have this particular form? Should not the parameters involved have any calculability? What is the origin of CP violation? Even in the SM, what is the origin of the CP violating CKM phase? Can it be computed? Is there any more efficient mechanism able to account for the matter-antimatter asymmetry in the Universe? What is the mechanism that generates masses and mixing angles for neutrinos? Is there CP violation in leptons? What would be its role in the evolution of the Universe? How dark matter and dark energy can be incorporated? Unfortunately, up to now, the experimental efforts were not able to indicate exactly what the physics beyond the SM should be.

In the framework of gauge theories, one way of introducing new physics is to consider a gauge symmetry group larger than the SM one. Some years ago models with the  $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$  gauge symmetry were proposed [1–4], which are having considerable further developments. The so-called 3-3-1 models present interesting features concerning the asked questions above. One of them is that, depending on the representation content, the triangle anomalies cancel out, and the number of families has to be a multiple of three. More precisely, it must be just three due to the asymptotic freedom. A version of this kind of models called minimal [1–4] presents a Landau-like pole when  $\sin^2 \theta_W = 1/4$  at energies of the order of a few TeVs [5]. This particular behavior stabilizes the electroweak scale avoiding the hierarchy problem and also explains why it is observed  $\sin^2 \theta_W < 1/4$ . This model also accounts for the electric charge quantization independently of the nature of the massive neutrinos, i.e. whether they are Dirac or Majorana particles [6]. The model has also interesting features concerning the strong CP problem. In the minimal 3-3-1 model there is an almost automatic Peccei-Quinn (PQ) symmetry, and an automatic one in the so called economical version of the model, as we will show here below. In both versions there

are ways of solving the strong CP problem while keeping the corresponding axion invisible and protected against gravitational effects [7, 8]. Due to a larger gauge symmetry group and a rich scalar sector, this kind of model has called some attention in many other subjects like new sources of CP violation, active neutrino mass generation and mixing, dark matter candidates and  $Z'$ -boson physics.

In this paper we are concerned with the quark mass generation, in the context of the economical 3-3-1 model (E331, for short). In particular we investigate the capabilities of the model in generating realistic quark masses. The quark sector of this model was already considered in literature and conflicting results were found [8, 9]. In this work, in order to clarify this important issue, we do a detailed study of the symmetries (local and global symmetries) of the entire E331-model Lagrangian. Once we have identified all the symmetries, and after the spontaneous symmetry breaking (SSB) of the scalar potential, we investigate which are the remaining symmetries (if any) of the vacuum state. In other words, we seek which are (if any) the independent linear combinations of the group generators that annihilate the vacuum state, in order to know if the corresponding symmetries are realized *à la* Wigner-Weyl (WW) or Nambu-Goldstone (NG). That is of fundamental importance since it will affect the physical particle spectrum. If the total Lagrangian and vacuum state are both invariant under a symmetry transformation, this is a WW realization of that symmetry. On the other hand, if the vacuum is not invariant this is a NG realization and this implies a massless NG scalar boson. We find that there is a WW realization of a subgroup of the initial symmetry group that protects some quarks from getting mass at all orders in perturbation theory, as expected from quantum field theory.

The paper is organized as follows. In Sec. II we briefly review the economical 3-3-1 model. In Sec. III we make a detailed study of the symmetries of the model, in both situations before and after the spontaneous symmetry breakdown, and its implication for the quark masses. Non-perturbative effects contributing to quark masses are also briefly considered. Our conclusions are presented in Sec. IV.

## II. A BRIEF REVIEW OF THE ECONOMICAL 3-3-1 MODEL

The model considered has a matter content given by [10]

$$\begin{aligned}
\Psi_{aL} &= \left( \nu_a, e_a, (\nu_{aR})^C \right)_L^T \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad e_{aR} \sim (\mathbf{1}, \mathbf{1}, -1), \\
Q_{\alpha L} &= (d_\alpha, u_\alpha, d'_\alpha)_L^T \sim (\mathbf{3}, \mathbf{3}^*, 0), \quad Q_{3L} = (u_3, d_3, u'_3)_L^T \sim (\mathbf{3}, \mathbf{3}, 1/3), \\
u_{aR} &\sim (\mathbf{3}, \mathbf{1}, 2/3), \quad u'_{3R} \sim (\mathbf{3}, \mathbf{1}, 2/3), \quad d_{aR} \sim (\mathbf{3}, \mathbf{1}, -1/3), \quad d'_{\alpha R} \sim (\mathbf{3}, \mathbf{1}, -1/3), \\
\chi &= (\chi^0, \chi^-, \chi_1^0)^T \sim (\mathbf{1}, \mathbf{3}, -1/3), \quad \rho = (\rho^+, \rho^0, \rho_1^+)^T \sim (\mathbf{1}, \mathbf{3}, 2/3),
\end{aligned} \tag{1}$$

where  $a = 1, 2, 3$ ,  $\alpha = 1, 2$  and the values in parentheses denote respectively the quantum numbers corresponding to the  $(\text{SU}(3)_C, \text{SU}(3)_L, \text{U}(1)_X)$  groups. From now on Latin and Greek letters always take the values 1, 2, 3 and 1, 2, respectively.

With the quark, lepton and scalar multiplets in Eq. (1) we have that the most general Yukawa interactions allowed by the gauge symmetries and renormalizability are

$$\begin{aligned}
\mathcal{L}_Y &= Y_{ab} \overline{\Psi}_{aL} e_{bR} \rho + Y'_{ab} \epsilon^{ijk} (\overline{\Psi}_{aL})_i (\Psi_{bL})_j^C (\rho^*)_k \\
&\quad + G^1 \overline{Q}_{3L} u'_{3R} \chi + G^2_{\alpha\beta} \overline{Q}_{\alpha L} d'_{\beta R} \chi^* + G^3_a \overline{Q}_{3L} d_{aR} \rho + G^4_{\alpha a} \overline{Q}_{\alpha L} u_{aR} \rho^* \\
&\quad + G^5_a \overline{Q}_{3L} u_{aR} \chi + G^6_{\alpha a} \overline{Q}_{\alpha L} d_{aR} \chi^* + G^7_\alpha \overline{Q}_{3L} d'_{\alpha R} \rho + G^8_\alpha \overline{Q}_{\alpha L} u'_{3R} \rho^* + \text{h.c.},
\end{aligned} \tag{2}$$

where  $G^i$ ,  $Y_{ab}$  and  $Y'_{ab}$  are arbitrary complex matrices and  $Y'_{ab}$  is also antisymmetric. We use the convention that addition over repeated indices is implied.

The  $\chi$  and  $\rho$  scalar multiplets break down spontaneously the  $\text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_X$  gauge symmetry. The vacuum expectation values, VEVs, in this model satisfy  $\langle \text{Re } \rho^0 \rangle \equiv v$ ,  $\langle \text{Re } \chi^0 \rangle \equiv u \ll \langle \text{Re } \chi_1^0 \rangle \equiv w$ . The most general scalar potential, that is both invariant under the gauge symmetry and renormalizable, is

$$V = \mu_\chi^2 \chi^\dagger \chi + \mu_\rho^2 \rho^\dagger \rho + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi) (\rho^\dagger \rho) + \lambda_4 (\chi^\dagger \rho) (\rho^\dagger \chi). \tag{3}$$

With only two scalar multiplets the scalar sector is simple and it is, in principle, an appealing feature of this model comparing to other 3-3-1 models [1, 2, 11].

Finally, the electric charge operator is written as

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X, \tag{4}$$

where  $T_3$  and  $T_8$  are the diagonal generators of the  $\text{SU}(3)_L$  group and  $X$  refers to the quantum number of the  $\text{U}(1)_X$  group.

### III. SPONTANEOUS SYMMETRY BREAKING AND MASSLESS QUARKS

Before considering which symmetries are broken down, we look for all exact symmetries, local and global, this model actually has. Doing so, we realize that apart from the local gauge symmetry  $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ , this model has two extra global  $U(1)$  symmetries which we denoted generically by  $U(1)_\zeta$ . In order to see that, we write down the relations that these symmetries have to obey in order to keep the entire Lagrangian invariant. From Eq. (2) we obtain the following relations

$$-\zeta_{Q_3} + \zeta_{u'_{3R}} + \zeta_\chi = 0, \quad -\zeta_Q + \zeta_{d'_R} - \zeta_\chi = 0, \quad -\zeta_{Q_3} + \zeta_{u_R} + \zeta_\chi = 0, \quad (5)$$

$$-\zeta_Q + \zeta_{d_R} - \zeta_\chi = 0, \quad -\zeta_{Q_3} + \zeta_{d_R} + \zeta_\rho = 0, \quad -\zeta_Q + \zeta_{u_R} - \zeta_\rho = 0, \quad (6)$$

$$-\zeta_{Q_3} + \zeta_{d'_R} + \zeta_\rho = 0, \quad -\zeta_Q + \zeta_{u'_{3R}} - \zeta_\rho = 0, \quad -\zeta_\Psi + \zeta_{e_R} + \zeta_\rho = 0, \quad (7)$$

$$2\zeta_\Psi + \zeta_\rho = 0, \quad (8)$$

where the  $\zeta_{\psi_i}$ 's above denote the  $U(1)_\zeta$  charges of the  $\psi_i$  fields. Solving Eqs. (5-8), we find that all charges,  $\zeta_{\psi_i}$ , can be written in terms of three independent ones. It means that the model has only three independent  $U(1)_\zeta$  symmetries. In principle, we can choose whatever three independent  $U(1)_\zeta$  symmetries as basis. However, some physical considerations can be done to appropriately choose them. First, we note that one of these symmetries is the  $U(1)_X$  gauge symmetry which is anomaly free by construction and which has an associated gauge boson. The other two are global symmetries and they can be divided into a vectorial and a axial symmetry acting on the quarks. The vectorial one is the well known baryon number symmetry, denoted here as  $U(1)_B$ , which is an accidental symmetry in this model as it is in the SM. The other one is a axial symmetry also acting on the quarks, which we denote as  $U(1)_{PQ}$ . The last symmetry is a PQ one since it is anomalous and  $A_{PQ}$ , the coefficient of the  $[SU(3)_C]^2 U(1)_{PQ}$  anomaly, is  $\propto -3$ . Also, notice that the  $U(1)_{PQ}$  is a natural symmetry in the sense that it is not imposed, it follows from the gauge local symmetry and renormalizability, instead. In other words, the economical model naturally has a PQ symmetry. The assignment of the three independent  $U(1)$  quantum charges is shown in Table I (these quantum charges appeared for the first time in Ref. [8], we have written them here for the sake of completeness and clearness). Thus, the model actually has a larger symmetry:  $G \equiv SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_B \otimes U(1)_{PQ}$ , where the last two ones are accidental and global symmetries.

	$Q_{\alpha L}$	$Q_{3L}$	$(u_{aR}, u'_{3R})$	$(d_{aR}, d'_{\alpha R})$	$\Psi_{aL}$	$e_{aR}$	$\rho$	$\chi$
$U(1)_X$	0	1/3	2/3	-1/3	-1/3	-1	2/3	-1/3
$U(1)_B$	1/3	1/3	1/3	1/3	0	0	0	0
$U(1)_{PQ}$	-1	1	0	0	-1/2	-3/2	1	1

Table I: Assignment of the three independent  $U(1)$  quantum charges in the economical 3-3-1 model.

Now, let us search for the remaining symmetries after the  $\chi$  and  $\rho$  scalar triplets obtain their VEVs,  $\langle\chi\rangle \equiv V_\chi = \frac{1}{\sqrt{2}}(u, 0, w)^T$  and  $\langle\rho\rangle \equiv V_\rho = \frac{1}{\sqrt{2}}(0, v, 0)^T$ . To do that, we consider an infinitesimal transformation of the total group  $G$  on the vacuum states to find the generators of the unbroken subgroups as a linear combination of the  $T_i$ ,  $X$ ,  $PQ$ , and  $B$  generators. Here, it is important to note that the  $SU(3)_C \otimes U(1)_B$  subgroups are clearly unbroken and thus we can omit them in the following analysis without affecting our conclusions. Then, under an infinitesimal transformation on the vacuum we have

$$\left( \sum_{i=1}^8 \alpha_i T_i + \gamma X_\chi 1_{3 \times 3} + \delta PQ_\chi 1_{3 \times 3} \right) V_\chi = 0, \quad (9)$$

$$\left( \sum_{i=1}^8 \alpha_i T_i + \gamma X_\rho 1_{3 \times 3} + \delta PQ_\rho 1_{3 \times 3} \right) V_\rho = 0, \quad (10)$$

where  $\alpha_i$ ,  $\gamma$  and  $\delta$  are independent real constants and  $1_{3 \times 3}$  denotes the  $3 \times 3$  identity matrix. Also, we have from Table I that  $X_\chi = -1/3$ ,  $X_\rho = 2/3$  and  $PQ_\chi = PQ_\rho = 1$ . Since the  $\chi$  and  $\rho$  scalar triplets are in the fundamental representation of  $SU(3)_L$ , the  $T_i$  generators in Eqs. (9) and (10) are given by  $\lambda_i/2$ , where  $\lambda_i$  are the well known Gell-Mann matrices. From Eqs. (9-10) follows

$$v(\alpha_1 - i\alpha_2) = 0, \quad (11)$$

$$v(\alpha_6 + i\alpha_7) = 0, \quad (12)$$

$$u(\alpha_1 + i\alpha_2) + w(\alpha_6 - i\alpha_7) = 0, \quad (13)$$

$$v(-3\alpha_3 + \sqrt{3}\alpha_8 + 4\gamma + 6\delta) = 0, \quad (14)$$

$$3u(\alpha_4 + i\alpha_5) - 2w(\sqrt{3}\alpha_8 + \gamma - 3\delta) = 0, \quad (15)$$

$$u(3\alpha_3 + \sqrt{3}\alpha_8 - 2\gamma + 6\delta) + 3w(\alpha_4 - i\alpha_5) = 0, \quad (16)$$

with  $i = \sqrt{-1}$ . Solving simultaneously Eqs. (11-16) (with  $u \neq 0$ ,  $v \neq 0$  and  $w \neq 0$ ) we have

that  $\alpha_1 = \alpha_2 = \alpha_5 = \alpha_6 = \alpha_7 = 0$  and

$$\alpha_4 = -\frac{6uw}{u^2 + w^2}\delta \equiv -3\sin(2\theta)\delta, \quad (17)$$

$$\alpha_8 = \frac{6}{\sqrt{3}}\left(\frac{2w^2}{u^2 + w^2} - 1\right)\delta - \frac{\alpha_3}{\sqrt{3}} \equiv \frac{6\cos(2\theta)}{\sqrt{3}}\delta - \frac{\alpha_3}{\sqrt{3}}, \quad (18)$$

$$\gamma = -\frac{3w^2}{u^2 + w^2}\delta + \alpha_3 \equiv -\frac{3}{2}(1 + \cos(2\theta))\delta + \alpha_3, \quad (19)$$

where  $\tan\theta \equiv u/w$ . Since the parameters  $\alpha_3$  and  $\delta$  are independent, this implies that from the ten generators only two linearly independent combinations, say  $g_1$  and  $g_2$ , remain unbroken. At a first glance, the choice of these generators is arbitrary. However, we take into consideration that one of them has to be the anomaly-free electric charge generator, which is achieved by taken  $\delta = 0$  and  $\alpha_3 = 1$ . Doing so,  $g_1 = Q$ . The other generator,  $g_2$ , must have  $\delta \neq 0$  in order to be linearly independent of  $g_1$  (since all generator with  $\delta = 0$  will be proportional to  $Q$ ). Hence, the unbroken generators are written as:

$$g_1 = T_3 - \frac{1}{\sqrt{3}}T_8 + X, \quad (20)$$

$$g_2 = \left[3\cos^2(\theta)T_3 - 3\sin(2\theta)T_4 + \frac{1}{2}\sqrt{3}(3\cos(2\theta) - 1)T_8 + \text{PQ}\right]\delta, \quad (21)$$

with  $\delta \neq 0$  in the last equation. The symmetry associated to  $g_1$ ,  $\text{U}(1)_Q$ , is anomaly free as it is well known. The  $g_2$  generator, which is independent from  $g_1$ , is a linear combination of  $T_3$ ,  $T_4$ ,  $T_8$ , PQ generators. We refer to the symmetry associated to  $g_2$  as  $\text{U}(1)_H$ . The key point here is that a part of the initial axial symmetry,  $\text{U}(1)_{\text{PQ}}$ , remains unbroken because the coefficient  $\delta$ , in Eq. (21), is always different from zero. In conclusion, the existence of  $g_1$  and  $g_2$  implies that the  $\text{U}(1)_Q \otimes \text{U}(1)_H$  subgroup of the  $\text{SU}(3)_L \otimes \text{U}(1)_X \otimes \text{U}(1)_{\text{PQ}}$  remains unbroken.

Now, since  $G$  is an exact symmetry, i.e.  $[G, \mathcal{L}_T] = 0$  (where  $\mathcal{L}_T$  is the total Lagrangian of the model) from the Goldstone's theorem [12], we have exactly eight NG scalar bosons (a NG scalar boson for each broken generator), which in this model will become the longitudinal component of the eight massive gauge vector bosons via the Higgs mechanism. In the physical scalar spectrum this model has only massive scalar bosons,  $H_1^0$ ,  $H_2^0$ ,  $H^+$ ,  $H^-$ , as it is shown in [13]. If the  $g_2$  was broken, a NG scalar boson would appear in the physical scalar spectrum. Because  $g_2$  has a component in PQ generator, that physical NG scalar boson would be an axion. However, it does not happen and the model has three massless quarks (one  $u$ -type quark and two  $d$ -type quarks) instead. This can be easily seen from



the mass matrices because a couple of rows in the  $u$ -quark mass matrix and two couples of rows in the  $d$ -quark mass matrix are proportional to each other, see Eqs. (18) and (19) in Ref. [13]. The exact form of those massless quarks is neither clarifying nor relevant for our analysis, thus, we do not write them here. These massless quarks are fully expected because an exact axial symmetry that is realized in the WW manner implies massless fermions.

The action of the remaining  $U(1)_H$  symmetry preventing some quarks to gain mass becomes obvious when we change basis to work with the mass eigenstates instead of the symmetry eigenstates. The symmetry eigenstates  $U_{iL,R}, D_{jL,R}$  (with  $i = 1, \dots, 4$  and  $j = 1, \dots, 5$ ) and the mass eigenstates  $(U_M)_{iL,R}, (D_M)_{jL,R}$  are related by  $U_{L,R} = (V_{L,R}^U)^\dagger (U_M)_{L,R}$  and  $D_{L,R} = (V_{L,R}^D)^\dagger (D_M)_{L,R}$ , where  $V_{L,R}^{U,D}$  are independent unitary matrices such that  $V_L^U M^U V_R^{U\dagger} = \hat{M}^U$  and  $V_L^D M^D V_R^{D\dagger} = \hat{M}^D$ , where  $\hat{M}^U = \text{diag}(m_{u_{M1}}, m_{u_{M2}}, m_{u_{M3}}, m_{u_{M4}})$  and  $\hat{M}^D = \text{diag}(m_{d_{M1}}, m_{d_{M2}}, m_{d_{M3}}, m_{d_{M4}}, m_{d_{M5}})$ . Since the mass matrix of the  $u$ - and  $d$ -quark types are not hermitian matrices, in order to obtain the mass eigenstates we have to solve the matrix equations:  $V_L^q M^q M^{q\dagger} V_L^{q\dagger} = V_R^q M^{q\dagger} M^q V_R^{q\dagger} = (\hat{M}^q)^2$ ,  $q = U, D$ . More specifically, we have to find the base-rotation matrices  $V_{L,R}^U$  and  $V_{L,R}^D$  to be able to write the Yukawa interactions in terms of the quark-mass eigenstates. This task can be done by standard procedures. Unfortunately, exact analytical expressions for these matrix are enormous so that it is worthless to show them here. The diagonalization study shows that we have one vanishing eigenvalue in the  $u$ -quark sector and two in the  $d$ -quark sector, as expected. The respective zero-mass eigenstates are clearly identified. Let's call them as  $u_{M1}$ ,  $d_{M1}$ , and  $d_{M2}$ . It means that there are no mass terms of the form  $m_{u_{M1}} \overline{u_{M1L}} u_{M1R} + m_{d_{M1}} \overline{d_{M1L}} d_{M1R} + m_{d_{M2}} \overline{d_{M2L}} d_{M2R} + \text{h.c.}$ , i.e.  $m_{u_{M1}} = m_{d_{M1}} = m_{d_{M2}} = 0$ .

We find more important results looking at the quark-scalar field interactions coming from the Yukawa interactions in Eq. 2. Here we find that there is no interactions involving the right component of these massless quark states. The right states  $u_{M1R}$ ,  $d_{M1R}$ , and  $d_{M2R}$  disappear from the Yukawa interactions. In other words, no left quark state are coupled with these massless states through neutral- or charged-scalar fields. Nonetheless, these zero-mass right components should have some interaction after all. In fact, from the quark kinetic terms we find that they interact only with the neutral vector bosons,  $A_\mu$ ,  $Z_\mu$ , and  $Z'_\mu$ . These interactions couple only quarks with the same chirality. It means that each one of the right components  $u_{M1R}$ ,  $d_{M1R}$ , and  $d_{M2R}$  can be transformed by an arbitrary  $U(1)_H$  phase without affecting any other term in the Lagrangian. Hence, looking at the Yukawa and

the neutral vector boson interactions, written in terms of the quark-mass eigenstates, we can identify the  $U(1)_H$  symmetry responsible for preventing these quark states to get mass: the massless right fields  $u_{M1R}$ ,  $d_{M1R}$ , and  $d_{M2R}$  transform as  $e^{i\alpha}u_{M1R}$ ,  $e^{i\alpha}d_{M1R}$ ,  $e^{i\alpha}d_{M2R}$  and all other fields transform trivially under  $U(1)_H$  (note that this symmetry is anomalous with  $[SU(3)_C]^2 U(1)_H$  anomaly  $\propto -3$ ). This is a clear and undoubtable manifestation of the remaining symmetry we have found. If we consider perturbation theory, these massless quarks can not get mass through radiative corrections to their propagators since the right component of these fields disappeared from the Yukawa interactions and they only couple to neutral vector bosons which conserve chirality. Therefore, there is no way to form loop diagrams to give mass for these particular fields, at all orders in perturbation theory.

Now, let us discuss the possibility of generating masses for those massless quarks through non-perturbative corrections and the viability of this model to explain the low-energy hadron phenomenology. Roughly speaking, that can be seen as follows. From both chiral QCD and lattice calculations the ratio  $\mu_u/\mu_d$  is  $0.410 \pm 0.036$  [14–16], where  $\mu_u$  and  $\mu_d$  are the “low-energy quark masses”. These should be distinguished from the quark mass parameters,  $m_i$  of the QCD Lagrangian at high scale [17]. In particular,  $\mu_u = \beta_1 m_u + \beta_2 \frac{m_d m_s}{\Lambda_{\chi SB}}$  where  $\Lambda_{\chi SB} \sim 1$  TeV (where we have identified naturally the massless quarks as  $u = u_{M1}$  and  $d = d_{M1}$  and  $s = d_{M2}$ ). It means,  $\mu_u$  receives an additive non-perturbative contribution of order  $m_d m_s$  in addition to the perturbative one,  $\beta_1 m_u$ , which is zero because both  $m_u$  is zero. The non-perturbative contribution is also zero because  $m_d$  and  $m_s$  are both zero and  $\beta_2$  is estimated to be a number of order one. Thus,  $\mu_u = 0$ , which is in complete disagreement with the ratio  $\mu_u/\mu_d$ . A similar analysis is also valid for the  $\mu_d$  and  $\mu_s$  [18].

#### IV. CONCLUSIONS

The scalar content in the E331 model is not enough to break down the initial symmetry,  $G$  to  $U(1)_Q \otimes U(1)_B$ . Instead, an extra generator  $g_2$  remains unbroken and thus the model has a  $U(1)_H$  axial symmetry after the spontaneous symmetry breaking. As we have explicitly shown above,  $g_2$  is a linear combination of the  $T_3$ ,  $T_4$ ,  $T_8$ , PQ generators and it is linearly independent of the generators of the electric charge and baryonic number,  $g_1$  and  $B$ , respectively. Because of the PQ component in the  $g_2$  generator, we have that the initial axial  $U(1)_{PQ}$  symmetry is not completely broken. In other words the  $U(1)_Q \otimes U(1)_H$  subgroup

of the  $SU(3)_L \otimes U(1)_X \otimes U(1)_{PQ}$  group remains unbroken. Therefore, the model has three massless quarks. The  $U(1)_H$  symmetry acts on the mass eigenstates as an axial symmetry,  $u_{M1R} \rightarrow e^{i\alpha} u_{M1R}$ ,  $d_{M1R} \rightarrow e^{i\alpha} d_{M1R}$ ,  $d_{M2R} \rightarrow e^{i\alpha} d_{M2R}$ , and this will protect these massless quarks to acquire mass at any level of perturbation theory. Furthermore, we recall that the unbroken  $U(1)_H$  subgroup has its origin in an axial symmetry,  $U(1)_{PQ}$ , which, although anomalous, is an accidental symmetry in the sense that it follows from the gauge symmetries and renormalizability. Therefore, the remaining axial symmetry acting on quarks will only be broken down by non-perturbative QCD processes [19]. However, these effects are not enough to provide the necessary low-energy quark masses,  $\mu_i$ , to the three massless quarks to make the model be in agreement with both chiral QCD and lattice calculations, which give the ratio  $\mu_u/\mu_d$  is  $0.410 \pm 0.036$  [14–16]. Hence, the economical version of the 3-3-1 model can not be considered a realistic description of the electroweak interaction. The original 3-3-1 models, as the model I presented in Ref. [4], for instance, do not suffer from such illness.

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- [1] F. Pisano and V. Pleitez. An  $SU(3) \times U(1)$  model for electroweak interactions. *Phys.Rev.*, D46:410–417, 1992.
  - [2] P.H. Frampton. Chiral dilepton model and the flavor question. *Phys.Rev.Lett.*, 69:2889–2891, 1992.
  - [3] Robert Foot, Oscar F. Hernandez, F. Pisano, and V. Pleitez. Lepton masses in an  $SU(3)$ -L x  $U(1)$ -N gauge model. *Phys.Rev.*, D47:4158–4161, 1993.
  - [4] J. C. Montero, F. Pisano, and V. Pleitez. Neutral currents and glashow-iliopoulos-maiani mechanism in  $SU(3)_L \otimes U(1)_N$  models for electroweak interactions. *Phys. Rev. D*, 47:2918–2929, Apr 1993.

- [5] A.G. Dias, R. Martinez, and V. Pleitez. Concerning the landau pole in 3-3-1 models. *The European Physical Journal C - Particles and Fields*, 39(1):101–107, 2005.
- [6] C. A. de S. Pires and O. P. Ravenez. Electric charge quantization in a chiral bilepton gauge model. *Phys. Rev. D*, 58:035008, Jul 1998.
- [7] Alex G. Dias and V. Pleitez. Stabilizing the invisible axion in 3-3-1 models. *Phys. Rev. D*, 69:077702, Apr 2004.
- [8] J. C. Montero and B. L. Sánchez-Vega. Natural peccei-quinn symmetry in the 3-3-1 model with a minimal scalar sector. *Phys. Rev. D*, 84:055019, Sep 2011.
- [9] P.V. Dong, H.N. Long, and H.T. Hung. Question of Peccei-Quinn symmetry and quark masses in the economical 3-3-1 model. *Phys.Rev.*, D86:033002, 2012.
- [10] William A. Ponce, Yithsbey Giraldo, and Luis A. Sanchez. Minimal scalar sector of 3-3-1 models without exotic electric charges. *Phys.Rev.*, D67:075001, 2003.
- [11] Robert Foot, Hoang Ngoc Long, and Tuan A. Tran. SU(3)-L x U(1)-N and SU(4)-L x U(1)-N gauge models with right-handed neutrinos. *Phys.Rev.*, D50:34–38, 1994.
- [12] Jeffrey Goldstone, Abdus Salam, and Steven Weinberg. Broken Symmetries. *Phys.Rev.*, 127:965–970, 1962.
- [13] J.C. Montero and B.L. Sanchez-Vega. Natural PQ symmetry in the 3-3-1 model with a minimal scalar sector. *Phys.Rev.*, D84:055019, 2011.
- [14] J. Gasser and H. Leutwyler. Quark Masses. *Phys.Rept.*, 87:77–169, 1982.
- [15] Daniel R. Nelson, George Tamminga Fleming, and Gregory W. Kilcup. Is strong CP due to a massless up quark? *Phys.Rev.Lett.*, 90:021601, 2003.
- [16] Jihn E. Kim and Gianpaolo Carosi. Axions and the Strong CP Problem. *Rev.Mod.Phys.*, 82:557–602, 2010.
- [17] Kiwoon Choi, C. W. Kim, and W. K. Sze. Mass renormalization by instantons and the strong CP problem. *Phys. Rev. Lett.*, 61:794–797, Aug 1988.
- [18] T. Banks, Y. Nir, and N. Seiberg. Missing (up) mass, accidental anomalous symmetries, and the strong cp problem. In P. Ramond, editor, *Yukawa Couplings and the origins of mass*. International Press, 1996.
- [19] Gerard 't Hooft. Symmetry Breaking Through Bell-Jackiw Anomalies. *Phys.Rev.Lett.*, 37:8–11, 1976.