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Unitarity Constraints on Dimension-Six Operators

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We obtain the partial-wave unitarity constraints on dimension-six operators stemming from the analyses of vector boson and Higgs scattering processes as well as the inelastic scattering of standard model fermions into electroweak gauge bosons. We take into account all coupled channels, all possible helicity amplitudes, and explore a six-dimensional parameter space of anomalous couplings. Our analysis shows that for those operators affecting the Higgs couplings, present 90% confidence level constraints from global data analysis of Higgs and electroweak data are such that unitarity is not violated if $\sqrt{s} \leq 3.2$ TeV. For the purely gauge-boson operator O_{WWW}, the present bounds from triple-gauge boson analysis indicate that within its presently allowed 90% confidence level range unitarity can be violated in $f\bar{f}' \to VV'$ at center-of-mass energy $\sqrt{s} \geq 2.4$ TeV.

I. INTRODUCTION

The standard model (SM) of electroweak interactions has been extremely successful in the description of the available data, and up to now there is no clear experimental evidence that challenges its predictions. As long as no new state has been observed, effective lagrangians provide a well defined systematic way to parametrize departures from the standard model. Furthermore, the recent discovery of a particle resembling a light Higgs boson indicates that the $SU(2)_L \otimes U(1)_Y$ gauge symmetry might be realized linearly in the effective theory. Therefore, we can parametrize the effects of new physics by adding to the SM lagrangian higher dimension operators made up of the SM fields. Within the global symmetries of the SM the lowest dimension of the new operators is six, hence we include those dimension-six operators:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2}\mathcal{O}_n \tag{1}$$

where, in general, the dimension-six operators $\mathcal{O}_n$ involve gauge-bosons, Higgs doublets, fermionic fields, and (covariant-) derivatives of these fields. Each operator has a corresponding coupling f_n and Λ is the characteristic energy scale at which new physics (NP) becomes apparent.

It is well known that nonrenormalizable higher dimensional operators give rise to rapid growth of the scattering amplitudes with energy, leading to partial-wave unitarity violation. This fact constrains the energy range where the low energy effective theory is valid once the coefficients f_n are fixed. With this aim in mind in this work we revisit the bounds from partial-wave unitarity on $\mathcal{L}_{\text{eff}}$ arising from vector boson and Higgs boson scattering, as well as inelastic processes $f\bar{f}' \to VV'$ where $f^{(\prime)}$ is a SM fermion and $V^{(\prime)}$ is an electroweak gauge boson.

Previous works in the literature studied the unitarity bounds on some of the dimension-six operators either considering only one non-vanishing coupling at a time, and/or they did not take into account coupled channels, or they worked in the framework of effective vertices [1–7]. Here, we complete these previous analyses by considering the

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effects of coupled channels leading to the strongest constraints, including both elastic and inelastic channels, and also by analyzing the general six-dimensional parameter space of relevant anomalous couplings. Moreover, we only consider contributions up to order $1/\Lambda^2$ to apply systematically the effective field theory approach.

The outline of this article is as follows. We summarize the formalism employed in Sec.II, in particular Sec.II A contains the basis of operators considered, and in Sec.II B we briefly present the standard partial-wave unitarity constraints from elastic gauge boson scattering and inelastic $f\bar{f}' \to VV'$ processes. Section III contains our results from the unitarity analysis and we compare those with the presently allowed range from collider searches. In particular we conclude that, even in the most general case, those operators affecting the Higgs couplings do not violate unitarity for center-of-mass energies $\sqrt{s} \leq 3.2$ TeV within the range presently allowed from global data analysis of Higgs and electroweak data at 90% CL.

II. ANALYSES FRAMEWORK

In this section we present the effective interactions considered in this work, as well as the unitarity relations that we use to constrain them

A. Effective Lagrangian

We parametrize deviations from the Standard Model (SM) in terms of dimension-six effective operators as in Eq. (1). The dimension-six basis contains 59 independent operators, up to flavor and Hermitian conjugation, which are sufficient to generate the most general S-matrix elements given the SM gauge symmetry and that baryon and lepton number symmetries are obeyed by the NP [8]. Exploiting the freedom in the choice of basis, we work in that of Hagiwara, Ishihara, Szalapski, and Zeppenfeld (HISZ) [9, 10].

In what follows we consider bosonic operators relevant to two-to-two scattering processes involving Higgs and/or gauge bosons at tree level, and will impose C- and P-evenness on the operators, which leaves us with ten dimension-six operators. These operators can be classified into three groups according to their field content[1]

- pure gauge operators, in this class there is just one operator

$$\mathcal{O}_{WWW} = \mathrm{Tr}[\widehat{W}_\mu^{\nu}\widehat{W}_\nu^{\rho}\widehat{W}_\rho^{\mu}] \;; \tag{2}$$

- gauge-Higgs operators which include

$$\begin{aligned}
\mathcal{O}_{WW} &= \Phi^\dagger \widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\Phi \;, && (3)\\
\mathcal{O}_{BB} &= \Phi^\dagger \widehat{B}_{\mu\nu}\widehat{B}^{\mu\nu}\Phi \;, && (4)\\
\mathcal{O}_{BW} &= \Phi^\dagger \widehat{B}_{\mu\nu}\widehat{W}^{\mu\nu}\Phi \;, && (5)\\
\mathcal{O}_{W} &= (D_\mu\Phi)^\dagger \widehat{W}^{\mu\nu}(D_\nu\Phi) \;, && (6)\\
\mathcal{O}_{B} &= (D_\mu\Phi)^\dagger \widehat{B}^{\mu\nu}(D_\nu\Phi) \;, && (7)\\
\mathcal{O}_{\Phi,1} &= (D_\mu\Phi)^\dagger \Phi\Phi^\dagger(D^\mu\Phi) \;, && (8)\\
\mathcal{O}_{\Phi,4} &= (D_\mu\Phi)^\dagger (D^\mu\Phi)(\Phi^\dagger\Phi) \;; && (9)
\end{aligned}$$

- and pure Higgs operators:

$$\begin{aligned}
\mathcal{O}_{\Phi,2} &= \frac{1}{2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) \;, && (10)\\
\mathcal{O}_{\Phi,3} &= \frac{1}{3}(\Phi^\dagger\Phi)^3 \;, && (11)
\end{aligned}$$

[1] We do not consider operators with higher derivative kinetic term for the Higgs and gauge bosons. They can be traded by a combination of the operators considered plus some fermionic operators and hence do not lead to new unitarity violating effects in the scattering amplitudes studied here; see [14] for the explicit derivation for the case of the operator with a higher derivative kinetic term for the Higgs.

	VVV	$VVVV$	HVV	$HVVV$	$HHVV$	HHH	$HHHH$	$H\bar{f}f$
$\mathcal{O}_{WWW}$	X	X						
$\mathcal{O}_{WW}$			X	X	X			
$\mathcal{O}_{BB}$			X		X			
$\mathcal{O}_{BW}$	X	X	X	X	X			
$\mathcal{O}_{W}$	X	X	X	X	X			
$\mathcal{O}_{B}$	X		X	X	X			
$\mathcal{O}_{\Phi,1}$	X	X	X		X	X	X	X
$\mathcal{O}_{\Phi,2}$			X		X	X	X	X
$\mathcal{O}_{\Phi,3}$						X	X	
$\mathcal{O}_{\Phi,4}$			X		X	X	X	X

TABLE I: Couplings relevant for our analysis that are modified by the dimension–six operators in Eqs. (2)–(11). Here, V stands for any electroweak gauge boson, H for the Higgs and f for SM fermions.

where Φ stands for the Higgs doublet and we have adopted the notation $\widehat{B}_{\mu\nu} \equiv i(g'/2)B_{\mu\nu}$, $\widehat{W}_{\mu\nu} \equiv i(g/2)\sigma^a W^a_{\mu\nu}$, g with g' being the $SU(2)_L$ and $U(1)_Y$ gauge couplings respectively, and σ^a the Pauli matrices.

The dimension-six operators given in Eqs. (2)–(11) modify the triple and quartic gauge boson couplings, the Higgs couplings to fermions and gauge bosons, and the Higgs self-couplings.; see Table I. Further details are presented in the appendix A. A thorough discussion of the effects of the operators relevant to Higgs physics and anomalous gauge couplings in the basis here employed can be found in [11–13].

We notice first that operators $\mathcal{O}_{BW}$ and $\mathcal{O}_{\phi,1}$ contribute at tree level to the oblique electroweak precision parameters T (or $\Delta\rho$) and S [15–18]

$$\alpha\Delta S = e^2 \frac{v^2}{\Lambda^2} f_{BW} \quad \text{and} \quad \alpha\Delta T = \frac{1}{2}\frac{v^2}{\Lambda^2} f_{\Phi,1} \quad . \tag{12}$$

Taking into account that the present available data impose strong bounds on these parameters [19], the couplings f_{BW} and $f_{\Phi,1}$ are severely constrained, consequently we neglect $\mathcal{O}_{BW}$ and $\mathcal{O}_{\phi,1}$ in our analyses. This leaves us with a basis of 8 operators. Furthermore for large center–of–mass energies ($\sqrt{s}$), which we will take to mean $\sqrt{s} \gg M_{W,Z,H}$ for our analysis, the behavior of $\mathcal{O}_{\phi,2}$ and $\mathcal{O}_{\phi,4}$ is the same up to a sign for the scattering processes considered and as such for our discussion we can quantify their behavior by a single operator coefficient:

$$\frac{f_{\Phi 2,4}}{\Lambda^2} \equiv \frac{f_{\Phi,2} - f_{\Phi,4}}{\Lambda^2} \quad . \tag{13}$$

This is expected since $\mathcal{O}_{\Phi,2} + \mathcal{O}_{\Phi,4}$ can be traded via equations of motion by a combination of Yukawa-like operators which do not contribute to the $2 \to 2$ scattering processes considered.

Additionally $\mathcal{O}_{\Phi,3}$ modifies the Higgs self-couplings as well as the relation between the Higgs mass, its vev and the potential term λ (see Appendix A). However these effects do not induce unitarity violation in the $2 \to 2$ scattering processes.

In summary our study will be carried out in terms of the six relevant operator coefficients f_W, f_B, f_{WW}, f_{BB}, f_{WWW}, and $f_{\Phi 2,4}$.

B. Partial-wave unitarity

In the two-to-two scattering of electroweak gauge bosons (V)

$$V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4} \tag{14}$$

the corresponding helicity amplitude can be expanded in partial waves in the in the center–of–mass system as [20]

$$\mathcal{M}(V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4}) = 16\pi \sum_J \left(J + \frac{1}{2}\right) \sqrt{1 + \delta^{V_{2\lambda_2}}_{V_{1\lambda_1}}} \sqrt{1 + \delta^{V_{4\lambda_4}}_{V_{3\lambda_3}}} d^J_{\lambda\mu}(\theta)\, e^{iM\varphi}\, T^J(V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4}) \ \ , \tag{15}$$

where $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$, $M = \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4$, and θ (φ) is the polar (azimuth) scattering angle. d is the usual Wigner rotation matrix. In the case one of the vector bosons is replaced by the Higgs we can still employ this expression by setting the correspondent λ to zero.

Partial-wave unitarity for the elastic channels requires that

$$|T^J(V_{1\lambda_1}V_{2\lambda_2} \to V_{1\lambda_1}V_{2\lambda_2})| \leq 2 \ , \tag{16}$$

where we have assumed $s \gg (M_{V_1} + M_{V_2})^2$. More stringent bounds can be obtained by diagonalizing T^J in the particle and helicity space and then applying the condition in Eq. (16) to each of the eigenvalues.

We have also studied unitarity constraints from fermion annihilation processes [6]

$$f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4} \ . \tag{17}$$

In this case the partial-wave expansion is given by

$$\mathcal{M}(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4}) = 16\pi \sum_J \left(J + \frac{1}{2}\right) \ \delta_{\sigma_1,-\sigma_2} d^J_{\sigma_1-\sigma_2,\lambda_3-\lambda_4}(\theta) \ T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4}) \ , \tag{18}$$

where, for simplicity, we have set $\varphi = 0$. These processes proceed via s-channel exchange of a $J = 1$ vector boson and therefore in the limit of massless fermions those must appear in opposite helicity states, a condition which is explicitly enforced in the expression above by the inclusion of the term $\delta_{\sigma_1,-\sigma_2}$.

Following the procedure presented in Ref. [6] the unitarity bound on the inelastic production of gauge boson pairs in Eq. (17) is found by relating the corresponding amplitude to that of the elastic process

$$f_{1\sigma_1}\bar{f}_{2\sigma_2} \to f_{1\sigma_1}\bar{f}_{2\sigma_2} \ . \tag{19}$$

In this case the unitarity relation is given by

$$\begin{aligned} 2\mathrm{Im}[T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to f_{1\sigma_1}\bar{f}_{2\sigma_2})] \ &= \ \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to f_{1\sigma_1}\bar{f}_{2\sigma_2})\right|^2 \\ &+ \sum_{V_{3\lambda_3},V_{4\lambda_4}} \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4})\right|^2 + \sum_N \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to N)\right|^2 \ , \end{aligned} \tag{20}$$

where as before we take the limit $s \gg (M_{V_1} + M_{V_2})^2$. N represents any state into which $f_{1\sigma_1}\bar{f}_{2\sigma_2}$ can annihilate which also does not consists of two gauge bosons. Eq. (20) is a quadratic equation for $\mathrm{Im}[T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to f_{1\sigma_1}\bar{f}_{2\sigma_2})]$ which only admits a solution if

$$\sum_{V_{3\lambda_3},V_{4\lambda_4}} \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4})\right|^2 \leq 1 \ . \tag{21}$$

The strongest bound can be found by considering some optimized linear combination

$$|X\rangle = \sum_{f_1,\sigma_1} x_{f_2,\sigma_2}|f_{1\sigma_1}\bar{f}_{2\sigma_2}\rangle \tag{22}$$

with the normalization condition $\sum_{f\sigma}|x_{f\sigma}|^2 = 1$, for which the amplitude $T^J(X \to V_{3\lambda_3}V_{4\lambda_4})$ is largest.

III. RESULTS

Let us start by considering all two-to-two Higgs and electroweak gauge-boson scattering processes. We have calculated the scattering amplitudes for all possible combinations of particles and helicities generated by the SM extended with the dimension-six operators presented in Sec.II A. In doing so we have consistently kept the anomalous terms induced by the dimension-six terms in linear order. It is interesting to notice that to this order there is no amplitude that diverges as s^2. This is a result of gauge invariance enforcing that the corresponding triple and quartic vertices satisfy the requirements for the cancellation of the s^2 terms to take place [21].

All together we find 26 processes (in particle space) which yield some helicity amplitude that grows as s for some of the dimension-six operators while the rest are constant or vanishing at large energies. We give the corresponding expressions of the parts of the amplitudes which grow as s in Tables II–VI. Table II displays the terms in the amplitudes that grow as s at high energies due to the contributions of the operators $\mathcal{O}_{\Phi,4}$ and $\mathcal{O}_{\Phi,2}$. It is interesting to notice that these operators lead to unitarity violation only for the scattering of longitudinal gauge bosons. This is expected as these operators do not generate higher derivative terms beyond those already present in the SM in the triple and quartic couplings. The amplitudes that violate unitarity at high energies due to the presence of $\mathcal{O}_W$

($\mathcal{O}_B$) are presented in Table III (IV), the results for $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$ are contained in Table V, and those for $\mathcal{O}_{WWW}$ are shown in Table VI. As we can see from these tables, for these five operators the growth as s of the amplitudes occurs not only for the scattering of longitudinal gauge bosons but also for transversely polarized ones. Notice also that all amplitudes which grow with s generated by $\mathcal{O}_{\Phi,4}$, $\mathcal{O}_{\Phi,2}$, $\mathcal{O}_W$, $\mathcal{O}_B$, $\mathcal{O}_{WW}$, and $\mathcal{O}_{BB}$ have only $J=0$ or $J=1$ partial-wave projections. $\mathcal{O}_{WWW}$ leads to violation of unitarity also in helicity amplitudes with projections over $J\geq 2$ partial waves. Notwithstanding, as the bounds are weakened for increasing J, we compute the constraints using only the amplitudes in $J=0$ and $J=1$ partial waves.

With the results in Tables II–VI we proceed to build the T^0 and T^1 amplitude matrices in particle and parameter space. These matrices are formed with the s-divergent amplitudes corresponding to all combinations of gauge boson and Higgs pairs with a given total charge $Q=2,1,0$ with possible projections on a given partial wave J which are:

(Q,J)	States	Total
$(2,0)$	$W^+_\pm W^+_\pm$ $W^+_0 W^+_0$	3
$(2,1)$	$W^+_\pm W^+_\pm$ $W^+_\pm W^+_0$ $W^+_0 W^+_\pm$	6
$(1,0)$	$W^+_\pm Z_\pm$ $W^+_0 Z_0$ $W^+_\pm \gamma_\pm$ $W^+_0 H$	6
$(1,1)$	$W^+_0 Z_0$ $W^+_\pm Z_0$ $W^+_0 Z_\pm$ $W^+_\pm Z_\pm$ $W^+_0 \gamma_\pm$ $W^+_\pm \gamma_\pm$ $W^+_0 H$ $W^+_\pm H$	14
$(0,0)$	$W^+_\pm W^-_\pm$ $W^+_0 W^-_0$ $Z_\pm Z_\pm$ $Z_0 Z_0$ $Z_\pm \gamma_\pm$ $\gamma_\pm \gamma_\pm$ $Z_0 H$ HH	12
$(0,1)$	$W^+_0 W^-_0$ $W^+_\pm W^-_0$ $W^+_0 W^-_\pm$ $W^+_\pm W^-_\pm$ $Z_\pm Z_0$ $Z_0 Z_\pm$ $Z_0 \gamma_\pm$ $Z_0 H$ $Z_\pm H$ $\gamma_\pm H$	18

(23)

where upper indices indicate charge and lower indices helicity, and taking into account the relation

$$T^J(V_{1\lambda_1}V_{2\lambda_2}\to V_{3\lambda_3}V_{4\lambda_4}) = (-1)^{\lambda_1-\lambda_2-\lambda_3+\lambda_4} T^J(V_{1-\lambda_1}V_{2-\lambda_2}\to V_{3-\lambda_3}V_{4-\lambda_4}) \ . \tag{24}$$

We present in the right-hand side of Eq. (23) the dimensionality of the corresponding T^J. For example for $Q=2$, T^0 in the basis $\left(W^+_+W^+_+, W^+_0W^+_0, W^+_-W^+_-\right)$ is the 3×3 matrix[2]

$$\frac{1}{8\pi}s\begin{pmatrix} 0 & 0 & \frac{3}{s_W^2}e^4 f_{WWW} \\ 0 & -\frac{3}{8c_W^2}e^2 f_B - \frac{3}{8s_W^2}e^2 f_W - \frac{1}{2}f_{\Phi_{2,4}} & 0 \\ \frac{3}{s_W^2}e^4 f_{WWW} & 0 & 0 \end{pmatrix} \ . \tag{25}$$

In order to obtain the most stringent bounds on the coefficients f_n/Λ^2 we diagonalize the six T^J matrices and impose the constraint Eq. (16) on each of their eigenvalues. We find that there are 50 possible nonzero eigenvalues of the total 59. Considering only one operator different from zero at a time, we find that the strongest constraint arise from the following eigenvalues:

$$\begin{aligned}
\left|\frac{3}{16\pi}\frac{f_{\Phi 2,4}}{\Lambda^2}s\right| &\leq 2 \quad\Rightarrow\quad \left|\frac{f_{\Phi 2,4}}{\Lambda^2}s\right| \leq 33 \ , \\
\left|1.4\frac{g^2}{8\pi}\frac{f_W}{\Lambda^2}s\right| &\leq 2 \quad\Rightarrow\quad \left|\frac{f_W}{\Lambda^2}s\right| \leq 87 \ , \\
\left|\frac{g^2 s_W(\sqrt{9+7c_W^2}+3s_W)}{128 c_W^2\pi}\frac{f_B}{\Lambda^2}s\right| &\leq 2 \quad\Rightarrow\quad \left|\frac{f_B}{\Lambda^2}s\right| \leq 617 \\
\left|\sqrt{\frac{3}{2}}\frac{g^2}{8\pi}\frac{f_{WW}}{\Lambda^2}s\right| &\leq 2 \quad\Rightarrow\quad \left|\frac{f_{WW}}{\Lambda^2}s\right| \leq 99 \ , \\
\left|.20\frac{g^2}{8\pi}\frac{f_{BB}}{\Lambda^2}s\right| &\leq 2 \quad\Rightarrow\quad \left|\frac{f_{BB}}{\Lambda^2}s\right| \leq 603 \ , \\
\left|(1+\sqrt{17-16c_W^2 s_W^2})\frac{3g^4}{32\pi}\frac{f_{WWW}}{\Lambda^2}s\right| &\leq 2 \quad\Rightarrow\quad \left|\frac{f_{WWW}}{\Lambda^2}s\right| \leq 82 \ .
\end{aligned} \tag{26}$$

[2] Notice that we have introduce in Eq. (15) the symmetry factors $\sqrt{1+\delta^{V_2\lambda_2}_{V_1\lambda_1}}$ and $\sqrt{1+\delta^{V_4\lambda_4}_{V_3\lambda_3}}$ in the definition of the corresponding T^J amplitude while in some other conventions they are included in the definition of the two equal gauge boson states.

Next we consider the effects of $\mathcal{O}_W$, $\mathcal{O}_B$, and $\mathcal{O}_{WWW}$ on fermion scattering into gauge bosons pairs. They are due to the induced modification of the triple gauge boson couplings. We considered the total charge $Q = 0$ processes

$$l\bar{l} \to W^+W^- \quad , \quad \nu\bar{\nu} \to W^+W^- \quad \text{and} \quad q\bar{q} \to W^+W^- \quad ,$$

where l (ν, q) stand for SM charged leptons (neutrinos, quarks), as well as the $Q = 1$ reactions

$$l\nu \to W^+Z \quad , \quad q_u\bar{q}_d \to W^+Z \quad , \quad q_u\bar{q}_d \to W^+\gamma \quad , \text{and} \quad l\nu \to W^+\gamma \quad .$$

Taking into account that the operators $\mathcal{O}_W$, $\mathcal{O}_B$, and $\mathcal{O}_{WWW}$ do not give rise to anomalous triple neutral gauge boson vertices we did not consider the $\gamma\gamma$ and ZZ final states.

Table VII contains the unitarity violating terms for the inelastic processes above. As we can see, the operator $\mathcal{O}_{WWW}$ does not contribute to the helicity amplitudes for which $\mathcal{O}_W$ and $\mathcal{O}_B$ do due to their different tensor structures. In order to impose unitarity constraints on these inelastic processes we will follow the procedure described in the previous section [6]; see Eq. (21). We find that strongest constraints can be imposed by using two fermion states in the $Q = 0$ ($V_aV_b = W^+W^-$) combination

$$|x1\rangle = \frac{1}{\sqrt{24}}\left|N_f\left(-e^-_-e^+_+ + \nu_{e-}\bar{\nu}_{e+} + N_c u_-\bar{u}_+ - N_c d_-\bar{d}_+\right)\right\rangle \quad , \tag{27}$$

$$|x2\rangle = \frac{1}{\sqrt{21}}\left|N_f\left(-e^-_+e^+_- + N_c u_+\bar{u}_- - N_c d_+\bar{d}_-\right)\right\rangle \quad , \tag{28}$$

where $N_f = 3$ is the number of generations and $N_C = 3$ the number of colours. They yield the bounds

$$\frac{1}{24}\left[\left|6\frac{g^4}{8\pi}\frac{f_{WWW}}{\Lambda^2}s\right|^2 + \left|1.41\frac{g^2}{8\pi}\frac{f_W}{\Lambda^2}s\right|^2\right] \le 1 \quad \Rightarrow \quad \left|\frac{f_{WWW}}{\Lambda^2}s\right| \le 122 \,, \quad \left|\frac{f_W}{\Lambda^2}s\right| \le 211 \quad , \tag{29}$$

$$\frac{1}{21}\left|\sqrt{2}\frac{s_w^2}{c_w^2}\frac{g^2}{8\pi}\frac{f_B}{\Lambda^2}s\right|^2 = \left|0.053\frac{g^2}{8\pi}\frac{f_B}{\Lambda^2}s\right|^2 \le 1 \quad \Rightarrow \quad \left|\frac{f_B}{\Lambda^2}s\right| \le 664$$

respectively.

Without further information on the parameters f_is/Λ^2, we must consider the case where more than one of the parameters is non-vanishing. Therefore, we should search for the largest allowed value of a given parameter while varying over the others. Technically we obtain these generalized bounds by searching in a six-dimensional grid the widest range of the parameters which satisfy both the elastic and inelastic partial-wave unitarity constraints. We get:

$$\begin{aligned}
\left|\frac{f_{\Phi 2,4}}{\Lambda^2}s\right| &\le 105 \quad , \\
\left|\frac{f_W}{\Lambda^2}s\right| &\le 205 \quad , \\
\left|\frac{f_B}{\Lambda^2}s\right| &\le 640 \quad , \\
\left|\frac{f_{WW}}{\Lambda^2}s\right| &\le 200 \quad , \\
\left|\frac{f_{BB}}{\Lambda^2}s\right| &\le 880 \quad , \\
\left|\frac{f_{WWW}}{\Lambda^2}s\right| &\le 82 \quad .
\end{aligned} \tag{30}$$

It is important to stress that these results do not mean that the largest ranges for each parameter can all simultaneously be realized but rather they are the most conservative constraints on a given parameter allowing for all possible cancellations with the others in the scattering amplitudes.

Comparing the results in Eq. (30) with those in Eqs. (26) and (29) we find that working in the most general six-dimensinal space the bounds become weaker, but not substantially. Thus, even when allowing for all possible cancellations between the contribution of the relevant dimension-six operators, partial-wave unitarity still imposes constraints on their range of validity.

The bounds in Eq. (30) must be understood as providing the maximum center–of–mass energy ($\sqrt{s_{\rm max}}$) for which unitarity holds for a given value of the f_i/Λ^2. One may argue that for not too small values of f_i the bounds in

Eq. (30) correspond to $s_{\rm max}$ larger than Λ^2 for which the quadratic contribution of dimension–six operators to the scattering amplitudes at order $f_i^2(s/\Lambda^2)^2$ can be sizeable and could substantially change the bounds. One must notice, however, that at the same order in $(s/\Lambda^2)^2$ the scattering amplitudes receive linear contributions from dimension–eight operators. Thus, the results in Eq. (30) can be interpreted as the bounds that partial wave unitarity imposes on the effects of dimension–six operators uniquely, irrespective of possible cancellations due to higher order contributions. Furthermore, we expect the appearance of some new state or a strongly interacting phase for center–of–mass energies approaching the bounds in Eq. (30). In this respect, we can also interpret $\sqrt{s_{\rm max}}$ as a generous upper limit for the validity of the description provided by the lowest term of the effective theory.

We can now compare the unitarity constraints in Eq. (30) with the bounds on the corresponding coefficients from the global analysis of the available data from Tevatron and LHC Higgs results as well as from triple anomalous gauge coupling bounds as updated from Ref. [12]. Mapping the allowed ranges at 90%CL of the six dimensional space from that analysis onto the unitarity constraints in Eq. (30) we find the lowest energy for which presently allowed values of the coefficients of operators affecting Higgs physics would lead to unitarity violation. For the operator O_{WWW} which only affects gauge boson self-couplings we can naively estimate the bound by using the presently allowed range on the effective parameter λ_γ [15] from the PDG [22], $\lambda_\gamma = -0.022 \pm 0.019$, which in the framework of the dimension-six operators is related to the coefficient of O_{WWW} by $\lambda_\gamma = \lambda_Z = \frac{3}{2} M_W^2 g^2 \frac{f_{WWW}}{\Lambda^2}$. Altogether we find:

$$
\begin{aligned}
-10 \le \frac{f_{\Phi,2}}{\Lambda^2}(\text{TeV}^{-2}) \le 8.5 \;&\Rightarrow\; \sqrt{s_{\rm max}} \le 3.2 \text{ TeV} \,, \\
-5.6 \le \frac{f_{W}}{\Lambda^2}(\text{TeV}^{-2}) \le 9.6 \;&\Rightarrow\; \sqrt{s_{\rm max}} \le 4.6 \text{ TeV} \,, \\
-29 \le \frac{f_{B}}{\Lambda^2}(\text{TeV}^{-2}) \le 8.9 \;&\Rightarrow\; \sqrt{s_{\rm max}} \le 4.7 \text{ TeV} \,, \\
-3.2 \le \frac{f_{WW}}{\Lambda^2}(\text{TeV}^{-2}) \le 8.2 \;&\Rightarrow\; \sqrt{s_{\rm max}} \le 4.9 \text{ TeV} \,, \\
-7.5 \le \frac{f_{BB}}{\Lambda^2}(\text{TeV}^{-2}) \le 5.3 \;&\Rightarrow\; \sqrt{s_{\rm max}} \le 11 \text{ TeV} \,, \\
-15 \le \frac{f_{WWW}}{\Lambda^2}(\text{TeV}^{-2}) \le 3.9 \;&\Rightarrow\; \sqrt{s_{\rm max}} \le 2.4 \text{ TeV} \,.
\end{aligned}
\tag{31}
$$

In summary, in this work we have consistently derived the partial-wave unitarity bounds on the general space of dimension-six operators affecting Higgs and/or electroweak gauge boson interactions from two-to-two scattering processes including vector boson and Higgs boson scattering channels, as well as inelastic processes $f\bar{f}' \to VV'$ where $f^{(\prime)}$ is a SM fermion and $V^{(\prime)}$ is an electroweak gauge boson. We have found that the relevant set reduces to six operators and gauge invariance enforces that the corresponding amplitudes only diverge as s in the large s limit. The most general bounds obtained in this framework are given in Eq. (30). They can be translated on the maximum center-of-mass energy for which the presently allowed range of the coefficients of the corresponding operators from the analysis of Higgs and gauge-boson data will satisfy partial-wave unitarity. For energies above these limits, it is expected that the interactions become strong [23] or the appearance of new physics. We find that for those operators affecting the Higgs couplings, present 90% constrains from global data analysis of Higgs and electroweak data are such that unitarity is not violated if $\sqrt{s} \le 3.2$ TeV. For the purely gauge-boson operator O_{WWW}, naive translation of the present bounds from triple-gauge boson analysis indicate that within its presently allowed 90% range unitarity can be violated in $f\bar{f}' \to VV'$ at center-of-mass energy $\sqrt{s} \ge 2.4$ TeV.

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Appendix A: Anomalous interactions

Here we present the anomalous interactions that are generated by the dimension–six operators in Eqs. (2)–(11). For simplicity of discussion we make use of the unitary gauge in which the Higgs doublet becomes:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{A1}$$

where v is the Higgs vacuum expectation value (vev).

We first note that $\mathcal{O}_{\phi,1}$, $\mathcal{O}_{\phi,2}$, and $\mathcal{O}_{\phi,4}$ lead to corrections of the kinetic term for the Higgs field, therefore, we make a field redefinition to obtain a canonical form for the kinetic Higgs term:

$$H = h\sqrt{1 + \frac{v^2}{2\Lambda^2}(f_{\Phi,1} + 2f_{\Phi,2} + f_{\Phi,4})}\ , \tag{A2}$$

resulting, together with $\mathcal{O}_{\phi,3}$, in corrections to the Higgs mass given by (to linear order)

$$M_H^2 = 2\lambda v^2 \left(1 - \frac{v^2}{2\Lambda^2}(f_{\Phi,1} + 2f_{\Phi,2} + f_{\Phi,4} + \frac{f_{\Phi,3}}{\lambda})\right)\ , \tag{A3}$$

where λ is the quartic scalar coupling. Additionally $\mathcal{O}_{BW}$ affects $Z\gamma$ mixing giving corrected mass eigenstates of the form:

$$Z_\mu = \left[1 - \frac{g^2 g'^2}{2(g^2+g'^2)}\frac{v^2}{\Lambda^2} f_{BW}\right]^{-1/2} Z_\mu^{SM} \tag{A4}$$

$$A_\mu = \left[1 + \frac{g^2 g'^2}{2(g^2+g'^2)}\frac{v^2}{\Lambda^2} f_{BW}\right]^{-1/2} A_\mu^{SM} - \left[\frac{gg'(g^2-g'^2)}{4(g^2+g'^2)}\frac{v^2}{\Lambda^2} f_{BW}\right] Z_\mu^{SM} \tag{A5}$$

where:

$$Z_\mu^{SM} = \frac{1}{\sqrt{g^2+g'^2}}(gW_\mu^3 - g'B_\mu) \ \text{ and } \ A_\mu^{SM} = \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu)\ . \tag{A6}$$

Furthermore, the operator $\mathcal{O}_{\Phi,4}$ simultaneously affects the W and Z boson masses, while $\mathcal{O}_{\Phi,1}$ and $\mathcal{O}_{BW}$ only affect the Z mass. Again to linear order we have:

$$M_Z^2 = \frac{g^2+g'^2}{4}v^2\left(1+\frac{v^2}{2\Lambda^2}\left(f_{\Phi,1}+f_{\Phi,4}-\frac{g^2g'^2}{g^2+g'^2}f_{BW}\right)\right), \tag{A7}$$

$$M_W^2 = \frac{g^2}{4}v^2\left(1+\frac{v^2}{2\Lambda^2}f_{\Phi,4}\right) \quad . \tag{A8}$$

Notice that in all expressions above v represents the vev of the Higgs field at the minimum of the potential including the effect of $O_{\Phi,3}$.

We will use in our analysis as inputs the measured values of G_F, M_Z, and α, the so called Z-scheme [18], and for convenience we absorb the tree-level renormalization factors mentioned in equations (A7) and (A8) into the measured value of M_W. Through the relation $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$ and equations (A7) and (A8), we obtain the relations:

$$v = (\sqrt{2}G_F)^{-1/2}\left(1-\frac{v^2}{4\Lambda^2}f_{\Phi,4}\right) \quad , \tag{A9}$$

$$M_Z^2 = (\sqrt{2}G_F)^{-1}\frac{g^2}{4c_W^2}\left(1+\frac{v^2}{2\Lambda^2}f_{\Phi,1}-\frac{g^2g'^2}{2(g^2+g'^2)}\frac{v^2}{\Lambda^2}f_{BW}\right) \quad , \tag{A10}$$

where we have introduced the tree level weak mixing angle, $c_W \equiv g/\sqrt{g^2+g'^2}$.

The dimension-six effective operators give rise to triple Higgs-gauge interactions, taking the following forms:

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{HVV} &= g_{H\gamma\gamma}HA_{\mu\nu}A^{\mu\nu} + g^{(1)}_{HZ\gamma}A_{\mu\nu}Z^{\mu}\partial^{\nu}H + g^{(2)}_{HZ\gamma}HA_{\mu\nu}Z^{\mu\nu} \\
&+ g^{(1)}_{HZZ}Z_{\mu\nu}Z^{\mu}\partial^{\nu}H + g^{(2)}_{HZZ}HZ_{\mu\nu}Z^{\mu\nu} + g^{(3)}_{HZZ}HZ_{\mu}Z^{\mu} \\
&+ g^{(1)}_{HWW}(W^+_{\mu\nu}W^{-\mu}\partial^{\nu}H + \text{h.c.}) + g^{(2)}_{HWW}HW^+_{\mu\nu}W^{-\mu\nu} + g^{(3)}_{HWW}HW^+_{\mu}W^{-\mu} \ ,
\end{aligned} \tag{A11}$$

where we have defined $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, for $V = A, Z, W$ and

$$\begin{aligned}
g_{H\gamma\gamma} &= -\left(\frac{g^2vs_W^2}{2\Lambda^2}\right)\frac{f_{BB}+f_{WW}-f_{BW}}{2} \\
g^{(1)}_{HZ\gamma} &= \left(\frac{g^2v}{2\Lambda^2}\right)\frac{s_W(f_W-f_B)}{2c_W} \\
g^{(2)}_{HZ\gamma} &= \left(\frac{g^2v}{2\Lambda^2}\right)\frac{s_W[2s_W^2f_{BB}-2c_W^2f_{WW}+(c_W^2-s_W^2)f_{BW}]}{2c_W} \\
g^{(1)}_{HZZ} &= \left(\frac{g^2v}{2\Lambda^2}\right)\frac{c_W^2f_W+s_W^2f_B}{2c_W^2} \\
g^{(2)}_{HZZ} &= -\left(\frac{g^2v}{2\Lambda^2}\right)\frac{s_W^4f_{BB}+c_W^4f_{WW}+c_W^2s_W^2f_{BW}}{2c_W^2} \\
g^{(3)}_{HZZ} &= \left(\frac{g^2v}{4c_W^2}\right)\left[1+\frac{v^2}{4\Lambda^2}\left(3f_{\Phi,1}+3f_{\Phi,4}-2f_{\Phi,2}-\frac{2g^2g'^2}{(g^2+g'^2)}f_{BW}\right)\right] \\
&= M_Z^2(\sqrt{2}G_F)^{1/2}\left[1+\frac{v^2}{4\Lambda^2}(f_{\Phi,1}+2f_{\Phi,4}-2f_{\Phi,2})\right] \\
g^{(1)}_{HWW} &= \left(\frac{g^2v}{2\Lambda^2}\right)\frac{f_W}{2} \\
g^{(2)}_{HWW} &= -\left(\frac{g^2v}{2\Lambda^2}\right)f_{WW} \\
g^{(3)}_{HWW} &= \left(\frac{g^2v}{2}\right)\left[1+\frac{v^2}{4\Lambda^2}(3f_{\Phi4}-f_{\Phi,1}-2f_{\Phi,2})\right] \\
&= 2M_W^2(\sqrt{2}G_F)^{1/2}\left[1+\frac{v^2}{4\Lambda^2}(2f_{\Phi,4}-f_{\Phi,1}-2f_{\Phi,2})\right]
\end{aligned} \tag{A12}$$

Quartic vertices involving Higgs and gauge bosons read:

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{HHV_1V_2} &= g^{(1)}_{HHWW}H^2W^+_{\mu\nu}W^{-\mu\nu} + g^{(2)}_{HHWW}H(\partial_\nu H)(W^-_\mu W^{+\mu\nu} + \text{h.c.}) \\
&\quad + g^{(3)}_{HHWW}H^2W^+_\mu W^{-\mu} \\
&+ g^{(1)}_{HHZZ}H^2Z_{\mu\nu}Z^{\mu\nu} + g^{(2)}_{HHZZ}HZ_\nu(\partial_\mu H)Z^{\mu\nu} + g^{(3)}_{HHZZ}H^2Z_\mu Z^\mu \\
&+ g^{(1)}_{HHZA}H(\partial_\mu H)Z_\nu A^{\mu\nu} + g^{(2)}_{HHZA}H^2A_{\mu\nu}Z^{\mu\nu} \\
&+ g^{(1)}_{HHAA}H^2A_{\mu\nu}A^{\mu\nu} \ ,
\end{aligned} \tag{A13}$$

with

$$
\begin{aligned}
g^{(1)}_{HHWW} &= -\frac{g^2}{4\Lambda^2} f_{WW} \\
g^{(2)}_{HHWW} &= \frac{g^2}{4\Lambda^2} f_{W} \\
g^{(3)}_{HHWW} &= \frac{g^2}{4}\left[1+\frac{v^2}{2\Lambda^2}(5f_{\Phi,4}-f_{\Phi,1}-2f_{\Phi,2})\right] \\
&= M_W^2\sqrt{2}G_F\left[1+\frac{v^2}{2\Lambda^2}(5f_{\Phi,4}-f_{\Phi,1}-2f_{\Phi,2})\right] \\
g^{(1)}_{HHZZ} &= -\frac{g^2}{8c_W^2\Lambda^2}(c_W^4 f_{WW}+s_W^4 f_{BB}+c_W^2 s_W^2 f_{BW}) \\
g^{(2)}_{HHZZ} &= -\frac{g^2}{4c_W^2\Lambda^2}(c_W^2 f_{W}+s_W^2 f_{B}) \\
g^{(3)}_{HHZZ} &= \frac{g^2}{8c_W^2}\left[1+\frac{v^2}{2\Lambda^2}(5f_{\Phi,1}+5f_{\Phi,4}-2f_{\Phi,2}-\frac{g^2g'^2}{(g^2+g'^2)}f_{BW})\right] \\
&= M_Z^2\sqrt{2}G_F\left[1+\frac{v^2}{2\Lambda^2}(4f_{\Phi,1}+5f_{\Phi,4}-2f_{\Phi,2})\right] \\
g^{(1)}_{HHZA} &= -\frac{g^2 s_W}{4c_W\Lambda^2}(f_W-f_B) \\
g^{(2)}_{HHZA} &= -\frac{g^2 s_W}{4c_W\Lambda^2}(c_W^2 f_{WW}-s_W^2 f_{BB}-\frac{1}{2}(c_W^2-s_W^2)f_{BW}) \\
g^{(1)}_{HHAA} &= -\frac{g^2 s_W^2}{8\Lambda^2}(f_{WW}+f_{BB}-f_{BW})
\end{aligned}
\tag{A14}
$$

and

$$
\begin{aligned}
\mathcal{L}_{\rm eff}^{HV_1V_2V_3} &= g^{(1)}_{HZWW}H(W^-_\mu W^+_\nu-{\rm h.c.})Z^{\mu\nu}+g^{(2)}_{HZWW}HZ_\mu(W^+_\nu W^{-\mu\nu}-{\rm h.c.})+g^{(3)}_{HZWW}(\partial_\mu H)Z_\nu(W^{-\mu}W^{+\nu}-{\rm h.c.}) \\
&+ g^{(1)}_{HAWW}H(W^-_\mu W^+_\nu-{\rm h.c.})A_{\mu\nu}+g^{(2)}_{HAWW}HA_\nu(W^{+\nu}_\mu W^{-\mu}-{\rm h.c.}) \\
&+ g^{(3)}_{HAWW}(\partial_\mu H)A_\nu(W^{-\mu}W^{+\nu}-{\rm h.c.})\ ,
\end{aligned}
\tag{A15}
$$

with

$$
\begin{aligned}
g^{(1)}_{HZWW} &= \frac{ig^3v}{8c_W\Lambda^2}(c_W^2 f_W-s_W^2 f_B+4c_W^2 f_{WW}+2s_W^2 f_{BW}) \\
g^{(2)}_{HZWW} &= -\frac{ig^3v}{4c_W\Lambda^2}(f_W+4c_W^2 f_{WW}) \\
g^{(3)}_{HZWW} &= \frac{ig^3v}{4c_W\Lambda^2}s_W^2 f_W \\
g^{(1)}_{HAWW} &= \frac{ig^3vs_W}{8\Lambda^2}(f_W+f_B+4f_{WW}-2f_{BW}) \\
g^{(2)}_{HAWW} &= -\frac{ig^3s_Wv}{\Lambda^2}f_{WW} \\
g^{(3)}_{HAWW} &= -\frac{ig^3vs_W}{4\Lambda^2}f_W
\end{aligned}
\tag{A16}
$$

Triple gauge boson couplings are:

$$
\begin{aligned}
\mathcal{L}_{\rm eff}^{WWV} &= g^{(1)}_{WWZ}(W^+_\nu W^-_\mu-{\rm h.c.})Z^{\mu\nu}+g^{(2)}_{WWZ}(W^+_{\mu\nu}W^{-\mu}Z^\nu-{\rm h.c.})+g^{(3)}_{WWZ}(W^+_{\mu\nu}W^{-\nu}_\rho-{\rm h.c.})Z^{\rho\mu} \\
&+ g^{(1)}_{WWA}(W^+_\nu W^-_\mu-{\rm h.c.})A^{\mu\nu}+g^{(2)}_{WWA}(W^+_{\mu\nu}W^{-\nu}_\rho-{\rm h.c.})A^{\rho\mu}\ ,
\end{aligned}
\tag{A17}
$$

where

$$
\begin{aligned}
g^{(1)}_{WWZ} &= \frac{ig^3v^2c_W}{16\Lambda^2}(f_W+\frac{s_W^2}{c_W^2}f_B+\frac{4s_W^2}{c_{2W}}f_{BW}-\frac{2s_W^2}{e^2c_{2W}}f_{\Phi,1})\equiv\frac{igc_W}{2}\Delta\kappa_Z \\
g^{(2)}_{WWZ} &= -\frac{ig^3v^2}{8c_W\Lambda^2}(f_W+\frac{2s_W^2}{c_{2W}}f_{BW}-\frac{s_{2W}^2}{2e^2c_{2W}}f_{\Phi,1})\equiv-igc_W\Delta g_Z \\
g^{(3)}_{WWZ} &= -\frac{3ig^3c_W\Lambda^2}{2}f_{WWW}\equiv\frac{-igc_W}{M_W^2}\lambda_Z \\
g^{(1)}_{WWA} &= \frac{ig^3v^2s_W}{16\Lambda^2}(f_W+f_B-2f_{BW})\equiv\frac{igs_W}{2}\Delta\kappa_\gamma \\
g^{(2)}_{WWA} &= -\frac{3ig^3s_W}{2\Lambda^2}f_{WWW}\equiv\frac{-igs_W}{M_W^2}\lambda_\gamma
\end{aligned}
\tag{A18}
$$

where we have defined $c_{2W}=\cos(2\theta_w)$ and $s_{2W}=\sin(2\theta_w)$.

Quartic gauge boson vertices read:

$$
\begin{aligned}
\mathcal{L}_{\rm eff}^{WWV_1V_2} &= g^{(1)}_{WWWW}W^-_\mu W^+_\nu(W^{-\mu}W^{+\nu}-{\rm h.c.})+g^{(2)}_{WWWW}W^+_{\mu\nu}W^{-\nu\rho}(W^{+\mu}W^-_\rho-W^+_\rho W^{-\mu}) \\
&+ g^{(1)}_{WWZZ}Z_\mu Z^\mu W^+_\nu W^-_\nu+g^{(2)}_{WWZZ}Z_\mu Z_\nu(W^+_\nu W^-_\mu+{\rm h.c.})+g^{(3)}_{WWZZ}\left(W^+_{\mu\nu}Z_\rho{}^\mu(Z^\nu W^{-\rho}-Z^\rho W^{-\nu})+{\rm h.c.}\right) \\
&+ g^{(3)}_{WWAA}\left(W^+_{\mu\nu}A_\rho{}^\mu(A^\nu W^{-\rho}-A^\rho W^{-\nu})+{\rm h.c.}\right) \\
&+ g^{(1)}_{WWZA}W^-_\mu W^{+\mu}Z_\mu A^\mu+g^{(2)}_{WWZA}(W^-_\nu W^+_\mu+{\rm h.c.})A^\nu Z^\mu \\
&+ g^{(3)}_{WWZA}\left(W^+_{\mu\nu}Z_\rho{}^\mu(A^\nu W^{-\rho}-A^\rho W^{-\nu})+W^+_{\mu\nu}A_\rho{}^\mu(Z^\nu W^{-\rho}-Z^\rho W^{-\nu})+{\rm h.c.}\right)
\end{aligned}
\tag{A19}
$$

with

$$
\begin{aligned}
g^{(1)}_{WWWW} &= \frac{e^2}{2s^2_W} + \frac{g^4v^2}{8\Lambda^2}(f_W + 2\frac{s^2_W}{c_{2W}}f_{BW} - \frac{s^2_{2W}}{2c_{2W}e^2}f_{\Phi,1}) \\
g^{(2)}_{WWWW} &= \frac{-3g^4}{2\Lambda^2}f_{WWW} \\
g^{(1)}_{WWZZ} &= -e^2\frac{c^2_W}{s^2_W} - \frac{g^4v^2\Lambda^2}{4c^2_W}(c^2_W f_W + \frac{2s^2_{2W}}{c_{2W}}f_{BW} - \frac{s^2_{2W}c^2_W}{2e^2c_{2W}}f_{\Phi,1}) \\
g^{(2)}_{WWZZ} &= \frac{e^2c^2_W}{2s^2_W} + \frac{g^4v^2\Lambda^2}{8c^2_W}(c^2_W f_W + \frac{s^2_{2W}}{2c_{2W}}f_{BW} - \frac{s^2_{2W}c^2_W}{2e^2c_{2W}}f_{\Phi,1}) \\
g^{(3)}_{WWZZ} &= \frac{-3g^4v^2c^2_W}{2\Lambda^2}f_{WWW} \\
g^{(3)}_{WWAA} &= -\frac{3g^4v^2s^2_W}{2\Lambda^2}f_{WWW} \\
g^{(1)}_{WWZA} &= -e^2 - \frac{g^4v^2s_W}{4c_W\Lambda^2}(f_W + 2\frac{s^2_W}{c_{2W}}f_{BW} - \frac{s^2_{2W}}{2c_{2W}e^2}f_{\Phi,1}) \\
g^{(2)}_{WWZA} &= \frac{e^2}{2} + \frac{g^4v^2s_W}{8c_W\Lambda^2}(f_W + 2\frac{s^2_W}{c_{2W}}f_{BW} - \frac{s^2_{2W}}{2c_{2W}e^2}f_{\Phi,1}) \\
g^{(3)}_{WWZA} &= \frac{-3g^4s_Wc_W}{2\Lambda^2}f_{WWW}
\end{aligned}
\tag{A20}
$$

Finally Higgs self interactions take the form:

$$\mathcal{L}^{HHH}_{\text{eff}} = g^{(1)}_{HHH}H^3 + g^{(2)}_{HHH}H(\partial_\mu H)(\partial^\mu H)\ , \tag{A21}$$

$$\tag{A22}$$

$$\mathcal{L}^{HHHH}_{\text{eff}} = g^{(1)}_{HHHH}H^4 + g^{(2)}_{HHHH}H^2(\partial_\mu H)(\partial^\mu H)\ , \tag{A23}$$

where

$$
\begin{aligned}
g^{(1)}_{HHH} &= -\lambda v + \frac{v^3}{\Lambda^2}(\frac{3\lambda}{4}f_{\Phi,1} + \frac{5}{6}f_{\Phi,3} + \frac{3\lambda}{2}f_{\Phi,2} + \frac{3\lambda}{4}f_{\Phi,4}) \\
&= -\frac{M^2_H}{2}(\sqrt{2}G_F)^{1/2}\left[1 - \frac{v^2}{4\Lambda^2}(f_{\Phi,1} + 2f_{\Phi,2}\frac{4}{3\lambda}f_{\Phi,3})\right] \\
g^{(2)}_{HHH} &= \frac{v}{\Lambda^2}(\frac{1}{2}f_{\Phi,1} + f_{\Phi,2} + \frac{1}{2}f_{\Phi,4}) \\
g^{(1)}_{HHHH} &= -\frac{\lambda}{4} + \frac{v^2}{4\Lambda^2}(\lambda f_{\Phi,1} + \frac{5}{2}f_{\Phi,3} + 2\lambda f_{\Phi,2} + \lambda f_{\Phi,4}) \\
&= -\frac{M^2_H}{8}(\sqrt{2}G_F)\left[1 + \frac{v^2}{2\Lambda^2}(f_{\Phi,1} + \frac{4}{\lambda}f_{\Phi,3} + f_{\Phi,2})\right] \\
g^{(2)}_{HHHH} &= \frac{1}{4\Lambda^2}(f_{\Phi,1} + 2f_{\Phi,2} + f_{\Phi,4})
\end{aligned}
\tag{A24}
$$

Appendix B: Helicity Amplitudes

We present here the list of unitarity violating amplitudes for all the $2 \to 2$ scattering processes considered in the evaluation of the unitarity constraints.

	$(\times \frac{f_{\Phi,2,4}}{\Lambda^2} \times s)$
$W^+W^+ \to W^+W^+$	-1
$W^+Z \to W^+Z$	$-\frac{1}{2}X$
$W^+H \to W^+H$	$-\frac{1}{2}X$
$W^+W^- \to W^+W^-$	$\frac{1}{2}Y$
$W^+W^- \to ZZ$	1
$W^+W^- \to HH$	-1
$ZZ \to HH$	-1
$ZH \to ZH$	$-\frac{1}{2}X$

TABLE II: Unitarity violating (growing as s) terms of the scattering amplitudes $\mathcal{M}(V_{1\lambda_1}V_{2\lambda_2} \to V_{3\lambda_3}V_{4\lambda_4})$ for longitudinal gauge bosons generated by the operators $\mathcal{O}_{\Phi,2}$ and $\mathcal{O}_{\Phi,4}$ where $X = 1 - \cos\theta$ and $Y = 1 + \cos\theta$. The overall factor extracted from all amplitudes is given at the top of the table.

	$(\times e^2 \frac{f_W}{\Lambda^2} \times s)$						
	0000	$00++$	$0+0-$	$0+-0$	$+00-$	$+0-0$	$++00$
$W^+W^+ \to W^+W^+$	$-\frac{3}{4s_W^2}$	0	$\frac{1}{8s_W^2}X$	$-\frac{1}{8s_W^2}Y$	$-\frac{1}{8s_W^2}Y$	$\frac{1}{8s_W^2}X$	0
$W^+Z \to W^+Z$	$-\frac{3}{8s_W^2}X$	$-\frac{1}{8c_W}$	$\frac{c_W^2-s_W^2}{8s_W^2}X$	$-\frac{1}{16c_W}Y$	$-\frac{1}{16c_W}Y$	$\frac{1}{8c_W^2}X$	$-\frac{1}{8c_W}$
$W^+\gamma \to W^+\gamma$	–	–	$\frac{1}{4}X$	–	–	–	–
$W^+Z \to W^+\gamma$	–	$\frac{1}{8s_W}$	$\frac{(3c_W^2-s_W^2)}{16c_Ws_W}X$	–	$\frac{1}{16s_W}Y$	–	–
$W^+Z \to W^+H$	0	–	–	$-\frac{1}{16c_W}Y$	–	0	$\frac{1}{8c_W}$
$W^+\gamma \to W^+H$	–	–	–	$\frac{1}{16s_W}Y$	–	–	$-\frac{1}{8s_W}$
$W^+H \to W^+H$	$-\frac{3}{8s_W^2}X$	–	–	–	–	$\frac{1}{8s_W^2}X$	–
$W^+W^- \to W^+W^-$	$\frac{3}{8s_W^2}Y$	$-\frac{1}{4s_W^2}$	$\frac{1}{8s_W^2}X$	0	0	$\frac{1}{8s_W^2}X$	$-\frac{1}{4s_W^2}$
$W^+W^- \to ZZ$	$\frac{3}{4s_W^2}$	$\frac{s_W^2-c_W^2}{4s_W^2}$	$\frac{1}{16c_W}X$	$-\frac{1}{16c_W}Y$	$-\frac{1}{16c_W}Y$	$\frac{1}{16c_W}X$	$-\frac{1}{4s_W^2}$
$W^+W^- \to \gamma\gamma$	–	$-\frac{1}{2}$	–	–	–	–	–
$W^+W^- \to Z\gamma$	–	$\frac{1-4c_W^2}{8c_Ws_W}$	$-\frac{1}{16s_W}X$	–	$\frac{1}{16s_W}Y$	–	–
$W^+W^- \to ZH$	0	–	–	$-\frac{1}{16c_W}Y$	–	$-\frac{1}{16c_W}X$	0
$W^+W^- \to \gamma H$	–	0	–	$\frac{1}{16s_W}Y$	–	$\frac{1}{16s_W}X$	–
$W^+W^- \to HH$	$-\frac{3}{4s_W^2}$	–	–	–	–	–	$\frac{1}{4s_W^2}$
$ZZ \to ZZ$	0	$-\frac{1}{4s_W^2}$	$\frac{1}{8s_W^2}X$	$-\frac{1}{8s_W^2}Y$	$-\frac{1}{8s_W^2}Y$	$\frac{1}{8s_W^2}X$	$-\frac{1}{4s_W^2}$
$ZZ \to Z\gamma$	–	$-\frac{1}{8c_Ws_W}$	$\frac{1}{16c_Ws_W}X$	–	$-\frac{1}{16c_Ws_W}Y$	–	–
$ZZ \to HH$	$-\frac{3}{4s_W^2}$	–	–	–	–	–	$\frac{1}{4s_W^2}$
$Z\gamma \to ZZ$	–	–	$\frac{1}{16c_Ws_W}X$	$-\frac{1}{16c_Ws_W}Y$	–	–	$-\frac{1}{8s_Wc_W}$
$Z\gamma \to HH$	–	–	–	–	–	–	$\frac{1}{8s_Wc_W}$
$ZH \to ZH$	$-\frac{3}{8s_W^2}X$	–	–	–	–	$\frac{1}{8s_W^2}X$	–
$ZH \to \gamma H$	–	–	–	–	–	$\frac{1}{16c_Ws_W}X$	–

TABLE III: Unitarity violating (growing as s) terms of the scattering amplitudes $\mathcal{M}(V_{1\lambda_1}V_{2\lambda_2} \to V_{3\lambda_3}V_{4\lambda_4})$ for gauge bosons with the helicities $\lambda_1\lambda_2\lambda_2\lambda_4$ listed on top of each column, generated by the operator $\mathcal{O}_W$. $X = 1 - \cos\theta$ and $Y = 1 + \cos\theta$. The overall factor extracted from all amplitudes is given on the top of the table. An entry marked as 0 means that there is no s growth for the amplitude, while we denote as $-$ an amplitude that does not exist.

	$(\times e^2 \frac{f_B}{\Lambda^2}) \times s$						
	0000	$00++$	$0+0-$	$0+-0$	$+00-$	$+0-0$	$++00$
$W^+W^+ \to W^+W^+$	$-\frac{3}{4c_W^2}$	0	0	0	0	0	0
$W^+Z \to W^+Z$	0	$-\frac{1}{8c_W}$	$\frac{s_W^2-c_W^2}{8c_W^2}X$	$-\frac{1}{16c_W}Y$	$-\frac{1}{16c_W}Y$	0	$-\frac{1}{8c_W}$
$W^+\gamma \to W^+\gamma$	$-$	$-$	$\frac{1}{4}X$	$-$	$-$	$-$	$-$
$W^+Z \to W^+\gamma$	$-$	$\frac{1}{8s_W}$	$\frac{c_W^2-3s_W^2}{16s_W c_W}X$	$-$	$\frac{1}{16s_W}Y$	$-$	$-$
$W^+Z \to W^+H$	$-\frac{2+Y}{8c_W^2}$	$-$	$-$	$-\frac{1}{16c_W}Y$	$-$	0	$\frac{1}{8c_W}$
$W^+\gamma \to W^+H$	$-$	$-$	$-$	$\frac{1}{16s_W}Y$	$-$	$-$	$-\frac{1}{8s_W}$
$W^+W^- \to W^+W^-$	$\frac{3}{8c_W^2}Y$	0	0	0	0	0	0
$W^+W^- \to ZZ$	0	$\frac{c_W^2-s_W^2}{4c_W^2}$	$\frac{1}{16c_W}X$	$-\frac{1}{16c_W}Y$	$-\frac{1}{16c_W}Y$	$\frac{1}{16c_W}X$	0
$W^+W^- \to \gamma\gamma$	$-$	$-\frac{1}{2}$	$-$	$-$	$-$	$-$	$-$
$W^+W^- \to Z\gamma$	$-$	$\frac{3-4c_W^2}{8c_W s_W}$	$-\frac{1}{16s_W}X$	$-$	$\frac{1}{16s_W}Y$	$-$	$-$
$W^+W^- \to ZH$	$\frac{1-Y}{4c_W^2}$	$-$	$-$	$-\frac{1}{16c_W}Y$	$-$	$-\frac{1}{16c_W}X$	0
$W^+W^- \to \gamma H$	$-$	0	$-$	$\frac{1}{16s_W}Y$	$-$	$\frac{1}{16s_W}X$	$-$
$ZZ \to ZZ$	0	$-\frac{1}{4c_W^2}$	$\frac{1}{8c_W^2}X$	$-\frac{1}{8c_W^2}Y$	$-\frac{1}{8c_W^2}Y$	$\frac{1}{8c_W^2}X$	$-\frac{1}{4c_W^2}$
$ZZ \to Z\gamma$	$-$	$\frac{1}{8c_W s_W}$	$-\frac{1}{16c_W s_W}X$	$-$	$\frac{1}{16c_W s_W}Y$	$-$	$-$
$ZZ \to HH$	$-\frac{3}{4c_W^2}$	$-$	$-$	$-$	$-$	$-$	$\frac{1}{4c_W^2}$
$Z\gamma \to ZZ$	$-$	$-$	$-\frac{1}{16c_W s_W}X$	$\frac{1}{16c_W s_W}Y$	$-$	$-$	$\frac{1}{8s_W c_W}$
$Z\gamma \to HH$	$-$	$-$	$-$	$-$	$-$	$-$	$-\frac{1}{8s_W c_W}$
$ZH \to ZH$	$-\frac{3}{8c_W^2}X$	$-$	$-$	$-$	$-$	$\frac{1}{8c_W^2}X$	$-$
$ZH \to \gamma H$	$-$	$-$	$-$	$-$	$-$	$-\frac{1}{16c_W s_W}X$	$-$

TABLE IV: Same as Table III for the operator $\mathcal{O}_B$.

	$(\times e^2 \frac{f_{WW}}{\Lambda^2} \times s)$						$(\times e^2 \frac{f_{BB}}{\Lambda^2}) \times s$					
	$00++$	$0+0-$	$0+-0$	$+00-$	$+0-0$	$++00$	$00++$	$0+0-$	$0+-0$	$+00-$	$+0-0$	$++00$
$W^+W^+ \to W^+W^+$	0	$-\frac{1}{4s_W^2}X$	$\frac{1}{4s_W^2}Y$	$\frac{1}{4s_W^2}Y$	$-\frac{1}{4s_W^2}X$	0	0	0	0	0	0	0
$W^+Z \to W^+Z$	0	$-\frac{c_W^2}{4s_W^2}X$	0	0	$-\frac{1}{4s_W^2}X$	0	0	$-\frac{s_W^2}{4c_W^2}X$	0	0	0	0
$W^+\gamma \to W^+\gamma$	–	$-\frac{1}{4}X$	–	–	–	–	–	$-\frac{1}{4}X$	–	–	–	–
$W^+Z \to W^+\gamma$	0	$-\frac{c_W}{4s_W}X$	–	0	–	–	0	$\frac{s_W}{4c_W}X$	–	0	–	–
$W^+H \to W^+H$	–	–	–	–	$-\frac{1}{4s_W^2}X$	–	–	–	–	–	0	–
$W^+W^- \to W^+W^-$	$\frac{1}{2s_W^2}$	$-\frac{1}{4s_W^2}X$	0	0	$-\frac{1}{4s_W^2}X$	$\frac{1}{2s_W^2}$	0	0	0	0	0	0
$W^+W^- \to ZZ$	$\frac{c_W^2}{2s_W^2}$	0	0	0	0	$\frac{1}{2s_W^2}$	$\frac{s_W^2}{4c_W^2}$	0	0	0	0	0
$W^+W^- \to \gamma\gamma$	$\frac{1}{2}$	–	–	–	–	–	$\frac{1}{2}$	–	–	–	–	–
$W^+W^- \to Z\gamma$	$\frac{c_W}{2s_W}$	0	–	0	–	–	$-\frac{s_W}{2c_W}$	0	–	0	–	–
$W^+W^- \to HH$	–	–	–	–	–	$-\frac{1}{2s_W^2}$	–	–	–	–	–	0
$ZZ \to ZZ$	$\frac{c_W^2}{2s_W^2}$	$-\frac{c_W^2}{4s_W^2}X$	$\frac{c_W^2}{4s_W^2}Y$	$\frac{c_W^2}{4s_W^2}Y$	$-\frac{c_W^2}{4s_W^2}X$	$\frac{c_W^2}{2s_W^2}$	$\frac{s_W^2}{2c_W^2}$	$-\frac{s_W^2}{4c_W^2}X$	$\frac{s_W^2}{4c_W^2}Y$	$\frac{s_W^2}{4c_W^2}Y$	$-\frac{s_W^2}{4c_W^2}X$	$\frac{s_W^2}{2c_W^2}$
$ZZ \to \gamma\gamma$	$\frac{1}{2}$	–	–	–	–	–	$\frac{1}{2}$	–	–	–	–	–
$ZZ \to Z\gamma$	$\frac{c_W}{2s_W}$	$-\frac{c_W}{4s_W}X$	–	$\frac{c_W}{4s_W}Y$	–	–	$-\frac{s_W}{2c_W}$	$\frac{s_W}{4c_W}X$	–	$-\frac{s_W}{4c_W}Y$	–	–
$ZZ \to HH$	–	–	–	–	–	$-\frac{c_W^2}{2s_W^2}$	–	–	–	–	–	$-\frac{s_W^2}{2c_W^2}$
$Z\gamma \to ZZ$	–	$-\frac{c_W}{4s_W}X$	$\frac{c_W}{4s_W}Y$	–	–	$\frac{c_W}{2s_W}$	–	$\frac{s_W}{4c_W}X$	$-\frac{s_W}{4c_W}Y$	–	–	$-\frac{s_W}{2c_W}$
$Z\gamma \to Z\gamma$	–	$-\frac{1}{4}X$	–	–	–	–	–	$-\frac{1}{4}X$	–	–	–	–
$Z\gamma \to HH$	–	–	–	–	–	$-\frac{c_W}{2s_W}$	–	–	–	–	–	$\frac{s_W}{2c_W}$
$\gamma\gamma \to HH$	–	–	–	–	–	$-\frac{1}{2}$	–	–	–	–	–	$-\frac{1}{2}$
$ZH \to ZH$	–	–	–	–	$-\frac{c_W^2}{4s_W^2}X$	–	–	–	–	–	$-\frac{s_W^2}{4c_W^2}X$	–
$\gamma H \to \gamma H$	–	–	–	–	$-\frac{1}{4}X$	–	–	–	–	–	$-\frac{1}{4}X$	–
$ZH \to \gamma H$	–	–	–	–	$-\frac{c_W}{4s_W}X$	–	–	–	–	–	$\frac{s_W}{4c_W}X$	–

TABLE V: Same as Table III for the operators $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$.

	$(\times 2e^4 \frac{f_{WWW}}{\Lambda^2} \times s)$							
	$00++$	$0+0-$	$0+-0$	$+00-$	$+0-0$	$++00$	$+++-$ $+---$ $+-++$ $++-+$	$++--$
$W^+W^+ \to W^+W^+$	0	$-\frac{3(2+Y)}{32s_W^4}$	$\frac{3(2+X)}{32s_W^4}$	$\frac{3(2+X)}{32s_W^4}$	$-\frac{3(2+Y)}{32s_W^4}$	0	$-\frac{3}{4s_W^4}$	$\frac{3}{2s_W^4}$
$W^+Z \to W^+Z$	$\frac{3(Y-X)c_W}{32s_W^4}$	0	$\frac{3(X+2)c_W}{32s_W^4}$	$\frac{3(X+2)c_W}{32s_W^4}$	0	$\frac{3(Y-X)c_W}{32s_W^4}$	$-\frac{3c_W^2}{8s_W^4}X$	$\frac{3c_W^2}{4s_W^4}X$
$W^+\gamma \to W^+\gamma$	–	0	–	–	–	–	$-\frac{3}{8s_W^2}X$	$\frac{3}{4s_W^2}X$
$W^+Z \to W^+\gamma$	$-\frac{3(Y-X)}{32s_W^3}$	0	–	$\frac{3(X+2)}{32s_W^3}$	–	–	$-\frac{3c_W}{8s_W^3}X$	$\frac{3c_W}{4s_W^3}X$
$W^+Z \to W^+H$	–	–	$\frac{3(X+2)c_W}{32s_W^4}$	–	$\frac{3(2+Y)}{32s_W^4}$	$-\frac{3(Y-X)c_W}{32s_W^4}$	–	–
$W^+\gamma \to W^+H$	–	–	$\frac{3(X+2)}{32s_W^3}$	–	–	$-\frac{3(Y-X)}{32s_W^3}$	–	–
$W^+W^- \to W^+W^-$	$\frac{3(Y-X)}{32s_W^4}$	$\frac{3(2+Y)}{32s_W^4}$	0	0	$\frac{3(2+Y)}{32s_W^4}$	$\frac{3(Y-X)}{32s_W^4}$	$\frac{3}{8s_W^4}Y$	$-\frac{3}{4s_W^4}Y$
$W^+W^- \to ZZ$	0	$\frac{3(2+Y)c_W}{32s_W^4}$	$-\frac{3(X+2)c_W}{32s_W^4}$	$-\frac{3(X+2)c_W}{32s_W^4}$	$\frac{3(2+Y)c_W}{32s_W^4}$	0	$\frac{3c_W^2}{4s_W^4}$	$-\frac{3c_W^2}{2s_W^4}$
$W^+W^- \to \gamma\gamma$	0	–	–	–	–	–	$\frac{3}{4s_W^2}$	$-\frac{3}{2s_W^2}$
$W^+W^- \to Z\gamma$	0	$\frac{3(2+Y)}{32s_W^3}$	–	$-\frac{3(2+X)}{32s_W^3}$	–	–	$\frac{3c_W}{4s_W^3}$	$-\frac{3c_W}{2s_W^3}$
$W^+W^- \to ZH$	–	–	$-\frac{3(2+X)c_W}{32s_W^4}$	–	$-\frac{3(2+Y)c_W}{32s_W^4}$	$\frac{3(Y-X)}{32s_W^4}$	–	–
$W^+W^- \to \gamma H$	–	–	$-\frac{3(X+2)}{32s_W^3}$	–	$-\frac{3(2+Y)}{32s_W^3}$	–	–	–

TABLE VI: Same as Table III for the operator $\mathcal{O}_{WWW}$.

Process	$\sigma_1, \sigma_2, \lambda_3, \lambda_4$	Amplitude
$e^+e^- \to W^-W^+$	$-+00$	$-\frac{ig^2 s\sin\theta}{8}\frac{\mathrm{c}_\mathrm{W}^2 f_\mathrm{W}+\mathrm{s}_\mathrm{W}^2 f_\mathrm{B}}{\mathrm{c}_\mathrm{W}^2\Lambda^2}$
	$+-00$	$-\frac{ig^2 s\sin\theta}{4}\frac{\mathrm{s}_\mathrm{W}^2 f_\mathrm{B}}{\mathrm{c}_\mathrm{W}^2\Lambda^2}$
	$-+--$	$-\frac{3ig^4 s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
	$-+++$	$-\frac{3ig^4 s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
$\nu\bar{\nu} \to W^-W^+$:	$-+00$	$\frac{ig^2 s\sin\theta}{8}\frac{\mathrm{c}_\mathrm{W}^2 f_\mathrm{W}-\mathrm{s}_\mathrm{W}^2 f_\mathrm{B}}{\mathrm{c}_\mathrm{W}^2}$
	$+-00$	0
	$-+--$	$\frac{3ig^4 s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
	$-+++$	$\frac{3ig^4 s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
$u\bar{u} \to W^-W^+$	$-+00$	$\frac{ig^2 \mathrm{N_c} s\sin\theta}{8}\frac{3\mathrm{c}_\mathrm{W}^2 f_\mathrm{W}+\mathrm{s}_\mathrm{W}^2 f_\mathrm{B}}{3\mathrm{c}_\mathrm{W}^2}$
	$+-00$	$\frac{ig^2 \mathrm{N_c} s\sin\theta}{6}\frac{\mathrm{s}_\mathrm{W}^2}{\mathrm{c}_\mathrm{W}^2} f_\mathrm{B}$
	$-+--$	$\frac{3ig^4 \mathrm{N_c} s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
	$-+++$	$\frac{3ig^4 \mathrm{N_c} s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
$d\bar{d} \to W^-W^+$	$-+00$	$-\frac{ig^2 \mathrm{N_c} s\sin\theta}{8}\frac{3\mathrm{c}_\mathrm{W}^2 f_\mathrm{W}-\mathrm{s}_\mathrm{W}^2 f_\mathrm{B}}{3\mathrm{c}_\mathrm{W}^2}$
	$+-00$	$-\frac{ig^2 \mathrm{N_c} s\sin\theta}{12}\frac{\mathrm{s}_\mathrm{W}^2 f_\mathrm{B}}{\mathrm{c}_\mathrm{W}^2\Lambda^2}$
	$-+--$	$-\frac{3ig^4 \mathrm{N_c} s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
	$-+++$	$-\frac{3ig^4 \mathrm{N_c} s\sin\theta}{8}\frac{f_{WWW}}{\Lambda^2}$
$e^+\bar{\nu} \to W^+Z$	$-+00$	$\frac{ig^2 s\sin\theta}{4\sqrt{2}}\frac{f_\mathrm{W}}{\Lambda^2}$
	$+-00$	0
	$-+--$	$\frac{3i\mathrm{c}_\mathrm{W} g^4 s\sin\theta}{4\sqrt{2}}\frac{f_{WWW}}{\Lambda^2}$
	$-+++$	$\frac{3i\mathrm{c}_\mathrm{W} g^4 s\sin\theta}{4\sqrt{2}}\frac{f_{WWW}}{\Lambda^2}$
$e^+\bar{\nu} \to W^+A$:	$-+00$	0
	$+-00$	0
	$-+--$	$\frac{3i\mathrm{s}_\mathrm{W} g^4 s\sin\theta}{4\sqrt{2}}\frac{f_{WWW}}{\Lambda^2}$
	$-+++$	$\frac{3i\mathrm{s}_\mathrm{W} g^4 s\sin\theta}{4\sqrt{2}}\frac{f_{WWW}}{\Lambda^2}$

TABLE VII: Unitarity violating (growing as s) terms of the scattering amplitudes $\mathcal{M}(f_{1\sigma_1}\bar{f}_{2\lambda_2} \to V_{3\lambda_3}V_{4\lambda_4})$ for fermions and gauge bosons with the helicities $\sigma_1\sigma_2\lambda_3\lambda_4$ given in the second column.