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No Firewalls or Information Problem for Black Holes Entangled with Large Systems

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We discuss how under certain conditions the black hole information puzzle and the (related) arguments that firewalls are a typical feature of black holes can break down. We first review the arguments of AMPS favoring firewalls, focusing on entanglements in a simple toy model for a black hole and the Hawking radiation. By introducing a large and inaccessible system entangled with the black hole (representing perhaps a de Sitter stretched horizon or inaccessible part of a landscape) we show complementarity can be restored and firewalls can be avoided throughout the black hole's evolution. Under these conditions black holes do not have an "information problem". We point out flaws in some of our earlier arguments that such entanglement might be generically present in some cosmological scenarios, and call out certain ways our picture may still be realized.

I. INTRODUCTION AND BACKGROUND

The prediction that black holes emit Hawking Radiation led to the understanding that black holes weren't eternal and would eventually evaporate [1]. This presented an apparent contradiction between quantum mechanics and general relativity with respect to the information stored within the black hole [2, 3]. The so-called "information paradox" concerns the information encoded in the initial quantum state of whatever collapsed to form the black hole (as well as anything that passed through the black hole's horizon). From the no-hair theorem, the properties of the black hole are independent of the details of the collapse. This would suggest that the Hawking Radiation which depends on properties of the black hole horizon would carry with it no information from the initial quantum state. Once the black hole has evaporated away completely the information would be irreversibly lost which would suggest non-unitary evolution for black holes.

An apparent solution to restore unitarity was to present corrections to the horizon allowing the radiation to carry information away. In a coarse grained description, information would seem to disappear but in a more fine grained description the correlations between the radiation in the end state would still exist. This solution is realized in Black Hole Complementarity [4] which is constructed such that no observer can see any contradictions. The postulates of Black Hole Complementarity are:

Postulate 1: The process of formation and evaporation of a black hole, as viewed by a distant observer, can be described entirely within the context of standard quantum theory. In particular, there exists a unitary S-matrix which describes the evolution from in-falling matter to outgoing Hawking-like radiation.

Postulate 2: Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations.

Postulate 3: To a distant observer, a black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states describing a black hole of mass, M is the exponential of the Bekenstein entropy $S(M)$.

In [5] it was pointed out that there is an implicit additional postulate which is a realization of the equivalence principle from classical GR:

Postulate 4: "No drama" at the horizon.

In other words, from the perspective of an observer freely falling into a black hole, the horizon would not be a special location. Spacetime would appear to be locally flat and for a massive enough black hole, the tidal forces would not be especially large.

A key concept in complementarity is that it is not possible to write down a quantum state that simultaneously describes the interior and the exterior of the black hole. From the viewpoint of complementarity, this would be considered unphysical since those regions are causally disconnected. Different frames such as ones that just describe the exterior (an outside observer's frame) and a frame that describes the interior (a freely falling frame) would have complementary descriptions. For the outside observer there exists an object, the quantum mechanical stretched horizon which stands in for the classical event horizon. Classical notions of spacetime are expected to break down at the stretched horizon. The stretched horizon exists just outside where the classical event horizon would have been and from the perspective of external observers is a real physical object with a temperature, coarse grained thermodynamic entropy and quantum state. From the perspective of the outside observer, the stretched horizon thermalizes with infalling matter and eventually radiates away the information. From the perspective of a freely falling observer, there is no stretched horizon. Instead, the horizon is just like any other location in space and the matter simply falls into the black hole's interior.

Complementarity seems to solve the information problem by preventing a simultaneous description of the interior and exterior. An observer falling into the black hole would observe a seemingly contradictory description to what the observer outside observes. However it has been argued since the two can never communicate, no issues arise from allowing both to observe different but self-consistent physical descriptions.

In a paper by Almheiri, Marolf, Polchinski and Sully (AMPS) [5], a thought experiment showed that the four postulates of Black Hole Complementarity seemed to be inconsistent. By considering a potential experiment that could be done by an observer collecting radiation throughout the history of the black hole's evaporation and then falling through the horizon, they have discovered a frame where "quantum monogamy" (a basic technical feature of quantum mechanics) would seem to be violated. They considered the least radical modification would be to do away with "no-drama" and propose a firewall, a barrier of high energy quanta which in-falling matter and observers would encounter.[12]

In our discussion, we consider a black hole that is entangled with a large *inaccessible* system. The system could be a de Sitter stretched horizon, as proposed for example in de Sitter Equilibrium (dSE) cosmology [8–11] which use as our main illustration. Equally well, the entangled system could be a string landscape or multiverse as long as it exhibits the key properties of being very large and inaccessible to the observer.[3]

In a de Sitter equilibrium universe, the observed universe evolves from a fluctuation in the Gibbons-Hawking radiation of de Sitter space. One could equally well simply consider a single black hole that forms in this manner. In a picture where the de Sitter horizon is a quantum stretched horizon, one would expect the state of the fluctuation to exhibit entanglement with it. Unlike entanglements that have previously been considered [14–16], the de Sitter horizon is a system that could not be measured. Specifically, we do not expect there is any way for an observer to find the information in the de Sitter stretched horizon which is entangled with a black hole and then jump into the hole. This feature allows us to evoke complementarity to avoid the possibility of observing quantum inconsistencies. The size of the de Sitter horizon system also dwarfs anything in the interior de Sitter space, allowing there to always be plenty of states to provide entanglement. We will show explicitly how these features help in a toy model. The large dimension of the de Sitter stretched horizon space is part of the reason we expect the entanglement information to be inaccessible (arguing along similar lines to [17]).

The question of whether our analysis is applicable in realistic cosmologies is a tricky one which we discuss in Section V.

II. TOY MODEL FOR AN EVAPORATING BLACK HOLE

We consider a very simple toy model with a small number of states that we can write out explicitly. The full toy model space "U" can be written as

$$U = A \otimes B \otimes C \otimes D \otimes E \otimes F \tag{1}$$

where each of the subspaces is a two state system. Taking the exterior viewpoint, these subspaces will either represent degrees of freedom of the stretched horizon or "particles" of emitted Hawking radiation. These roles will change as the hole evaporates, and we will often lump together the subspaces that make up the stretched horizon under the label "H".

First we'll use this toy model to review the conventional information problem for just a black hole, not in the presence of a de Sitter stretched horizon. Assuming the entire evaporation process is unitary, if the black hole begins in a pure state it will evolve into a pure state of radiation. Our toy model system will end with only the six (one bit) "particles" in the final state. By unitarity, the initial black hole state need not be described by a larger Hilbert space. Thus our toy model has dimension 2^6.

Generically chosen pure states are known to have entanglement properties of subsystems related to subsystem sizes. For subdivisions into a large and small subsystem, the small subsystem will tend to have little entanglement within itself (that is, entanglement among its own subsystems). In contrast, the large subsystem will have a lot of entanglement among *its* subsystems. The small subsystem will tend to be "maximally entangled" with the large one, but not vice versa. In the Appendix we explain some of these points using simple illustrations. Page [18] developed these insights and used them to argue the following: Consider the time when the initially pure black hole has radiated roughly half its entropy (often referred to as the Page time). Not long after the Page time the already emitted radiation will represent a larger subspace than the black hole. Thus, most of the entanglement for the black hole will be with the previously emitted radiation (referred to as the "early radiation"). Also, there will be very little entanglement between subsystems of the black hole. Upon completion of the black hole decay, this translates to there being very little entanglement between subsystems of the "late radiation". These arguments are a result of the assumption that once radiation has gotten sufficiently far from the black hole, it no longer interacts with the black hole system and therefore entanglements cannot change. The toy model is not

[1] Braunstein comes to similar conclusions, reasoning that vanishing entanglement across the horizon results in high energy particles, an "energetic curtain" [6].

[2] Marolf and Polchinski have presented firewalls as typical features of black holes in dual field theories independent of the quantum monogamy arguments [7]. We do not address these arguments in this paper and focus on the original AMPS thought experiment.

[3] In some pictures of the string theory landscape the inaccessibility property is not realized, for example [12, 13].

complex enough to give a detailed account of the evolution that exhibits these entanglement properties. Instead we simply write down toy model states "by hand" that reflect appropriate entanglements for different stages of the evolution.

We now consider the time (after the Page time) when the black hole has emitted four particles (represented by subsystems F, B, E and A, given in order of emission[4]). These four particles are the early radiation. We expect the remaining black hole subsystem ($H = C \otimes D$) to be highly entangled with the early radiation. A maximally entangled state between the two can be expressed in terms of entanglement between just two of the early radiation particles and H, and we use subsystems E and F for this purpose. The same general arguments from Page lead us to expect the remaining early radiation particles to be maximally entangled with each other. While the Page arguments require a larger space to operate generically, we enforce this feature explicitly on the toy model by choosing a maximally entangled state for the remaining early radiation particles A and B.

The toy model state

$$(\frac{1}{\sqrt{2}}\left|\uparrow_A\downarrow_B\right\rangle - \frac{1}{\sqrt{2}}\left|\downarrow_A\uparrow_B\right\rangle) \otimes \left|\Psi_{H,E,F}\right\rangle \qquad (2)$$

has all the properties listed above to represent a black hole in late stages of decay. In particular, particles A and B appear in a Bell state which exhibits maximal entanglement. Maximal entanglement between the stretched horizon in space H, and the system of $E \otimes F$ requires a pure state describing H, E and F combined. This state is written as $\left|\Psi_{H,E,F}\right\rangle$.

Looking at the time when particle A is just leaving the stretched horizon, we can consider an observer that has collected all the radiation up to this point who then falls into the black hole, collecting particle A and attempting to view what lies beyond the horizon. For this observer there is no stretched horizon and instead, according to complementarity she finds in the interior of the black hole a re-expression of the wave function that was used to describe the horizon (contained within $\left|\Psi_{H,E,F}\right\rangle$).

A vacuum state should be a pure state. The purity of the low energy vacuum we expect the falling observer to experience in the region of the horizon implies that localized subsystems where one is inside and the other outside the horizon will typically be entangled. To the infalling observer particle A represents modes just outside the horizon. To encounter a vacuum, this observer would need to see those modes entangled with modes just inside the horizon (call them C, which is part of H). This is where the problem arises. We have already enforced that A be maximally entangled with B. Those particles could have been measured or brought into a black hole by this observer. Complementarity requires local unitarity which would not allow us to change the entanglement between A and B. However, the form of Eqn. 2 (established to meet other requirements as detailed above) explicitly does not allow *any* entanglement between A and H. This feature is an aspect of quantum monogamy. With C part of H, there is thus also no entanglement between A and C. Since we have just argued that such an entanglement is required of a vacuum state, complimentarity appears to be in conflict with the no drama we expect for the infalling observer.

Complementarity allows for quantum monogamy to be broken as long as it is not violated within a single causal patch. A quantum system could be maximally entangled with multiple systems as long as no observer could ever encounter this contradiction. The AMPS thought experiment provided an example where to enforce "no drama" quantum monogamy would have to be violated within a single causal patch (seen by the falling observer). Once maximal entanglement between A and B has been established, nothing else can be entangled with A. Since systems A, B and C can all be encountered by a single observer this flexibility afforded by complementarity does not help.

III. TOY MODEL FOR AN EVAPORATING BLACK HOLE IN DS

We consider a black hole (far from the Nariai limit[5]) formed by a fluctuation in the Gibbons-Hawking radiation in de Sitter space. We think of the de Sitter space as having a stretched horizon much as we did for the black hole. We expect the Gibbons-Hawking radiation to be strongly entangled with the stretched horizon. We note that the entropy associated with the de Sitter horizon is much greater than that of the Gibbons-Hawking radiation or black hole we are considering within the space, and thus the stretched de Sitter Horizon subspace will have dimension many orders of magnitude greater than that of the black hole or any other system. Consider a black hole forming from Gibbons-Hawking radiation which begins maximally entangled with the de Sitter horizon. An observer outside of and in the rest frame of the hole will eventually observe a time after the black hole has decayed but before the decay products reach the de Sitter stretched horizon. It might seem that the decay products would have similar properties to Hawking radiation discussed in the previous section. But in fact unitarity will ensure that entanglement with the de

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[4] The particles are not emitted in alphabetical order so as to conform with other conventions used for this topic.

[5] The Nariai limit places the largest possible black hole in de Sitter space such that the black hole's horizon area approaches the area of the de Sitter horizon. We will consider black holes much smaller than this limit, giving horizons that are clearly separated and vastly different in size. Very close to the Nariai limit our arguments may break down.

Sitter stretched horizon will persist through black hole formation and decay, and become a feature of the decay products of the black hole once the decay is complete. In this case we expect the final state to be a mixed state of the radiation entangled with the de Sitter stretched horizon. The Hawking radiation produced by the decaying black hole will always be maximally mixed.

We can consider similar thought experiments as before. Here the Hawking radiation is always maximally mixed, so the focus on times after the Page time (which we used in the previous section) will not be needed in this discussion. With this simplification we only need to consider *two* radiated particles (A and B), and again we chose B to be emitted earlier than A. Since they are part of a maximally mixed system A and B will not have any entanglement with each other and the combined system of A and B will be in a mixed state. Thus the density matrix that describes A and B combined will be diagonal with equal valued entries on the diagonal. All this is a consequence of basic facts about quantum states (connected to the topic of monogamy) which are reviewed in the Appendix. A completely general way to write the state of the entire system including the de Sitter horizon, black hole horizon and radiation (which all together are in a pure state) is

$$\begin{aligned}&\tfrac{1}{2}\left|1_H\right\rangle \otimes \left(\tfrac{1}{\sqrt{2}}\left|\uparrow_A\downarrow_B\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_A\uparrow_B\right\rangle\right)\\ &+\tfrac{1}{2}\left|2_H\right\rangle \otimes \left(\tfrac{1}{\sqrt{2}}\left|\uparrow_A\downarrow_B\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_A\uparrow_B\right\rangle\right)\\ &+\tfrac{1}{2}\left|3_H\right\rangle \otimes \left(\tfrac{1}{\sqrt{2}}\left|\uparrow_A\uparrow_B\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_A\downarrow_B\right\rangle\right)\\ &+\tfrac{1}{2}\left|4_H\right\rangle \otimes \left(\tfrac{1}{\sqrt{2}}\left|\uparrow_A\uparrow_B\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_A\downarrow_B\right\rangle\right). \end{aligned} \tag{3}$$

Here the space H is the combined space of the black hole and de Sitter stretched horizons. The states $\left|n_H\right\rangle$ are orthogonal states in H. These four states do not span H which has a very large dimension, but maximal entanglement between two systems can be expressed using only a number of states equal to the dimension of the smaller of the two spaces.

By inspection one can see that the state in Eqn. 3 gives a density matrix for the $A\otimes B$ space which has Bell states as eigenstates. A single line of this equation would be a state where A and B were maximally entangled (as in previous toy model), but overall there exists no entanglement between A and B. One way to see this by is examining the correlations that would be observed between A and B. The density matrix of $A\otimes B$ describes a uniform statistical mixture of the states given in each line of the equation. The first two lines show states where A and B are perfectly anti-correlated and the last two are states where they are perfectly correlated. Since half of the states are anti-correlated and half are correlated, you expect no overall correlation between systems A and B and therefore no entanglement.

Now consider an observer that has collected the radiation and wishes to pass through the black hole's horizon. Just as before, to ensure smooth spacetime we need a subsystem of H to be entangled with system A. We label the subsystem of H that participates in this entanglement "C". A state for the whole system which gives such entanglement is

$$\begin{aligned}&\tfrac{1}{4}(|\tilde{1}_H\rangle\left|c_1\right\rangle + |\tilde{2}_H\rangle\left|c_2\right\rangle)\otimes(\left|\uparrow_A\downarrow_B\right\rangle - \left|\downarrow_A\uparrow_B\right\rangle) +\\ &\tfrac{1}{4}(|\tilde{1}_H\rangle\left|c_3\right\rangle + |\tilde{2}_H\rangle\left|c_4\right\rangle)\otimes(\left|\uparrow_A\downarrow_B\right\rangle + \left|\downarrow_A\uparrow_B\right\rangle) +\\ &\tfrac{1}{4}(|\tilde{1}_H\rangle\left|c_5\right\rangle + |\tilde{2}_H\rangle\left|c_6\right\rangle)\otimes(\left|\uparrow_A\uparrow_B\right\rangle - \left|\downarrow_A\downarrow_B\right\rangle) +\\ &\tfrac{1}{4}(|\tilde{1}_H\rangle\left|c_7\right\rangle + |\tilde{2}_H\rangle\left|c_8\right\rangle)\otimes(\left|\uparrow_A\uparrow_B\right\rangle + \left|\downarrow_A\downarrow_B\right\rangle). \end{aligned} \tag{4}$$

The $|\tilde{n}_H\rangle$'s are new orthogonal states in H which now only includes the de Sitter stretched horizon. The $\left|c_m\right\rangle$'s are linearly dependent states are in C. Line by line, the apparent entanglement between A and B does not seem to be disturbed (appearing just as in Eqn. 3). This is as we expect since particles A and B have not interacted with anything. To see where the entanglement lies, we need to consider the entire state.

To get the required entanglement between C and A we choose

$$\begin{aligned}\left|c_1\right\rangle &= -\tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle, \left|c_2\right\rangle = -\tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle\\ \left|c_3\right\rangle &= \tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle, \left|c_4\right\rangle = \tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle\\ \left|c_5\right\rangle &= -\tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle, \left|c_6\right\rangle = \tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle\\ \left|c_7\right\rangle &= \tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle, \left|c_8\right\rangle = -\tfrac{1}{\sqrt{2}}\left|\uparrow_c\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_c\right\rangle. \end{aligned} \tag{5}$$

Then the full state can be written as

$$\begin{aligned}&\left(\tfrac{1}{\sqrt{2}}\left|\uparrow_C\downarrow_A\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_C\uparrow_A\right\rangle\right)\otimes\\ &\left(\tfrac{1}{\sqrt{2}}(\left|\uparrow_B\right\rangle + \left|\downarrow_B\right\rangle)|\tilde{1}_H\rangle + \tfrac{1}{\sqrt{2}}(\left|\uparrow_B\right\rangle - \left|\downarrow_B\right\rangle)|\tilde{2}_H\rangle\right). \end{aligned} \tag{6}$$

Now we can see explicitly that systems C and A are in a Bell state and are therefore maximally entangled. The entanglement with the horizons has conspired to protect the local physics required to meet Postulate 4 in addition to the other three postulates, thus evading the AMPS argument.

This basic argument holds at any point during the black hole's evolution. The Page time, when the black hole has lost half its entropy in our scenario hold no significance. The overall evolution of the black hole is evolving from a mixed state to a mixed state.

Where this differs from simply starting the black hole in some entangled state (such as in [14, 16]) is that by having the black hole be entangled with the de Sitter horizon, we gain two useful features of that horizon; its size and inaccessibility. The very high dimension of de Sitter stretched horizon subspace means that typical states for the whole system will have the other subsystems highly entangled with the de Sitter horizon. While the states hidden from outside observers in the black hole stretched horizon eventually are revealed as the black hole decays, as long as the de Sitter space is stable we maintain a very large space of hidden degrees of freedom throughout time in this scenario. The fact that no single observer would encounter both the black hole stretched horizon and the de Sitter stretched horizon allows complementarity to never encounter the AMPS contradictions.

IV. THE ROLE OF MEASUREMENT

A. Background discussion

A potentially confusing concept in this framework is the effect of measurement in these thought experiments. We will mostly default to the "many worlds" interpretation of quantum measurement (where the only evolution of the wavefunction is the unitary evolution determined by the Schroedinger equation, with no explicit "collapse") but much of the discussion is not dependent on that choice. Understanding the role of measurements in these thought experiments requires an understanding of entanglements when measurements are made. One way to describe the measurement is from the perspective of the observer. When a good measurement is made the observer is only aware of a single outcome (or "Everett branch") and doesn't see interference with other possible outcomes that, from the perspective of the observer only "could have happened but didn't" but which are still represented in the complete wavefunction. Another description includes the observer as a part of the total wave function. In that description, the observer becomes entangled with the system being observed through the interactions that facilitate the measurement. It is this entanglement (and the *stability* of this entanglement) that enables the first "observer-centered" description because the observer is correlated with the measured system. Both descriptions will be important in what follows.

In the original firewall paper [5], a thought experiment is presented in which a measurement is being performed by an observer. In agreement with those authors [19] we believe that an actual observer making measurements is not needed for their argument to work. The important thing is to clearly identify the frame from which the situation is being analyzed, and it is convenient to identify frames by specifying observers. If the observer actually does make certain measurements that can alter some details of the discussion but not the main points. An observer that encounters emitted particles outside the black hole for long enough and then falls in describes a frame in which the arguments given in Sect. II produce a conflict between quantum mechanics and the "no drama" postulate. If the no drama postulate holds, quantum monogamy would seem to be contradicted since the early radiation should have parts that are entangled with each other as well as the now revealed interior of the black hole. This argument holds regardless of whether or not the observer measures the radiation. In the case of no measurement, entanglement of the black hole with the radiation presents the contradiction. If the radiation is measured then the problem results from entanglement of the black hole with the measurement apparatus and observer.

B. Quantum Teleporting a Firewall

Other experiments can be performed that would rearrange entanglements. One such experiment is a Bell measurement, which projects arbitrary states onto a Bell state. This is the basis of quantum teleportation. Consider two subsystems, one which an observer interacts with (or "controls") and another with which the observer does not interact at all. If the two subsystems are maximally entangled, performing Bell measurements will create entangled Bell states in the observed subsystem. As a result the observer will be certain that the non-observed subsystem will also be in a particular Bell state related (via the properties of the initial state) to the outcome of the measurement of the observed subsystem.[6] Performing the proper coherent operation (or "quantum computation") on the controlled subsystem can "teleport" a particular quantum state: The state of the non-controlled system can be modified without any direct interactions, only exploiting the initial entanglement between the two systems [20].

This concept can be applied to black holes as well. Consider in the AMPS thought experiment, an observer that has collected enough radiation such that she possesses a system that the black hole is maximally entangled with. She could make measurements that would entangle radiation particles with each other resulting in the black hole's subsystems to become entangled with themselves. For particular measurements, the entanglements desired for "no drama" can be created and the black hole would be projected (or "teleported") into a state with no firewall. This does not serve as a counter argument to firewalls. It simply shows that through particular measurements a system can have its state projected onto a particular subspace. Much of the firewall discussion necessarily revolves around which states one thinks are typical [7, 19, 21], since there is general agreement that states with and without firewalls exist. The AMPS paper stands by the notion that firewalls are the typical state for black holes. But sufficiently elaborate measurements to teleport to a non-firewall state should be possible in principle. Likewise, even if you don't believe firewalls are typical [22–25] you could envision specific measurements that would project the black hole into a firewall state. Maldacena and Susskind have examined quantum computations performed on a pair of entangled black holes, sending signals between them, and have analyzed the features needed to send (or in our words "teleport") a firewall [14, 15].

In our thought experiment in de Sitter space, the radiation emitted from the black hole will have practically no entanglement with the black hole. The absence of this

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[6] Although "Bell states" are technically described as states in a four-state Hilbert space these points can be generalized to larger spaces.

sort of entanglement eliminates the possibility of teleportation and means that measurements of the radiation after the black hole has formed and begun to decay will leave the black hole basically unchanged. These measurements will affect the states of systems that *are* entangled with the emitted radiation, that is, small subsystems of the de Sitter stretched horizon.

One can also consider measurements that can be made on the initial Gibbons-Hawking radiation that came from the de Sitter horizon to form the black hole. A careful measurement of this incoming radiation can result in a pure state of radiation that will become a pure black hole (described by states modeled by Eqn. 2). This would be projecting onto a state that will evolve into a firewall black hole state. This is an example of using teleportation to project onto what we consider in our picture to be an atypical firewall state.

V. DISCUSSION AND CONCLUSIONS

The concept of black hole firewalls has arisen from the desire to ensure completely unitary evolution, where pure states evolve into pure states. Our toy model has not discarded unitary quantum mechanics, nor have we abandoned the idea that the entire system is in a pure state. But in our picture the black hole starts in a mixed state due to entanglement with inaccessible states of the de Sitter space stretched horizon. In this case the information problem is nullified. The initially completely mixed black hole would contain no quantum information and as a result there is no information for the radiation to carry away (the information would instead reside in the entanglements, as discussed for example in [26]) . The mixed nature of our states removes all entanglement among subsystems of the black hole and its decay products and as a result saves us from worrying about quantum monogamy provided we cannot access the system that these states are actually entangled with. While we have used the stretched horizon of de Sitter as a simple illustration, our conclusions should apply to any situation where a black hole is fully entangled with a large inaccessible system.

We note that all the discussions of the firewall issue revolve around the question of what is a typical state for a black hole.[7] In an earlier preprint version of this paper we tried to argue that the maximal entanglement we discuss here would be typical in certain cosmological scenarios. We have since changed our minds about this. The fact that we, and the black holes we see around us are all correlated with a cosmological state of the universe which is very far from equilibrium prevents the maximal entanglement we require for our analysis from being realistic in the universe we observe. The vast majority of black holes formed in our universe would originate not from fluctuations like we've considered in this paper, but instead from the gravitational collapse of "ordinary" matter. The entropy of the matter which will form a black hole of mass, M will be on the order of $M^{3/2}$ which is much smaller than entropy needed to start a black hole in a maximally mixed state, M^2. For this reason, we might expect black holes formed in many cosmological descriptions to begin essentially pure, so much of the discussion in this paper will not apply. In the language of this paper, even very basic information about the universe (which for example supports a simple FRW description) constitutes measurements which take us essentially all the way to the point of "teleporting" a firewall into the black holes around us, even if the entire observed universe is entangled with something larger. An error in our earlier thinking was to assume that the measurements required for this teleporting would be extremely complicated ones of the sort discussed in [14, 15]. Now we have realized that such measurements have already been done.[8]

Our approach may still be relevant in a dSE description of cosmology [8–11]. If there was some strong form of holography in place, enforcing the Hilbert space of our entire universe to be small (dimension of order $e^{10^{12}}$) resulting in smaller Hilbert spaces actually needed to describe black holes, radiation and everything else we observe. However, such a concept has not been rigorously realized, and simple things we know about the properties (such as heat capacities and entropies) of everyday matter suggest that such a construction would be in conflict with known physics.

We wrap up by considering briefly a more ordinary implementation of our ideas. Consider a standard semiclassical picture of the universe with quantum fields on a Friedmann-Robertson-Walker (FRW) background (with no de Sitter stretched horizon, perhaps having an unstable form of dark energy). If we consider the state of the entire universe to be pure, a black hole of the sort we actually observe will have a space of states that make up a small portion of the total universe's very large Hilbert space. Using the Page arguments [18] you'd expect subsystems that are less than half the space (or much less, as is the case for a typical black hole in our universe) to be essentially maximally mixed. At first glance it appears that this mixing will provide all the same technical features we have used in this paper to argue that firewalls

[7] Separately we have considered whether a very strong form of complementarity and holography could exist to enforce entanglements of horizons to absolutely forbid firewalls in all cases. But that would involve adding a new principle regarding non-local interactions between the stretched horizons of different objects, with some seemingly exotic consequences (such as black holes evolving from pure to mixed states, even as the whole system evolves unitarily). Such a new principle is certainly not needed for the main points of this paper which are that with a large enough external space black hole firewalls can plausibly avoided based only on the statistical arguments about entanglement developed by Page.

[8] We thank Don Marolf for pointing out this error to us.

are not likely, without evoking something as exotic as a de Sitter stretched horizon.

However, a key feature of the de Sitter stretched horizon is its persistent inaccessibility to the relevant observers. We do not expect there is any way for an observer to find the information in the de Sitter stretched horizon which is entangled with the black hole (a tiny fraction of the bits in that stretched horizon) and then jump into the hole. This feature allows us to evoke complementarity to avoid the possibility of observing quantum inconsistencies. In the FRW case it is not at all clear we have this same feature. Indeed, in a realistic universe there will be lots of "inaccessible" states in other black hole horizons. These holes will eventually decay, but others will form. The question of whether the stretched horizons of other black holes (or other aspects of a realistic FRW picture) could play a role similar to the one we have assigned to the de Sitter stretched horizon is an interesting one which we do not address here.

Our basic point is that if a black hole is entangled with a large inaccessible space, the usual discussions of the firewall and information problems are dramatically changed. We have argued that in this case there no longer is an information problem, and that one can satisfy the four postulates of complementarity without any contradictions. A de Sitter stretched horizon appears to have the necessary features to play the role of this large space, and we have used that for most of this paper to make our points. Our considerations may only be applicable in the case of the largest space of possible black hole initial states such as black holes forming from a fluctuation in de Sitter space. That would make our considerations inapplicable to cosmologically realistic cases. However, we have also raised the possibility that each black hole we observe could have sufficient entanglement with other existing black holes to allow our arguments to go through. If that were the case, observed black holes would not have an information problem and the AMPS analysis would not apply.

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APPENDIX

A. Entanglement and Quantum Monogamy

In this section we review some basic properties of quantum pure and mixed states and entanglement that we have utilized in this paper. A pure quantum state is a state which can be represented by a single vector in the Hilbert space (denoted by a ket). A mixed state is a statistical mixtures of pure states and cannot be represented by a single ket, instead it is described by a density matrix. The maximal deviation from a pure state is called a maximally mixed state, which appears as a diagonal density matrix with equal entries on the diagonal in every basis.

Quantum mixed states for subsystems can occur due to entanglement. Page has argued that such entanglement is typical when examining a subsystem of a larger system which is in a pure state. If a system in a pure state is divided into two equal sized subsystems, A and B, that are both maximally mixed then systems A and B are said to be maximally entangled with each other. The simplest examples of maximal entanglement are the Bell states which span a four dimensional Hilbert space, a product space of two two-state systems:

$$\begin{aligned}
&\tfrac{1}{\sqrt{2}}\left|\uparrow_A\downarrow_B\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_A\uparrow_B\right\rangle \\
&\tfrac{1}{\sqrt{2}}\left|\uparrow_A\downarrow_B\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_A\uparrow_B\right\rangle \\
&\tfrac{1}{\sqrt{2}}\left|\uparrow_A\uparrow_B\right\rangle - \tfrac{1}{\sqrt{2}}\left|\downarrow_A\downarrow_B\right\rangle \\
&\tfrac{1}{\sqrt{2}}\left|\uparrow_A\uparrow_B\right\rangle + \tfrac{1}{\sqrt{2}}\left|\downarrow_A\downarrow_B\right\rangle . \qquad (7)
\end{aligned}$$

Each state appears to be a pure state in $A \otimes B$ but for both A and B subsystems their states are maximally mixed. It is easy to see the high level of correlations between systems A and B for these states. For each of the above states taken individually, when spin is measured in the basis shown, a measurement of spin for A always correlates with a specific spin for B. These states represent the strongest possible correlations between two two state systems which is why they are described as being maximally entangled.

One can consider a larger space by adding subsystem C to the picture. One can construct product states between the Bell states for $A \otimes B$ and pure states in C without disturbing the entanglement between A and B, for example:

$$\left(\frac{1}{\sqrt{2}}\left|\uparrow_A\downarrow_B\right\rangle - \frac{1}{\sqrt{2}}\left|\downarrow_A\uparrow_B\right\rangle\right) \otimes \left|\psi_C\right\rangle . \qquad (8)$$

System A can also be described as being maximally entangled with system $B \otimes C$ but not vice-versa. In general when a system is maximally entangled with a larger system, a *subsystem* of the large system can be identified which is maximally entangled with the small system. For two subsystems to be mutually maximally entangled they need to be the same size (as is the case with our illustration in Eqn. 7).

Quantum monogamy can be stated as follows: If system A is maximally entangled with B then A cannot share any entanglement with another system C. Assuming this is not true leads to a contradiction. We will next demonstrate the weakest form of this statement (which we use in this paper), namely that A and C cannot be maximally entangled.

By definition, A and B being maximally entangled means $A \otimes B$ is in a pure state, and state of $A \otimes B$ can written as a single ket, $|\Psi_{AB}\rangle$. Also, A and B will each be in maximally mixed states.

Suppose A and C were maximally entangled which would imply $A \otimes C$ is in a pure state and A and C are both maximally mixed. Then the state of $A \otimes C$ can written as a single ket, $|\Phi_{AC}\rangle$.

Combining the above two assumptions means the state for $A \otimes B \otimes C$ can be written as $|\Psi_{AB}\rangle \otimes |\Phi_{AC}\rangle$. Since this state can be written as a single ket, $A \otimes B \otimes C$ is in a pure state. However, $A \otimes B$ being in a pure state while C is in a mixed state means $(A \otimes B) \otimes C$ is in a mixed state which is in direct conflict with the previous statement. Simply put, the linearity of quantum mechanics prevents any state that can be written down for $A \otimes B \otimes C$ that will give maximal entanglement for A and B as well as the needed correlations between A and C.

These properties of entanglement, combined with statistical arguments about what is typical when a system is divided into subspaces lie at root of the results developed by Page. The statistical arguments are used to make the case that certain entanglements are likely to be maximal, a feature we have simply put in by hand by using Bell states in our illustration here. A key result is that typically a pure state system divided into a large and small subsystem results in the small subsystem being very mixed and entangled with the large subsystem. The large subsystem is more pure and most of its entanglements are between its own subsystems and not with the other small subsystem.

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