Chaotic inflation with kinetic alignment of axion fields

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Phys. Rev. D **91**, 023520 — Published 28 January 2015

DOI: 10.1103/PhysRevD.91.023520

A New Angle on Chaotic Inflation

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N-flation is a radiatively stable scenario for chaotic inflation in which the displacements of $N \gg 1$ axions with decay constants $f_1 \leq \ldots \leq f_N < M_P$ lead to a super-Planckian effective displacement equal to the Pythagorean sum $f_{\rm Py}$ of the f_i. We show that mixing in the axion kinetic term generically leads to the phenomenon of kinetic alignment, allowing for effective displacements as large as $\sqrt{N} f_N \geq f_{\rm Py}$, even if $f_1, \ldots, f_{N-1}$ are arbitrarily small. At the level of kinematics, the necessary alignment occurs with very high probability, because of eigenvector delocalization. We present conditions under which inflation can take place along an aligned direction. Our construction sharply reduces the challenge of realizing N-flation in string theory.

INTRODUCTION

Inflationary scenarios producing detectable primordial gravitational waves are extraordinarily sensitive to Planck-scale physics, motivating the understanding of these models in theories of quantum gravity. The recent observation of B-mode polarization at degree angular scales by the BICEP2 collaboration [1] provides the prospect of direct experimental study of large-field inflation, if the signal is established as primordial in origin (cf. [2–4]).

Among the best-motivated scenarios for large-field inflation in string theory are axion inflation models, including string-theoretic variants of natural inflation [5], in which shift symmetries protect the inflaton potential (for a recent review, see [6]). In the effective field theory description, axionic shift symmetries with large periodicities, i.e. with decay constants $f \gg M_P$, can ensure radiative stability of large-field inflation, but whether such symmetries admit completions in quantum gravity is a delicate question that requires knowledge of the ultraviolet theory. By embedding axion inflation in string theory one can address this problem through well-defined computations.

A general finding about axions in presently-understood string vacua is that the decay constants f are small, $f \ll M_P$, in all regions in which the perturbative and nonperturbative corrections to the effective action are under parametric control [7]. At the same time, axions are very numerous, with $\mathcal{O}(10^2) - \mathcal{O}(10^3)$ independent axions appearing in typical compactifications. To achieve large-field inflation in string theory, one could therefore consider a collective excitation of $N \gg 1$ axions ϕ_i, $i = 1, \ldots, N$, with effective displacement $\Delta\Phi$ larger than the displacements $\Delta\phi_i$ of the individual fields. This proposal, known as *N-flation* [8], builds on the idea of assisted inflation [9].

If the fields ϕ_i are canonically-normalized axions with periodicity $2\pi f_i$ then the diameter of the field space is

$$\mathrm{Diam} = 2\pi \sqrt{\sum f_i^2} \equiv 2\pi f_{\rm Py}\,, \tag{1}$$

where $f_{\rm Py}$ denotes the Pythagorean sum of the f_i. To achieve N-axion inflation in string theory using (1), one must find a string compactification containing $N \gg 1$ axions with $f_N \geq \ldots \geq f_1 \gtrsim 0.1 M_P$. Because this is not technically feasible at present, N-flation's status as a scenario for chaotic inflation in string theory has been questioned [10, 11].

The field range (1) has been widely understood as the maximum attainable in a theory with N axions. In this Letter we show instead that for *generic* axion kinetic terms, the field range is actually parametrically larger than (1), and one can achieve a large displacement $\Delta\Phi \approx \sqrt{N} f_N$ even if $f_1, \ldots, f_{N-1}$ are very small. This general result transforms and sharply simplifies the problem of realizing N-flation, both in field theory and in string theory.

THE ACTION FOR N AXIONS

Consider a field theory containing N axion fields θ_i corresponding to N independent shift symmetries $\theta_i \to \theta_i + c_i$. We take the symmetries to be broken nonperturbatively by the potential

$$V(\theta_1, \ldots \theta_N) = \sum_i \Lambda_i^4 \big[1 - \cos(\theta_i)\big] + \ldots \tag{2}$$

to discrete shifts $\theta_i \to \theta_i + 2\pi$, where Λ_i are dynamically-generated scales, and the ellipses indicate terms at higher orders in the instanton expansion. The potential (2) breaks the $GL(N, \mathbb{R})$ symmetry of the perturbative Lagrangian: the periodic identifications define a lattice in the field space. Without loss of generality, we have taken the periodicities of the dimensionless fields θ_i to be 2π, so that a fundamental domain is a hypercube of side length 2π in $\mathbb{R}^N$. We refer to the corresponding basis, which

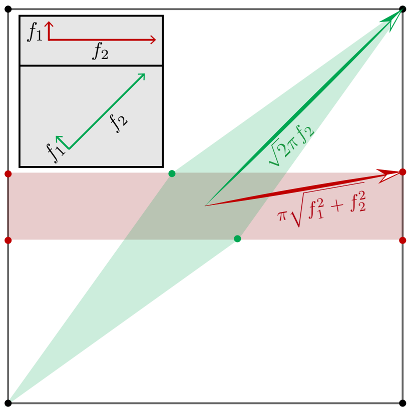

FIG. 1. N-flation with and without alignment, for $N = 2$. The outer box shows a fundamental domain in the dimensionless θ coordinates with period 2π. The shaded rhombus and rectangle depict the physical size of the fundamental domain with and without alignment, respectively. The green and red vectors show the semidiameters in the aligned and unaligned cases. The inset is not to scale.

is simply the usual Cartesian basis of $\mathbb{R}^N$, as the *lattice basis*.

Including the kinetic term, the Lagrangian takes the form

$$\mathcal{L} = \frac{1}{2} K_{ij} \partial\theta^i \partial\theta^j - \sum_i \Lambda_i^4 \left[1 - \cos(\theta_i)\right] , \qquad (3)$$

where K_{ij} has mass dimension two. We will refer to the dimensionless field space parameterized by the θ_i as $\mathcal{M}_\theta$, and the physical field space with metric K_{ij} as $\mathcal{M}_K$. We refer to the basis in which K_{ij} is diagonal as the *kinetic basis*. There is no reason to expect that K_{ij} should be diagonal in the lattice basis: instead, the lattice basis and the kinetic basis will typically be related by a rotation.[1]

In a model with a single axion θ, the axion decay constant f can be defined by changing coordinates to write the Lagrangian in the form $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \Lambda^4[1-\cos(\phi/f)]$. The periodicity of the canonically-normalized field ϕ is then $2\pi f$. In a model with N axions, but for which the lattice basis is proportional to the kinetic basis (a nongeneric circumstance), the Lagrangian can be put in the form

$$\mathcal{L} = \frac{1}{2} (\partial\phi_i)^2 - \sum_i \Lambda_i^4 \left[1 - \cos(\phi_i/f_i)\right] , \qquad (4)$$

[1] Although our considerations are general, a concrete example may be helpful. In flux compactifications of type IIB string theory, the kinetic terms for the axions appearing in Kähler moduli multiplets are determined by the intersection numbers, while a nonperturbative potential arises from Euclidean D-branes. The four-cycles associated to the leading instanton terms are generally nontrivial linear combinations of the coordinates in which the Kähler metric is diagonal, so that the lattice basis differs from the kinetic basis.

where f_i^2 is the ith eigenvalue of K_{ij}. But in a general model with N axions, where the lattice and kinetic bases are not proportional, the Lagrangian cannot be brought to the simple form (4). Instead, if the kinetic term is written as $\frac{1}{2}(\partial\phi_i)^2$, the cosines will depend on nontrivial linear combinations of the canonical fields ϕ_i. In this case we will still define f_i^2 to be the ith eigenvalue of K_{ij}, even though none of the single cosine terms has period $2\pi f_i$.

KINETIC ALIGNMENT

The fundamental limitation on field displacements in the theory (3) comes from the periodicity of the axion potential. In the absence of monodromy, which we will not invoke, the maximum rectilinear displacement that could be used for inflation is half the diameter of the fundamental domain in the field space. Here we have in mind a fine-tuned trajectory moving simultaneously from the maximum to the minimum of each independent cosine in V. More realistically, validity of the quadratic expansion around a minimum permits displacements $\Delta\Phi \sim 0.1 \cdot \mathrm{Diam}$.

Displacements in $\mathcal{M}_\theta$ are related to meaningful physical displacements in $\mathcal{M}_K$ by the metric on field space, K_{ij}. If no rotation is required to relate the lattice basis to the kinetic basis, the hypercube of side length 2π in $\mathcal{M}_\theta$ is mapped by the metric information to a rectangular box with side lengths $2\pi f_i$: see figure 1. The maximum (rectilinear) displacement of a canonical field in $\mathcal{M}_K$ is then πf_{Py}.

We will be interested instead in the generic situation where the lattice basis and the kinetic basis are related by a nontrivial rotation. For achieving a large field range, it is advantageous to have the metric assign as much physical distance as possible to a given dimensionless displacement along the particular direction in which $\mathcal{M}_\theta$ has its maximum diameter (namely, $2\pi\sqrt{N}$). The optimal case is then that the eigenvector ψ_N of K_{ij} with the largest eigenvalue, f_N^2, points along a long diagonal, e.g. along $\frac{1}{\sqrt{N}}(1, 1, \ldots 1)$ in the lattice basis. We refer to this circumstance as (perfect) *kinetic alignment*. In this case the physical diameter of the fundamental region of $\mathcal{M}_K$ is $2\pi\sqrt{N} f_N$.

MECHANISMS FOR KINETIC ALIGNMENT

Although kinetic alignment might appear unlikely at first glance, it is essentially inevitable in a wide range of systems. Consider an ensemble of theories of the form (3); the associated K_{ij} then form an ensemble $\mathcal{E}$ of $N \times N$ matrices. Suppose that $\mathcal{E}$ is statistically rotationally invariant, so that the corresponding normalized eigenvectors ψ_a point in directions that are uniformly distributed on S^{N-1}. Then in the large N limit, the components $\psi_a^{(i)}$,

$i = 1, \ldots, N$, are distributed as $\psi_a^{(i)} \in \frac{1}{\sqrt{N}}\mathcal{N}(0,1)$, with $\mathcal{N}(0,1)$ denoting the normal distribution. Intuitively, a single component of order unity is possible only if many other components are atypically small. More geometrically, nearly all eigenvectors point approximately along a diagonal direction in some hyperoctant, rather than being nearly parallel to a Cartesian basis vector. This is not surprising, since there are 2^N diagonals but just N basis vectors.

This general phenomenon is known as *eigenvector delocalization* in random matrix theory, and has been proved to hold in a number of random matrix ensembles [12, 13]. We will focus on the well-motivated case in which K_{ij} belongs to a canonical ensemble of positive definite random matrices known as the Wishart ensemble: we take

$$K = A^\top A, \quad \text{with } A_{ij} \in \Omega(0,\sigma)\,, \tag{5}$$

where $\Omega(0,\sigma)$ is a statistical distribution with mean zero and variance σ^2. Given suitable bounds on the moments of $\Omega(0,\sigma)$, Tao and Vu have proved that with overwhelming probability, the eigenvectors ψ_a of $A^\top A$ have components $\psi_a^{(i)} \lesssim \mathcal{O}(1/\sqrt{N})$, up to corrections logarithmic in N [13]. This result is universal, in the sense that it does not depend on the details of $\Omega(0,\sigma)$. We conclude that essentially every eigenvector of a Wishart matrix is nearly parallel to a diagonal direction in a hyperoctant.

If the kinetic matrix K_{ij} is well-approximated by a Wishart matrix — an assumption that is natural on the grounds of universality and symmetries, and is substantiated by investigations of random Kähler metrics, both in mathematics [14] and in string compactifications [15] — then the eigenvector ψ_N with eigenvalue f_N^2 points along a nearly diagonal direction [13], along which $\mathcal{M}_\theta$ has diameter $2\pi\sqrt{N}$. As a result, the diameter of the physical field space $\mathcal{M}_K$ is

$$\text{Diam} \approx 2\pi f_N \sqrt{N}\,, \tag{6}$$

where the $\approx$ becomes an equality in the case of perfect alignment. This is one of our main results. We emphasize that the diameter of the field space is given by (6) unless K_{ij} violates eigenvector delocalization: the Wishart model given as a concrete example is well-motivated, but (6) applies to a much wider range of kinetic terms.

MASSES AND MISALIGNMENT

Thus far we have established that displacements of order $\sqrt{N} f_N$ are generically possible in systems with N axions. Whether large-field inflation can occur along such a direction depends on the form of the scalar potential, which we now consider.

Expanding (2) to quadratic order around the minimum at $\theta_i = 0$, we have $V \approx \frac{1}{2}\Lambda_i^4\theta_i^2 \equiv \frac{1}{2}M^{\mathrm{L}}_{ij}\theta^i\theta^j$ with $M^{\mathrm{L}} = \mathrm{diag}(\Lambda_i^4)$. Rotating and rescaling to canonical fields $\vec{\phi} = \mathrm{diag}(f_i)S_K^\top\vec{\theta}$, where $S_K^\top K S_K = \mathrm{diag}(f_i^2)$, the Lagrangian takes the form

$$\mathcal{L} = \frac{1}{2}(\partial\phi_i)^2 - \frac{1}{2}M^{\mathrm{C}}_{ij}\phi^i\phi^j\,. \tag{7}$$

Diagonalizing M^{C} via $S_{M^{\mathrm{C}}}^\top M^{\mathrm{C}} S_{M^{\mathrm{C}}} = \mathrm{diag}(m_i^2)$, we have

$$\mathcal{L} = \frac{1}{2}(\partial\Phi_i)^2 - \frac{1}{2}m_i^2\Phi_i^2\,, \tag{8}$$

with $\vec{\theta} = S_K \mathrm{diag}(1/f_i) S_{M^{\mathrm{C}}}\vec{\Phi}$. In the special case that $\Lambda_i = \Lambda\ \forall i$, so that $M^{\mathrm{L}} = \Lambda^4\mathbb{1}$, the masses of the Φ_i are given by $m_i^2 = \Lambda^4/f_i^2$, while $\vec{\Phi} = \mathrm{diag}(f_i)S_K^\top\vec{\theta}$. The maximal displacement then corresponds, in this simple case, to motion predominantly by the lightest field.

The extent to which the potential disrupts alignment depends on the precise relation between $\vec{\theta}$ and $\vec{\Phi}$, and hence on the relation between K_{ij} and M^{L}_{ij}. We will begin by presenting an explicit example with $N = 2$, assuming perfect alignment in the absence of mass terms, and keeping both the eigenvalues of the kinetic matrix and the diagonal entries of M^{L} general: $K = S_K \mathrm{diag}(f_1^2, f_2^2) S_K^\top$ and $M^L = \mathrm{diag}(\Lambda_1^4, \Lambda_2^4)$, where S_K is a rotation by $\pi/4$. Assuming that $f_1 < f_2$ and $\Lambda_2^4 > \Lambda_1^4$, and writing $\alpha = f_1^2/f_2^2$, we find that for

$$\frac{8\alpha + \sqrt{2}\alpha^2 - \sqrt{2}}{1 + 6\alpha + \alpha^2} \leq \frac{\Lambda_1^4}{\Lambda_2^4} \leq 1\,, \tag{9}$$

a displacement proportional to $(0,1)$ has a cosine $\gtrsim 0.86$ with the diagonal direction. This implies that for $f_1/f_2 \sim \mathcal{O}(1)$ there is a tight constraint on the hierarchy allowed in the mass matrix in order to preserve alignment. However, for f_2/f_1 sufficiently large, there is no constraint on the hierarchy of Λ_1^4/Λ_2^4. We conclude that large hierarchies in the Λ_i can spoil the alignment mechanism: for some ranges of masses, the dynamical trajectory may not be aligned with a diagonal direction in field space.

In general models with $N \gg 1$ axions, approximate alignment[2] persists at the level of (8) if the hierarchies in the f_i are sufficiently large compared to the hierarchies in the dynamically-generated scales Λ_i. We have checked numerically that for K_{ij} a Wishart matrix, the effective number of fields contributing to inflation equals the number of the Λ_i^4 within a factor ~ 2 of the smallest of the Λ_i^4. While a closely-spaced spectrum of this form is not a generic expectation in field theories with N axions, approximate equality of many Λ_i could be achieved through moderately fine-tuned choices of flux in a string compactification. We leave a systematic study of this point for the future.

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[2] A sufficient condition for perfect alignment is that M^{L} is diagonalized by the eigenvectors of K_{ij}^{-1}.

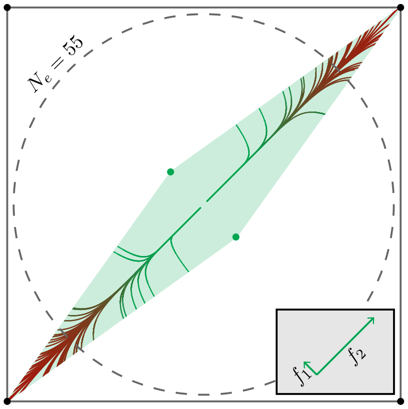

FIG. 2. Attractor behavior for $N = 2$, with axes as in fig. 1. The red (green) curves are inflating (non-inflating) trajectories. Curves beginning outside the circle yield $N_e \geq 55$.

DYNAMICS AND PREDICTIONS

The dynamics in aligned N-flation depends on the mass matrix, and in particular on the relative sizes of the Λ_i. In the simple case where $\Lambda_i = \Lambda\ \forall i$, the field that admits the largest displacement is automatically the lightest field. Inflation arises very naturally in this setting, for a wide range of initial conditions: the heavier fields relax toward their minima, leaving the lightest field, which then drives single-field inflation: see figure 2. For $f_1, \dots f_{N-1} \ll f_N$, the heavy fields relax quickly, and the resulting effective description is simply $\frac{1}{2}m^2\phi^2$ chaotic inflation [16], albeit underpinned by N shift symmetries. We then have $n_s \approx 0.967$ and $r \approx 0.13$ for $N_e = 60$.

More general realizations of aligned N-flation will manifest multi-field behavior. Provided the slow-roll approximations hold and an adiabatic limit is reached before the end of inflation, we have [17–20]

$$\mathcal{P}_\zeta = \frac{H^2}{4\pi^2}N_e, \qquad n_s - 1 = -2\epsilon - \frac{1}{N_e},$$
$$\alpha = -8\epsilon^2 - \frac{1}{N_e^2} + 4\epsilon_i\eta_i, \qquad r = \frac{8}{N_e}\,, \tag{10}$$

where we have set $M_P = 1$, all quantities are to be evaluated at horizon crossing, $\epsilon \equiv \sum \epsilon_i$, $\epsilon_i \equiv \frac{1}{2}(V_i'/V)^2$, $\eta_i \equiv V_i''/V$ and $3H^2 \approx V = \sum V_i$, where $V_i \equiv \frac{1}{2}m_i^2\Phi_i^2$ (no sum), primes represent differentiation with respect to the corresponding field, and the index i indicates summation over the light fields active during inflation. When a few fields are active, the power spectrum, spectral index and running depend on the mass hierarchy and on the initial conditions, but the single field result still corresponds to an attractor [20]. Excitations of heavier fields provide the prospect of novel signatures.

CONCLUSIONS

We considered a theory containing N axions with decay constants $f_1 \leq \ldots \leq f_N$, and asked whether the maximum collective displacement $\Delta\Phi$ can exceed the Pythagorean sum $f_{\rm Py}$ of the f_i (in the absence of fine-tuning of the relative sizes of the f_i, as in [21–23]).

The allowed region of field space is defined by the fundamental periodicities of the dimensionless axions; without loss of generality, this region can be taken to be a hypercube in $\mathbb{R}^N$. To convert dimensionless displacements to physical displacements requires the use of the metric on field space. If the metric on field space has eigenvectors that align with the edges of the hypercube then the diameter is $2\pi f_{\rm Py}$, and $\Delta\Phi \approx f_{\rm Py}$. But if instead the eigenvector of the metric with largest eigenvalue f_N^2 is aligned with a (long) diagonal of the hypercube, the diameter is $2\pi\sqrt{N}f_N$, so that $\Delta\Phi \approx \sqrt{N}f_N$, which is considerably larger than $f_{\rm Py}$ in the generic case in which the f_i are distinct. We referred to this situation as *kinetic alignment*. Approximate kinetic alignment is equivalent to the phenomenon of eigenvector delocalization in random matrix theory, which has been proved to hold in a number of relevant cases [13]. At the level of kinematics, kinetic alignment is almost inevitable in a system with $N \gg 1$ axions and a general kinetic term. Although our arguments did not rely on string theory, and hold in an effective field theory with generic axion kinetic mixing, the necessary structures readily arise in string theory.

We then argued that the axion potential (2) can be compatible with large-field inflation along an aligned direction. As an example, if K_{ij} is a Wishart matrix and $P \leq N$ of the dynamically-generated scales Λ_i^4 fall within a range of size ~ 2, then P fields participate in the alignment, and the effective range is $\sqrt{P}f_N$. Alignment is possible for more general potentials, but we leave a systematic analysis for the future.

Despite the simplicity of these observations, the implications are profound. Achieving N-flation with the range (1) by arranging for $N \gg 1$ axions to have decay constants f_i as large as $\mathcal{O}(0.1)M_P$ is a serious challenge for the construction of models of N-flation in string theory (cf. e.g. [8, 10, 24, 25]): perturbative control of the g_s and α' expansions generally enforces $f_i \ll M_P$ [7], and while accidental cancellations may permit a few of the f_i to be larger, points in moduli space with many f_i large are not presently computable. However, kinetic alignment allows for successful large-field inflation even if only *one* axion has large (but sub-Planckian) decay constant, dramatically reducing the difficulty of embedding N-flation in string theory. The only price is that one must adjust the Λ_i to achieve dynamic alignment; since this amounts to changing the volumes of cycles that are well inside the Kähler cone, it is readily computable in the supergravity approximation.

Beyond the context of inflation, our finding implies a significant modification of the geometry of field space in theories with many axions. Characterizing the consequences of kinetic alignment for the astrophysical and gravitational signatures of multiple axions [26] is an important problem for the future.

ACKNOWLEDGEMENTS

We thank Connor Long, Paul McGuirk, John Stout, and Timm Wrase for very useful conversations on related topics. The research of T. C. B. and L. M. was supported by NSF grant PHY-0757868. M. D. acknowledges support from the European Research Council under the European Unions Seventh Framework Programme (FP/20072013)/ERC Grant Agreement No. [308082]. J. F. is supported by IKERBASQUE, the Basque Foundation for Science.

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