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Phys. Rev. D **91**, 023503 — Published 6 January 2015

DOI: [10.1103/PhysRevD.91.023503](https://doi.org/10.1103/PhysRevD.91.023503)

# Eternal Inflation with Arrival Terminals

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We analyze the cosmological role of terminal vacua in the string theory landscape, and point out that existing work on this topic makes very strong assumptions about the properties of the terminal vacua. We explore the implications of relaxing these assumptions (by including “arrival” as well as “departure” terminals) and demonstrate that the results in earlier work are highly sensitive to their assumption of no arrival terminals. We use our discussion to make some general points about tuning and initial conditions in cosmology.

## I. INTRODUCTION

Cosmology seeks to provide a history of our universe, describing the dynamical evolution from its initial state to the world we observe today. The question of fine tuning of initial conditions in cosmology was one of the original motivations for cosmic inflation theory [1], and it remains a difficult problem [2]. Thanks to our everyday experience with the 2nd law of thermodynamics, it seems very natural to accept initial conditions that have low entropy. However, low entropy initial conditions are necessarily fine tuned. In this paper we make some specific points about how tuning assumptions can slip in essentially unnoticed into cosmological discussions.

One challenging related issue is the possibility that our observations do not result from a natural cosmological evolution but instead are the result of a thermal (or other) fluctuation. The observations of this fluctuation-based observer (or “Boltzmann Brain” [3]) need not reflect the universe or its physical laws. Cosmological theories which predict we are more likely to be these Boltzmann Brains instead of “normal” observers are unsatisfactory and such a prediction is usually considered grounds for rejection of the theory in question.

In an eternally inflating multiverse in a landscape there are many different types of vacua with different physics. Throughout the history of this “multiverse” transitions among these different vacua will take place, typically via bubble transitions which form “pocket universes” in the new vacuum. According to standard treatments<sup>1</sup>, a landscape with only vacua with positive cosmological constants (de Sitter-like vacua) would reach an equilibrium fixed point regardless of initial conditions. This equilibrium would be dominated by Boltzmann Brain observers.

Page [4] showed that if our vacuum decays at a fast enough rate then Boltzmann Brain type observers would be in the minority. Bousso and Freivogel [5] as well as Linde [6] applied this concept to a local viewpoint allowing de Sitter-like vacua to decay to “terminal” vacua

(Anti-de Sitter-like vacua which collapse to a singularity). Bousso et al. [7, 8] and Harlow et al. [9] have taken this further, showing that introducing “terminal” vacua can remove the problematic equilibrium fixed point. A terminal vacuum will crunch to a singularity in a finite time and thus no longer participate in any further transitions. If the rate of tunneling out of a vacuum that supports normal observers to terminals is significantly greater than the rate for producing Boltzmann Brains, the Boltzmann Brain problem would be resolved. Furthermore, the state of the universe will approach a special steady state that is determined by the values of the various tunneling rates.

Many assumptions about the nature of terminals and the initial state of the universe have gone into this description. These assumptions can be explored by taking a closer look at how the terminals are described: A bubble with negative cosmological constant will crunch at a rate faster than an unlikely up-tunneling which is why transitions out of terminals are typically not considered.<sup>2</sup> In addition the physics of the Anti-de Sitter (AdS) pocket universe as it nears a singularity is not well understood, so one could not even apply something like Coleman-de Luccia tunneling rates as one approaches the singularity.

But if we believe that physics is fundamentally unitary one expects that the crunching spacetime continues to evolve after the crunch. There is no clear theory of what this evolution would look like, and no strong reason to expect it would be described in terms of the local field theoretic ideas on which the rest of the picture is based. Nonetheless, if one believes the theory is unitary, the evolution must continue. For this paper we simply consider a “completion” of the theory where we say there is some hidden part of the Hilbert space in which the unitary evolution continues on the other side of the singularity. Since we cannot access it directly and have no

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<sup>1</sup> See [2] for an alternative

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<sup>2</sup> Transitions from AdS vacua to dS has been considered by Gariga and Vilenkin [10] by removing the singularity from AdS space allowing for a bounce during the crunch with an arrow of time arising from special initial conditions. We do not use this particular mechanism for transitions in this paper.

clear picture what that evolution is like, we just call it the “hidden Hilbert space” (HHS). Again evoking unitarity, one needs to consider the time reverse of the AdS crunch. Such evolution would describe an allowed process in the theory where a subsystem would start in the HHS and pop out through a time reversed singularity into the time reverse of an AdS crunch. This time reversed picture could be extended further to describe time reversed tunneling from AdS out into another vacuum. In this paper we refer to the usual terminals considered in [6–9] as “departure terminals”, and the time reverse of these (which can describe transitions from AdS to dS vacua) as “arrival terminals”. Note that since departure terminals can occur at any time in the cosmological evolution, unitarity would allow arrival terminals to appear at any time as well.

This work is different from past treatments in that the rate of arrival terminals popping in is not described by known physics like the Coleman-de Luccia tunneling. Instead, it is related to unknown physics in the HHS which feeds into the time reverse of the tunneling process. Thus we can no longer assume the rate of transitions from AdS to dS to be negligible. Comoving volume and probability that leaks into departure terminals in principle can return from arrival terminals. In this paper, we ignore AdS bubbles which arrive then crunch without tunneling to a dS vacuum and use an effective rate of transitioning from arrival terminal to dS type vacuum in Eqn 8. Figure 1 shows a standard picture of landscape evolution with only departure terminals present (meant to correspond to Fig. 4 from [9]), while our Fig. 2 adds in arrival terminals as well.

Terminals with zero vacuum energy which evolve asymptotically to Minkowski space at late times were also considered in [9] (following [11] these are called “hats”). Our arguments about allowing time reversed behavior also apply to hat-type terminals (in this case the arrival terminals would correspond to special states coming in from infinitely early times and ultimately tunneling to a different vacuum state). For the purposes of our discussion we lump these hats (and their time reverse) together with the AdS terminals. While the special assumptions and properties of the AdS crunch discussed here do look very different from the asymptotic Minkowski behaviors in the zero vacuum energy, they can still be lumped together to make our main points.

There could exist some special subset of initial quantum states of the multiverse which for the entirety of it’s evolution will completely avoid sectors of the HHS that create arrival terminals. We frame past treatments such as the analysis in [9] with only departure type terminals as specially chosen “initial conditions” in our picture. We put “initial conditions” in quotes to acknowledge that the complicated dynamics we are contemplating for “beyond the AdS crunch” may well not lend themselves to a simple initial time slice picture for the entire system. But still, the presence or absence of various arrival terminals certainly counts as the technical equivalent to assump-

tions about initial conditions. The term initial may be misleading however since it does not only describe the initial volume of AdS type vacua but also states in the HHS which may lead to future AdS arrivals not on the initial time slice.

In this paper we show that the special steady state attractor found in [9] depends heavily on their choice of an arrival-free state. The steady state found in [9] is a feature put in “by hand” via the choice of initial state, rather than a feature determined intrinsically by the tunneling rates alone, as it might first appear. We demonstrate this by making different assumptions about arrival terminals in a simple toy model and show that these lead to different steady state attractors. Thus we disagree with the claim in [9] that their attractor “depends on the existence of an initial condition but not on its details”<sup>3</sup>.

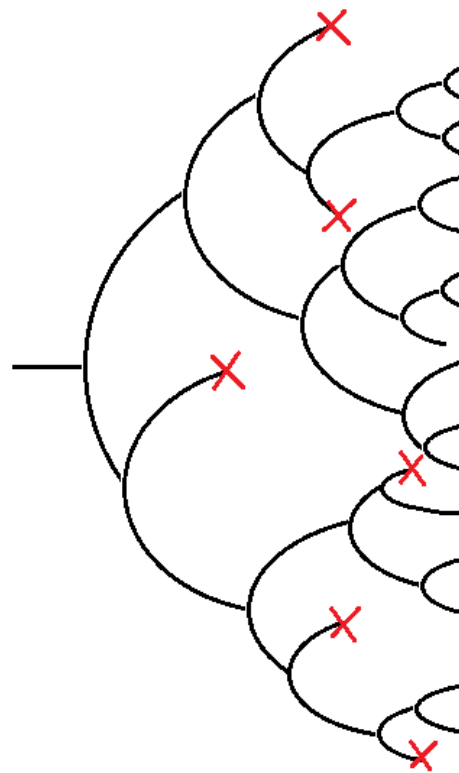


FIG. 1. The tree like structure represents a discrete model of eternal inflation like the one presented in [9]. The tree follows the causal future of an inflating bubble with time flowing from left to right. Branching of the tree represents the exponential expansion of de Sitter space as well as bubble nucleation, with separated branches falling out of causal contact. Terminal vacua are marked with an X, where we no longer follow their evolution once they have crunched.

<sup>3</sup> Technically this disagreement depends on whether their assumption of no arrivals should be attributed to initial conditions or something else (a purely semantic question). Here we emphasize the non-semantic result that the attractor depends sensitively on assumptions about the state which are put in by hand.

The topics of equilibrium and the arrow of time are closely tied to our discussion. Without *any* terminals, the system is understood to reach equilibrium (maximum entropy) which means that the arrow of time only appears as a transient, before the equilibrium sets in. It is the equilibrium behavior that leads to the Boltzmann Brain problem. In the picture with only departure terminals proposed in [9] the system does exhibit an arrow of time, and would never reach equilibrium. The reason for this behavior is the specially tuned choice of initial state (specifically, the assumption that arrival terminals are absent for all eternity). In this respect the arrow of time identified in [9] originates in the usual way, as the result of a special choice of low entropy initial conditions (see [2, 12, 13] for discussions of this point in the context of cosmology).

This paper is organized as follows: Section II reviews the standard formalism used in earlier work to analyze the no-arrivals case, and then extends that formalism to include arrival terminals. Section III applies the formalism to a toy model, and demonstrates the dependence of the steady state attractor on the assumptions about arrival terminals. Section IV develops our discussion of the toy model further, presenting specific solutions that illustrate our main point. We also summarize our main

points and conclusions in Sect. IV.

## II. TRANSITION RATES AND STEADY STATES

### A. No arrival terminals

This section reviews the formalism developed in [14] (and utilized in [7–9]) as applied only to departure terminals. One can define a dimensionless transition rate from vacuum  $j$  to vacuum  $i$ , as

$$\kappa_{ij} = \frac{4\pi\Gamma_{ij}}{3H_j^4} \quad (1)$$

Where  $\Gamma_{ij}$  is the bubble nucleation rate per physical volume for bubbles of type  $i$  in bubbles of type  $j$ . Here  $H_j$  is the expansion rate of bubble  $j$ . The  $\Gamma'_{ij}$ 's could be given by the Coleman-de Luccia tunneling rates in the thin wall approximation but their exact form isn't important for this discussion.

In the absence of arrival terminals, in a particular measure we can define probabilities from the fraction of co-moving volume occupied by vacuum  $i$ ,  $f_i$ . Since we are interested in observers that form in a space with a positive cosmological constant, we will only keep track of volume fractions that correspond to de Sitter-like vacua (which are non-terminal). The equation that describes how these volume fractions evolve with time is given by:

$$\frac{df_i}{dt} = \sum_j \kappa_{ij} f_j - \kappa_i f_i \quad (2)$$

$$\kappa_i \equiv \kappa_i^D + \sum_j \kappa_{ji}. \quad (3)$$

The quantity  $\kappa_i$  is defined to be the total transition rate out vacuum  $i$  which includes rates to transition into other de Sitter type vacua,  $\kappa_{ji}$  and rates to transition to departure terminals,  $\kappa_i^D$ . Since these equations do not keep track of the volume fraction that is lost to departure terminals,  $f$  would have to be renormalized to define a probability.

This equation can be rewritten using a matrix,  $M$ :

$$\frac{df_i}{dt} = \sum_j M_{ij} f_j \quad (4)$$

$$M_{ij} \equiv \kappa_{ij} - \kappa_i \delta_{ij} \quad (5)$$

Solutions to this are of the form:

$$f_i = \sum_l s_i^l e^{-q^l t} \quad (6)$$

where  $-q^l$  are the eigenvalues of  $M$  and  $s_i^l$  are the corresponding eigenvectors. Since the  $f_i$ 's as we have defined them do not include terminal vacua there is an effective leak of volume fraction. This property and

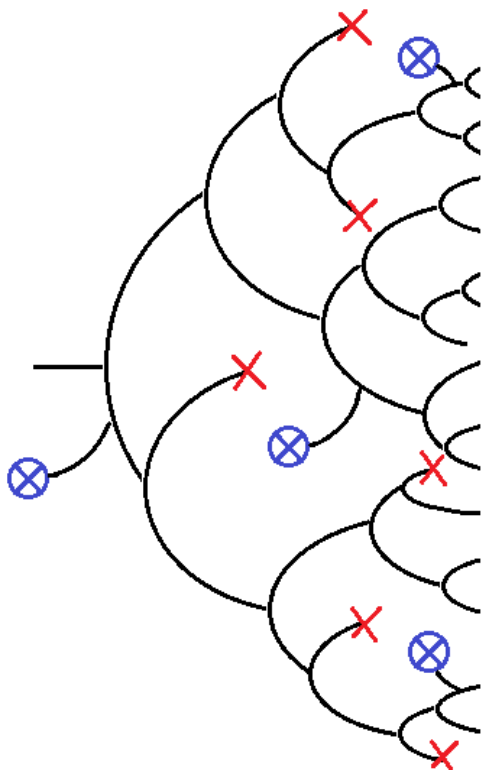


FIG. 2. We modify the tree model shown in Fig. 1 by allowing arrival terminals represented by  $\otimes$ 's. For unitary theories such arrival terminals represent allowed processes, and the choice to exclude them is effectively a choice about initial conditions.

the assumption that each de Sitter vacuum is accessible from each other in one or more transitions result in all eigenvalues being negative and non-zero. The least negative eigenvalue is non-degenerate (shown in the appendix of [14]), which is called the dominant eigenvalue. The dominant eigenvalue (along with the corresponding eigenvector) will dominate late time behavior. We denote the dominant eigenvector/eigenvalue without the  $l$  superscript and write our late time solution as:

$$f_i = s_i e^{-qt} \quad (7)$$

This attractor solution is seen in [9] as giving rise to an emergent arrow of time, as probability flows through the dominant steady state given by  $q$  and out into the departure terminals.

### B. Adding arrival terminals

We now extend the above formalism to include arrival terminals. The timescale for tunneling out of an AdS terminal vacuum before it has crunched is much longer than the timescale for nearing the singularity and approaching (poorly understood) Planck scale physics. Transitions of the sort which could be described with the typical Coleman de-Luccia rates would be highly sub-dominant. Here we focus on allowing transitions that are simply the time reverse of departure terminal crunches (in the AdS case, or alternatively the time reverse of “hats” in the zero vacuum energy case). We do not presume to understand these processes at a technical level (since we don’t have a detailed theory of the crunch) but we expect such processes to be allowed if the full theory is unitary.

To allow transitions from “arrival terminals” we introduce an additional term in Eqn. 2 to get:

$$\frac{df_i}{dt} = \sum_j \kappa_{ij} f_j - \kappa_i f_i + \kappa_i^A f^A(t) \quad (8)$$

This equation is constructed such that it mirrors the form of Eqn. 2 which has a clear space-time description. Here, the quantity  $f^A(t)$  looks like an effective comoving volume fraction for the arrival terminals in which case  $\kappa_i^A$  would be like a rate to transition from arrival terminal to vacuum  $i$ . However we want to emphasize that the for HHS states we do not necessarily expect to have a space-time interpretation. We leave  $\kappa_i^A f^A(t)$  as an effective rate that could vary with time.

The function  $f^A(t)$  could in principle be anything due to our lack of knowledge of the physics of AdS vacua once they have crunched, and similar open questions about “incoming hats”. We just consider a simple case where in some sense the arrival terminal vacua are being depleted as transitions occur. We will characterize this depletion with a simple decaying exponential with some time constant,  $R$ :

$$f^A(t) = e^{-Rt} \quad (9)$$

We emphasize that this approach does not consider a fully generic initial state, but just one that is a bit more generic than one gets by assuming no arrivals at all. This choice is meant to easily fit methods of previous work while demonstrating our main point (the sensitivity of the steady state to choices about initial conditions). In the general case one would not expect to even have an arrow of time or a steady state, but our modifications are not sufficiently general to demonstrate that aspect. In that sense our modification keeps the “temporal provincialism” [15] exhibited in [9].

### III. TOY MODEL

To make our main points we now apply our formalism to a simple toy model. Our toy model landscape has three de-Sitter vacua and two anti de-Sitter vacua very similar to the case considered in [8]. The potential for this landscape is shown in Fig. 3. The original purpose of this toy landscape was to show how terminal vacua could resolve Boltzmann Brain problems in certain cases by producing an attractor solution with the right properties. We will show that allowing arrival terminals can change the properties of the attractor solution, and can thus reintroduce the Boltzmann Brain problem depending on the specific choices made about arrivals.

Starting with the case of departure terminals but no arrival terminals, Eq. 5 is:

$$M_{ij} = \begin{pmatrix} -\kappa_1 & \kappa_{12} & 0 \\ \kappa_{21} & -\kappa_2 & \kappa_{23} \\ 0 & \kappa_{32} & -\kappa_3 \end{pmatrix}. \quad (10)$$

$$(11)$$

Since in this model we have assumed vacuum 1 has a much lower  $\Lambda$  than vacuum 2, the probability of up tun-

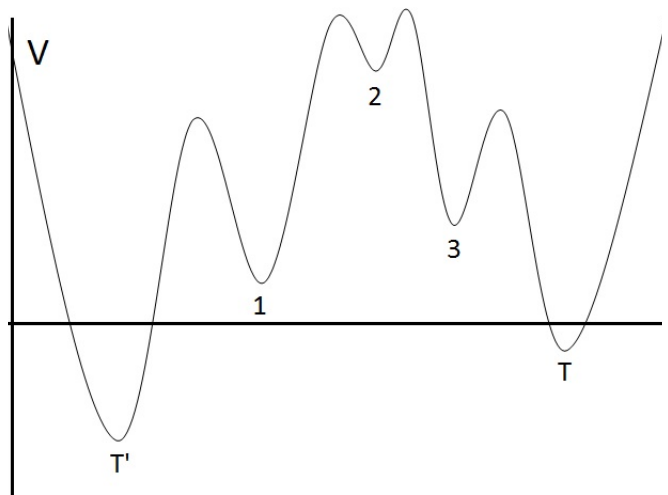


FIG. 3. A sketch of our toy model landscape. Vacua 2, 3 and 1 (in order of decreasing  $\Lambda$ ) are the only vacua with positive cosmological constant. Vacua  $T$  and  $T'$  are terminals and we assume are only accessible from vacua 1 and 3 respectively.

neling from vacuum 1 to 2 is much lower than the total decay rate out of 1.

$$\epsilon \equiv \frac{\kappa_{21}}{\kappa_1} \ll 1 \quad (12)$$

To first order in  $\epsilon$ , the dominant eigenvalue ( $Q$ ) and eigenstate  $\vec{s}$  are given by

$$q = \kappa_1 - \frac{\epsilon(\kappa_3 - \kappa_1)\kappa_{12}\kappa_1}{(\kappa_3 - \kappa_1)(\kappa_2 - \kappa_1) - \kappa_{32}\kappa_{23}} \quad (13)$$

$$\vec{s} = \begin{pmatrix} 1 \\ \frac{\epsilon(\kappa_3 - \kappa_1)\kappa_1\kappa_1}{(\kappa_3 - \kappa_1)(\kappa_2 - \kappa_1) - \kappa_{32}\kappa_{23}} \\ \frac{\epsilon\kappa_{32}\kappa_1}{(\kappa_3 - \kappa_1)(\kappa_2 - \kappa_1) - \kappa_{32}\kappa_{23}} \end{pmatrix} \quad (14)$$

For convenience, the dominant eigenvector,  $\vec{s}$  is normalized such that the entry for the longest lived vacuum is unity.

Now let's modify this to include arrival terminals. For simplicity we will only allow arrival terminals to tunnel into vacua 1 and 3. The solution given by using Eqns. 14 in Eqn. 7 will now be the homogeneous solution to Eqn. 2. Including the contributions from arrival terminals will give the general solution:

$$f_i(t) = b(Cs_i e^{-qt} + \frac{\kappa_i^A}{\kappa_i - R} e^{-Rt}) \quad (15)$$

where  $C$  is an arbitrary constant determined by initial conditions and  $b$  is an overall normalization factor.

In the case where  $R \ll q$ , the second term in the equation will dominate at late times. The ratio of volume fractions are then strongly governed by the arrival rates.

#### IV. DISCUSSION AND CONCLUSIONS

In this paper we try to implement unitarity which we believe should be fundamental in the full theory. That leads us to add a sector to the Hilbert space (which we call the HHS) to account for evolution on the other side of singularities. We do not assume the HHS has a space-time description and suspect it may not. We have chosen to describe interactions between the spacetime sector and the HHS through additional terms of similar form to the volume fractions and transition rates used to describe transitions among actual vacua. This choice is made for convenience and does not describe the most general case. In our formalism, from the point of view of the space-time sector the considerations presented in this paper do not seem to alter certain known features for the multiverse. In this view, the departure/arrival terminals may look effectively like just another type of vacuum that dS vacua can transition to and from. The result (discussed in [9, 14, 16]) that the final attractor is independent of the initial volume fractions and depends only on the transition rates is unchanged. However, our point is that including information about all the actual vacua in the landscape is not enough. One also has to

include terms to represent transitions from the HHS to the space-time sector (the arrival terminals). While in a certain formal sense the earlier results carry over, our point is that the earlier work has implicitly made assumptions about the HHS which have not been justified and are not at all general. In fact, one can imagine more general scenarios which give arrival terminals that arrive more randomly in the space-time sector (not describable by the extended equations we have developed here) in which the HHS interactions with the space-time sector prevent any attractor from appearing. Admittedly, understanding which assumptions and scenarios are realistic depends on achieving much greater clarity on the nature of the HHS than anyone has at present.

Even sticking to our extended equations (and despite formal similarities with earlier work) we find that the inclusion of arrival terminals can dramatically change the evolution of the multiverse. It can change the steady state attractor that emerges and can significantly modify the probability of forming Boltzmann Brains. Our toy model is adapted from [8], where it is assumed that vacuum 3 is the only vacuum capable of supporting any type of observer. Comparing the probability of ordinary observers to Boltzmann Brain observers is done using the dominant eigenvector (or dominant history) method, which evaluates the relative abundance of Boltzmann Brains vs. legitimate observers in the steady state which emerges after an initial transient. In the previous section we showed that with arrival terminals, the dominant eigenvector would be heavily dependent on the arrival rates. While we do not have a formalism like the Coleman-De Luccia tunneling rates to characterize the arrival rates, one still cannot make them disappear without effectively making an assumption about either the initial quantum state or not allowing the theory to be unitary.

To give a concrete illustration on how the dominant eigenvector can be affected, we numerically solved Eqn. 8 for some arbitrarily chosen transition rates that are characteristic of our toy model:

$$M = \begin{pmatrix} -0.01 & 0.03 & 0 \\ 0.003 & -0.055 & 0.002 \\ 0 & 0.025 & -0.02 \end{pmatrix} \quad (16)$$

We considered the case where the arrival rates are exactly zero and compared to the case where they are non-zero (specific values are given in the caption to Fig. 4). From the resulting volume fractions shown in Fig. 4 we can see for initial conditions that differ only on the treatment of arrival terminals, the dominant eigenvector achieved in the two scenarios are different. Without knowledge of what the arrival rates should be, we are unable to make predictions on the dominant eigenvector which prevents us from making predictions of the late-time behavior of the theory.

Our experience with a strongly expressed 2nd law (and the associated low entropy past) in the world around us

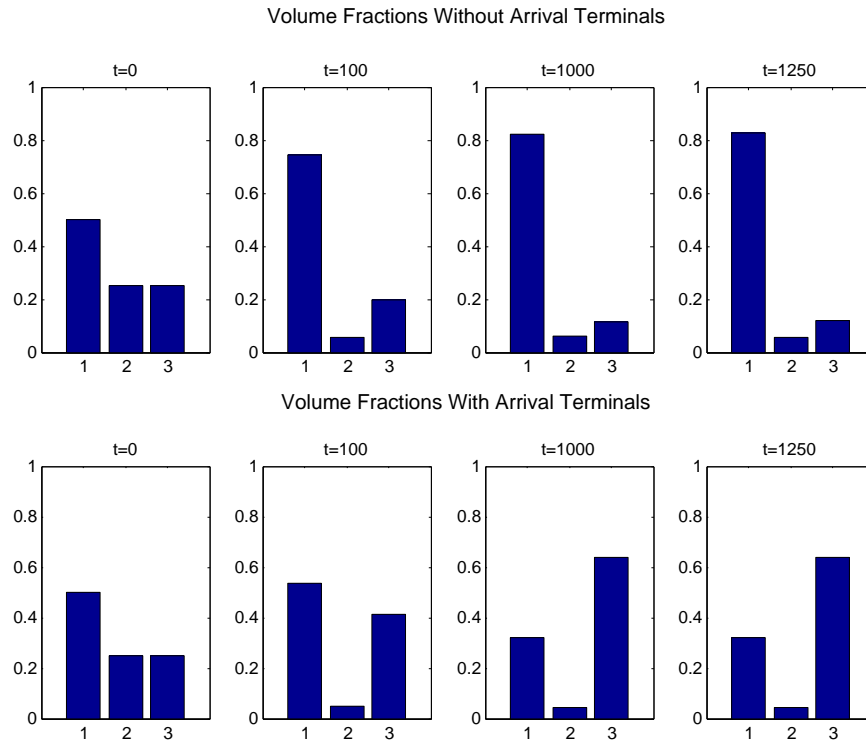


FIG. 4. Renormalized volume fractions for de Sitter vacua 1, 2 and 3 from our toy model at various times. The top row corresponds to the scenario where the arrival rates are exactly zero and the bottom row has arrival rates:  $\kappa_A^1 = 0.0005$ ,  $\kappa_A^2 = 0$ ,  $\kappa_A^3 = 0.004$  and time constant:  $R = 0.002$ . Initially there is transient behavior but by  $t=1000$  both scenarios have reached the dominant eigenvector and the renormalized volume fractions no longer evolve with time. These results illustrate how the properties of the dominant eigenvector depend on assumptions about arrival terminals.

can make it seem very natural to assume similar properties must be true in the general cosmological case (a phenomenon aptly dubbed “temporal provincialism” in [15]). When exploring the fundamental origins of the arrow of time in the universe one must take great care to avoid, or at least clearly identify the extent to which we are allowing a temporally provincial perspective. We argue that the intuition that one is free to exclude arrival terminals in landscape cosmologies, even while allowing departure terminals, is an example of this temporally provincial phenomenon. This conclusion does not detract from important points made in [8, 9]. Those papers show the significant implications of imposing the (highly tuned) condition that arrival terminals are absent, while departure terminals are allowed. In particular, they show how adding only the departure terminals can eliminate the Boltzmann Brain problem in models which would otherwise have one.

While we have focused our comments on [8, 9], our points are relevant more generally, as reviewed in [2]. For example in [17], the presence of an arrow of time is deeply dependent on one’s willingness to accept the assumption that the time reverse of their tunneling process out of the background de Sitter space is entirely absent. The de Sitter equilibrium model [2] (which has many similarities with the model presented in [17]) is very different in this

regard, since as a fundamentally equilibrium model the time reverse of all processes are included, and their rates are given by the detailed balance property of equilibrium systems.

The origin of the thermodynamic arrow of time is a very important and puzzling aspect of cosmology. There appear to be only two ways to incorporate it in our theories: One can put the arrow of time in by hand, and accept the finely tuned initial conditions this approach requires, or consider an equilibrium model which requires exotic dynamics to avoid the Boltzmann Brain problem [2]. Our familiarity with a world that strongly exhibits an arrow of time can make it easy to overlook the strong assumptions that go into the first approach. In this paper we carefully analyze the tuning that appears in [8, 9] via the the assumed absence of arrival terminals. While the authors of [8, 9] understand this tuning and embrace it for the purposes of that work<sup>4</sup>, we still find it useful to call out these aspects more explicitly than has been done in those papers. We feel the sort of analysis presented here is helpful in clarifying our thinking about the arrow of time in cosmology, and may lead us ultimately to a more satisfactory understanding.

<sup>4</sup> R. Bousso and L. Susskind, private communication

## ACKNOWLEDGMENTS

We thank R. Bousso and L. Susskind for helpful discussions. This work was supported in part by DOE Grant DE-FG02-91ER40674.

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