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A natural Little Hierarchy for SUSY from radiative breaking of PQ symmetry

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Abstract

While LHC8 Higgs mass and sparticle search constraints favor a multi-TeV value of soft SUSY breaking terms, electroweak naturalness favors a superpotential higgsino mass $\mu \sim 100 - 200$ GeV: the mis-match results in an apparent Little Hierarchy characterized by $\mu \ll m_{\rm soft}$ (with $m_{\rm soft} \sim m_{3/2}$ in gravity-mediation). It has been suggested that the Little Hierarchy arises from a mis-match between Peccei-Quinn (PQ) and hidden sector intermediate scales $v_{PQ} \ll m_{\rm hidden}$. We examine the Murayama-Suzuki-Yanagida (MSY) model of radiatively-driven PQ symmetry breaking which not only generates a weak scale value of μ but also produces intermediate scale Majorana masses for right-hand neutrinos. For this model, we show ranges of parameter choices with multi-TeV values of $m_{3/2}$ which can easily generate values of $\mu \sim 100 - 200$ GeV so that the apparent Little Hierarchy suggested from data emerges quite naturally. In such a scenario, dark matter would be comprised of an axion plus a higgsino-like WIMP admixture where the axion mass and higgsino masses are linked by the value of the PQ scale. The required light higgsinos should ultimately be detected at a linear e^+e^- collider with $\sqrt{s} > 2m$(higgsino).

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1 Introduction

While the recent discovery of the Higgs boson with mass $m_h = 125.5 \pm 0.5$ GeV at the CERN LHC [1, 2] confirms the particle content of the Standard Model (SM), many physicists nonetheless expect new physics beyond the SM to yet emerge. This expectation arises theoretically from two fine-tuning problems that afflict the SM: one in the electroweak sector arising from quadratically divergent contributions to the Higgs mass while the other arises in the QCD sector and is known as the strong CP problem [3].

The latter of these is solved elegantly by hypothesizing the existence of a global $U(1)_{PQ}$ symmetry [4] valid at some high energy scale [5, 6], $v_{PQ} \sim f_a \sim 10^9 - 10^{16}$ GeV, where v_{PQ} is the scale of the PQ symmetry breaking and f_a is the axion decay constant.[1] Upon breaking of PQ symmetry, the axion field emerges as the associated massless Goldstone boson [8]. The axion field acquires a mass and hence a potential due to QCD instanton effects. In this case, then the offending CP-violating term

$$\mathcal{L} \ni \left(\bar{\theta} - \frac{a}{f_a}\right) G^{A\mu\nu}\tilde{G}^{A}_{\mu\nu} \tag{1}$$

can dynamically settle to tiny values. In the process, the universe is filled with a cold axion fluid –via the mis-alignment mechanism– which acts as cold dark matter (CDM) [9].

The EW fine-tuning (or big hierarchy) problem is elegantly solved by introducing supersymmetry (SUSY) which guarantees cancellation of quadratic divergences [10]. The softly broken minimal supersymmetric SM (MSSM) then requires superpartners for all SM states which are expected to lie at or around the weak scale, since indeed some soft masses and the superpotential μ parameter contribute directly to the Higgs, W and Z masses [11, 12]. While indirect support for SUSY exists via gauge coupling unification and the measured values of the top quark and Higgs boson mass, so far no superparticles have been seen at LHC. This latter situation is summarized by mass limits $m_{\tilde{g}} \gtrsim 1.3$ TeV (for $m_{\tilde{g}} \ll m_{\tilde{q}}$) and $m_{\tilde{g}} \gtrsim 1.8$ TeV (for $m_{\tilde{g}} \sim m_{\tilde{q}}$) in the context of simple models such as mSUGRA/CMSSM [13, 14]. In models of gravity-mediated SUSY breaking, one expects SUSY to be broken in a hidden sector so that the gravitino gains a mass $m_{3/2} \sim m^2_{\rm hidden}/M_P$ where $m_{\rm hidden}$ is some mass scale associated with the hidden sector and M_P is the reduced Planck scale [15]. The effect of hidden sector SUSY breaking on the observable sector is to induce soft SUSY breaking terms of order $m_{3/2}$ in the Lagrangian so that the gravitino mass sets the scale for the sparticle masses [16]. Based on recent LHC8 search limits, we thus expect $m(\text{sparticle}) \sim m_{3/2} \gtrsim$ TeV which would then imply $m_{\rm hidden} \gtrsim 10^{11}$ GeV.

In contrast to the expectations for soft term masses given above, it is important to note that the W, Z and h masses also depend on soft SUSY breaking terms and the superpotential μ term via the shape of the (radiatively corrected) scalar potential which determines the Higgs field vevs v_u and v_d. For the Z mass, we have

$$\frac{m_Z^2}{2} = \frac{(m^2_{H_d} + \Sigma^d_d) - (m^2_{H_u} + \Sigma^u_u)\tan^2\beta}{(\tan^2\beta - 1)} - \mu^2 \simeq -m^2_{H_u} - \mu^2 \tag{2}$$

[1]It is model-dependent to determine the exact relation between v_{PQ} and f_a. In most cases, $v_{PQ} \sim f_a$. We show the exact relation for MSY model (Ref. [7]) in Sec. 3.

where $m_{H_u}^2$ and $m_{H_d}^2$ are the *weak scale* soft SUSY breaking Higgs masses, μ is the *supersymmetric* higgsino mass term and Σ_u^u and Σ_d^d contain an assortment of loop corrections to the scalar potential [17]. To avoid large, unnatural cancellations between $m_{H_u}^2$ and μ^2 in obtaining the measured value of m_Z, one then expects that $|m_{H_u}^2|$ and μ^2 are both $\sim m_Z^2$ [17, 18, 19, 20, 21, 22]. The mis-match between LHC8 search limits and naturalness implies a puzzling Little Hierarchy [23] characterized by

$$\mu \sim |m_{H_u}| \sim 100 \text{ GeV} \ll m_{3/2} \sim 2 - 20 \text{ TeV}. \tag{3}$$

The soft term $m_{H_u}^2$ is expected to be $\sim m_{3/2}^2$ at some high scale (usually taken to be $m_{\rm GUT} \simeq 2 \times 10^{16}$ GeV). However, $m_{H_u}^2$ is driven radiatively through zero to negative values in the heralded radiative electroweak symmetry breaking (REWSB) mechanism due to the large top-quark Yukawa coupling [24]. One simple way to accommodate naturalness is to accept that $m_{H_u}^2$ has been driven to small rather than large negative values. Such a scenario has been dubbed "radiatively-driven natural SUSY" or RNS for short [17, 18].

In addition to $m_{H_u}^2$, naturalness also expects that $\mu^2 \sim m_Z^2$. However, since the μ parameter arises in the superpotential (*i.e.* it is supersymmetric and not SUSY breaking), naively one expects it to be of order the reduced Planck mass $M_P \simeq 2.4 \times 10^{18}$ GeV. This mis-match in expectations is known as the supersymmetric μ problem [25, 26]. Solutions to the μ problem first invoke some symmmetry to forbid the appearance of μ in the superpotential. Next, the up- and down- Higgs multiplets are coupled to new singlet fields either via renormalizable (NMSSM [27]) or non-renormalizable (KN [25] or GM [26]) operators suppressed by powers of M_P. Finally, one arranges for the singlets to gain suitable vevs so that an effective weak scale value of μ is induced.

In the Kim-Nilles solution [25] to the μ problem, one introduces PQ charges for the Higgs fields H_u and H_d along with a PQ-charged field $\hat{X}$ coupled via

$$\hat{f}_{KN} \ni \lambda_\mu \hat{X}^2 \hat{H}_u \hat{H}_d / M_P \tag{4}$$

which is in fact just the supersymmetrized DFSZ axion model which solves the strong CP problem [28]. The KN superpotential also includes the term

$$\hat{f}_{KN} \ni \lambda_{PQ} \hat{Z} \left(\hat{X}\hat{Y} - v_{PQ}^2/2 \right) \tag{5}$$

which causes the scalar components ϕ_X and ϕ_Y to gain vevs of order the PQ breaking scale $v_{PQ}/\sqrt{2}$ where $v_{PQ} = f_a/\sqrt{2}$ Then a μ term is induced with

$$\mu \sim \lambda_\mu f_a^2 / M_P. \tag{6}$$

Originally Kim and Nilles had sought to relate the scales f_a and $m_{\rm hidden}$. Instead, we see that the emerging Little Hierarchy characterized by $\mu \ll m_{3/2}$ may just be a consequence of a disparity between intermediate mass scales

$$f_a \ll m_{\rm hidden}. \tag{7}$$

While it is sufficient phenomenologically to accommodate the PQ/hidden sector hierarchy by hand, it would be more satisfying to see such a hierarchy emerge naturally from a particle physics model.

A model which accomplishes such a goal has in fact been proposed some time ago by Murayama, Suzuki and Yanagida (MSY) [7, 29]. In the MSY model, the PQ scale v_{PQ} emerges quite naturally in that PQ symmetry is radiatively broken as a consequence of SUSY breaking, much like the case where EWSB emerges as a consequence of SUSY breaking. The question then is: does the MSY model (or other similar models) generate a μ value comparable to $m_{3/2}$, or one that is comparable to m_Z or m_h as expected by naturalness? We will show in this paper that the latter possibility emerges easily for generic model parameters, showing that values of μ comparable to m_Z can be generated from TeV-scale values of $m_{3/2}$ (as seemingly required by LHC8 constraints). Thus, the Little Hierarchy seems to lose some of its mystery, and one can reconcile naturalness with the Higgs mass m_h and LHC8 sparticle mass bounds.

To this end, in Sec. 2 we review features of the MSY model which are relevant for our calculations. In Sec. 3 we present our numerical results showing that natural values of μ can be easily generated from multi-TeV values of $m_{3/2}$. Since the PQ scale v_{PQ} is related to μ, then the Higgs mass, and better yet the higgsino masses if they are discovered, would provide an important clue as to the value of the axion mass. An additional feature of the MSY model is that it generates simultaneously a third intermediate mass scale– the Majorana mass scale M associated with the neutrino see-saw mechanism.

In such a model, we expect dark matter to be composed of a mixture of higgsino-like WIMPs (but with non-negligible gaugino components) along with axions. The exact abundances of each depend on details of the SUSY axion model [30] (such as axino and saxion masses, PQ breaking scale and saxion field strength) and computations for the SUSY DFSZ model have been presented previously in Ref. [31].[2] In Sec. 5, we briefly discuss some experimental implications of a radiatively-generated Little Hierarchy in supersymmetric models. In Sec. 6 we present our conclusions: mainly that the Little Hierarchy Problem is no problem at all, but a *feature* to be expected in SUSY axion models which simultaneously address the gauge hierarchy problem, the strong CP problem and the SUSY μ problem.

2 MSY model of radiatively broken PQ symmetry

The MSY model assumes a MSSM superpotential of the form

$$\hat{f}_{\rm MSSM} = \sum_{i,j=1,3} \left[(\mathbf{f}_u)_{ij}\epsilon_{ab}\hat{Q}^a_i\hat{H}^b_u\hat{U}^c_j + (\mathbf{f}_d)_{ij}\hat{Q}^a_i\hat{H}_{da}\hat{D}^c_j + (\mathbf{f}_e)_{ij}\hat{L}^a_i\hat{H}_{da}\hat{E}^c_j + (\mathbf{f}_\nu)_{ij}\epsilon_{ab}\hat{L}^a_i\hat{H}^b_u\hat{N}^c_j \right]. \quad (8)$$

where $\hat{N}^c$ is the SM gauge singlet field containing a right-hand neutrino. The PQ charges are assumed to be 1/2 and -1 for matter and Higgs fields, respectively. The MSSM superpotential is augmented by an additional set of terms containing new PQ charged fields $\hat{X}$ and $\hat{Y}$ with charges -1 and $+3$:

$$\hat{f}' = \frac{1}{2}h_{ij}\hat{X}\hat{N}^c_i\hat{N}^c_j + \frac{f}{M_P}\hat{X}^3\hat{Y} + \frac{g}{M_P}\hat{X}\hat{Y}\hat{H}_u\hat{H}_d. \quad (9)$$

[2] In Ref. [31], the effective theory was considered so that only axion superfield remains light among fields in PQ breaking sector (*e.g.* Kim-Nilles). In the MSY model, there is one light fermion and one complex scalar in addition to axion, saxion and axino. Although the decay processes of PQ particles are more complicated than those in Ref. [31], the big picture is almost the same since all the couplings are still of order μ/f_a.

For simplicity, h_{ij} is taken as diagonal in generation space: $h_{ij} = h_i\delta_{ij}$ and we will also assume $h_1 = h_2 = h_3 \equiv h$.

The corresponding soft SUSY breaking terms are given by

$$\begin{aligned} V_{\rm soft} &= m_X^2|\phi_X|^2 + m_Y^2|\phi_Y|^2 + m_{N_i^c}^2|\phi_{N_i^c}|^2 \\ &+ \left(\frac{1}{2}h_iA_i\phi_{N_i^c}^2\phi_X + \frac{f}{M_P}A_f\phi_X^3\phi_Y + \frac{g}{M_P}A_gH_uH_d\phi_X\phi_Y + h.c.\right). \end{aligned} \tag{10}$$

From these, one may compute the two-loop renormalization group equations (RGEs) by using the recipe in Ref. [32]. Neglecting neutrino Yukawa couplings, we find

$$\frac{dh_i}{dt} = \frac{h_i}{(4\pi)^2}\left(2|h_i|^2 + \frac{1}{2}\sum_j|h_j|^2\right) - \frac{h_i}{(4\pi)^4}\left(2|h_i|^4 + \sum_j|h_j|^4 + |h_i|^2\sum_j|h_j|^2\right) \tag{11}$$

$$\begin{aligned} \frac{dA_i}{dt} &= \frac{2}{(4\pi)^2}\left(2|h_i|^2A_i + \frac{1}{2}\sum_j|h_j|^2A_j\right) \\ &-\frac{4}{(4\pi)^4}\left(2|h_i|^4A_i + \sum_j|h_j|^4A_j + \frac{1}{2}|h_i|^2A_i\sum_j|h_j|^2 + \frac{1}{2}|h_i|^2\sum_j|h_j|^2A_j\right) \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{dm_X^2}{dt} &= \frac{1}{(4\pi)^2}\sum_i|h_i|^2\left(m_X^2 + 2m_{N_i^c}^2 + |A_i|^2\right) \\ &-\frac{4}{(4\pi)^4}\sum_i|h_i|^4\left(m_P^2 + 2m_{N_i^c}^2 + 2|A_i|^2\right) \end{aligned} \tag{13}$$

$$\frac{dm_Y^2}{dt} = 0 \tag{14}$$

$$\begin{aligned} \frac{dm_{N_i^c}^2}{dt} &= \frac{2|h_i|^2}{(4\pi)^2}\left(2m_{N_i^c}^2 + m_X^2 + |A_i|^2\right) - \frac{4|h_i|^4}{(4\pi)^4}\left(2m_{N_i^c}^2 + m_X^2 + 2|A_i|^2\right) \\ &-\frac{|h_i|^2}{(4\pi)^4}(2m_{N_i^c}^2\sum_j|h_j|^2 + 2m_X^2\sum_j|h_j|^2 + 2\sum_j m_{N_j^c}^2|h_j|^2 \\ &+2A_i\sum_j A_j|h_j|^2 + \sum_j|h_j|^2|A_j|^2 + |A_i|^2\sum_j|h_j|^2) \end{aligned} \tag{15}$$

with $t = \ln(Q/M_P)$. For simplicity, we will take all soft terms equal to $m_{3/2}$ or $m_{3/2}^2$ at $Q = M_P$ although this simplification need not apply.

One may then evolve the couplings and soft terms from $Q = M_P$ the reduced Planck scale $M_P \simeq 2.4\times 10^{18}$ GeV down to the scale $Q \sim v_{PQ}$ of PQ symmetry breaking. The essential feature here is that the soft mass m_X^2 gets driven radiatively to negative values, resulting in the spontaneous breaking of PQ symmetry.

The scalar potential consists of the terms $V = V_F + V_D + V_{\rm soft}$. For now, we can ignore the Higgs field directions since these develop vevs at much lower energy scales in radiatively-driven natural SUSY. Then the relevant part of the scalar potential is just

$$V_F \ni \frac{|f|^2}{M_P^2}|\phi_X^3|^2 + \frac{9|f|^2}{M_P^2}|\phi_X^2\phi_Y|^2. \tag{16}$$

Augmenting this with $V_{\rm soft}$, we minimize V at a scale $Q = v_{PQ}$ to find the vevs of ϕ_X and ϕ_Y (v_X and v_Y):

$$0 = \frac{9|f|^2}{M_P^2}|v_X^2|^2 v_Y + f^* \frac{A_f^*}{M_P} v_X^{*3} + m_Y^2 v_Y \quad (17)$$

$$0 = \frac{3|f|^2}{M_P^2}|v_X^2|^2 v_X + \frac{18|f|^2}{M_P^2}|v_X|^2|v_Y|^2 v_X + 3f^* \frac{A_f^*}{M_P} v_X^{*2} v_Y^* + m_X^2 v_X. \quad (18)$$

The first of these may be solved for v_Y. Substituting into the second, we find a polynomial for v_X which may be solved for numerically. The potential has two minima in the v_X and v_Y plane symmetrically located with respect to the origin. For practical purposes, we use the notation v_X=$|v_X|$ and v_Y=$|v_Y|$ in the rest of the paper.

At this point one may generate the Majorana neutrino mass scale

$$M_{N_i^c} = v_X\, h_i|_{Q=v_X} \quad (19)$$

and the SUSY μ term:

$$\mu = g\frac{v_X v_Y}{M_P}\ . \quad (20)$$

Note that since the μ term depends on an arbitrary coupling g, one may obtain any desired value of μ for particular v_X and v_Y vevs by suitably adjusting g. However, if the required values of g are very different from unity, *i.e.* $g \gg 1$ or $g \ll 1$, we might need to introduce an additional physical scale to explain the μ term.

The QCD axion field a is now the corresponding Goldstone boson of the broken PQ symmetry and is a combination of the phases of the ϕ_X and ϕ_Y fields. Along with the axion, one gains a corresponding saxion s and axino $\tilde{a}$ with masses $\sim m_{3/2}$ but with superweak couplings suppressed by $1/v_{PQ}$. In addition, one obtains an orthogonal combination of a super-weakly coupled singlet field ϕ_s plus a singlino $\tilde{s}$ also with masses $\sim m_{3/2}$.

3 Numerical results

In this section, we report on results of our numerical solution of the coupled RGEs (11)-(15) and subsequent determination of the PQ scalar vevs via Eq's. (17) and (18). The vevs v_X and v_Y allow us to determine the values of the Majorana neutrino intermediate scale M, Eq. (19), and the SUSY μ parameter, Eq. (20).

In Fig. 1 we show a case of the coupled RG evolution of PQ soft terms and couplings versus renormalization scale Q starting from the reduced Planck mass M_P down to the scale of PQ breaking.[3] In the figure, we adopt a PQ-neutrino coupling value $h_i = 2$ and assume universal SUSY breaking parameters set equal to $m_{3/2}$ at M_P with value $m_{3/2} = 5$ TeV. While m_Y^2 remains constant, $m_{N_i^c}^2$ is suppressed by RG running. Meanwhile, the value of m_X^2 is pushed from an initial value of 5 TeV down through zero to negative values so that PQ symmetry is radiatively broken. Solving the scalar potential minimization conditions (with canonical choices $f = 1$ and

[3] Although Fig. 1 and Fig. 2 show the evolution of parameters down to 10^{10} GeV, we find solutions for g and M_N at $Q = v_{PQ}$ for each set of parameters.

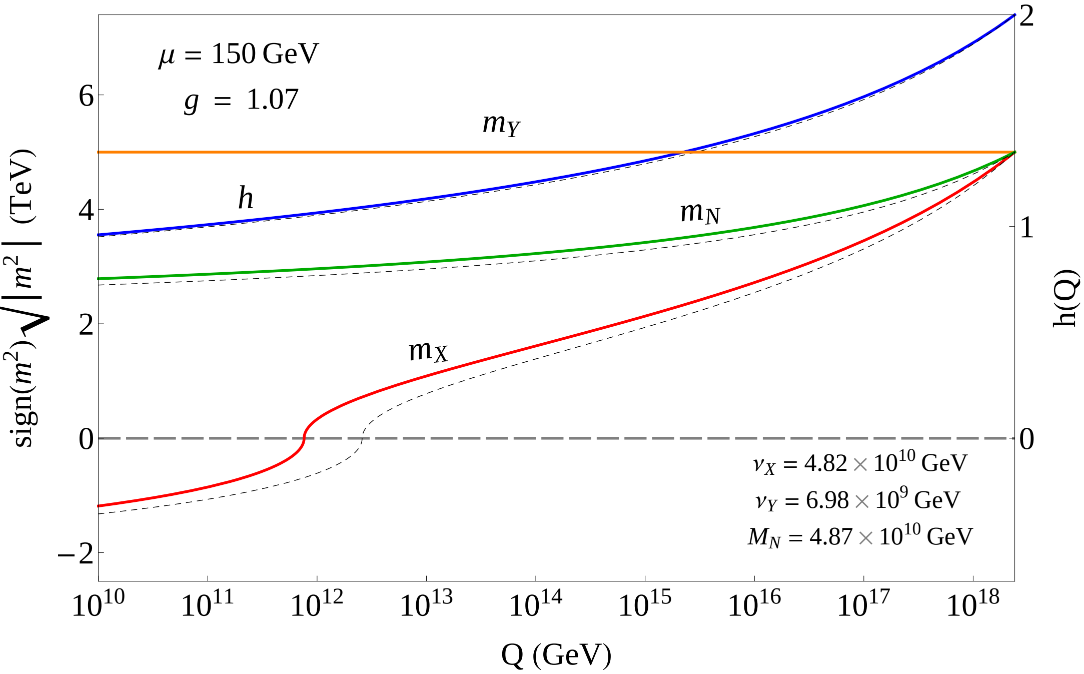


Figure 1: Plot of the running values of various soft terms and couplings versus Q for $h = 2$. We take a common value of SUSY breaking parameters, *i.e.* $m_X = m_Y = m_{N_i^c} = A_i = 5$ TeV. Black dashed lines show RG evolution without the 2-loop corrections.

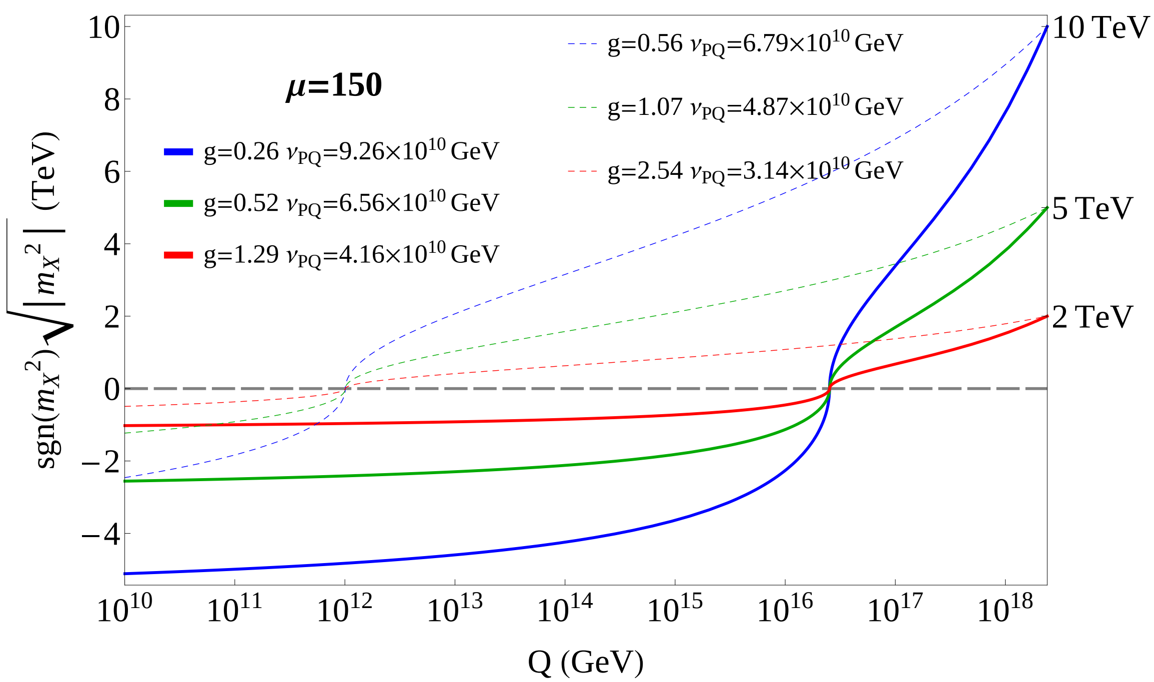


Figure 2: Plot of the running values of m_X^2 versus Q for various values of $m_{3/2}$ and $h = 2$ (dashed) and $h = 4$ (solid).

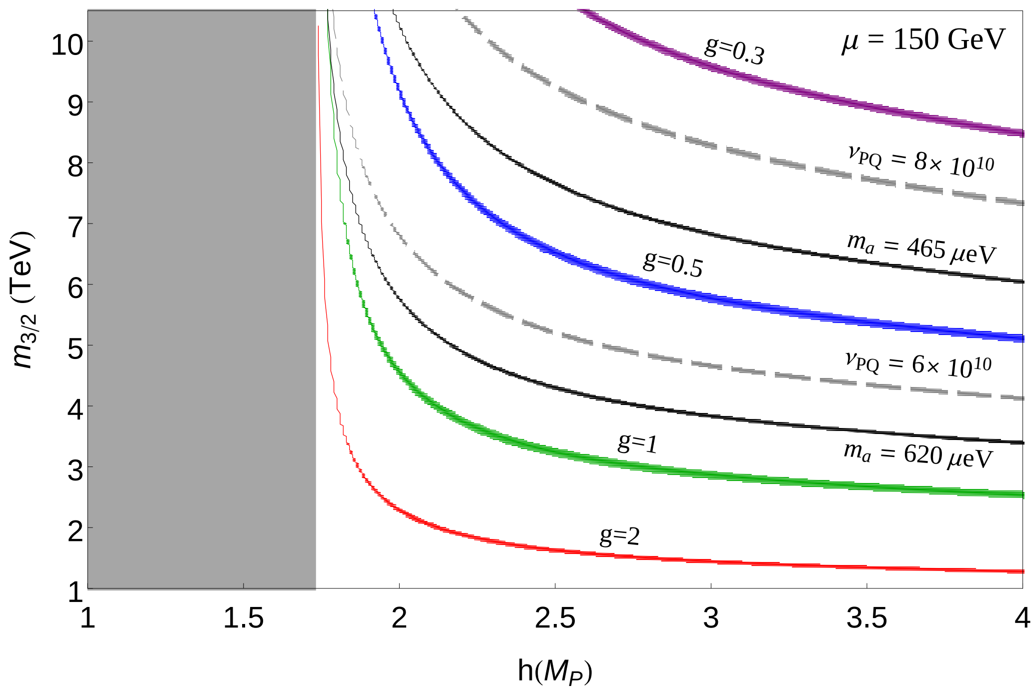


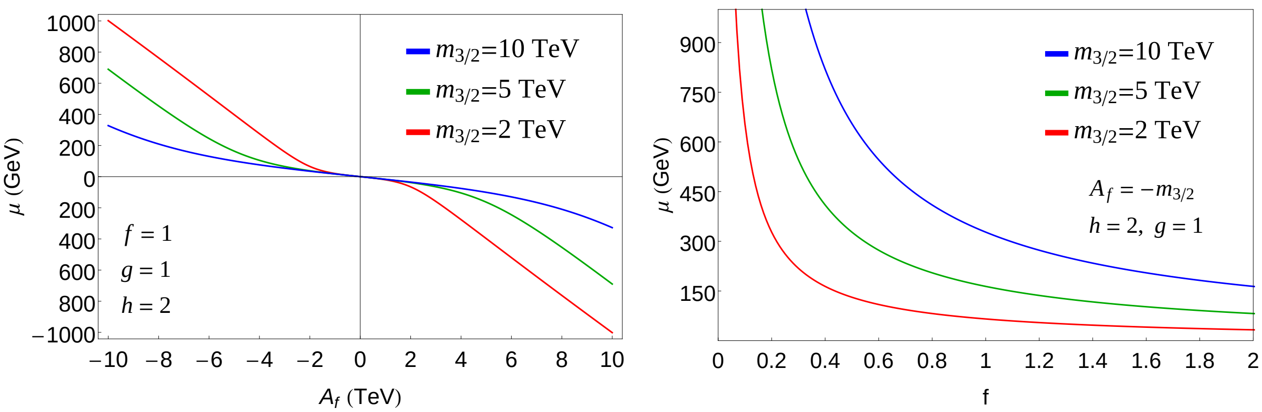
Figure 3: Values of g which are needed to generate $\mu = 150$ GeV in the h vs. $m_{3/2}$ plane. Dashed gray lines show contours of constant v_{PQ} and black lines show contours of constant axion mass, m_a.

$A_f = -m_{3/2}$) implies values of $v_X = 4.82 \times 10^{10}$ GeV and $v_Y = 6.98 \times 10^9$ GeV. The PQ scale $v_{PQ} = \sqrt{v_X^2 + v_Y^2} = 4.87 \times 10^{10}$ GeV so indeed an intermediate scale PQ breaking is generated. In this case, the axion decay constant is $f_a = \sqrt{v_X^2 + 9v_Y^2} = 5.26 \times 10^{10}$ GeV.[4] Furthermore, a right-hand (RH) Majorana neutrino scale is generated to be $M_{N_i^c} = 4.78 \times 10^{10}$ GeV. Finally, a SUSY μ term is also generated. In this case, a value of $g = 1.07$ in the PQ superpotential $\hat{f}'$ allows for a value of $\mu = 150$ GeV which is the expected region from naturalness.

In Fig. 2, we show the RG running of the critical soft breaking mass m_X^2 versus energy scale Q for several initial values of $m_X = 2$, 5 and 10 TeV. We also take values of $h_i = 2$ (dashed curves) and 4 (solid curves) at M_P. In the case of the dashed curves with $h_i = 2$, we see that for each case of m_X, the value of m_X^2 gets driven negative at exactly the same value of Q so that PQ symmetry is broken in each case. By solving the minimization conditions, we are able to generate a value of $\mu = 150$ GeV by adopting values of $g = 2.54$, 1.07 and 0.56 for $m_X(Q = M_P) = 2$, 5 and 10 TeV respectively. Thus, indeed a multi-TeV value of SUSY breaking soft parameters can generate a value of $\mu \sim m_Z$ as required by naturalness and resulting in a Little Hierarchy. If instead we take $h_i = 4$, then values of $g = 1.29$, 0.52 and 0.26 are required to generate $\mu = 150$ GeV for $m_X(M_P) = 2$, 5 and 10 TeV.

In Fig. 3, we plot contours of the value of g which is required to generate a μ parameter of 150 GeV in the $h(M_P)$ versus $m_{3/2}$ plane. The first point to note is that if h_i is too small, then m_X^2 will not get driven negative. In the case in which parameters run down to 10^{10} GeV

[4]The axion model is of DFSZ type so the axion interaction is determined by $f_a/N_{\rm DW}$ where $N_{\rm DW} = 6$.



(a) (b)

Figure 4: Plot of value of μ for three values of $m_{3/2}$ vs. (a) variation in A_f for $f = g = 1$ and $h = 2$ and (b) variation in f for $g = 1$, $h = 2$ and $A_f = -m_{3/2}$.

$(\sim v_{PQ})$, this region occurs for $h_i \lesssim 1.73$ and is shaded gray. Typically, in the $h(M_P)$ versus $m_{3/2}$ plane, large values of $g > 1$ are required for rather low values of $m_{3/2} \lesssim 2.5$ TeV. For much higher values of $m_{3/2} \sim 5$ TeV, then typically $g \sim 0.5$ is required to generate the Little Hierarchy. Values of $g \sim 0.2 - 0.3$ can generate $\mu = 150$ GeV for $m_{3/2}$ in the 10 TeV range. We also show contours of $v_{PQ} = 6 \times 10^{10}$ GeV, $v_{PQ} = 8 \times 10^{10}$ GeV (dashed gray lines), $m_a = 465$ μeV and $m_a = 620$ μeV (black lines) on the same plane. In the region above the $g = 2$ line, f_a and v_{PQ} can take a range of values such as $3.7 \times 10^{10} \lesssim f_a \lesssim 1.1 \times 10^{11}$ GeV and $3.4 \times 10^{10} \lesssim v_{PQ} \lesssim 9.4 \times 10^{10}$ GeV.

While the solution for m_X^2 is independent of f and A_f, the vevs v_X and v_Y do depend on these quantities, and hence so does μ. In our previous plots, we have taken a canonical choice $f = 1$ and $A_f = -m_{3/2}$. By choosing $A_f < 0$ we get vevs with the same sign which generate positive μ values, this choice has no other effect on any results.[5] In Fig. 4a, we show the value of μ which is generated versus A_f by taking $m_X = m_Y = m_{3/2}$ for $m_{3/2} = 2$, 5 and 10 TeV with $f = g = 1$ and $h = 2$. With $A_f = 0$, we generate $v_Y = 0$ so that $\mu = 0$ while for $A_f \sim m_{3/2}$, then we generate natural values for $\mu \sim 100 - 200$ GeV. For very large $|A_f| \gg m_{3/2}$, then unnaturally large values of μ develop for the lower range of $m_{3/2} \sim 1 - 2$ TeV. In Fig. 4b, we show the variation of μ versus f. In this case, we see that very small values of f result in large v_X and v_Y and hence large μ values. For $f \sim 1$, then natural values of $\mu \sim 100 - 200$ GeV can develop.

A phenomenological aspect of the MSY model has been investigated by Martin [33]. Since two PQ fields $\hat{X}$ and $\hat{Y}$ have been hypothesized, then one combination gives the usual axion-axino-saxion supermultiplet while the other gives a super-weakly coupled singlet-singlino combination $(\phi_s, \tilde{s})$ with masses $\sim m_{3/2}$. While normally one would not expect such super-weakly coupled states to give rise to collider effects, in this case the singlino $\tilde{s}$ could be the LSP. Then each NLSP produced via sparticle production followed by cascade decays in collider experiments would ultimately decay to the singlino. These delayed NLSP decays could give rise to sparticle

[5]We can also obtain positive μ by taking $g < 0$ for $A_f > 0$.

production events with displaced vertices which might be easily seen in LHC detectors.

4 Some related models

In the previous section, we have seen that, starting with multi-TeV values of gravitino mass (as required by LHC constraints for models of gravity mediation) one can easily generate values of $\mu \sim m_Z$ as required by electroweak naturalness. In this case, a Little Hierarchy emerges quite naturally from radiatively-driven PQ symmetry breaking. While our results are illustrated in the MSY model of radiative PQ symmetry breaking, the overall phenomena may be more general. Here we comment on two related models.

A very similar model is written down by Choi, Chun and Kim (CCK) [34]. In the CCK model, the PQ part of the superpotential is given by

$$\hat{f}_{CCK} = \frac{1}{2} h_{ij} \hat{X} \hat{N}_i^c \hat{N}_j^c + \frac{f}{M_P} \hat{X}^3 \hat{Y} + \frac{g}{M_P} \hat{X}^2 \hat{H}_u \hat{H}_d. \tag{21}$$

While the PQ charge assignments will differ from the MSY case, this model also exhibits radiative PQ symmetry breaking for sufficiently large values of h_i. Thus, the resulting μ term is similar to the MSY case.

Martin has also written down similar models but with a different mechanism for PQ breaking [11, 35, 36]. In this case, the superpotential is given by

$$\hat{f}_{SPM} \ni \frac{g_1}{M_P} \hat{X}^2 \hat{H}_u \hat{H}_d + \frac{g_2}{M_P} \hat{X}^2 \hat{Y}^2. \tag{22}$$

Martin notes that the field directions $\hat{X}$ and $\hat{Y}$ give rise to nearly flat directions in the scalar potential. In such a case, then Planck-suppressed hard SUSY breaking quartic operators are expected to occur and can contribute to the scalar potential. Then one can achieve intermediate scale PQ breaking even without soft mass terms being driven to negative values [36]. It is also possible to break PQ symmetry in the MSY model by the large quartic coupling, *i.e.* large $|A_f|$ in Eq. (10). In this case, however, the PQ scale is rather large, *i.e.* $v_{PQ} \sim m_{\rm hidden}$, and thus we need much smaller g to generate a natural value of $\mu \sim m_Z$. Models with more than two PQ fields are of course also possible.

5 Consequences for experiment

The consequences of a Little Hierarchy $\mu \ll m_{3/2}$ for LHC sparticle searches has been examined in Ref's [37, 38]. While the LHC reach for gluino pairs extends to $m_{\tilde{g}} \sim 1.7$ TeV (for 300 fb^{-1} fb and in the case of heavier squarks), the requirements from naturalness are only that $m_{\tilde{g}} \lesssim 4$ TeV[17] (for the electroweak measure $\Delta_{EW} < 30$ corresponding to no worse than 3% electroweak fine-tuning). Also, a distinct same-sign diboson signature arising from wino pair production followed by decay to higgsinos– which is endemic to natural SUSY with light higgsinos– can extend this reach[37]. However, light higgsinos by themselves are very difficult to detect at LHC, in spite of reasonably large production cross sections, due to their compressed spectra and low visible energy release from their three-body decays. Tagging on initial state radiation from higgsino pair production may aid this enterprise[39, 40, 41, 42].

The smoking gun signature for natural SUSY with low $\mu \sim m_Z$ is pair production of light higgsinos at an e^+e^- collider [43] provided that $\sqrt{s} > 2m(\text{higgsino})$. The clean event environment should easily allow discovery of any higgsinos which are produced in spite of their compressed spectra. In addition, precision measurements of μ and the electroweak -ino masses can be made[43].

In this class of models, one expects dark matter to be composed of an axion plus higgsino-like WIMP admixture, and detection of both axions[44] and WIMPs[45] should ultimately be expected. In such a case, the measured value of $\mu \sim f_a^2/M_P$ at ILC will be related to the axion mass $m_a \sim 620\mu\text{eV}\,(10^{10}\text{ GeV}/(f_a/N_{DW}))$. The correlation between LHC, ILC, WIMP and axion detections should provide good corroborating evidence for MSY-like models which correlate the mass of the expected light higgsinos to the value of the axion mass.

6 Conclusions

In this paper, we have explored the case where the gauge hierarchy problem is solved via supersymmetry while the strong CP problem is solved by the introduction of PQ symmetry and its concommitant axion. In such models, three intermediate scales are present: the hidden sector mass scale m_{hidden}, the Majorana neutrinos scale M_N and the PQ scale v_{PQ}. We have explored consequences of the MSY SUSY axion model which is able to generate the neutrino and PQ scales as a consequence of radiative PQ symmetry breaking triggered by hidden sector SUSY breaking. In fact, in string theory the first expectation is that the PQ scale $f_a \sim M_{\text{GUT}} - M_P$ [46]. In the MSY model instead it naturally emerges at a phenomenologically more viable intermediate scale $\sim 10^{10} - 10^{12}$ GeV.

While LHC sparticle search limits plus the rather high value of the Higgs mass $m_h \sim 125.5$ GeV seem to indicate a sparticle mass scale $m_{3/2}$ in the multi-TeV range, electroweak naturalness requires the weak scale soft term $|m_{H_u}|$ and the μ parameter to be of order m_Z. While $m_{H_u}^2$ can be driven to small negative values via radiative electroweak symmetry breaking, the MSY model provides a similar mechanism to produce a value of $\mu \sim 100 - 200$ GeV via radiative PQ breaking. In this case, the Little Hierarchy characterized by $\mu \ll m_{3/2}$ emerges quite naturally and is in fact associated with the intermediate scale hierarchy $f_a \ll m_{\text{hidden}}$.

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