X(3872), X_{b}, and the χ_{b1}(3P) state

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$X(3872)$, X_b, and the $\chi_{b1}(3P)$ state

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ABSTRACT

We discuss the possible production and discovery channels in e^+e^- and pp machines of the X_b, the bottomonium counterpart of $X(3872)$ and the putative isoscalar analogue of the charged bottomonium-like states Z_b discovered by Belle. We suggest that the X_b may be close in mass to the bottomonium state $\chi_{b1}(3P)$, mixing with it and sharing its decay channels, just as $X(3872)$ is likely a mixture of a $\bar{D}D^*$ molecule and $\chi_{c1}(2P)$. Consequently, the experiments which reported observing $\chi_{b1}(3P)$ might have actually discovered the X_b, or a mixture of the two states.

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I   Introduction

The search for "exotic" mesons made of more than a quark and an antiquark, and for exotic baryons made of more than three quarks, is almost as old as the quark model itself [1, 2]. (For an early suggestion of exotic mesons in baryon-antibaryon channels see Ref. [3].) However, for many years no such states were conclusively observed [4]. The discovery of a neutral state $X(3872)$ [5] in 2003, where 3872 stands for the mass in MeV/c^2, suggested the possibility of richer structures, such as $c\bar{c}q\bar{q}$, where c is a charmed quark and $q = u$ or d. This particle is now most plausibly understood as a mixture of a P-wave charmonium $\chi_{c1}(2P)$ level of spin 1 and an S-wave molecule of $D^0\bar{D}^{*0}$ + c.c. [6] whose binding energy is so close to zero that its sign is not yet known [4]. Evidence for a charged counterpart of this particle at a mass of about 3900 MeV came several years later [7, 8].

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Molecular states of charmonium had been proposed a number of years ago [9–11]. The fact that the $X(3872)$ can decay both to $J/\psi\rho^0$ and $J/\psi\omega$ is accounted for by the substantial isospin splitting between $D^0\bar{D}^{*0}$+c.c. and D^+D^{*-}+c.c., so that the tetraquark configuration of $X(3872)$ is mainly $c\bar{c}u\bar{u}$, permitting equal coupling to ρ^0 and ω [12].

It was proposed in 2008 [13] that similar behavior in the bottomonium system could lead to resonant effects near $B\bar{B}^*$ + c.c. thresholds. (See also Refs. [14, 15].) In 2011 a charged candidate for such a state, $Z_b(10610)$, along with a candidate for a $B^*\bar{B}^*$ state $Z_b(10650)$, was observed by the Belle Collaboration [16].

In the present article we propose ways to look for the X_b, which plays the dual role of the bottomonium analogue of the $X(3872)$ and the isoscalar partner of the $Z_b(10610)$. In Section II we discuss some expected general features of X_b, including estimates of its mass if it is a bound state of $B\bar{B}^*$ + c.c., and a discussion of its likely mixing with a nearby bottomonium state, the recently observed P-wave $b\bar{b}$ excitation $\chi_{b1}(3P)$ [17–20]. Properties of the $\chi_{b1}(3P)$, especially its radiative decays, are treated in Sec. III. A corresponding discussion for the $\chi_{c1}(2P)$, expected to mix with the $X(3872)$, is given in Sec. IV. The isoscalar nature of the X_b is the subject of Sec. V. Consequences for observing the X_b are noted in Sec. VI, while Sec. VII concludes.

II   Expected general features of X_b

An accurate estimate of the mass of $X(3872)$ [11] was made long ago based on a calculation of the binding of D^0 and $\bar{D}^{*0}$ due to pion exchange and other forces. This estimate was extended to bottom mesons, leading to a predicted mass $M(X_b) = 10562$ MeV [11, 12] for a $B\bar{B}^*$ + c.c. state of $J^{PC} = 1^{++}$. This mass is quite close to that observed for the P-wave $b\bar{b}$ excitation $\chi_{b1}(3P)$, which ranges from about 10510 to 10550 MeV/c^2. One prediction [21] estimates $M(\chi_{b1}(3P)) = 10516$ MeV/c^2 but the spread in models is modest as the interquark potential is well known in this regime [22].

An independent estimate of $M(X_b)$, to be mentioned in more detail below, was based on the expected binding energy of a B and $\bar{B}^*$, yielding 10585 MeV/c^2 [23]. This is 23 MeV above Tornqvist's value, but still fairly close to the expected mass of $\chi_{b1}(3P)$. Thus, in either case, there should be two nearby states with $I = 0$, $J^{PC} = 1^{++}$, sharing common decay modes to some extent. Another estimate, appearing in an unpublished version of Ref. [24], is $M(X_b) = 10604$ MeV/c^2, just below $B\bar{B}^*$ threshold.

The radiative decay widths of a $B\bar{B}^*$+c.c. bound state to $\Upsilon(1S, 2S, 3S)$ have been estimated to be quite small, of order (1.5,1.8,0.4) keV or less [25]. Unfortunately they depend sensitively on unknown parameters; a set favored by the authors predicts (0.7,0.5,0.2) keV for these values. Note that the decay width to the 3S state is smaller than those to 1S or 2S. As we shall see, only a small admixture of $\chi_{b1}(3P)$ in the $B\bar{B}^*$+c.c. wave function can alter this pattern drastically.

Table I: Values of $M(\chi_{b1}(3P))$ observed in various experiments.

Collaboration	Reference	Value (MeV/c^2)
ATLAS	[17]	$10530 \pm 5 \pm 9$
D0	[18]	$10551 \pm 14 \pm 17$
LHCb (a)	[19]	$10511.3 \pm 1.7 \pm 2.5$
LHCb (b)	[20]	$10515.7^{+2.2+1.5}_{-3.9-2.1}$

(a) Using non-converted photons. (b) Using converted photons.

Table II: Dipole matrix elements, photon energies, and partial decay widths for the transitions $\chi_{b1}(3P) \to \gamma \Upsilon(n'S)$.

n'	$\langle n'|r|3 \rangle$ $(\text{GeV})^{-1}$	E_γ (MeV)	$\Gamma(\chi_{b1}(3P)) \to$ $\gamma\Upsilon(n'S))$ (keV)
1	0.101	1003	3.69
2	0.298	481	3.56
3	2.627	159	10.1

III Properties of the $\chi_{b1}(3P)$ bottomonium state

A Mass

The reported mass values of the $\chi_{b1}(3P)$ are summarized in Table I. These are compatible with the predicted value of 10516 MeV/c^2 in Ref. [21]. The differences between the first two values and the last two, if not due to limited statistics, may stem from different admixtures of $\chi_{b2}(3P)$ in central [17, 18] and forward [19, 20] production.

B Radiative decays

A key feature of the $\chi_{b1}(3P)$ is the expected dominance of $\Gamma(\chi_{b1}(3P) \to \gamma\Upsilon(3S))$ over $\Gamma(\chi_{b1}(3P) \to \gamma\Upsilon(2S))$ or $\Gamma(\chi_{b1}(3P) \to \gamma\Upsilon(1S))$, as a result of a much larger electric dipole amplitude. The rate for an E1 transition from a $^3P_1$ state with radial quantum number n to a $^3S_1$ state with radial quantum n' in a quarkonium $Q\bar{Q}$ system [21, 26–29] is

$$\Gamma(n^3P_1) \to \gamma n'^3 S_1) = \frac{4}{9}e_Q^2 \alpha E_\gamma^3 |\langle n'|r|n \rangle|^2 \ , \tag{1}$$

where e_Q is the charge of Q (2/3 for c, –1/3 for b), $\alpha = 1/137.036$, and E_γ is the photon energy. Dipole matrix elements predicted in Ref. [21], photon energies, and partial decay widths are summarized in Table II. The dominance of the transition to the 3S level is a key signature that one is dealing with a state with at least a substantial admixture of $\chi_{b1}(3P)$. Indeed, LHCb has observed the transition $\chi_b(3P) \to \gamma\Upsilon(3S)$ [19], but ratios of branching fractions of $\chi_b(3P) \to \gamma\Upsilon(1S, 2S, 3S)$ are not quoted.

Table III: Comparison of electric dipole matrix elements and decay rates for $\chi_{c1}(2P) \to (J/\psi, \psi(2S))\gamma$ transitions in various treatments.

Reference	$\langle 1S\|r\|2P \rangle$ $(\text{GeV})^{-1}$	$\Gamma(\chi_{c1}(2P) \to J/\psi\gamma)$ (keV)	$\langle 2S\|r\|2P \rangle$ $(\text{GeV})^{-1}$	$\Gamma(\chi_{c1}(2P) \to \psi(2S)\gamma)$ (keV)	$R(\chi_{c1}(2P))$
This work	0.416	84.7	3.315	94.1	1.11
[37]	0.348	59.2	3.196	87.5	1.48
[38]	0.150	11.0	2.723	63.9	5.81
[39]	0.412	83	3.468	103	1.24
[39]	(a)	45	(a)	60	1.33
[40]	0.260	33	4.128	146	4.42
[41]	0.150	11.0	2.859	70	6.36

(a) Relativistic calculation

IV   Properties of the $\chi_{c1}(2P)$

The hierarchy of electric dipole matrix elements, discussed in the previous Section for bottomonium transitions, also applies to charmonium transitions, and is likely the reason that the branching ratio of $X(3872)$ to $\gamma\psi(2S)$ is larger than that for $X(3872) \to \gamma J/\psi$.

We assume that $X(3872)$ is a mixture of the charmonium state $\chi_{c1}(2P)$ and a $D^0\bar{D}^{*0} +$ c.c. molecule [6, 30]. This assumption is supported by a measurement by the LHCb Collaboration [31] of the ratio

$$R_{\psi\gamma} \equiv \frac{\mathcal{B}(X(3872) \to \psi(2S)\gamma)}{\mathcal{B}(X(3872) \to J/\psi\gamma)} = 2.46 \pm 0.64 \pm 0.29 \ . \tag{2}$$

These authors quote numerous theoretical predictions for $R_{\psi\gamma}$: $(3\text{--}4)\times 10^{-3}$ for a $D\bar{D}^*$ molecule (see, e.g., [30]), 1.2–15 for a pure charmonium state, and 1.5–5 for a molecule-charmonium admixture. (Ref. [32] addresses the question of how to determine the relative fractions of $c\bar{c}$ and $D\bar{D}^*$ molecule in $X(3872)$ using the line shape in the decay $X(3872) \to (D^0\bar{D}^{*0}+\text{c.c.})$.) The large variation for charmonium is due mainly to uncertainty in the size of the electric dipole matrix element $\langle 1S|r|2P \rangle$, which is sensitive to cancellations between contributions from positive and negative values of the $2P$ wave function. Such a matrix element would vanish completely in a harmonic oscillator potential. The pattern of such suppressions has been discussed in [33].

We estimate the decay rates of a pure charmonium state at 3872 MeV. We found $\langle 1S|r|2P \rangle = 0.240$ GeV^{-1} and $\langle 2S|r|2P \rangle = 1.911$ GeV^{-1} for the bottomonium system in Ref. [21]. Using the similarity of the charmonium and bottomonium interaction to a logarithmic potential [34], one can then obtain $\langle 1S|r|2P \rangle$ and $\langle 2S|r|2P \rangle$ for charmonium by a simple rescaling [35]. For a $Q\bar{Q}$ bound state of quarks with mass m_Q in a potential $V(r) = \lambda r^a$, lengths scale as $\ell \sim m_Q^{-1/(2+a)}$, and hence as $\ell^{-1/2}$ in a logarithmic potential. We thus obtain $\langle 1S|r|2P \rangle = 0.416$ GeV^{-1} and $\langle 2S|r|2P \rangle = 3.315$ GeV^{-1} for charmonium, using the quark masses $m_c = 1663$ MeV and $m_b = 5004$ MeV from fits to spectra of mesons with one c or b quark [36].

Using Eq. (1) for electric dipole transitions from a n^3P_1 to a n'^3S_1 state, and photon energies of (697,181) MeV for the transitions from 3872 MeV to (3097,3686) MeV, we then

predict

$$\Gamma(\chi_{c1}(2P) \to J/\psi\gamma) = 84.7 \text{ keV} \ ; \ \Gamma(\chi_{c1}(2P) \to \psi(2S)\gamma) = 94.1 \text{ keV} \ ; R(\chi_{c1}(2P)) = 1.11 \ ,$$
$$(3)$$

where $R(\chi_{c1}(2P))$ is the ratio in Eq. (2) for a pure $\chi_{c1}(2P)$. These values are compared with some others [37–41] in Table III. (Several other calculations [42–45], based on varying assumptions beyond the use of Eq. (1), give values of $R(\chi_{c1}(2P))$ mostly within the same range.) Where electric dipole matrix elements are not quoted, we have extracted them from these references and rescaled predicted widths to photon energies appropriate for $X(3872)$. One sees considerably more variation in $\langle 1S|r|2P\rangle$ than in $\langle 2S|r|2P\rangle$. One should expect, similarly, to be less confident about estimates of $\langle 1S|r|3P\rangle$ and $\langle 2S|r|3P\rangle$ than of $\langle 3S|r|3P\rangle$ in bottomonium.

V Isoscalar nature of the X_b

Recently CMS and ATLAS have searched for the decay $X_b \to \Upsilon(1S)\pi^+\pi^-$ [46, 47]. The search in this particular channel was motivated by the seemingly analogous decay $X(3872) \to J/\psi\,\pi^+\pi^-$. For this particular decay channel the analogy is misguided [48, 49]. The null result of the search described in this paper does not tell us if the X_b exists, because for an isoscalar with $J^{PC} = 1^{++}$ such a decay is forbidden by G-parity conservation.

So how come $X(3872)$ which also has $J^{PC} = 1^{++}$ *does* decay into $J/\psi\,\pi^+\pi^-$? In the charm sector isospin is badly broken between D^+ and D^0. D^+ is 4.76 MeV heavier than D^0, while D^{*+} is 3.30 MeV heavier than D^{*0}. Since $X(3872)$ is right at the $\bar{D}D^*$ threshold, its decays break isospin, so $X(3872) \to J/\psi\,\pi^+\pi^-$ is allowed. In fact, $X(3872)$ decays roughly equally to $J/\psi\,\rho$ and to $J/\psi\,\omega$, which of course cannot happen if the decays conserve isospin.

On the other hand, in the bottom sector the $B^0 - B^+$ mass difference is tiny, 0.32 MeV, so isospin is very well conserved in the decays of $(\bar{B}B^*)$ "molecules" X_b, and the decay $X_b \to \Upsilon\pi^+\pi^-$ is forbidden.

A simple way to quantify the difference between isospin violation in decays of $X(3872)$ and X_b is to compare the binding energy with the isospin splitting in the two-meson sector. In order for $X(3872)$ to be pure $I = 0$, it would have to be an equal admixture of $\bar{D}^0D^{0*}$ and D^-D^{+*}. But the masses of these two components are very different:

$$M(\bar{D}^0D^{0*}) = (3871.80 \pm 0.12) \text{ MeV},$$
$$vs. \qquad\qquad\qquad\qquad\qquad\qquad (4)$$
$$M(D^-D^{+*}) = (3879.87 \pm 0.12) \text{ MeV}.$$

$X(3872)$, with a quoted mass [4] of (3871.69 ± 0.17) MeV, is essentially at $\bar{D}^0D^{0*}$ threshold, i.e., the binding energy is less than 1/2 MeV. The would-be D^-D^{+*} component is well above threshold and does not contribute. The $\bar{D}^0D^{0*}$ is a combination of $I = 0$ and $I = 1$ and therefore $X(3872)$ decays into both $J/\psi\,\omega$ and $J/\psi\,\rho$ with roughly equal branching fractions.

In Ref. [23] the X_b binding energy was estimated with the help of the existing data: $Z_b(10610)$, $Z_c(3900)$ in the $I = 1$ channel and $X(3872)$ in the $I = 0$ channel. Since the kinetic energy is inversely proportional to mass, the heavier the heavy quark, the deeper the binding. The upshot is that even in the most extreme case of infinitely heavy b-quark

analogue the binding energy is 35 MeV. For the real-world 5 GeV b-quark, the binding energy was found to be significantly smaller, about 20 MeV.

With X_b about 20 MeV below $\bar{B}B^*$ threshold the situation in the bottomonium sector is very different from the charmonium sector:

$$
\begin{aligned}
M(\bar{B}^0 B^{0*}) &= (10604.8 \pm 0.4) \text{ MeV}, \\
vs. & \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (5)\\
M(B^- B^{+*}) &= (10604.5 \pm 0.4) \text{ MeV}.
\end{aligned}
$$

In this case the isospin splitting is very small compared with the binding energy: (0.3 ± 0.4) MeV vs. at least 20 MeV, i.e., at most 1.5%. Therefore X_b will be an almost pure isoscalar. The estimate of Ref. [23] predicts its mass to be about 10585 MeV/c^2, about 23 MeV/c^2 above that of Tornqvist [11, 12].

VI   Strategies for observation

The likely mixing of X_b with the $\chi_{b1}(3P)$ bottomonium state suggests that decays of the latter (and of lighter χ_b states) will provide a good guide to isospin-conserving X_b decays. We focus on several final states.

A $X_b \to \Upsilon(1S)\omega = \Upsilon(1S)\pi^+\pi^-\pi^0$

This process has features in common with the decays $\chi_{b1,2}(2P) \to \Upsilon(1S)\omega$ observed by the CLEO Collaboration [50]:

$$
\begin{aligned}
\mathcal{B}(\chi_{b1}(2P) \to \Upsilon(1S)\omega) &= (1.63^{+0.35+0.16}_{-0.31-0.15})\% \ , \\
\mathcal{B}(\chi_{b2}(2P) \to \Upsilon(1S)\omega) &= (1.10^{+0.32+0.11}_{-0.28-0.10})\%. \quad\quad (6)
\end{aligned}
$$

An estimate of the rate for $\chi_{b1}(3P) \to \Upsilon(1S)\omega$ is difficult because the increased Q-value may be offset by a smaller transition matrix element. The total width predicted for $\chi_{b1}(2P)$ [21] is 79 keV, so the branching fraction quoted above corresponds to an expected partial decay width $\Gamma(\chi_{b1}(2P) \to \Upsilon(1S)\omega) \simeq 1.3$ keV, about 1/3 of the partial decay rates for $\chi_{b1}(3P) \to \gamma\Upsilon(1S, 2S)$ predicted in Table II. No significant signal for $X_b \to \Upsilon(1S)\pi^+\pi^-\pi^0$ has been seen by the Belle Collaboration [51].

B $X_b \to \Upsilon(2S)\omega^* = \Upsilon(2S)\pi^+\pi^-\pi^0$

The Q-value of this decay is too small to permit the production of a real ω, but the three-pion system can still be produced with an effective mass up to 540–560 MeV, depending on the exact mass of X_b.

C $X_b \to \chi_{b1}\pi^+\pi^-$

This decay has features in common with the decays $\chi_b(2P) \to \chi_b(1P)\pi\pi$ observed by the CLEO [52] and BaBar [53] Collaborations. The Particle Data Group [4] quotes the averages

$$
\mathcal{B}(\chi_{b1}(2P) \to \chi_{b1}(1P)\pi\pi) = (9.1\pm1.3)\times10^{-3} \, , \ \mathcal{B}(\chi_{b2}(2P) \to \chi_{b2}(1P)\pi\pi) = (5.1\pm0.9)\times10^{-3} \, .
$$
$$
(7)
$$

Note that the total spin of the bottomonium system is preserved in these decays, so it is reasonable to assume that will also be the case in X_b decays. There is just barely enough Q-value to permit the decay $X_b \to \chi_{b1}(2P)\pi\pi$, so it makes more sense to look for $X_b \to \chi_{b1}(1P)\pi\pi$, followed of course by $\chi_{b1}(1P) \to \gamma\Upsilon(1S)$.

D $X_b \to \Upsilon(3S)\gamma$

We have argued that if X_b contains a substantial amount of $\chi_{b1}(3P)$ in its wave function, this decay is likely to dominate over $X_b \to \Upsilon(1S, 2S)\gamma$.

E $X_b \to \Upsilon(1S, 2S)\gamma$

As the decays $\chi_b(3P) \to \Upsilon(1S, 2S)\gamma$ have been observed [17–20], and the X_b is expected to mix strongly with the $\chi_{b1}(3P)$, it is worth while to examine the $\Upsilon(1S, 2S)\gamma$ mass spectra for any departures from single Breit-Wigner behavior.

VII   Conclusions

We have offered several suggestions for identifying the X_b, the bottomonium analogue of $X(3872)$. We have noted the close proximity of its predicted mass to the bottomonium state $\chi_{b1}(3P)$ recently identified at hadron colliders. Thus, we expect a molecular state of $B\bar{B}^* + \mathrm{c.c.}$ to mix strongly with a bottomonium state, and many of the decay modes of X_b to mirror those of a pure $\chi_{b1}(3P)$ state. The most promising of these include $\Upsilon(1S)\omega$, $\chi_{b1}(1P)\pi\pi$, and $\Upsilon(3S)\gamma$. The mass spectra in the latter final state (as well as $\Upsilon(1S, 2S)\gamma$) should show some departures from a pure Breit-Wigner shape if examined with sufficient resolution.

Note added: After this work was submitted for publication we became aware of the work of Ref. [54], which finds $X(3872)$ to be a mixture of $c\bar{c}$ ($\sim 65\%$), $D^0\bar{D}^{*0} + \mathrm{c.c.}$ ($\sim 27\%$) and $D^+D^{*-} + \mathrm{c.c.}$ (7%) with sub-percent contributions from other states. It would be interesting to see what a similar approach gives for X_b.

Acknowledgments

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