Puzzles in hadronic transitions of heavy quarkonium with two pion emission

Jorge Segovia, D. R. Entem, and F. Fernández

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Jorge Segovia[*]

Physics Division, Argonne National Laboratory,
Argonne, Illinois 60439-4832, USA

D.R. Entem[†] and F. Fernández[‡]

Departamento de Física Fundamental and IUFFyM,
Universidad de Salamanca, E-37008 Salamanca, Spain

We study the anomalously large rates of some hadronic transitions observed in heavy quarkonia using a constituent quark model which has been successful in describing meson and baryon phenomenology. QCD multipole expansion (QCDME) is used to described the hadronic transitions. The hybrid intermediate states needed in the QCDME method are calculated in a natural, parameter-free extension of our constituent quark model based on the Quark Confining String (QCS) scheme. Some of the anomalies are explained due to the presence of a hybrid state with a mass near the one of the decaying resonance whereas others are justified by the presence of molecular components in the wave function. Certain unexpected results are pointed out.

I. INTRODUCTION

The observation in the last decade of many new $c\bar{c}$ and charmonium-like states has re-opened interest in charmonium spectroscopy. This interest has been also extended to the bottomonium sector. Hadronic transitions of heavy quarkonia such as $\psi(nS)$ or $\Upsilon(nS)$ to lower states with emission of two pions are important means to study these new states and understanding both the heavy quarkonium dynamics and the light hadron(s) formation.

Many new results have been collected by the electron-positron colliders using the initial state radiation technique. In particular, the $e^+e^- \to J/\psi\pi^+\pi^-$ and $e^+e^- \to \psi(2S)\pi^+\pi^-$ reactions have been recently analyzed in a mass distribution region between 3.5 and $5.5\,\mathrm{GeV/c^2}$. As the entrance channel fixes the quantum numbers $J^{PC} = 1^{--}$, one expects to obtain information of the possible vector charmonium resonances in this energy region. However, the recent experimental data on hadronic transitions of charmonium states which are above the open-flavor threshold show a puzzling behavior. In the $J/\psi\pi^+\pi^-$ channel only one resonance appears, attributed to the $X(4260)$, whereas in the $\psi(2S)\pi^+\pi^-$ channel two resonances, compatible with the $X(4360)$ and $X(4660)$, show up. A recent re-analysis of the $\psi(2S)\pi^+\pi^-$ data including the $X(4260)$ resonance shows a non-significant contribution $(2.1\,\sigma)$. No signals of the $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ appears in the data. Furthermore, the $X(4360)$ and $X(4660)$ resonances show an anomalous large width in the $\psi(2S)\pi^+\pi^-$ channel [1–3]. In the bottom sector, compared to the ordinary $\Upsilon(nS) \to \Upsilon(mS)$ $(m < n)$ transitions, the partial widths of the $\Upsilon(10860)$ decaying into $\Upsilon(1S)$, $\Upsilon(2S)$ or $\Upsilon(3S)$ plus two pions are out of line by two orders of magnitude [4].

The anomalous widths can be due to several mechanisms: Contribution of hadron loops [5]; four-quark components in the quarkonium wave functions [6]; or, as we will see later, the existence of hybrid mesons with a mass near the one of the decaying resonance.

Hadronic transitions can be described using the QCD multipole expansion (QCDME) approach [7]. In the single channel picture the light hadrons are converted from the gluons emitted by the heavy quarks in the transition. The typical momentum of the emitted gluons is too low for perturbative QCD to be applicable. Then nonperturbative approaches, like QCDME, are needed.

In this approach the heavy quarkonium system serves as a compact color source which emits two soft gluons that hadronize, for instance, into two pions. After the emission of the first gluon and before the emission of the second one, there exists an intermediate state where the $Q\bar{Q}$ pair together with the gluon forms a hybrid state. The width of the transition depends critically on the position of this state, therefore it is important to describe consistently the $Q\bar{Q}$ states and the hybrids using as few parameters as possible. Apart from lattice calculations [8, 9], hybrid meson properties has been calculated in different models: the flux-tube model [10, 11], constituent gluons [12], Coulomb gauge QCD [13] and quark confining string model (QCS) [14–16] or QCD string model [17].

In this work we will address the description of the new data of the hadronic transitions in heavy quarkonium within the framework of a constituent quark model (see references [18] and [19] for reviews) which has been successful in describing the hadron phenomenology and the hadronic reactions. Hybrid states are consistently

[*] jsegovia@anl.gov
[†] entem@usal.es
[‡] fdz@usal.es

generated in the original quark model using the QCS scheme. In this way, we minimize the number of free parameters describing both conventional and hybrid states.

The paper is organized as follows. In Sec. II we will review the main properties of the constituent quark model and give its prediction for the vector charmonium and bottomonium states. Sec. III is devoted to the description of the QCDME approach and the hybrid model we use. We will present our results in Sec. IV. The work will be summarized in Sec. V.

II. CONSTITUENT QUARK MODEL AND ITS UPDATED RESULTS OF VECTOR QUARKONIUM RESONANCES

Spontaneous chiral symmetry breaking of the QCD Lagrangian together with the perturbative one-gluon exchange (OGE) and the nonperturbative confining interaction are the main pieces of constituent quark models. Using this idea, Vijande *et al.* [20] developed a model of the quark-quark interaction which is able to describe meson phenomenology from the light to the heavy quark sector.

In the heavy quark sector chiral symmetry is explicitly broken and Goldstone-boson exchanges do not appear. Thus, OGE and confinement are the only interactions remaining. The one-gluon exchange potential is given by

$$V_{\text{OGE}}^{\text{C}}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\left[\frac{1}{r_{ij}} - \frac{1}{6m_im_j}(\vec{\sigma}_i \cdot \vec{\sigma}_j)\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}r_0^2(\mu)}\right],$$

$$V_{\text{OGE}}^{\text{T}}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_im_j}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}}\left(\frac{1}{r_{ij}^2} + \frac{1}{3r_g^2(\mu)} + \frac{1}{r_{ij}r_g(\mu)}\right)\right]S_{ij},$$

$$V_{\text{OGE}}^{\text{SO}}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_i^2m_j^2}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3}\left(1 + \frac{r_{ij}}{r_g(\mu)}\right)\right] \times$$
$$\times \left[((m_i + m_j)^2 + 2m_im_j)(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(\vec{S}_- \cdot \vec{L})\right],$$

(1)

where $r_0(\mu) = \hat{r}_0\frac{\mu_{nn}}{\mu_{ij}}$ and $r_g(\mu) = \hat{r}_g\frac{\mu_{nn}}{\mu_{ij}}$ are regulators which depend on μ_{ij}, the reduced mass of the $q\bar{q}$ pair. The contact term of the central potential has been regularized as

$$\delta(\vec{r}_{ij}) \sim \frac{1}{4\pi r_0^2}\frac{e^{-r_{ij}/r_0}}{r_{ij}}.$$

(2)

The wide energy range needed to provide a consistent description of light, strange and heavy mesons requires an effective scale-dependent strong coupling constant. We use the frozen coupling constant of Ref. [20]

$$\alpha_s(\mu) = \frac{\alpha_0}{\ln\left(\frac{\mu^2+\mu_0^2}{\Lambda_0^2}\right)},$$

(3)

in which μ is the reduced mass of the $q\bar{q}$ pair and α_0, μ_0 and Λ_0 are parameters of the model determined by a global fit to the meson spectra.

One characteristic of the model is the use of a screened linear confinement potential. This has been able to reproduce the degeneracy pattern observed for the higher excited states of light mesons [21]. As we assume that confining interaction is flavor independent, we hope that this form of the potential will be useful in our case because we are focusing on the high energy region of the vector charmonium spectrum.

The different pieces of the confinement potential are

$$V_{\text{CON}}^{\text{C}}(\vec{r}_{ij}) = \left[-a_c(1 - e^{-\mu_cr_{ij}}) + \Delta\right](\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c),$$

$$V_{\text{CON}}^{\text{SO}}(\vec{r}_{ij}) = -(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\frac{a_c\mu_ce^{-\mu_cr_{ij}}}{4m_i^2m_j^2r_{ij}} \times$$
$$\times \left[((m_i^2 + m_j^2)(1 - 2a_s) + 4m_im_j(1 - a_s))(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(1 - 2a_s)(\vec{S}_- \cdot \vec{L})\right].$$

(4)

Table I shows the model parameters used herein and relevant for the heavy quark sector. Further details about the quark model and the fine-tuned model parameters can be found in Ref. [22], where an attempt to describe the different properties of vector charmonium resonances was done.

In Tables II and III we summarize the model results for vector charmonium and bottomonium states, respectively. We also compare with the existing experimental data. In the bottomonium sector, only S-wave states are shown because: i) 1^{--} D-wave states have not yet been observed; ii) S–D splittings are expected to be very small in the bottomonium sector; and iii) they are not very relevant in the course of this article.

As one can see in the case of $J^{PC} = 1^{--}$ $c\bar{c}$ states, the agreement with the experimental data is remarkable

Quark masses	m_c (MeV)	1763
	m_b (MeV)	5110
OGE	$\hat{r}_0$ (fm)	0.181
	$\hat{r}_g$ (fm)	0.259
	α_0	2.118
	Λ_0 (fm^{-1})	0.113
	μ_0 (MeV)	36.976
Confinement	a_c (MeV)	507.4
	μ_c (fm^{-1})	0.576
	Δ (MeV)	184.432
	a_s	0.81

TABLE I. Quark model parameters.

except for one state: the $X(4260)$ which do not fit in the $Q\bar{Q}$ scheme.

Usually the $\psi(4415)$ state has been assigned as a $4S$ state. Our particular choice of the potential includes the new $X(4360)$ as a $4S$ state between the well established $\psi(4160)$ and $\psi(4415)$ which are both predicted as D-wave states. Whether or not this assignment is correct can be tested with the e^+e^- leptonic widths. From Table II one can see that the width of the $4S$ state is $0.78\,\text{keV}$, whereas the last experimental value for the $\psi(4415)$ is $\Gamma_{e^+e^-} = 0.35 \pm 0.12\,\text{keV}$ [23], in excellent agreement with the result for the $3D$ state $(0.33\,\text{keV})$. The measurement of the leptonic width for the $X(4360)$ is very important and will clarify the situation.

Furthermore, it has been shown in Ref. [25] that the assignment of the $X(4360)$ and $\psi(4415)$ resonances as the $4S$ and $3D$ $J^{PC} = 1^{--}$ $c\bar{c}$ states, respectively, is compatible with the data of the exclusive reactions $e^+e^- \to D^0D^-\pi^+$ and $e^+e^- \to D^0D^{*-}\pi^+$.

The total decay widths shown in Table II differ with respect to those calculated in Ref. [22]. The reason for that is the following. The $^3P_0$ model presents a parameter, the strength γ of the decay interaction, which is regarded as a free flavor independent constant and is fitted to the data. In Ref. [22] the parameter γ of the $^3P_0$ model was adjusted to reproduce the decay rate $\psi(3770) \to D^+D^-$, which is the only decay rate well established in the vector charmonium sector. However, in Ref. [26], we performed a calculation of the strong decay widths of the mesons which belong to charmed, charmed-strange, hidden charm and hidden bottom sectors. A global fit of the experimental data shows that, contrarily to the usual wisdom, the γ depends on the flavor of the quark-antiquark in the decaying meson. We proposed a γ strength parameter that depends on the scale through the reduced mass of the $q\bar{q}$ pair in the decaying meson. The results predicted by the $^3P_0$ model with the suggested running of the γ parameter are in a global agreement with the experimental data, being remarkable in most of the cases studied.

Above the $\psi(4415)$ our model predicts two states which have been assigned as the $X(4640)$ and $X(4660)$ despite of the total decay widths are not so well reproduce. The

lower value of the total width of the $X(4660)$ favors the $4D$ assignment for this state although interference between the two states can be the origin of the poor description of the total widths. A similar situation has been recently observed by the LHCb experiment [27] in which the $D_{s1}(2860)$ signal has resulted in a couple of resonances that compete and obscure the experimental measurement of the total decay width. Moreover, it is well known that couple channels effects may have important consequences on the width [28, 29].

The model predicts reasonably well the masses and total decay widths of the $b\bar{b}$ states. Our results for the leptonic widths are suppressed. It is noteworthy that the experimental value of the leptonic width of the $\Upsilon(10860)$ does not follow the pattern elucidated by the other resonances.

III. QCD MULTIPOLE EXPANSION

Hadronic transitions between color singlet states involve, at least, the emission of two gluons. These gluons are rather soft because the energy difference between the initial and final quarkonium state are usually small. Gottfried [30] has pointed out that this gluon radiation can be treated in a multipole expansion since the wavelengths of the emitted gluons are large compared with the size of the $Q\bar{Q}$ states. After the expansion of the gluon field, the Hamiltonian of the system can be decomposed as

$$\mathcal{H}_{\text{QCD}}^{\text{eff}} = \mathcal{H}_{\text{QCD}}^0 + \mathcal{H}_{\text{QCD}}^1 + \mathcal{H}_{\text{QCD}}^2, \tag{5}$$

with $\mathcal{H}_{\text{QCD}}^0$ the sum of the kinetic and potential energies of the heavy quarks, and $\mathcal{H}_{\text{QCD}}^1$ and $\mathcal{H}_{\text{QCD}}^2$ defined by

$$\begin{aligned} \mathcal{H}_{\text{QCD}}^1 &= Q_a A_0^a(x,t), \\ \mathcal{H}_{\text{QCD}}^2 &= -d_a E^a(x,t) - m_a B^a(x,t), \end{aligned} \tag{6}$$

in which Q_a, d_a and m_a are the color charge, the color electric dipole moment and the color magnetic dipole moment, respectively. As the $Q\bar{Q}$ is a color singlet, there is no contribution of the $\mathcal{H}_{\text{QCD}}^1$ and only E_l and B_m transitions occur. The lowest order term between two color singlets involves two gluons and therefore the lowest multipole is E_1E_1.

The gauge-invariant formulation of multipole expansion within QCD was given by Tung-Mow Yan in Ref. [31]. We will follow the updated review [7] and references therein to calculate the hadronic transitions in which we are interested.

A. Two pion hadronic transitions

These processes are dominated by double electric-dipole transitions (E1E1). The transition amplitude is

(nL)	States	$M_{\text{The.}}$ (MeV)	$M_{\text{Exp.}}$ (MeV)	$\Gamma^{e^+e^-}_{\text{The.}}$ (keV)	$\Gamma^{e^+e^-}_{\text{Exp.}}$ (keV)	$\Gamma^{\text{S}}_{\text{The.}}$ (MeV)	$\Gamma^{\text{S}}_{\text{Exp.}}$ (MeV)
$(1S)$	J/ψ	3096	3096.916 ± 0.011	3.93	5.55 ± 0.14	-	-
$(2S)$	$\psi(2S)$	3703	3686.09 ± 0.04	1.78	2.43 ± 0.05	-	-
$(1D)$	$\psi(3770)$	3796	3772 ± 1.1	0.22	0.22 ± 0.05	26.5	27.6 ± 1.0
$(3S)$	$\psi(4040)$	4097	4039 ± 1	1.11	0.83 ± 0.20	111.3	80 ± 10
$(2D)$	$\psi(4160)$	4153	4153 ± 3	0.30	0.48 ± 0.22	116.0	103 ± 8
	$X(4260)$	-	4260 ± 10	-	-	-	-
$(4S)$	$X(4360)$	4389	4361 ± 9	0.78	-	113.9	74 ± 18
$(3D)$	$\psi(4415)$	4426	4421 ± 4	0.33	$0.58 \pm 0.07 \quad 0.35 \pm 0.12$ [23]	159.0	$62 \pm 20 \quad 119 \pm 16$
$(5S)$	$X(4640)$	4614	4634^{+8+5}_{-7-8}	0.57	-	206.4	92 ± 52
$(4D)$	$X(4660)$	4641	$4664 \pm 11 \pm 5$	0.31	-	135.1	48 ± 15

TABLE II. Model results for $J^{PC} = 1^{--}$ $c\bar{c}$ states compared with the experimental data reported in PDG [24]. The last measurement for the leptonic width of the $\psi(4415)$ resonance can be found in Ref. [23].

(nL)	States	$M_{\text{The.}}$ (MeV)	$M_{\text{Exp.}}$ (MeV)	$\Gamma^{e^+e^-}_{\text{The.}}$ (keV)	$\Gamma^{e^+e^-}_{\text{Exp.}}$ (keV)	$\Gamma^{\text{S}}_{\text{The.}}$ (MeV)	$\Gamma^{\text{S}}_{\text{Exp.}}$ (MeV)
$(1S)$	$\Upsilon(1S)$	9502	9460 ± 0.26	0.71	1.34 ± 0.018	-	-
$(2S)$	$\Upsilon(2S)$	10015	10023.2 ± 0.31	0.37	0.612 ± 0.011	-	-
$(3S)$	$\Upsilon(3S)$	10349	10355.2 ± 0.5	0.27	0.443 ± 0.008	-	-
$(4S)$	$\Upsilon(4S)$	10607	10579.4 ± 1.2	0.21	0.272 ± 0.029	20.59	20.5 ± 2.5
$(5S)$	$\Upsilon(10860)$	10818	10876 ± 11	0.18	0.31 ± 0.07	27.89	55 ± 28
$(6S)$	$\Upsilon(11020)$	10995	11019 ± 8	0.15	0.130 ± 0.030	79.16	79 ± 16

TABLE III. Model results for $J^{PC} = 1^{--}$ S-wave $b\bar{b}$ states compared with the experimental data reported in PDG [24].

given by [7]

$$\mathcal{M}_{E1E1} = i\frac{g_E^2}{6} \left\langle \Phi_F h \left| \vec{x} \cdot \vec{E} \frac{1}{E_I - H_{QCD}^{(0)} - iD_0} \vec{x} \cdot \vec{E} \right| \Phi_I \right\rangle,$$
(7)

where $\vec{x}$ is the separation between Q and $\bar{Q}$, and $(D_0)_{bc} \equiv \delta_{bc}\partial_0 - g_s f_{abc} A_0^a$.

Inserting a complete set of intermediate states the transition amplitude (7) becomes

$$\mathcal{M}_{E1E1} = i\frac{g_E^2}{6} \sum_{KL} \frac{\langle \Phi_F | x_k | KL \rangle \langle KL | x_l | \Phi_I \rangle}{E_I - E_{KL}} \langle \pi\pi | E_k^a E_l^a | 0 \rangle,$$
(8)

where E_{KL} is the energy eigenvalue of the intermediate state $|KL\rangle$ with the principal quantum number K and the orbital angular momentum L.

The intermediate states in the hadronic transition are those produced after the emission of the first gluon and before the emission of the second gluon. They are states with a gluon and a color-octet $Q\bar{Q}$ and thus these states are the so-called hybrid states. It is difficult to calculate these hybrid states from first principles of QCD. So we take a reasonable model, which will be explained below, to describe them.

The transition amplitude (8) split into two factors. The first one concerns to the wave functions and energies of the initial and final quarkonium states as well as those of the intermediate hybrid mesons. All these quantities can be calculated using suitable quark models. The second one describes the conversion of the emitted gluons into light hadrons. As the momenta involved are very low this matrix element cannot be calculated using perturbative QCD and one needs to resort to a phenomenological approach based on soft-pion techniques [32]. In the center-of-mass frame, the two pion momenta q_1 and q_2 are the only independent variables describing this matrix element which, in the nonrelativistic limit, can be parametrized as [7, 31, 32]

$$\frac{g_E^2}{6} \langle \pi_\alpha(q_1)\pi_\beta(q_2) | E_k^a E_l^a | 0 \rangle = \frac{\delta_{\alpha\beta}}{\sqrt{(2\omega_1)(2\omega_2)}} \times$$
$$\times \left[C_1 \delta_{kl} q_1^\mu q_{2\mu} + C_2 \left(q_{1k}q_{2l} + q_{1l}q_{2k} - \frac{2}{3}\delta_{kl}\vec{q}_1 \cdot \vec{q}_2 \right) \right],$$
(9)

where C_1 and C_2 are two unknown constants.

Finally, the transition rate is given by [33]

$$\Gamma(\Phi_I(^{2s+1}l_{I\,J_I}) \to \Phi_F(^{2s+1}l_{F\,J_F})\pi\pi) = \delta_{l_I l_F}\delta_{J_I J_F}(G|C_1|^2 - \frac{2}{3}H|C_2|^2)\left|\sum_L (2L+1)\begin{pmatrix} l_I & 1 & L \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} L & 1 & l_I \\ 0 & 0 & 0 \end{pmatrix}f_{IF}^L\right|^2$$

$$+ (2l_I+1)(2l_F+1)(2J_F+1)\sum_k(2k+1)(1+(-1)^k)\begin{Bmatrix} s & l_F & J_F \\ k & J_I & l_I \end{Bmatrix}^2 H|C_2|^2\times$$

$$\times\left|\sum_L(2L+1)\begin{pmatrix} l_F & 1 & L \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} L & 1 & l_I \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} l_I & L & 1 \\ 1 & k & l_F \end{Bmatrix}f_{IF}^L\right|^2, \tag{10}$$

with

$$f_{IF}^L = \sum_K \frac{1}{M_I - M_{KL}}\left[\int dr\, r^3 R_F(r)R_{KL}(r)\right]\left[\int dr'\, r'^3 R_{KL}(r')R_I(r')\right], \tag{11}$$

where $R_I(r)$, $R_F(r)$ and $R_{KL}(r)$ are the radial wave functions of the initial, final and intermediate vibrational states, respectively. M_I is the mass of the decaying meson and M_{KL} are the masses of the intermediate vibrational states. The quantities G and H are the phase-space integrals

$$G = \frac{3}{4}\frac{M_F}{M_I}\pi^3\int dM_{\pi\pi}^2\, K\left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}\right)^{1/2}(M_{\pi\pi}^2 - 2m_\pi^2)^2,$$

$$H = \frac{1}{20}\frac{M_F}{M_I}\pi^3\int dM_{\pi\pi}^2\, K\left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2}\right)^{1/2}\times$$

$$\times\left[(M_{\pi\pi}^2 - 4m_\pi^2)^2\left(1 + \frac{2}{3}\frac{K^2}{M_{\pi\pi}^2}\right)\right.$$

$$\left. + \frac{8K^4}{15M_{\pi\pi}^4}(M_{\pi\pi}^4 + 2m_\pi^2 M_{\pi\pi}^2 + 6m_\pi^4)\right], \tag{12}$$

with K given by

$$K = \frac{\sqrt{[(M_I+M_F)^2 - M_{\pi\pi}^2][(M_I-M_F)^2 - M_{\pi\pi}^2]}}{2M_I}. \tag{13}$$

B. A model for hybrid mesons

From the generic properties of QCD, we might expect to have states in which the gluonic field itself is excited and carries J^{PC} quantum numbers. A bound-state is called glueball when any valence quark content is absent, the addition of a constituent quark-antiquark pair to an excited gluonic field gives rise to what is called a hybrid meson. The gluonic quantum numbers couple to those of the $q\bar{q}$ pair. This coupling may give rise to so-called exotic J^{PC} mesons, but also can produce hybrid mesons with natural quantum numbers. We are interested on the last ones because they are involved in the calculation of hadronic transitions within the QCDME approach.

As stated in the introduction, estimates of hybrid meson properties have traditionally followed from different models. Among them, we adopt the QCS model since it was used in the early works of QCDME and it incorporates finite quark mass corrections. The QCS model is defined by a relativistic-, gauge- and reparametrization-invariant action describing quarks interacting with color $SU(3)$ gauge fields in a two dimensional world sheet. It is assumed that the meson is composed of a quark and antiquark linked by an appropriate color electric flux line (the string).

The string can carry energy-momentum only in the region between the quark and the antiquark. The string and the quark-antiquark pair can rotate as a unit and also vibrate. Ignoring its vibrational motion, the equation which describes the dynamics of the quark-antiquark pair linked by the string should be the usual Schrödinger equation with a confinement potential. Gluon excitation effects are described by the vibration of the string. These vibrational modes provide new states beyond the naive meson picture.

A complete description of the model can be found in Refs. [14–16]. We will give here only a brief description of it. The dynamics of the string is defined by the action

$$S = \int_{-\infty}^{\infty} d^2u\,\sqrt{-g}\times$$

$$\times\left\{\sum_j \bar{\psi}_j\left[\gamma_\mu\tau^{\alpha\mu}\left(\frac{i}{2}\bar{\partial}_\alpha - eB_{a\alpha}T^\alpha\right) - M_j\right]\psi_j \right.$$

$$\left. - \frac{1}{4}F_{a\alpha\beta}F_a^{\alpha\beta}\right\}, \tag{14}$$

where $\psi_j(u)$ is a four-component Dirac field, $d^2u\sqrt{-g}$ is the invariant volume element, T^a are the eight matrix generators of $SU(3)$ color and $B_{a\alpha}$ are de color gauge fields. From this action, in the nonrelativistic limit, one obtains the effective Hamiltonian [15] composed of three

terms (the quark, the string and the Coulomb):

$$\mathcal{H} = \mathcal{H}_q + \mathcal{H}_s + \mathcal{H}_c$$
$$= \int d\sigma \chi^+ \left(M\beta - i\alpha_1 \partial_1 \right) \chi$$
$$+ \int d\sigma \chi^+ \beta \chi \frac{Mv^2}{2}$$
$$+ \frac{e^2}{2} \int d\sigma d\sigma' \chi^+(\sigma) T^a \chi(\sigma) G(\sigma, \sigma') \chi^+(\sigma') T^a \chi(\sigma'),$$
(15)

which, in absence of vibrations and after quantization of the rotational modes, leads to the following Schrödinger equation for the meson bound-states in the center-of-mass frame

$$\left[2M - \frac{1}{M} \frac{\partial^2}{\partial r^2} + kr - \frac{l(l+1)}{Mr^2} \right] \psi(r) = E\psi(r). \quad (16)$$

The coupled equations that describe the dynamics of the string and the quark sectors are very nonlinear so that there is no hope of solving them completely. Then, to introduce the vibrational modes, we use the following approximation scheme. First, we solve the string Hamiltonian (via de Bohr-Oppenheimer method) to obtain the vibrational energies as functions of r, the interquark distance. These are then inserted into the meson equation as an effective potential, $V_n(r)$, that depends on the distance between the quark and the antiquark.

Assuming the quark mass to be very heavy so that the ends of the string are fixed, the vibrational potential energy can be estimated using the Bohr-Sommerfeld quantization to be [15]

$$V_n(r) = \sigma r \left\{ 1 + \frac{2n\pi}{\sigma \left[(r - 2d)^2 + 4d^2 \right]} \right\}^{1/2}, \quad (17)$$

where d is the correction due to the finite quark mass

$$d(m_Q, r, \sigma, n) = \frac{\sigma r^2 \alpha_n}{4(2m_Q + \sigma r \alpha_n)}, \quad (18)$$

being α_n a parameter related with the shape of the vibrating string [15], and can take the values $1 \le \alpha_n^2 \le 2$. For $n = 0$, $V_n(r)$ reduces to the naive $Q\bar{Q}$ one.

In our quark model, the central part of the confining potential has the following form

$$V_{\mathrm{CON}}^{\mathrm{C}}(r) = \frac{16}{3} [a_c(1 - e^{-\mu_c r}) - \Delta], \quad (19)$$

and can be written as

$$V_{\mathrm{CON}}^{\mathrm{C}}(r) = \sigma(r)r + \mathrm{cte}, \quad (20)$$

where

$$\sigma(r) = \frac{16}{3} a_c \left(\frac{1 - e^{-\mu_c r}}{r} \right),$$
$$\mathrm{cte} = -\frac{16}{3} \Delta. \quad (21)$$

This means that our effective string tension, $\sigma(r)$, is not a constant but depends on the interquark distance, r. In fact, it decreases with respect to r until it reaches the string breaking region.

Following the ideas of Ref. [16], the potential for hybrid mesons derived from our constituent quark model has the following expression

$$V_{\mathrm{hyb}}(r) = V_{\mathrm{OGE}}^{\mathrm{C}}(r) + V_{\mathrm{CON}}^{\mathrm{C}}(r) + [V_n(r) - \sigma(r)r], \quad (22)$$

where we have not taken into account the spin-dependent terms. $V_{\mathrm{OGE}}^{\mathrm{C}}(r) + V_{\mathrm{CON}}^{\mathrm{C}}(r)$ is the naive quark-antiquark potential and $V_n(r)$ is the vibrational one. We must subtract the term $\sigma(r)r$ because it appears twice, one in $V_{\mathrm{CON}}^{\mathrm{C}}(r)$ and the other one in $V_n(r)$. This potential does not include new parameters besides those of the original quark model. In that sense the calculation of the hybrids states is parameter-free. More explicitly, our different contributions are

$$V_{\mathrm{OGE}}^{\mathrm{C}}(r) = -\frac{4\alpha_s}{3r},$$
$$V_{\mathrm{CON}}^{\mathrm{C}}(r) = \frac{16}{3} [a_c(1 - e^{-\mu_c r}) - \Delta],$$
$$V_n(r) = \sigma(r)r \left\{ 1 + \frac{2n\pi}{\sigma(r) \left[(r - 2d)^2 + 4d^2 \right]} \right\}^{1/2}, \quad (23)$$

where

$$d(m_Q, r, \sigma, n) = \frac{\sigma(r)r^2 \alpha_n}{4(2m_Q + \sigma(r)r\alpha_n)}. \quad (24)$$

One can realize that, just like the naive quark model, the hybrid potential has a threshold defined by

$$V_{\mathrm{hyb}}(r) \xrightarrow{r \to \infty} \frac{16}{3} (a_c - \Delta). \quad (25)$$

IV. RESULTS

All the parameters of the quark model are taken from [22] and shown in Table I for completeness. Then, we only need to fix the two parameters C_1 and C_2 of Eq. (9).

For a given $M_{\pi\pi}$ invariant mass C_1 term is isotropic and therefore contributes to the S-state into S-state transitions while the C_2 term has a $L = 2$ angular dependence and contributes to the D-wave into S-wave transitions. Then, we can use the well established $\psi(2S) \to J/\psi\pi^+\pi^-$ and $\psi(3770) \to J/\psi\pi^+\pi^-$ transitions to fix the two parameters. It is argued sometimes that to reproduce the leptonic decay width of the $\psi(3770)$, this state should be a mixture like $\psi(3770) = |2S\rangle \sin\vartheta + |1D\rangle \cos\vartheta$ being ϑ an adjustable parameter. In our model the angle ϑ is determined by the dynamics to be $\vartheta \approx 1°$ although a correct value of the leptonic width is obtained (see Table II). Therefore, we will consider the $\psi(3770)$ as a

K	$c\bar{c}g$	$b\bar{b}g$
1	4351	10785
2	4639	10999
3	4855	11175
4	5020	11325
5	5145	11452
6	5238	11562

TABLE IV. $J^{PC} = 1^{--}$ hybrid meson masses, in MeV, calculated in the $c\bar{c}$ and $b\bar{b}$ sectors. The variation of the parameter α_n which range between $1 < \alpha_n < \sqrt{2}$ modifies the energy as much as 30 MeV, we have taken $\alpha_n = \sqrt{1.5}$.

Initial Meson	$R_{J/\psi}^{\text{th}}$ (eV)	$R_{J/\psi}^{\text{ex}}$ (eV)	$R_{\psi(2S)}^{\text{th}}$ (eV)	$R_{\psi(2S)}^{\text{ex}}$ (eV)
$\psi(4040)$	1.20	-	0.11	-
$\psi(4160)$	0.40	-	6×10^{-2}	-
$X(4360)$	52.5	-	5.05	7.4 ± 0.9
$X(4415)$	3×10^{-6}	-	0.27	-
$X(4660)$	0.58	-	1.08	1.04 ± 0.5

TABLE V. $R_{\psi(nS)} = \mathcal{B}_{\pi^+\pi^-\psi(nS)} \times \Gamma_{e^+e^-}$ for the $J^{PC} = 1^{--}$ S-wave charmonium states. Experimental data are from Ref. [3].

pure D-wave state. Taken the experimental values of the widths of these two resonances from PDG [24] we obtain

$$|C_1|^2 = (9.396 \pm 0.503) \times 10^{-5},$$
$$|C_2|^2 = (3.051 \pm 0.430) \times 10^{-4}. \tag{26}$$

The value of the ratio $C_2/C_1 \sim 1.8$ is compatible with the literature [34].

From Eq. (8) one can see that the quantum numbers of hybrid states which participate in the two pion transitions are $J^{PC} = 1^{--}$. In Table IV we show the values of the masses of the hybrid states with these quantum numbers and different radial excitations in the charmonium and bottomonium sector.

The mass of the ground state in the $c\bar{c}g$ sector is $4.35\,\text{GeV}$ which agrees roughly with the results of the flux-tube model [11] $(4.1 - 4.2)$, Coulomb gauge QCD [13] (4.47), QCD string model [17] (4.397), potential model [35] (4.23) and lattice calculation [9] (4.40). The ground state mass in the $b\bar{b}g$ sector agrees with that of Ref. [35] $(10.79\,\text{GeV})$.

Typically, in experiments that cover hadronic transitions of heavy quarkonia with two-pion emission, the quantities which are measured are the cross section and the product $\mathcal{B}_{\pi^+\pi^-(Q\bar{Q})} \times \Gamma_{e^+e^-}$ where $\mathcal{B}_{\pi^+\pi^-(Q\bar{Q})}$ indicates the branching ratio of the decay and $\Gamma_{e^+e^-}$ is the leptonic width of the resonance. We will give results for the product $\mathcal{B}_{\pi^+\pi^-(Q\bar{Q})} \times \Gamma_{e^+e^-}$ and values of the cross section at peak.

Table V shows the calculated $\mathcal{B}_{\pi^+\pi^-\psi(nS)} \times \Gamma_{e^+e^-}$ for the $J^{PC} = 1^{--}$ charmonium states. As the decays $\psi(2S) \to J/\psi\pi^+\pi^-$ and $\psi(3770) \to J/\psi\pi^+\pi^-$ have

Initial Meson	$\sigma_{J/\psi}^{\text{th}}$ (pb)	$\sigma_{J/\psi}^{\text{ex}}$ (pb)	$\sigma_{\psi(2S)}^{\text{th}}$ (pb)	$\sigma_{\psi(2S)}^{\text{ex}}$ (pb)
$\psi(4040)$	13.46	-	1.25	-
$\psi(4160)$	3.32	-	0.50	-
$X(4360)$	329.7	-	31.69	52 ± 2
$X(4415)$	4×10^{-5}	-	3.38	-
$X(4660)$	10.97	-	20.29	28 ± 2

TABLE VI. The cross section at peak for the $J^{PC} = 1^{--}$ S-wave charmonium states. Experimental data are from Ref. [3].

been used to fit the C_1 and C_2 parameters they are not included in the table. One can see that in the case of the decay channel $\psi(2S)\pi^+\pi^-$ the only significant values correspond to the decays of the $X(4360)$ and $X(4660)$ which are also in agreement with the recent experimental data. These results justify why no signals of the $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ have been seen in the data. In the decay channel $J/\psi\pi^+\pi^-$ a high value of the $\mathcal{B}_{\pi^+\pi^-\psi(nS)} \times \Gamma_{e^+e^-}$ is obtained for the $X(4360)$ resonance. This result apparently contradicts the experimental data because this decay has not been reported in the reaction $e^+e^- \to J/\psi\pi^+\pi^-$ [1]. The cross section of this reaction shows a resonance in the $4.2 - 4.4$ energy region which has been attributed to the $X(4260)$, which does not appear in our calculation as a $c\bar{c}$ meson. However, an interference between the $X(4260)$ and $X(4360)$ resonances would be possible.

Same conclusions can be obtained from the values of the cross section at peak (Table VI). The two measured values in the $\psi(2S)\pi^+\pi^-$ channel are in agreement with our theoretical results and the only significant cross section at peak is obtained for the $X(4360)$ resonance in the $J/\psi\pi^+\pi^-$ channel. The values of the rest of peak cross sections are of the order of the experimental background and it is difficult to decide whether a resonance is present or not.

The results for the bottomonium sector are shown in Tables VII and VIII. One can see that the theoretical values agree reasonably well with the experimental ones except in the case of the $\Upsilon(10860)$. We do not find any hybrid state around the $\Upsilon(10860)$ mass region. Therefore, the mechanism which explains in our model the large widths in the charm sector seems not to work for this case. The Belle data on $e^+e^- \to \Upsilon(nS)\pi\pi$ has been recently analyzed in Ref. [36] being compatible with a tetraquark interpretation for the $\Upsilon(10890)$. Then, further studies are needed to understand the anomalous width of the $\Upsilon(10890)$.

V. SUMMARY

Hadronic transitions of heavy quarkonia with two-pion emission have been calculated in the framework of the QCD multipole expansion (QCDME). Charmonium and bottomonium states are described in a constituent quark

Initial Meson	$R^{\text{th}}_{\Upsilon(1S)}$ (eV)	$R^{\text{ex}}_{\Upsilon(1S)}$ (eV)	$R^{\text{th}}_{\Upsilon(2S)}$ (eV)	$R^{\text{ex}}_{\Upsilon(2S)}$ (eV)	$R^{\text{th}}_{\Upsilon(3S)}$ (eV)	$R^{\text{ex}}_{\Upsilon(3S)}$ (eV)
$\Upsilon(2S)$	98.34	105.4 ± 4.3	-	-	-	-
$\Upsilon(3S)$	23.94	18.5 ± 9.8	5.58	-		
$\Upsilon(4S)$	6×10^{-2}	$(2.3 \pm 0.9) \times 10^{-2}$	2.5×10^{-3}	$(2.3 \pm 0.4) \times 10^{-2}$	-	-
$\Upsilon(10860)$	4.1×10^{-2}	1.64 ± 0.40	5.8×10^{-2}	2.42 ± 0.64	1.8×10^{-2}	1.49 ± 0.65

TABLE VII. $R_{\Upsilon(nS)} = \mathcal{B}_{\pi^+\pi^-\Upsilon(nS)} \times \Gamma_{e^+e^-}$ for the $J^{PC} = 1^{--}$ S-wave bottomonium states. Experimental data are from Ref. [24].

Initial Meson	$\sigma^{\text{th}}_{\Upsilon(1S)}$ (pb)	$\sigma^{\text{ex}}_{\Upsilon(1S)}$ (pb)	$\sigma^{\text{th}}_{\Upsilon(2S)}$ (pb)	$\sigma^{\text{ex}}_{\Upsilon(2S)}$ (pb)	$\sigma^{\text{th}}_{\Upsilon(3S)}$ (pb)	$\sigma^{\text{ex}}_{\Upsilon(3S)}$ (pb)
$\Upsilon(2S)$	4.49×10^5	-	-	-	-	-
$\Upsilon(3S)$	1.61×10^5	-	0.38×10^5	-	-	-
$\Upsilon(4S)$	0.39	-	1.57×10^{-2}	-	-	-
$\Upsilon(10860)$	9.38×10^{-2}	2.27 ± 0.14	1.31×10^{-1}	4.07 ± 0.45	4.06×10^{-2}	1.46 ± 0.16

TABLE VIII. The cross section at peak for the $J^{PC} = 1^{--}$ S-wave bottomonium states. Experimental data are from Ref. [24].

model whereas the hybrid intermediate states, needed in the QCDME method, are calculated in a natural extension of the constituent quark model. This extension is based on the quark confining string scheme which does not include any new parameter.

We have analyzed the $J/\psi\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ channels for charmonium decays and the $\Upsilon(nS)\pi^+\pi^-$ with $n = 1$, 2 and 3 for bottomonium decays.

In the invariant mass distribution process $e^+e^- \to \psi(2S)\pi^+\pi^-$ two resonance structures appear at the masses of the $X(4360)$ and $X(4660)$ states. Our calculation shows that the only significant transition rates correspond to these resonances being the rest, at least, one order of magnitude smaller. This result is explained by the presence of two hybrid states, each of them located near the masses of the $X(4360)$ and $X(4660)$, which enhance the transition rates.

In the $e^+e^- \to J/\psi\pi^+\pi^-$ reaction, there is only a significant peak in the mass region of the $X(4260)$. As most of the potential models, our model is not able to describe this peculiar state which is only seen in this transition but not in any open-charm decay channel as the rest of the $J^{PC} = 1^{--}$ charmonium states. Our theoretical results indicate that the only significant transition rate in the $J/\psi\pi^+\pi^-$ channel corresponds to the $X(4360)$ resonance. This tension between the theoretical and experimental results requires more accurate studies.

In the bottomonium sector our results agree with the experimental data except in the case of the $\Upsilon(10860)$ resonance. This may suggest a more complex structure (tetraquark or molecule) for this state.

VI. ACKNOWLEDGMENTS

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