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Glashow resonance as a window into cosmic neutrino sources

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Abstract

The Glashow resonance at $E_\nu = 6.3$ PeV is a measure of the $\bar{\nu}_e$ content of the astrophysical neutrino flux. The fractional $\bar{\nu}_e$ content depends on the neutrino production model at the cosmic neutrino source, and the environment at the source. Thus, the strength of the Glashow resonance event rate is a potential window into astrophysical sources. We quantify the "Glashow resonometer" and comment on the significance that no Glashow events are observed in the IceCube three–year data.

I. INTRODUCTION

The rate of interaction of ν_e, ν_μ, ν_τ, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$, with electrons is mostly negligible compared to interactions with nucleons. However, the case of $\bar{\nu}_e$ is unique because of resonant scattering, $\bar{\nu}_e e^- \to W^- \to$ anything, at $E_\nu \simeq 6.3$ PeV. The W^- resonance in this process is commonly referred to as the Glashow resonance [1]. The signal for $\bar{\nu}_e$ at the Glashow resonance, when normalized to the total $\nu + \bar{\nu}$ flux, can be used to differentiate among the main primary mechanisms for neutrino-producing interactions in optically thin sources of cosmic rays [2].

In 2012, IceCube released the first two-year equivalent dataset, observing high-energy non-atmospheric neutrino events for the first time [3, 4]. The maximum neutrino energy inferred was 1–2 PeV. In 2014, IceCube reported its three-year dataset [5]. The maximum neutrino energy inferred to date remains at ~ 2 PeV. The energy resolution on the observed events is $\sim 25\%$. In particular, Glashow resonance events should produce showers that are not (yet) observed. The integrated cross section of the resonance is comparable for some flavor models to that of the non-resonant spectrum integrated above a PeV, which implies that the falling power law ($E_\nu^{-\alpha}$) of the incident neutrino spectrum is effectively canceled and that resonant events could have been seen [6].

In this Letter, we evaluate the ratio of the expected number of Glashow events at 6.3 PeV to the number of non-resonant events expected above various minimum energies ($\sim$ PeV) for six popular cosmic neutrino source models.

II. SIX ASTROPHYSICAL NEUTRINO SOURCE MODELS

We consider six possible source models:
(i) $pp \to \pi^\pm$ pairs $\to \nu_e + \bar{\nu}_e + 2\nu_\mu + 2\bar{\nu}_\mu$, referred to as the *"$\pi^\pm$ mode"*;
(ii) $pp \to \pi^\pm$ pairs $\to \nu_\mu,\ \bar{\nu}_\mu$ only, referred to as the *"damped $\mu^\pm$ mode"*;
(iii) $p\gamma \to \pi^+$ only, $\to \nu_e + \nu_\mu + \bar{\nu}_\mu$, referred to as the *"$\pi^+$ mode"*;
(iv) $p\gamma \to \pi^+ \to \nu_\mu$ only, referred to as the *"damped μ^+ mode"*;
(v) charm production and immediate decay to $\nu_e,\ \bar{\nu}_e,\ \nu_\mu,\ \bar{\nu}_\mu$, referred to as the *"prompt mode"*; and
(vi) β-decay of cosmic neutrons to $\bar{\nu}_e$, referred to as the *"neutron decay (or β decay) mode"*.
The initial flavor content of the produced neutrinos in these six models are summarized in the second column of Table I.

When the $\pi^\pm$ mode occurs in an astrophysical source, isospin invariance yields a roughly equal ratio of π^+, π^-, and π^0 production, followed by decay of the charged $\pi^\pm$s through the $\mu^\pm$ chain to produce equal numbers of ν_μ and $\bar{\nu}_\mu$, a number of ν_e plus $\bar{\nu}_e$ equal to a half of ν_μ plus $\bar{\nu}_\mu$, and roughly equal numbers of ν_e and $\bar{\nu}_e$. The rest-frame lifetimes of the charged pions and muons are 2.6×10^{-8} s and 2.2×10^{-6} s, respectively. Since the rest frame lifetime of the muon exceeds that of the charged pion by a factor of 85, it is possible for $\pi^\pm$ decay to take place but the subsequent $\mu^\pm$ decay to be inhibited [7]. This would happen if the muon in the decay chain loses energy in the source environment before it decays (e.g., by synchrotron radiation in a $\vec{B}$-field, or by scattering). In a falling spectrum, the decay of a lower-energy muon would make a negligible contribution. This damped $\mu^\pm$ mode results in only ν_μ and $\bar{\nu}_\mu$ being produced at the source; flavor mixing between the source and Earth then produces a small amount of $\bar{\nu}_e$.

In contrast to charged-pion production by pp scattering, charged pions may be produced by $p\gamma$ scattering. Here, the Δ^+ resonance contributes to produce $\pi^+ + n$ and $\pi^0 + p$, in

TABLE I: Neutrino flavor ratios at source, component of $\bar{\nu}_e$ in total neutrino flux at Earth after mixing and decohering, and consequent relative strength of Glashow resonance, for six astrophysical models. (Neutrinos and antineutrinos are shown separately, when they differ.)

<table>
<tr><th></th><th colspan="2">Source flavor ratio</th><th colspan="2">Earthly flavor ratio</th><th>$\bar{\nu}_e$ fraction in flux ($\mathcal{R}$)</th></tr>
<tr><td>$pp \to \pi^{\pm}$ pairs</td><td colspan="2">(1:2:0)</td><td colspan="2">(1:1:1)</td><td>$18/108 = 0.17$</td></tr>
<tr><td>w/ damped $\mu^{\pm}$</td><td colspan="2">(0:1:0)</td><td colspan="2">(4:7:7)</td><td>$12/108 = 0.11$</td></tr>
<tr><td>$p\gamma \to \pi^{+}$ only</td><td>(1:1:0)</td><td>(0:1:0)</td><td>(14:11:11)</td><td>(4:7:7)</td><td>$8/108 = 0.074$</td></tr>
<tr><td>w/ damped μ^{+}</td><td>(0:1:0)</td><td>(0:0:0)</td><td>(4:7:7)</td><td>(0:0:0)</td><td>0</td></tr>
<tr><td>charm decay</td><td colspan="2">(1:1:0)</td><td colspan="2">(14:11:11)</td><td>$21/108 = 0.19$</td></tr>
<tr><td>neutron decay</td><td>(0:0:0)</td><td>(1:0:0)</td><td>(0:0:0)</td><td>(5:2:2)</td><td>$60/108 = 0.56$</td></tr>
</table>

the ratio of 1 : 2. Since π^- production is suppressed and the π^+ mode produces no $\bar{\nu}_e$s at the source, only a small amount of $\bar{\nu}_e$ arises from mixing [8]. If, in addition, the μ^+s in $p\gamma$ mode are damped, then no antineutrinos are produced at all at the source, and so even with mixing there will be no $\bar{\nu}_e$s at Earth.

Charmed particles decay promptly (e.g. the $D^{\pm}$ has a lifetime of 1.0×10^{12} s) and semileptonically to $e^{\pm}$ or $\mu^{\pm}$ (e.g., the $D^{\pm}$ has a 34% branching ratio to these modes). Lepton universality ensures that equal numbers (modulo small mass differences) of ν_e, $\bar{\nu}_e$, ν_μ, and $\bar{\nu}_\mu$ are produced, while production of ν_τ and $\bar{\nu}_\tau$ is kinematically suppressed. Thus, $\bar{\nu}_e$s produced in charm decay will arrive at Earth.

Finally, there may be sources that inject a nearly pure neutron flux [9]. Such would be the case if Fe is emitted and subsequently dissociated to protons and neutrons, with the charged protons then degraded in energy, or swept aside, by a magnetic field at the source. Such would also be the case if the cosmic accelerator entrains and accelerates charged protons, with cosmic-ray escape occurring via $p_{\text{entrained}} \to n + \pi^+$. This escaping (and pointing) beta beam decays to pure $\bar{\nu}_e$, leading to a large amount of $\bar{\nu}_e$ arriving at Earth, even after mixing.

Each of these six models are possible, as are combinations of the six. For our purposes, we consider each model in isolation, and show how the rate for Glashow resonant events can serve as a barometer ("resonometer") distinguishing among these six source models.

A caveat is in order here. It has been shown, especially in Ref. [10], that multi-pion contributions can produce antineutrinos which via mixing ensure some $\bar{\nu}_e$s at Earth. These multi-pion contributions are not included in our discussion here. For certain source parameters, the "contamination" from multi-pion processes can be large. In addition, we assume that possible damping of muons at the sources is complete; it may be incomplete, in which case results will be intermediate between the cases considered here. We mention in passing that the effect of kaon decays on source neutrino flavor ratios is small in the energy range of interest [10]. All in all, our results must be treated as suggestive. If and when Glashow resonance events are observed, a more careful treatment than presented here will be warranted. Until Glashow resonance events are observed, our results can be considered motivational.

At this early stage of astrophysical data collection, it is a good approximation [11] to assume that tribimaximal mixing [12] holds. Then, the evolution $\nu_\alpha \to \nu_\beta$, with α and β any elements of the three-flavor set $\{e,\ \mu,\ \tau\}$, is described in terms of the PMNS matrix U, by the symmetric propagation matrix P whose positive definite elements are

$$P_{\alpha\beta} = \sum_j |U_{\alpha j}|^2\, |U_{\beta j}|^2 = \frac{1}{18}\begin{pmatrix} 10 & 4 & 4 \\ 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix} . \tag{1}$$

The element with the largest uncertainty is $P_{e\mu}$, which has an uncertainty of 20% at 2σ. As we show below, all but one of the fifteen combinations of the six flux models predict a difference much larger than 20% for the $\bar{\nu}_e$ fraction.

III. RESONANT AND NON-RESONANT EVENTS

The resonant cross section for $\bar{\nu}_e + e^- \to W^- \to$ hadrons is

$$\sigma_{\rm Res}(s) = 24\pi\, \Gamma_W^2\ {\rm B}(W^- \to \bar{\nu}_e e^-)\, {\rm B}(W^- \to {\rm had}) \frac{(s/M_W^2)}{(s - M_W^2)^2 + (M_W \Gamma_W)^2}\,, \tag{2}$$

where M_W is the W mass (80.4 GeV), Γ_W is the W's FWHM (2.1 GeV), and ${\rm B}(W^- \to \bar{\nu}_e e^-)$ and ${\rm B}(W^- \to {\rm had})$ are W^- branching ratios to the $\bar{\nu}_e e^-$ state (11%) and the hadronic state (67%), respectively. At the peak,

$$\sigma_{\rm Res}^{\rm peak}(s) = \frac{24\pi\, {\rm B}(W^- \to \bar{\nu}_e e^-)\, {\rm B}(W^- \to {\rm had})}{M_W^2} = 3.4 \times 10^{-31} {\rm cm}^2\,. \tag{3}$$

Consequently, the resonant cross section may be written as

$$\sigma_{\rm Res} = \left[\frac{\Gamma_W^2\, s}{(s - M_W^2)^2 + (M_W \Gamma_W)^2}\right] \sigma_{\rm Res}^{\rm peak}(s)\,. \tag{4}$$

The W's width is small compared to the W's mass ($\frac{\Gamma_W}{M_W} = 2.6\%$), and the experimental resolution will always exceed by far the W width. Thus, we are justified in using the "narrow width approximation" (NWA) throughout. A contour integration in s over the s-dependent bracketed expression in Eq. (4), and the residue theorem, yields the value $\pi\, M_W \Gamma_W$. Thus, the resulting NWA is simply

$$\sigma(s)_{\rm Res} = (\pi M_W \Gamma_W)\, \sigma_{\rm Res}^{\rm peak}(s)\, \delta(s - M_W^2)\,, \tag{5}$$

and the number of resonant events per unit time and unit steradian is

$$\left(\frac{N}{T\Omega}\right)_{\rm Res} = N_e\,(\pi M_W \Gamma_W)\,\sigma^{\rm peak}_{\rm Res}\int dE_{\bar\nu_e}\,\left(\frac{dF_{\bar\nu_e}}{dE_{\bar\nu_e}}\right)\,\delta(s-M_W^2)$$

$$= \frac{N_p}{2m_e}\,(\pi M_W \Gamma_W)\,\sigma^{\rm peak}_{\rm Res}\,\left.\frac{dF_{\bar\nu_e}}{dE_{\bar\nu_e}}\right|_{E_{\bar\nu_e}=6.3{\rm PeV}}\,, \qquad (6)$$

where $N_e = N_p$ is the number of electrons or protons in the detector volume.

In contrast, the continuum (non-resonant) neutrino event rate between $E_\nu^{\rm min} \sim$ PeV to $E_\nu^{\rm max}$ is given by

$$\left(\frac{N}{T\Omega}\right)_{\rm non-Res} = N_{n+p}\int_{E_\nu^{\rm min}}^{E_\nu^{\rm max}} dE_\nu\,\left(\frac{dF_\nu}{dE_\nu}\right)\,\sigma^{\rm CC}_{\nu N}(E_\nu)$$

$$\approx \frac{N_{n+p}}{(\alpha-1.40)}\left[\left.\left(\sigma^{\rm CC}_{\nu N}\,E_\nu\left(\frac{dF_\nu}{dE_\nu}\right)\right)\right|_{E_\nu^{\rm min}} - \left.\left(\sigma^{\rm CC}_{\nu N}\,E_\nu\left(\frac{dF_\nu}{dE_\nu}\right)\right)\right|_{E_\nu^{\rm max}}\right]$$

$$= \frac{N_{n+p}}{(\alpha-1.40)}\left[\left(\frac{6.3\,{\rm PeV}}{E_\nu^{\rm min}}\right)^{(\alpha-1.40)} - \left(\frac{6.3\,{\rm PeV}}{E_\nu^{\rm max}}\right)^{(\alpha-1.40)}\right]\left.\left(\sigma^{\rm CC}_{\nu N}(E_\nu)\frac{E_\nu\,dF_\nu}{dE_\nu}\right)\right|_{E_\nu=6.3\,{\rm PeV}} \qquad (7)$$

where N_{n+p} is the number of nucleons in the detector volume, and $\frac{dF_\nu}{dE_\nu}$ is the total (summed over flavors) ν plus $\bar\nu$ flux. Here we have assumed an $E^{0.40}$ energy dependence for $\sigma_{\nu N}$ as predicted for the 1–10 PeV region in Ref. [13], and we have included only the charged-current cross section; in a falling spectrum, the neutral-current contribution is lower in average by $\frac{\sigma^{NC}_{\nu N}(E_{\rm obs})}{\sigma^{CC}_{\nu N}(E_{\rm obs})}\langle y\rangle^{\alpha-0.4}$, where $\langle y\rangle = \frac{E_{\rm obs}}{E}$ is the average fraction of energy transferred from the incident neutrino to the detector. The simple Fermi shock-acceleration mechanism yields $\alpha = 2.0$, whereas an earlier statistical study of the first-release dataset concluded that α was constrained by the absence of Glashow events in the IceCube data to $\alpha \geq 2.3$ [14, 15]. Taking $\langle y\rangle \sim 0.25$ and the NC to CC ratio to be 0.4, one finds less than a 5% contribution from the neutral-current even with the conservative spectral index of $\alpha = 2$. The resonant cross section and the non-resonant charged-current $\sigma_{\nu_e N}$ cross section are shown in Fig. 1.

From Eq. (7), it is seen that the integrated continuum event rate scales with the minimum energy as

$$\left(\frac{N}{T\Omega}\right)_{\rm non-Res} \propto \left[\,(E_\nu^{\rm min})^{-(\alpha-1.40)} - (E_\nu^{\rm max})^{-(\alpha-1.40)}\,\right]\,. \qquad (8)$$

Failure of future events to follow this energy-dependent rate equation would indicate a broken power-laaw spectrum, or in the extreme case, a cutoff spectrum. On the other hand, when $E_\nu^{\rm max}$ can be taken to infinity, as can be done when the neutrino energy spectrum is a power-law falling as fast or faster than E^{-2}, then we count all events that are initiated in the IceCube detector with energy exceeding $E_\nu^{\rm min}$.

We normalize the expected number of events in any energy interval to the expected number $\langle N^{\rm expected}_{\text{1-2 PeV}}\rangle$ for the highest-energy IceCube bin with nonzero number, the 1-2 PeV bin. Then, in the limit $E_\nu^{\rm max} \to \infty$, we have that the expected number of continuum events above $E_\nu^{\rm min}$ is

$$N^{\rm expect}(\geq E_\nu) = \left(\frac{E_\nu^{-(\alpha-1.40)}}{1-2^{-(\alpha-1.40)}}\right)\,\langle N^{\rm expected}_{\text{1-2 PeV}}\rangle\,, \qquad (9)$$

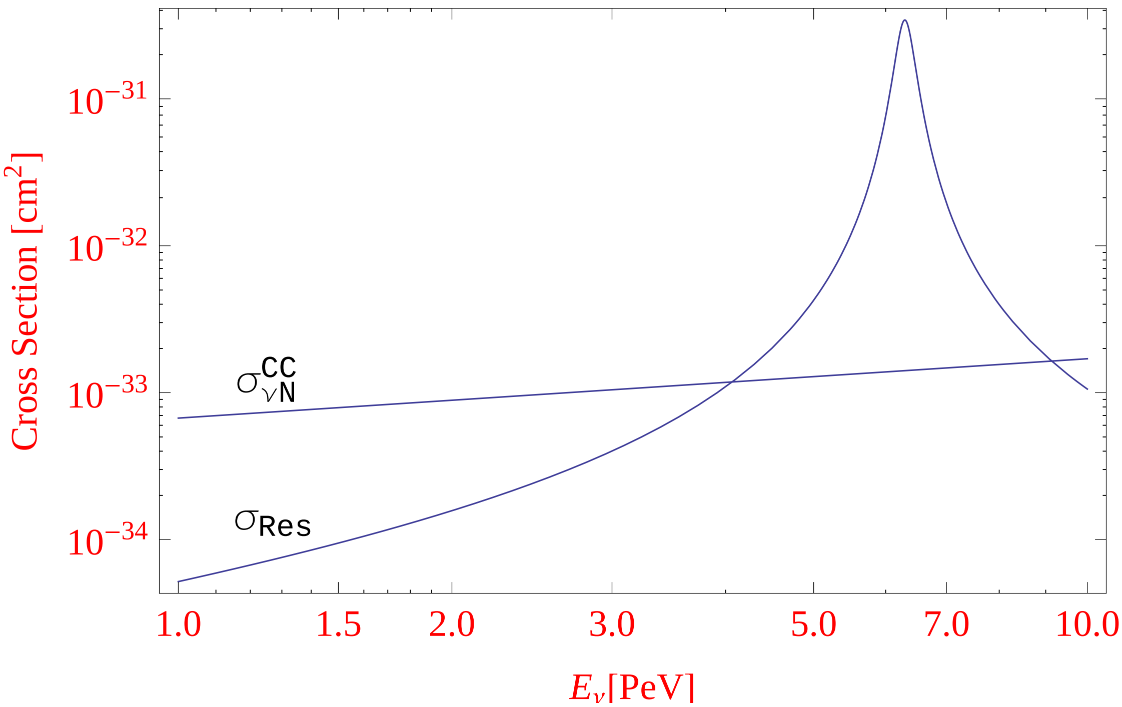

FIG. 1: Cross sections for the resonant process, $\bar{\nu}_e + e^- \to W^- \to$ hadrons, and the non-resonant process, $\nu_e + N \to e^- +$ hadrons, in the 1–10 PeV region.

which for $\alpha = 2.0$ and 2.3 is equal to $2.94\, E^{-0.6} \langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$ and $2.15\, E^{-0.9} \langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$, respectively. In turn, the number expected above 1 PeV is $2.94 \langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$ and $2.15 \langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$, respectively; the number of events expected above 2 PeV is $1.94 \langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$ and $1.15 \langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$, respectively.

The 1-2 PeV IceCube bin contains the three observed events. The expected event number for this bin is not known. The "Feldman-Cousins" [16] tables provide an estimate for the range of expected numbers of events, given an observed number of events, with or without background. (The zero-background case is the relevant one for us.) Given three events in the 1-2 PeV bin, the Feldman-Cousins expected number of events for this bin is 0.82 to 8.25 at 95% C.L. However, there is additional information in the IceCube data: no events are observed above $\sim$ 2 PeV. Thus, tension between the populated bin and the remaining unpopulated bins is minimized by investigating the lower numbers of expected events. Consider the integer expected values $\langle n\rangle = 1$, 2, and 3 as representative; the mean value 3 is appropriate if the observed value were spot on the mean, while the mean values 1 and 2 are appropriate if the observed value is an upward fluctuation. The Poisson probability to observe n events against an expected number $\langle n\rangle$ is $P(n|\langle n\rangle) = e^{-\langle n\rangle} \frac{\langle n\rangle^n}{n!}$. Thus, we have probabilities $P(3|3) = 22\%$ for the "spot-on" rate, and $P(3|2) = 18\%$ and $P(3|1) = 6.1\%$ for possible upward fluctuations. Since we discount the disfavored cases where $\langle n\rangle > n$, we do not have the general Poisson result that $\int_0^\infty d\langle n\rangle\, P(n|\langle n\rangle) = 1$. Thus, it is the relative rates 1, 0.82, and 0.28 for the expected values 3, 2, 1, respectively, that lead us to the obvious conclusion: with just three events, the unknown expected number spans a large range of possibilities and so is ill-determined.

For $\langle N^{\rm expected}_{\text{1-2\,PeV}} \rangle = 3,\, 2,\, 1$, we expect $N(\geq 2~\text{PeV}) = 5.8\,(3.5),\ 3.9\,(2.3),\ 1.9\,(1.15)$ events, for $\alpha = 2.0\ (2.3)$, respectively. No events above $\sim$ 2 PeV are observed. The Poisson probability for a downward fluctuation to no events in a bin where $\langle n \rangle$ are expected is $P(0|\langle n \rangle) = e^{-\langle n \rangle}$. Thus, the tension between observed events in the 1-2 PeV bin and the absence of events above 2 PeV is quantified in the probabilities to observe none of the expected continuum events above 2 PeV: 0.30% (3.0%), 2.0% (10%), and 15% (33%), respectively. Moreover, if one normalizes to the three observed events not in the 1-2 PeV interval, but rather in the 1-3 PeV interval, then the expected number of continuum events above 3 PeV is reduced to 3.2 (1.8), with Poisson probabilities to observe no events of 4.1% (17%). As discussed above, these odds are higher if the three observed events are themselves an upward fluctuation.

At face value, these results favor the more steeply falling spectrum, and may even suggest a broken power law or cutoff [17] in the neutrino spectrum. However, these results are not compelling at present.

Here we will assume that the absence of events is the result of a downward fluctuation, and continue the calculation with the unbroken power spectrum to assess possibilities for the Glashow resonance event rate. Since the event rate expected for the continuum and Glashow resonance depends on the expected rate determined with $\sim$PeV events, one cannot yet predict the number of expected events at higher energy. Nevertheless, in the ratio of expected Glashow events to expected continuum events, which we next present, the normalizing factor cancels out.

From Eqs. (6) and (7), we find the ratio of resonant Glashow events to non-resonant continuum events to be

$$
\begin{aligned}
&\frac{N_{\rm Res}}{N_{\rm non-Res}(E_\nu > E_\nu^{\rm min})} \\
&= \frac{10\,\pi}{18}\left(\frac{\Gamma_W}{M_W}\right)\left(\frac{\sigma^{\rm peak}_{\rm Res}}{\sigma^{\rm CC}_{\nu N}(E_\nu = 6.3\,{\rm PeV})}\right)\frac{(\alpha-1.40)\left(\frac{E_\nu^{\rm min}}{6.3\,{\rm PeV}}\right)^{\alpha-1.40}}{\left[1-\left(\frac{E_\nu^{\rm min}}{E_\nu^{\rm max}}\right)^{(\alpha-1.40)}\right]}\,\mathcal{R}\,, \qquad (10) \\
&= 11 \times \frac{(\alpha-1.40)\left(\frac{E_\nu^{\rm min}}{6.3\,{\rm PeV}}\right)^{\alpha-1.40}}{\left[1-\left(\frac{E_\nu^{\rm min}}{E_\nu^{\rm max}}\right)^{(\alpha-1.40)}\right]} \times \mathcal{R}\,, \quad \text{with } \mathcal{R} \equiv \left[\left(\frac{dF_{\bar\nu_e}}{dE_{\bar\nu_e}}\right)/\left(\frac{dF_\nu}{dE_\nu}\right)\right]_{E=6.3\,{\rm PeV}}\,.
\end{aligned}
$$

Here we have taken $\frac{N_p}{N_p+N_n} = \frac{10}{18}$ in the detector material (water), and set $\sigma^{\rm CC}_{\nu N} = \sigma^{\rm CC}_{\bar\nu_e N} = 1.42\times 10^{-33}~{\rm cm}^2$ at $E_\nu = 6.3$ PeV [13]. $\mathcal{R}$ is the ratio of the $\bar\nu_e$ flux that produces the resonance events to the total ν flux that produces the continuum events; $\mathcal{R}$ is model-dependent number, exhibited for each of our six models in the final column of Table I. We stress that the ratio in Eq. (10) is valid for down-coming events, but not for up-coming events. The reason is that the large resonant cross section at 6.3 PeV implies that 6.3 PeV neutrinos are strongly absorbed if transiting the Earth, thereby eliminating the possibility for up-coming Glashow events [13].

We list in Table II the ratio of Glashow events to continuum events above $E_\nu^{\rm min} = 1,\ 2,\ 3,\ 4,\ 5$ PeV, with $\alpha = 2.0$ (2.3) and $E_\nu^{\rm max} = \infty$, for the six models of cosmic neutrino production under consideration.[1] Note that we keep the value of $E_\nu^{\rm min}$ well below the energy region of the resonance: at the energy value of the peak minus one FWHM, the incident neutrino energy is $6.3\,{\rm PeV}\,(1-\Gamma_W/M_W)^2 \approx 6.3\,{\rm PeV}\,(1-0.052) = 6.0\,{\rm PeV}$.

We note that the numbers of expected resonant events presented in Table II is greatly reduced from the ratio of resonant to non-resonant cross sections by the additional factors. The cross section ratio at 6.3 PeV is 240: see Fig. 1. The $\frac{\Gamma_W}{M_W}$ ratio is 1/38. The α-dependent factor $\left[(\alpha-1.40)\left(\frac{1\,{\rm PeV}}{6.3\,{\rm PeV}}\right)^{\alpha-1.40}\right]$ yields about 0.2 for both α's of interest, 2.0 and 2.3. The end result is about $2\mathcal{R}$ for the ratio of resonant events to non-resonant events above 1 PeV.

Of course, the expected number of Glashow events does depend on $\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle$. The number of Glashow events is found by multiplying the first numerical column of Table II by $N(\geq 1\,{\rm PeV}) = 2.94\,\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle\,(2.15\,\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle)$. These expected resonant event numbers are 1.1 (0.69), 0.71 (0.43), 0.47 (0.28), 0 (0), 1.2 (0.77), and 3.5 (2.1), each times $\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle$, for the six models, and for $\alpha = 2.0\,(2.3)$. With increased statistics the Glashow event numbers may separate into values which discriminate among the astrophysical source models.

Since no 6.3 PeV events are observed, the Poisson probabilities for each model, based solely on the absence of resonance events, for $\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle = 3$, are, 3.8% (13%), 12% (28%), 24% (43%), large (large), 2.7% (9.9%), and 0.0025% (0.18%), respectively; and for $\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle = 1$, are 34% (50%), 49% (65%), 62% (76%), large (large), 30% (46%), 3.0% (12%), respectively. All models remain viable except perhaps the final one, where neutron decay to pure $\bar\nu_e$ predicts some resonance events at Earth. However, since the probabilities vary exponentially with $\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle$, more data is needed before reasonably-definite conclusions can be drawn. These "Glashow-event" probabilities should be multipled by the continuum probabilities to determine overall Poisson probabilities for a $\langle N^{\rm expected}_{\text{1-2}\,{\rm PeV}}\rangle$ value, and for the unbroken power law hypotheses with $\alpha = 2.0$ and 2.3.

For the sake of completeness, we briefly consider the possibility of exotic neutrino properties that modify the flavor mix of neutrinos, specifically neutrino decay and pseudo-Dirac neutrino oscillations. Neutrino decay [18] allows the flavor mix to deviate significantly from the democratic mix. Observation of a significant $\bar\nu_e$ flux from SN1987A precludes any observable effects of ν_1 decay on L/E scales of astrophysical interest. In the case of a normal hierarchy (with mass ordering $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$), the ν_2 and ν_3 mass eigenstates may decay completely to ν_1, whose flavor content ratios are $|U_{e1}|^2 : |U_{\mu1}|^2 : |U_{\tau1}|^2 = 4:1:1$ for both ν and $\bar\nu$. The $\bar\nu_e$ content of the neutrino flux at Earth is then 1/3 which may be an *enhancement*. On the other hand, if the mass hierarchy is inverted (with $m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$), then both ν_1 and ν_3 are stable and a variety of final flavor ratios are possible, depending on the intial ratios of ν_1, ν_2, and ν_3, and the decay mode of ν_2.

Another possibility for deviations from standard flavor mixes [19] arises in scenarios of pseudo-Dirac neutrinos [20], in which each of the three neutrino mass eigenstates is a doublet with tiny mass differences less than 10^{-6} eV (to evade detection so far).[2] The smallness of the

[1] The purpose of allowing for a finite $E_\nu^{\rm max}$ in Eq. (10) is to compare our ratios to the ratios that result from the effective areas provided in Ref. [4]. There an $E_\nu^{\rm max} = 10$ PeV. On including this $E_\nu^{\rm max}$ in our calculation, we find very good agreement with the IceCube numbers. Note that in the IceCube nomenclature for incident $\nu+\bar\nu$ fluxes, the ratio of down-coming Glashow events to continuum events is given by $\left(\frac{\nu_e-\nu_\mu}{3\nu_\mu}\right)_{south}$.

[2] In fact, observing an energy-dependence of flavor mixes of high energy cosmic neutrinos is the only known way to detect mass-squared differences in the range $10^{-18}-10^{-12}$ eV^2.

TABLE II: Ratio of resonant event rate around the 6.3 PeV peak to non-resonant event rate above $E_\nu^{\rm min} = 1,\ 2,\ 3,\ 4,\ 5$ PeV. The single power-law spectral index α is taken to be 2.0 and 2.3 for the non-parenthetic and parenthetic values, respectively. The single power-law extrapolation just above 1 PeV predicts a mean number of observed resonance events around 6.3 PeV equal to the first numerical column times $2.94\,\langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$ ($2.15\,\langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$), as calculated in the text.

$E_\nu^{\rm min}$ (PeV)	1	2	3	4	5
$pp \to \pi^\pm$ pairs	0.37 (0.32)	0.56 (0.59)	0.71 (0.85)	0.84 (1.1)	0.96 (1.3)
w/ damped $\mu^\pm$	0.24 (0.20)	0.37 (0.38)	0.47 (0.56)	0.54 (0.71)	0.62 (0.88)
$p\gamma \to \pi^+$ only	0.16 (0.13)	0.24 (0.26)	0.31 (0.37)	0.37 (0.48)	0.42 (0.59)
w/ damped μ^+	0 (0)	0 (0)	0 (0)	0 (0)	0 (0)
charm decay	0.41 (0.36)	0.62 (0.67)	0.80 (0.95)	0.94 (1.2)	1.1 (1.6)
neutron decay	1.2 (1.0)	1.9 (2.0)	2.3 (2.8)	2.8 (3.6)	3.2 (4.4)

mass difference tells us that the mixing angle between the active state with $SU(2)$ couplings, and the sterile state without, is necessarily maximal. For cosmically-large L/E, the flux of each active flavor is therefore reduced by a half. Of course, if all three flavors are reduced by a half, there is no change in the flavor ratios; however, at intermediate energies each flavor can be reduced or not, leading to a possible suppression of the absolute flavor ratio for $\bar{\nu}_e$ by $R^{\rm pD}_{\bar{\nu}_e}/R_{\bar{\nu}_e}$ of roughly 1/2, or an enhancement of the $\bar{\nu}_e$ flux ratio of roughly 2. (Note that the maximal suppression/enhancement will be a bit less than 1/2 or 2 if there is a ν_e flux present.)

IV. CONCLUSIONS

Normalized to the three down-coming IceCube events in the 1-2 PeV range, we find that the number of predicted resonant Glashow events ranges from zero (for the damped μ^+ mode, which generates no antineutrinos) to almost three (for the neutron decay mode which generates only antineutrinos) times $\langle N^{\rm expected}_{\rm 1\text{-}2\,PeV}\rangle$. The other four popular neutrino-generating modes give intermediate values. Thus we have demonstrated that the fraction of resonance events is a potential discriminator among the popular neutrino-generating astrophysical models.

Our calculations are done in a somewhat idealized approximation. For example, in pion production from $p\gamma$ collisions, we consider only the contribution of the Δ^+ intermediate states. Also, we do not consider the possibility that more than one neutrino source model may be contributing. When more data become available, refinements on our "Resonometer" will become necessary.

Until that day, we conclude that the absence of Glashow resonance events in IceCube favors the lower values of the fractional $\bar{\nu}_e$ flux. Should this non-observation of resonance events continue, the "damped μ^+ mode" $p\gamma \to \pi^+ n \to n + \mu^+ + \nu_\mu$ would become uniquely favored.[3] Caveats to this conclusion include the possibility of pseudo-Dirac neutrino oscillations, and the possibility of neutrino decay.

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