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Multiple scattering of heavy-quarks in dense matter and the parametric prominence of drag

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The case of heavy quark propagation in dense extended matter is studied in the multiple scattering formalism of the higher twist energy loss scheme. We consider the case of deep inelastic scattering off a large nucleus. The hard lepton scatters off a heavy quark fluctuation within one of the nucleons. This heavy quark then propagates through the dense medium, multiply scattering off the gluon field of the remaining nucleons in its path. We consider the fictitious process where a heavy quark propagates through the nucleus without radiation. Invoking Soft-Collinear Effective Theory power counting arguments, we consider the case of a "semi-hard" heavy-quark where the mass is of the order of the out-going momentum and larger than the transverse momentum imparted per unit length due to scattering. In this limit, it is found that longitudinal momentum exchanges (quantified by the transport coefficient $\hat{e}$) have a comparable effect on the off-shellness of the propagating quark, as the transverse momentum exchanges (quantified by $\hat{q}$) which constitute the leading cause of off-shellness for propagating light quarks or gluons. Consequences of this new hierarchy for the propagation of the heavy quark in dense matter are discussed.

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I. INTRODUCTION

With the advent of hard sector observables at the LHC [1, 2], the medium modification of high energy jets has become one of the forefront topics of research. The observed suppression in the light flavor sector, especially the dependence on the transverse momentum of the observed hadrons, is now well understood within a factorized perturbative QCD (pQCD) based approach [3]. However, the heavy quark sector has remained a bit of a mystery: There is an observed large suppression, both at the Relativistic Heavy-Ion Collider (RHIC) and at the Large Hadron Collider (LHC).

Simple minded extensions of the formalism applied to light quarks have yielded theoretical results that are in moderate agreement with experimental measurements [4–7]. The agreement with measurements of the nuclear modification factor and the azimuthal anisotropy of non-photonic leptons (from the decay of open heavy flavor hadrons) at RHIC is found to improve with increasing transverse momentum (p_T) of the detected lepton. At the LHC, one is not restricted to leptons from the decay of D and B hadrons, but instead has access to the nuclear modification factors of both D and B hadrons separately. The accuracy and p_T range of the new measurements of heavy-quark suppression at the LHC, along with the fact that the medium is now both larger and denser calls for a more sophisticated approach. It is the object of this paper to lay the ground work for such an approach based on pQCD, in particular, we will incorporate power-counting techniques borrowed from Soft-Collinear-Effective-Theory (SCET) [8–11] to address the issue.

This paper extends the effort started in Refs [12–18] which systematically extends the next-to-leading twist set up of Refs. [19, 20] to a scattering resummed formalism for light and (in this paper) to heavy-flavors. While gluon radiation from the heavy-quark will not be considered, the scatterings of the heavy quark will engender both longitudinal and transverse momentum transfer, leading to the simultaneous appearance of transverse diffusion, and longitudinal drag and straggling [14, 18]. While similar calculations have appeared for a light quark, the surprise in this case is the importance of longitudinal transfers codified by $\hat{e}$ to the stimulated off-shellness of the heavy-quark. This turned out to be important to the specific case of "semi-hard" heavy-quarks: a terminology that will be made clear in the subsequent sections.

To this end, we consider the theoretically well defined case of heavy-quark production and propagation in Deep-Inelastic Scattering on a large nucleus (A-DIS), where the produced heavy quark propagates through the dense extended and confined nuclear medium. The remaining sections are organized as follows: in Sec. II, we will setup the basic formalism of DIS on a large nucleus where the hard virtual photon strikes a heavy-quark created due to high Q^2 fluctuations inside a proton. In Sec. III, we consider the multiple scattering of the produced hard quark and introduce power-counting scales similar to SCET. In Sec. IV, we introduce the factorization of the final state scattering from the initial parton distribution function (PDF) followed by gradient expansion of the hadronic tensor and its resummation.

We offer concluding discussions and an outlook in Sec. V.

II. DEEP INELASTIC SCATTERING AND INTRINSIC HEAVY FLAVOR.

In this section, the formalism for the scattering of the heavy-quark will be set up, within the framework of DIS on a nucleus. The propagation of the heavy-quark will be factorized both from the initial hard scattering which produces the outgoing heavy-quark as well as from the many soft matrix elements which appear in the subsequent multiple scattering.

Consider the deep-inelastic scattering of a virtual photon with a heavy quark off a nucleon, within a large nucleus with mass number A. The nucleus possess a momentum $P = pA$, with p the average momentum of a nucleon in this nucleus. A frame is considered where the exchanged virtual photon has no transverse momentum, and has momentum components,

$$q \equiv [q^+, q^-, q_\perp] = \left[-\frac{-q^2}{2q^-}, q^-, 0, 0\right]. \quad (1)$$

We are interested in the reaction,

$$e(L_1) + A(p) \longrightarrow e(L_2) + J_{\mathcal{Q}}(\vec{l}\,) + X, \quad (2)$$

where, $e(L_1)\,[e(L_2)]$ represents the incoming (outgoing) electron with momentum $L_1\,(L_2)$, $A(p)$ represents the incoming nucleus and $J_{\mathcal{Q}}(\vec{l}\,)$ represents the outgoing jet which contains one heavy quark $\mathcal{Q}$ with mass M. Since there are no valence heavy-quarks within the nucleon, to produce a jet containing a single heavy-quark, the virtual photon will have to strike a heavy quark from within the sea of the nucleon, *i.e.*, from a $\mathcal{Q}\bar{\mathcal{Q}}$ fluctuation. As a result, the outgoing remnants of the nucleon, denoted by the X in the equation above, will contain a $\bar{\mathcal{Q}}$. It is of course equally likely that the $\bar{\mathcal{Q}}$ will be struck by the virtual photon and the remnants of the proton will contain the quark $\mathcal{Q}$; this will make very little difference to our discussion of multiple scattering of the hard parton as it passes through a nucleus. In this article, we will not discuss the dynamics of the production of the heavy quark, containing it within a parton distribution function. We high-light the power counting of the momentum components. We consider a quark mass $M \gg \Lambda_{QCD}$ and a final outgoing quark momentum which is larger, but of the order of the quark mass. In the frame where the proton is boosted by a large factor $\gamma = 1/\lambda$ in the '$+$' direction, we have the momentum components of the incoming heavy quark as,

$$p_{\mathcal{Q}} = [p^+_{\mathcal{Q}}, p^-_{\mathcal{Q}}, \vec{p}_{\mathcal{Q}\perp}] \equiv \left[\sqrt{2}\gamma M, \frac{M}{2\gamma\sqrt{2}}, 0\right]. \quad (3)$$

We assume that the quark, anti-quark fluctuation is almost stationary in the proton rest frame and thus $\vec{p}_{\mathcal{Q}\perp} \to 0$. The reader should note that the boost factor γ is simply a alternate variable to $p^+_{\mathcal{Q}}$ and carries no extra information other than the relation between a large $p^+_{\mathcal{Q}}$ and a small $p^-_{\mathcal{Q}}$.

The momentum components of the incoming photon are assumed to be,

$$q = \left[-\sqrt{2}\gamma M + \frac{M^2}{2q^-}, q^- - \frac{M}{2\gamma\sqrt{2}}, 0\right]. \quad (4)$$

In the equation above, we are assuming that $\gamma M \gg M \sim q^- \gg M/\gamma$. Thus we have, $-q^2 \simeq Q^2 \simeq 2\sqrt{2}\gamma M q^-$. As a result, we obtain the final outgoing quark to have momentum components

$$(p_{\mathcal{Q}} + q) \simeq \left[\frac{M^2}{2q^-}, q^-, 0\right]. \quad (5)$$

For concreteness, we may consider $M \lesssim q^- \sim \sqrt{\lambda}Q$ for slow heavy quarks.

A. Power Counting and the small λ parameter

Power corrections to hard processes in vacuum are generally suppressed in the presence of a hard scale $Q^2 \gg \Lambda_{QCD}$. However, there exist scenarios where power corrections to the operators with higher twist may be enhanced and become non-negligible compared to the leading process. To explore this possibility, in this discussion, we have introduced the dimensionless small parameter λ. We assume that the scaling variable λ is so chosen that perturbation theory may be applied down to momentum transfer scales at or above $\lambda^2 Q$. This is a concept borrowed from soft collinear effective theory (SCET) [8–11], and constitutes the power counting variable: Terms that are sub-leading in λ will be dropped, similar to [15],

$$\lambda^0 \gg \sqrt{\lambda} \gg \lambda \gg \lambda^{\frac{3}{2}}. \quad (6)$$

As mentioned above, in the remainder of this paper, we will consider the case of a heavy quark with a momentum $p \sim \sqrt{\lambda}Q$ and mass $M \sim \sqrt{\lambda}Q$. The off-shellness of the hard virtual photon Q is the hardest scale in the problem and hard momentum components are expected to scale as $\sqrt{\lambda}Q$. Softer momentum components may scale as λQ or even $\lambda^2 Q$. The meaning of $\sqrt{\lambda}$ is an intermediate suppression factor that may not be neglected. Note that at this level of approximation, there is no shift in the Bjorken variable x_B, given as

$$x_B = \frac{Q^2}{2p^+q^-} \sim \frac{Q^2}{[Q/\sqrt{\lambda}]\sqrt{\lambda}Q}. \quad (7)$$

Thus $\mathcal{O}(x_B) \sim 1$, *i.e.*, x_B is a large momentum ratio.

B. Mass modification to hadronic tensor

We may express the differential cross section to produce a hard parton with 3-momentum $\vec{l} \equiv l^-, l_\perp$ in the DIS on a large nucleus as,

$$\frac{E_{L_2} d\sigma}{d^3 L_2 dl^- d^2 l_\perp} = \frac{\alpha_e}{2\pi s} \frac{1}{Q^4} L_{\mu\nu} \frac{dW^{\mu\nu}}{dl^- d^2 l_\perp}, \quad (8)$$

where, the Mandelstam variable, $s = (p + L_1)^2$. All terms that contain the wave-functions of the incoming and outgoing leptons are included in the leptonic tensor,

$$L_{\mu\nu} = \frac{1}{2} \text{Tr}[\not{L}_1 \gamma_\mu \not{L}_2 \gamma_\nu]. \quad (9)$$

The entire strongly interacting part of the cross section is included in the hadronic tensor, defined as

$$\begin{aligned} W^{\mu\nu} &= \sum_X (2\pi)^4 \delta^4(q + P_A - p_X) \\ &\times \langle A; p | J^\mu(0) | X \rangle \langle X | J^\nu(0) | A; p \rangle \\ &= 2\text{Im}\left[\int d^4 y e^{iq\cdot y} \langle A; p | J^\mu(y) J^\nu(0) | A; p \rangle \right] \quad (10) \end{aligned}$$

The initial state of the incoming large nucleus, with A nucleons and an average momentum p per nucleon, is represented by the ket $|A; p\rangle$. The final unidentified hadronic or partonic state is defined as $|X\rangle$. The sum ($\sum_X$) runs over all possible hadronic states and J^μ is the hadronic current ($J^\mu = Q_{\mathcal{Q}} \bar{\psi}_{\mathcal{Q}} \gamma^\mu \psi_{\mathcal{Q}}$, where $Q_{\mathcal{Q}}$ is the charge of the heavy-quark of flavor $\mathcal{Q}$ in units of the electron charge e). Factors of the electromagnetic coupling constant have already been extracted and included in Eq. (8).

Ignoring all power corrections of the order of Λ_{QCD}/Q as well as factors of the heavy quark mass, the hadronic tensor may be expressed as (we also take the average over initial states and sum over final states to obtain)

$$\begin{aligned} W_0^{A\mu\nu} &= C_p^A\, W_0^{\mu\nu} = C_p^A \frac{2\pi}{2Q^2} \sum_{\mathcal{Q}} Q_{\mathcal{Q}}^2\, f_{\mathcal{Q}}(x_B) \quad (11) \\ &\times \text{Tr}\left[\not{p}_{\mathcal{Q}} \gamma^\mu (\not{p}_{\mathcal{Q}} + \not{q}) \gamma^\nu \right] \\ &= \sum_{\mathcal{Q}} Q_{\mathcal{Q}}^2 C_p^A\, 2\pi \left[g^{\mu -} g^{\nu +} + g^{\mu +} g^{\nu -} - g^{\mu\nu} \right] \\ &\times \int \frac{dy^-}{2\pi} e^{-ix_B p^+ y^-} \frac{1}{2} \langle p | \bar{\psi}(y^-) \gamma^+ \psi(0) | p \rangle. \end{aligned}$$

In the absence of quark mass corrections, the only surviving components of the hadronic tensor are $W_0^{A\perp\perp}$. However the situation is different for the case of heavy quarks. Here the components of the hadronic tensor can be expressed as follows,

$$\begin{aligned} & W_{0M}^{A\mu\nu} \\ &= C_p^A\, W_{0M}^{\mu\nu} \quad (12) \\ &\simeq C_p^A \frac{2\pi}{2Q^2} \sum_{\mathcal{Q}} Q_{\mathcal{Q}}^2\, f_{\mathcal{Q}}(x_B) \\ &\times \text{Tr}\left[(\not{p}_{\mathcal{Q}})\, \gamma^\mu\, (\not{p}_{\mathcal{Q}} + \not{q})\, \gamma^\nu \right] \\ &\simeq C_p^A \frac{2\pi\sqrt{2}\gamma M}{2Q^2} \left[-4q^- g^\mu{}_\perp g^\nu{}_\perp + \frac{4M^2}{q^-} g^\mu{}_+ g^\nu{}_+ \right] \\ &\times \sum_{\mathcal{Q}} Q_{\mathcal{Q}}^2 \int \frac{dy^-}{2\pi} e^{-ix_B p^+ y^-} \frac{1}{2} \langle p | \bar{\psi}(y^-) \gamma^+ \psi(0) | p \rangle. \end{aligned}$$

In the equation above, all terms suppressed by powers of λ have been neglected. We have two leading components of the hadronic tensor $W^{\perp\perp}$ and W^{++}. In the subsequent sections, we will consider both the hadronic tensor components, $W_0^{A\perp\perp}$ and W_0^{A++} and investigate how they evolve in the final state propagation of the heavy quark. Radiation will be ignored. In all cases, the initial state parton distribution function and hard cross section will be defined as above. The focus will be only on the final state 3-momentum distribution of the heavy quark, which, in the case without any final state scattering will possess a narrow width and can be approximated as a delta function, *i.e.*,

$$\begin{aligned} \frac{dW_0^{A\mu\nu}}{dl^- d^2 l_\perp} &= W_0^{A\mu\nu} \phi_0(l^-, l_\perp) \\ &= W_0^{A\mu\nu} \delta(l^- - q^-) \delta^2(l_\perp)\ . \quad (13) \end{aligned}$$

In the subsequent sections, we will consider only the modification to the final state momentum distribution $\phi(l^-, l_\perp)$.

III. MULTIPLE SCATTERING AND FINAL STATE POWER COUNTING.

In the preceding section, the final state momentum distribution of the outgoing heavy-quark has been identified and factorized from the hard cross section. In this section, the propagation of this quark without radiation will be considered. This is by no means a physical process. High momentum partons produced in hard processes are most often produced far off their mass shell *i.e.*, with a considerable virtuality μ^2, which, though small compared to the forward energy of the quark, is still much larger than Λ_{QCD}^2. The hard parton tends to shed this large virtuality through a series of gluon emissions. The emitted gluons are also virtual and will radiate further leading to the development of a partonic cascade. Each parton in this cascade will engender multiple scattering while radiating a gluon (or splitting into a quark antiquark pair). In order to incorporate the effect of multiple scattering on the development of a cascade, the propagation of single partons in a medium, without emission, has to be clearly understood [12, 14, 15, 18]. As the struck heavy quark propagates through the nucleus, it will scatter off the dense gluon field within nucleons in its path. Every scattering engenders an extra factor of the strong coupling constant α_s. As will be shown later, every scattering also contains an integration over the location of where the scattering took place. In a large nucleus ($A \gg 1$), these length integrals give large factors that counter the suppression due to the appearance of factors of α_s.

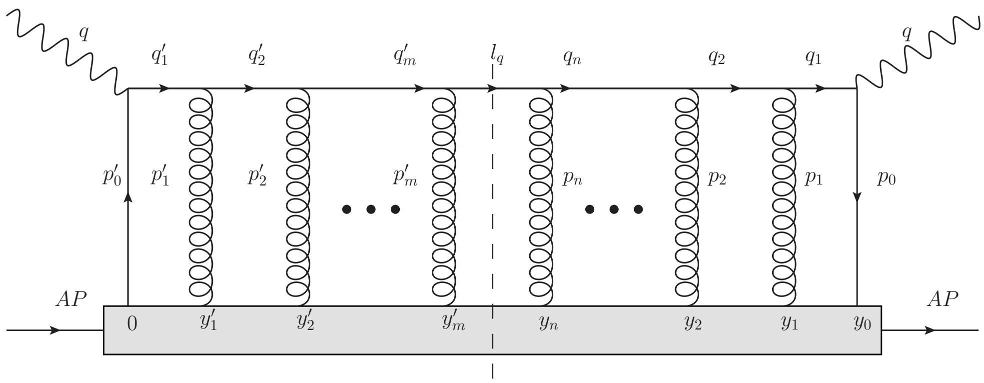

FIG. 1. A heavy-quark produced in DIS on a nucleon inside a large nucleus. The produced quark then propagates through the remaining nucleus multiply scattering off the soft gluon field of the nucleons behind the struck nucleon.

In order to remain consistent with previous calculations, we will denote p_Q as p_0 in the remainder of this paper. Thus the incoming heavy quark possesses a momentum, p_0 such that,

$$p_0 \equiv \left[p_0^+, \frac{M^2}{2p_0^+} + \delta p_0^-, 0, 0\right] \tag{14}$$
$$= \left[x_B p^+, \frac{M^2}{2x_B p^+} + \delta p_0^-, 0, 0\right].$$

In the equation above, $p_0^- = M^2/(2p_0^+) + \delta p_0^-$, where $M^2/2p_0^+ \sim \lambda^{3/2} Q$ and $\delta p_0^- \sim \lambda^2 Q$ is considered as a small correction. The incoming virtual photon has momentum components

$$q \simeq \left[-\frac{Q^2}{2q^-} + \frac{M^2}{2q^-}, q^-, 0, 0\right], \tag{15}$$

with $Q^2 \simeq 2\gamma M q^-$ and thus the final outgoing heavy quark has momentum components,

$$q_1 = p_0 + q$$
$$\simeq \left[x_B p^+ - \frac{Q^2}{2q^-} + \frac{M^2}{2q^-}, q^-, 0, 0\right]. \tag{16}$$

In the equation above, we have ignore the small corrections from p_0^-. Note that, the vanishing transverse components of the virtual photon are a choice of coordinate system, whereas the vanishing components of the incoming quark are an approximation. Insisting that the outgoing quark is close to mass shell, this yields x_B (identical to x_0, defined later) as given in Eq. (7).

The relevant Feynman diagram is shown in Fig. 1. This describes the hard scattering processes when a virtual photon strikes a heavy quark off the nucleus with momentum p'_0 (p_0 in amplitude) at space-time point $y'_0 = 0$ (y_0 in complex conjugate amplitude). It is then scattered back through the nucleus with momentum q'_1 (q_1 in the complex conjugate amplitude). While propagating through the large nucleus, the heavy quark scatters with the the gluon fields inside the nuclear medium at space-time points y'_j with $0 < j < m$ (y_i for the complex conjugate amplitude with $0 < i < n$). All calculations will be carried out in $A^- = 0$ gauge. The choice of gauge controls the power counting of the gluon vector potentials. In each scattering, the "semi hard" heavy quarks accumulates momenta p'_j (p_i in the complex conjugate). Momentum conservation, at each vertex, enable one to assign various momenta as follows (See Fig. 1):

$$q_{i+1} = q_i + p_i = q + \sum_{j=0}^{i} p_j = q + k_i,$$
$$q'_{i+1} = q'_i + p'_i = q + \sum_{j=0}^{i} p'_j = q + k'_i, \qquad (17)$$

where we have defined new momentum variables, for convenience, as $k_i = \sum_{j=0}^{i} p_j$, and $k'_i = \sum_{j=0}^{i} p'_j$, which denote the 'total' momentum exchanged between the propagating heavy quark and the nuclear medium. The hadronic tensor now can be written as,

$$W^{A\mu\nu}_{mn} = \sum_{\mathcal{Q}} Q^2_{\mathcal{Q}} g^{n+m} \frac{1}{N_c} \mathrm{Tr}\left[\left(\prod_{i=1}^{n} T^{a_i}\right)\left(\prod_{j=m}^{1} T^{a'_j}\right)\right] \int \frac{d^4 l_q}{(2\pi)^4}(2\pi)\delta^+(l_q^2 - M^2) \int d^4 y_0 e^{iq\cdot y_0}$$
$$\times \left(\prod_{i=1}^{n}\int d^4 y_i\right)\left(\prod_{j=1}^{m}\int d^4 y'_j\right)\left(\prod_{i=1}^{n}\int \frac{d^4 q_i}{(2\pi)^4} e^{-iq_i\cdot(y_{i-1}-y_i)}\right) e^{-il_q\cdot(y_n - y'_m)} \left(\prod_{j=1}^{m}\int \frac{d^4 q'_j}{(2\pi)^4} e^{-iq'_j\cdot(y'_j - y'_{j-1})}\right)$$
$$\times \langle A|\bar{\psi}(y_0)\gamma^\mu \left(\prod_{i=1}^{n} \frac{\gamma\cdot q_i + M}{q_i^2 - M^2 - i\epsilon}\gamma\cdot A^{a_i}(y_i)\right)\gamma\cdot l_q \left(\prod_{j=m}^{1} \gamma\cdot A^{a'_j}(y'_j)\frac{\gamma\cdot q'_j + M}{{q'_j}^2 - M^2 + i\epsilon}\right)\gamma^\nu \psi(0)|A\rangle. \qquad (18)$$

For further simplifications, we will now change the integral variables $q_{i+1} \to p_i$ and $q'_{j+1} \to p'_j$, and will incorporate one additional exchanged momentum p_n inside the complex conjugate amplitude by bringing the following δ-function,

$$1 = \int \frac{d^4 p_n}{(2\pi)^4}(2\pi)^4 \delta^4(l_q - q - p_n). \qquad (19)$$

A. Mass Modifications

Mass modifications to the case of a light quark occur from two sources:

- The spin structure of the propagators specified by the numerators of the propagators
- The pole structure of the propagators, as specified by the denominators of the propagators.

We will go for a power counting analysis to both of them in the following,

Numerators of the propagators: The sum over spins in the numerator of each quark propagator has a factor of M. This is an obvious M dependent correction compared to the case of mass-less quarks. However, to include factors of M require that there be at least 2 factors of M in the trace (else we will have an odd number of γ matrices) and, secondly, each factor be either preceded or followed by a $\gamma_\perp \cdot A_\perp$, as

$$\mathrm{Tr}\left[\ldots\gamma^- A^+ M \gamma^- A^+ \ldots \gamma^- A^+ M \gamma^- A^+ \ldots\right] = 0.$$

This is due to the fact that in light-cone coordinates $\{\gamma^+, \gamma^+\} = \{\gamma^-, \gamma^-\} = 0$. As a result, the first non-vanishing correction, from M dependent terms in the numerator, yields an additive contribution to the heavy-quark hadronic tensor, of the following form:

$$\delta W^{\mu\nu}_{\mathcal{Q}} \propto \mathrm{Tr}\left[\ldots\gamma_\perp A_\perp M \ldots \gamma_\perp A_\perp M \ldots\right] \sim \lambda^2 W^{\mu\nu}_{\mathcal{Q}}.$$

In the equation above $W^{\mu\nu}_{\mathcal{Q}}$ is the leading contribution to the heavy-quark hadronic tensor. The overall factor of λ^2 is due to the appearance of two factors of $A_\perp$ which scale as λA^+, in $A^- = 0$ gauge [15, 17].

Denominator of the propagators: To simplify the denominator, consider the denominator of the first propagator in Eq. (18), where $q_1 = q + p_0$. This can be expressed as

$$q_1^2 - M^2 = -Q^2 + 2q^+ p_0^- + 2q^- p_0^+. \qquad (20)$$

Contour integration on p_0^+ will set $p_0^+ = (Q^2 - 2q^+p_0^-)/(2q^-)$. Even if the incoming quark were on mass shell, $p_0^- = M^2/(2p_0^+)$, a term of order $\lambda^{3/2}Q$ and thus negligible compared to $q^- \sim \lambda^{1/2}Q$.

The fate of the remaining denominators, on contour integration, can now be easily surmised. For example, the second propagator yields the relation,

$$\begin{aligned} q_2^2 - M^2 &= (q_1 + p_1)^2 - M^2 \\ &= 2p_1^+p_1^- - {p_1}_\perp^2 + 2p_1^+(q^- + p_0^-) \\ &+ 2p_1^-(q^+ + p_0^+). \end{aligned} \tag{21}$$

As in previous calculations with light quarks in Refs. [12, 14, 18], we will assume that all rescattering of the produced quark with the soft gluons off the medium engender momentum exchanges where $p_\perp \sim \lambda Q$ and $p^- \sim \lambda^2 Q$, with p^+ is fixed by the requirement that the propagating quark be close to its mass shell. Neglecting all but the lowest power of λ, we obtain,

$$p_1^+ = p_1^- \frac{q^+ + p_0^+}{q^- + p_0^-} - \frac{{p_1}_\perp^2}{q^- + p_0^-}. \tag{22}$$

Comparing with the results for p_i^+ in Refs. [12, 14, 18], the above equation represents a remarkable departure from the case of a light quark. In this case of the heavy quark, if $p_i^- \sim \lambda^2 Q$, p_i^+ is comparably controlled by p_i^- and $p_{i\perp}$. If $p_i^- \sim \lambda^2 Q$ then both terms on the right hand side of Eq. (22) are present. Thus $p_i^+ \sim \lambda^{3/2}Q$, and depends, non-negligibly, on the value of the longitudinal exchange p_i^-. In the case of a light quark, p_i^+ was dominantly controlled by ${p_i}_\perp^2/(2q^-)$ (with sub-leading corrections from p_i^-) and in that case $p_i^+ \sim \lambda^2 Q$.

As a result of the above considerations, the (+)-components of all the exchanged gluons off which the heavy-quark scatters can be of the order of $\lambda^2 Q$ or even as high as $\lambda^{3/2}Q$. This implies that if the heavy-quark goes off shell and radiates a perturbatively resolvable radiation, it can go off shell by $\delta q \sim \lambda^2 Q$. Since the two light-cone components of the heavy-quark momentum are of the order $\sqrt{\lambda}Q$, the radiation pattern (and the energy loss) from a heavy- quark driven off shell by scattering may be somewhat different than for a light quark and gluon.

In the subsequent section, Eq. (18) will be expanded in a power series in λ, where we will find that the leading terms in the series expansion will yield both a transverse diffusion equation and a longitudinal drag equation, and not just a diffusion equation as in the case of a light quark [12].

We now study the structure of the denominator of an arbitary propagators. For the heavy-quark line after i^{th} scattering, we have,

$$\begin{aligned} & q_{i+1}^2 - M^2 \\ &= (q + k_i)^2 - M^2 \\ &= 2p^+q^- \left[\bar{x}_i - x_B - \bar{x}_{Di} + x_B \bar{y}_i\right] \end{aligned} \tag{23}$$

where we have introduced some momentum fraction variables and defined a few new variables. These have been defined purely for convenience,

$$\bar{x}_i = \sum_{j=0}^{i} x_j, \quad x_j = \frac{p_j^+}{p^+}, \tag{24}$$

$$x_B = \frac{Q^2}{2p^+q^-}, \tag{25}$$

$$\bar{x}_{Di} = \frac{(k_\perp^i)^2}{2p^+q^-} = \sum_{j=1}^{i} x_{Di}, \tag{26}$$

$$x_{Di} = \frac{{p_\perp^i}^2 + 2p_\perp^i \cdot \sum_{j=0}^{i-1} p_\perp^j}{2p^+q^-}, \tag{27}$$

$$\bar{y}_i = \sum_{j=1}^{i} y_j, \quad y_j = \frac{p_j^-}{q^-}. \tag{28}$$

For all cases where the mass M scales with a higher power of λ than λQ, the modifications to the case of massless quark traversing an extended nuclear medium is suppressed by a factor of λ^2. We seek only the largest corrections $\propto \lambda^0$ compared to the propagation of a light quark in an extended medium. Leading corrections to the propagation of a heavy quark occur for the case when the mass $M \sim Q$ or $M \sim q^- \sim \sqrt{\lambda}Q$, *i.e.* the mass is of the order of the largest momentum component. Note: this is not the non-relativistic limit where $M \gg p$ and $p \sim Q$. We refer to this regime where $M \sim p$ as the intermediate momentum region. If Q always refers to the hardest scale in the problem, then the high momentum regime is when $M \sim \lambda Q$, and the low momentum regime (equivalent to the non-relativistic regime) is where $M \sim Q/\lambda$. Physically speaking, the intermediate momentum regime for a b-quark corresponds to a total energy $E \sim M$; the high energy regime corresponds to the region where $E \gg M$. It is the intermediate momentum regime where all the somewhat surprising results regarding heavy-quark energy loss have been measured and this is the regime, that we will study in greater detail.

We recall that the exchanged momenta with the medium have momentum components $k \equiv [k^+, k^-, \vec{k}_\perp] \sim [\lambda^{3/2}, \lambda^2, \lambda]Q$. As such, these are somewhat removed from the scale of the mass of the heavy quark. As a result, we separate the terms containing the mass of the heavy-quark from the remaining terms and re-write the entire set of propagator denominators (both cut and uncut lines) as,

$$\mathcal{D}_q = \frac{2\pi}{(2p^+q^-)^{n+m+1}} \quad (29)$$
$$\times \prod_{i=0}^{n-1}\left(\frac{1}{\bar{x}_i - x_B(1+\frac{x_M}{x_0}) + x_B\bar{y}_i - \bar{x}_D^i - i\epsilon}\right)$$
$$\times \quad \delta\left[\bar{x}_n - x_B\left(1+\frac{x_M}{x_0}\right) + x_B\bar{y}_n - \bar{x}_D^n\right]$$
$$\times \prod_{j=0}^{m-1}\left(\frac{1}{\bar{x}'_j - x_B\left(1+\frac{x_M}{x'_0}\right) + x_B\bar{y}'_j - \bar{x}_D^{\prime j} + i\epsilon}\right).$$

In so doing, we have retained the leading corrections in λ coming from the longitudinal momentum loss experienced by the heavy quark from exchanged gluons with non-negligible k_i^-. The leading effect on the offshellness, due to the presence of mass, is from the terms $\bar{y}_i$, which are absent in the massless case. Note that in the denominator the term $x_M/x_0 \sim \lambda \ll 1$ can be ignored.

In the high energy ($Q \to \infty$) and collinear ($\lambda \to 0$) limit, we may approximate [13],

$$\langle\ \bar{\psi}(y)\ \hat{O}\ \psi(0)\ \rangle \ \approx\ \frac{\gamma^-}{2}\ \langle\ \bar{\psi}(y)\ \frac{\gamma^+}{2}\ \hat{O}\ \psi(0)\ \rangle \quad (30)$$

As, $A^+ \sim \lambda^2 Q$ and $A_\perp \sim \lambda^3 Q$, in $A^- = 0$ gauge [17], we approximate,

$$\gamma\cdot A(y) \approx \gamma^- A^+(y). \quad (31)$$

Though we have retained, in Eq. (29), the order λ^2 corrections to the propagators, nonetheless we neglect contributions to the vertices from $A_\perp$ in Eq. (31). However, retained them in the denominators as the order λ^2 terms are the leading terms in the denominators of the propagators. With all these simplifications, structure of the numerator will be as follows,

$$\langle A|\bar{\psi}(y_0)\gamma^\mu\left(\prod_{i=1}^{n}\gamma\cdot(q_i+M)\ \gamma\cdot A^{a_i}(y_i)\right)\gamma\cdot l_q\left(\prod_{j=n'}^{1}\gamma\cdot A^{a'_j}(y'_j)\ \gamma\cdot(q'_j+M)\right)\gamma^\nu\psi(0)|A\rangle$$
$$= \langle A|\bar{\psi}(y_0)\frac{\gamma^+}{2}\left(\prod_{i=1}^{n}A^{+a_i}(y_i)\right)\left(\prod_{j=n'}^{1}A^{+a'_j}(y'_j)\right)\psi(0)|A\rangle$$
$$\times\ \mathrm{Tr}\left[\frac{\gamma^-}{2}\gamma^\mu\left(\prod_{i=1}^{n}\gamma\cdot(q_i+M)\ \gamma^-\right)\gamma\cdot l_q\left(\prod_{j=n'}^{1}\gamma^-\ \gamma\cdot(q'_j+M)\right)\gamma^\nu\right]. \quad (32)$$

Following $q_{i+1} = q + k_i = q + \sum_{j=0}^{i} p_i$, the trace now becomes,

$$\mathcal{T} = \mathrm{Tr}\left[\frac{\gamma^-}{2}\gamma^\mu\left(\prod_{i=1}^{n}\gamma\cdot(q+k_{i-1})\gamma^-\right)\right. \quad (33)$$
$$\left.\times\ \gamma\cdot(q+k_n)\left(\prod_{j=m}^{1}\gamma^-\gamma\cdot(q+k'_{j-1})\right)\gamma^\nu\right],$$

where $\gamma\cdot(q+k_i) = \gamma^+(q^- + k_i^-) + \gamma^-(q^+ + k_i^+) - \vec{\gamma}_\perp\cdot\vec{k}_i^\perp$. As $\{\gamma^-,\gamma^-\} = 0$ the term in $\gamma\cdot(q+k_i)$ containing a γ^- vanishes in the trace.

We now obtain,

$$\mathcal{T} = \left(g^\mu{}_\perp g^\nu{}_\perp - \frac{M^2}{(q^-)^2}\ g^\mu{}_+ g^\nu{}_+\right)\left(2q^-\right)^{n+m+1} \quad (34)$$

In the equation above, we have ignored the suppressed factors of x_M as well as those of p_0^-. While such terms have been dropped from the numerator, they will remain in the denominators and in the overall δ-function until the contour integrations are carried out and the denominators expanded in λ. This is done, to clearly demonstrate that these terms are sub-leading in the determination of the pole structure and in the ensuing expansion.

One now obtains the contribution to the hadronic tensor from the term with m scatterings in the amplitude and n scatterings in the complex conjugate as (with both leading projections),

$$W_{mn}^{A\mu\nu} = \sum_q Q_Q^2 \, g^{n+m} \, \frac{1}{N_c} \, \mathrm{Tr}\left[\left(\prod_{i=1}^{n} T^{a_i}\right)\left(\prod_{j=m}^{1} T^{a'_j}\right)\right] \int \frac{d^3 l_q}{(2\pi)^3}(2\pi)^3\delta^3(\vec{l}_q - \vec{q} - \vec{k}_n)$$
$$\times \left(\prod_{i=0}^{n}\int dy_i^- \int d^3y_i\right)\left(\prod_{j=1}^{m}\int dy'^-_j \int d^3y'_j\right)\left(\prod_{i=0}^{n}\int \frac{dx_i}{2\pi}\int\frac{d^3p_i}{(2\pi)^3}\right)\left(\prod_{j=0}^{m-1}\int\frac{dx'_j}{2\pi}\int\frac{d^3p'_j}{(2\pi)^3}\right)$$
$$\times \left(\prod_{i=0}^{n} e^{-ix_ip^+(y_i^- - y'^-_m)} e^{-i\vec{p}_i\cdot(\vec{y}_i - \vec{y}'_m)}\right)\left(\prod_{j=0}^{m-1} e^{ix'_jp^+(y'^-_j - y'^-_m)} e^{i\vec{p}'_j\cdot(\vec{y}'_j - \vec{y}'_m)}\right)$$
$$\times (2\pi)\delta(-x_B\tau_M + \bar{x}_n - \bar{\Delta}_n)\left(\prod_{i=0}^{n-1}\frac{1}{-x_B\tau_M + \bar{x}_i - \bar{\Delta}_i - i\epsilon}\right)\left(\prod_{j=0}^{m-1}\frac{1}{-x_B\tau_M + \bar{x'}_j - \bar{\Delta}'_j + i\epsilon}\right)$$
$$\times \left(-g^{\mu\perp}g^{\nu\perp} + \frac{M^2}{(q^-)^2}\, g^{\mu-}g^{\nu-}\right)\langle A|\bar{\psi}(y_0)\frac{\gamma^+}{2}\left(\prod_{i=1}^{n}A^{+a_i}(y_i)\right)\left(\prod_{j=m}^{1}A^{+a'_j}(y'_j)\right)\psi(0)|A\rangle, \qquad (35)$$

where using the delta function $(2\pi)\delta(l_q^+ - q^+ - k_n^+)$ one can now able to perform the integration over l_q^+. Integration variables have also been changed for convenience, $p_i^+ \to x_i = p_i^+/p^+$, $p_j^{+'} \to x'_j = p_j^{+'}/p^+$. In the equation above,

$$\tau_M = 1 + x_M/x_0 \simeq 1,$$
$$\bar{\Delta}_i = \sum_{j=1}^{i}\Delta_j = \bar{x}_{Di} - x_B\bar{y}_i,$$
$$\bar{\Delta}'_i = \sum_{j=1}^{i}\Delta'_j = \bar{x}'_{Di} - x_B\bar{y}'_i. \qquad (36)$$

While the factor τ_M will eventually be set to unity, we retain it in the next few expressions. We have introduced additional notations for convenience: $\vec{p} = (p^-, \vec{p}_\perp)$ and $\vec{y} = (y^+, \vec{y}_\perp)$, with $\vec{p}\cdot\vec{y} = p^-y^+ - \vec{p}_\perp\cdot\vec{y}_\perp$.

The end delta function which constrains cut line to be on shell is needed to integrate over x_n,

$$\bar{x}_n = \tau_M x_B + \bar{\Delta}_n \; . \qquad (37)$$

i.e.,

$$x_n = -\bar{x}_{n-1} + \tau_M x_B + \bar{\Delta}_n$$
$$= -\sum_{i=1}^{n-1} x_i + \tau_M x_B + \bar{\Delta}_n \; . \qquad (38)$$

Now the (+)-component of the phase factor is as follows,

$$\Gamma^+ = e^{-i(\tau_M x_B + \bar{\Delta}_n)p^+y_n^-}\left(\prod_{i=0}^{n-1} e^{-ix_ip^+(y_i^- - y_n^-)}\right)$$
$$\times\, e^{i(\tau_M x_B + \bar{\Delta}'_m)p^+y'^-_{n'}}\left(\prod_{j=0}^{n'-1} e^{ix'_jp^+(y'^-_j - y'^-_m)}\right)$$
$$= \Gamma_n^+\Gamma_m^+. \qquad (39)$$

Two phase factors Γ_n^+ and Γ_m^+, respectively, are related to x_i integration and the x'_j integration. The rest of the integration may now be performed, similar to [18] (over the two momentum fractions x_i and x'_j). Starting from the propagators that are attached to the cut line one proceeds to the initial hard scattering electromagnetic vertex which will produce a string of θ-functions that mimics the fact that the heavy quark is traveling from y_{n-1}^- to y_n^- an so on.

After performing all integrations over the internal quark momentum components and taking the Dirac trace of the factors in the numerator, the expression for the hadronic tensor, Combining both the leading projections, is as follows,

$$
\begin{aligned}
&W^{A\mu\nu}_{mn} \\
&= \sum_q Q_Q^2\, g^{n+m}\, \frac{1}{N_c}\, \mathrm{Tr}\left[\left(\prod_{i=1}^{n} T^{a_i}\right)\left(\prod_{j=m}^{1} T^{a'_j}\right)\right] \int \frac{d^3 l_q}{(2\pi)^3}(2\pi)^3\delta^3(\vec{l}_q - \vec{q} - \vec{k}_n) \\
&\times \left(\prod_{i=0}^{n}\int dy_i^- \int d^3y_i\right)\left(\prod_{j=1}^{m}\int dy'^-_j \int d^3y'_j\right)\left(\prod_{i=0}^{n}\int\frac{d^3p_i}{(2\pi)^3}\right)\left(\prod_{j=0}^{m-1}\int\frac{d^3p'_j}{(2\pi)^4}\right)\left(\prod_{i=0}^{n} e^{-i\vec{p}_i\cdot\vec{y}_i}\right)\left(\prod_{j=1}^{m} e^{i\vec{p}'_j\cdot\vec{y}'_j}\right) \\
&\times e^{-ix_B p^+ y_0^-}\left(\prod_{i=1}^{n} e^{-i\Delta_i p^+ y_i^-}\right)\left(\prod_{j=1}^{m} e^{i\Delta'_j p^+ y'^-_j}\right) i^n(-i)^m \left(\prod_{i=1}^{n}\theta(y_i^- - y_{i-1}^-)\right)\left(\prod_{j=1}^{m}\theta(y'^-_j - y'^-_{j-1})\right) \\
&\times \left(-g_\perp^{\mu\nu} + \frac{M^2}{(q^-)^2}\, g^{\mu -} g^{\nu -}\right)\langle A|\bar\psi(y_0)\frac{\gamma^+}{2}\left(\prod_{i=1}^{n} A^{+a_i}(y_i)\right)\left(\prod_{j=m}^{1} A^{+a'_j}(y'_j)\right)\psi(0)|A\rangle. \qquad (40)
\end{aligned}
$$

From the above expression of $W^{A\mu\nu}_{mn}$ its evident that when $M \sim \sqrt{\lambda}Q$ ($q^- \sim \sqrt{\lambda}Q$), the two tensor projections $W^{A\perp\perp}_{mn}$ and W^{A++}_{mn} are of same order. In the subsequent section we will expand it as a series in the small scattering momenta around the hard part of the above expression.

IV. FACTORIZATION, GRADIENT EXPANSION AND RESUMMATION

After performing all integrations over the internal quark momentum components and taking the Dirac trace of the factors in the numerator, the expression for the hadronic tensor, Combining both the leading projections, is as follows,

$$
C_{p_0,p_1\cdots p_n} = AC_p^A\left(\frac{\rho}{2p^+}\right)^n, \qquad (41)
$$

here ρ being the parton density inside the large nucleus, and the factor $1/(2p^+)$ is require for the normalization of partonic state. One may now make an average over the colors of the gauge fields and quark fields, leading components of the hadronic tensor will now become,

$$
\begin{aligned}
W^{A\mu\nu}_{nn} = &\sum_q Q_q^2\left(-g_\perp^{\mu\nu} + \frac{M^2}{(q^-)^2}\, g^{\mu -} g^{\nu -}\right) \; A\, C_p^A\left(\frac{\rho}{2p^+}\right)^n g^{2n}\left(\frac{C_F}{N_c^2-1}\right)^n \\
&\times \int\frac{d^3 l_q}{(2\pi)^3}(2\pi)^3\delta^3(\vec{l}_q - \vec{q} - \vec{k}_n)\int dy_0^- \int d^3y_0 \int\frac{d^3p_0}{(2\pi)^3} e^{-i\vec{p}_0\cdot\vec{y}_0} e^{-i\tau x_B p^+ y_0^-}\langle p|\bar\psi(y_0)\frac{\gamma^+}{2}\psi(0)|p\rangle \\
&\times \left(\prod_{i=1}^{n}\int dy_i^- \int dy'^-_i\, \theta(y_i^- - y_{i-1}^-)\theta(y'^-_i - y'^-_{i-1})\right)\left(\prod_{i=1}^{n}\int d^3y_i\int d^3y'_i\int\frac{d^3p_i}{(2\pi)^3}\int\frac{d^3p'_i}{(2\pi)^3}\right. \\
&\times \left. e^{-i\vec{p}_i\cdot\vec{y}_i} e^{i\vec{p}'_i\cdot\vec{y}'_i} e^{-i\Delta_i p^+ y_i^-} e^{i\Delta'_i p^+ y'^-_i}\langle p|A^+(y_i)A^+(y'_i)|p\rangle\right), \qquad (42)
\end{aligned}
$$

where the integrating variable dp'_0 have been changed to dp'_n. Exploiting the homogeneity approximation, the expression should be further elucidated by the following transformation of variables $(y_i, y'_i) \to (Y_i, \delta y_i)$,

$$
Y_i = (y_i + y'_i)/2, \quad \delta y_i = y_i - y'_i. \qquad (43)
$$

It is now possible to perform the integration over the phase factor, which now depends only on the average values $\vec{Y}_i$, which produce the delta function $\delta^3(\vec{p}_i - \vec{p}_i)$ that actually fix the momentum fractions $x_{Di} = x'_{Di}$. Since both δy_i^- and δy_{i-1}^- belongs to the nucleon size (smaller compared to the size of the large nucleus $\sim Y^-$), we may simplify product of θ-functions as,

$$
\theta(y_i^- - y_{i-1}^-)\theta(y'^-_i - y'^-_{i-1}) = \theta(Y_i^- - Y_{i-1}^-). (44)
$$

The time-ordered product of θ-functions is now as follow,

$$\prod_{i=1}^{n}\int_0^{L^-} dY_i^- \theta(Y_i^- - Y_{i-1}^-) = \frac{\prod_{i=1}^{n}\int_0^{L^-} dY_i^-}{n!}, \quad (45)$$

where extent of the nuclear size is expressed by L^-. These terms, on integration, yield a factor of $(L^-)^n$ and cause the overall length enhancement of the process. Finally we obtain the leading components of the differential hadronic tensor for n scatterings both in the amplitude and complex conjugate as,

$$\begin{aligned}
\frac{dW_{nn}^{A\mu\nu}}{d^3l_q} &= \sum_q Q_q^2 \left(-g_\perp^{\mu\nu} + \frac{M^2}{(q^-)^2}\, g^{\mu -}g^{\nu -}\right) AC_p^A \int dy_0^- e^{-ix_B p^+ y_0^-}\langle p|\bar{\psi}(y_0)\frac{\gamma^+}{2}\psi(0)|p\rangle \\
&\times \frac{1}{n!}\prod_{i=1}^{n}\left(\int_0^{L^-} dY_i^- \int d\delta y_i^- \int d^3\delta y_i \int \frac{d^3p_i}{(2\pi)^3}\frac{\rho}{2p^+}g^2\frac{C_F}{N_c^2-1}e^{-i\vec{p}_i\cdot\delta\vec{y}_i}\langle p|A^+(\delta y_i)A^+(0)|p\rangle\right) \\
&\times \left(\prod_{i=1}^{n} e^{-i\Delta_i p^+\delta y_i^-}\right)\delta^3\left(\vec{l}_q - \vec{q} - \sum_{i=1}^{n}\vec{p}_i\right) \\
&= \left(W_{0\perp}^{A\mu\nu} + W_{0L}^{A\mu\nu}\right)\phi_n. \qquad (46)
\end{aligned}$$

In the equation above, $W_{0\perp}^{A\mu\nu}$ and $W_{0L}^{A\mu\nu}$ represent the leading order hadronic tensors without any rescattering of the produced heavy quarks. They are given as,

$$W_{0\perp}^{A\mu\nu} = \left(-g^{\mu\perp}g^{\nu\perp}\right)C_p^A \sum_q Q_q^2 \int dy_0^- e^{-ix_B p^+ y_0^-}\langle p|\bar{\psi}(y_0)\frac{\gamma^+}{2}\psi(0)|p\rangle, \quad (47)$$

and for the leading light-cone projection as,

$$W_{0L}^{A\mu\nu} = \left(g^{\mu -}g^{\nu -}\frac{M^2}{(q^-)^2}\right)C_p^A \sum_q Q_q^2 \int dy_0^- e^{-ix_B p^+ y_0^-}\langle p|\bar{\psi}(y_0)\frac{\gamma^+}{2}\psi(0)|p\rangle. \quad (48)$$

The factor ϕ_n in Eq. (46) represents the piece from n-scattering on the outgoing heavy quark in the final state. This contains both the "hard-part" which contains factors of the momentum of the heavy-quark, as well as the "soft-part" which contains phase factors and nucleon matrix elements.

A. Resummations

To carry out the resummation, we will assume the hadronic tensor is analytic around $\vec{p}_i = 0$. We will Taylor expand it around the soft exchanged momenta assuming the collinear/eikonal approximations,

$$H(q^-, p^+, p_0^-, p_i^\alpha) = \prod_{i=1}^{n}\left[(H)_{\vec{p}_1\cdots\vec{p}_n=0} + p_i^\alpha\left(\frac{\partial}{\partial p_i^\alpha}H\right)_{\vec{p}_1\cdots\vec{p}_n=0} + \frac{1}{2}p_i^\alpha p_i^\beta\left(\frac{\partial}{\partial p_i^\alpha}\frac{\partial}{\partial p_i^\beta}H\right)_{\vec{p}_1\cdots\vec{p}_n=0} + \cdots\right] \quad (49)$$

In the above expansion, the α, β represents both "$-$" and "$\perp$". Terms up to the second order have been retained for simplicity. The first terms (the term without any derivative) generally are gauge corrections for the diagrams with lower number of scatterings. After transforming the exchanged momentum into the appropriate derivatives over position, it is straightforward to perform the integrations over $\vec{p}_i$ and $\delta\vec{y}_i$ [18]. It will essentially result in spatial moments of the two gluon field products such as $\langle A^+(\delta y^-, \delta\vec{y}_\perp)\delta y^- A^+(0)\rangle$. The transverse projection of the differential hadronic tensor now reads as,

$$\frac{dW_{nn\perp}^{A\mu\nu}}{d^3l_q} = W_{0\perp}^{A\mu\nu}\frac{1}{n!}\left(\prod_{i=1}^{n}\int_0^{L^-}dY_i^-\left[-\mathcal{D}_{L1}\frac{\partial}{\partial p_i^-} + \frac{1}{2}\mathcal{D}_{L2}\frac{\partial^2}{\partial^2 p_i^-} + \frac{1}{2}\mathcal{D}_{D2}\nabla^2_{p_{i\perp}}\right]\right)\delta^3\left(\vec{l}_q - \vec{q} - \sum_{i=1}^{n}\vec{p}_i\right)_{\vec{p}_1\cdots\vec{p}_n=0}. \quad (50)$$

The hard sector transport coefficients $\mathcal{D}_{L1}$, $\mathcal{D}_{L2}$ and $\mathcal{D}_{T2}$ are defined as,,

$$\mathcal{D}_{L1} = g^2\frac{C_F}{N_c^2-1}\int dy^- \frac{\rho}{2p^+}\langle p|i\partial^- A^+(y^-)A^+(0)|p\rangle \left(\prod_{i=1}^{n} e^{-i\bar{\Delta}_i p^+\delta y_i^-}\right),$$
$$\mathcal{D}_{L2} = g^2\frac{C_F}{N_c^2-1}\int dy^- \frac{\rho}{2p^+}\langle p|\partial^- A^+(y^-)\partial^- A^+(0)|p\rangle \left(\prod_{i=1}^{n} e^{-i\bar{\Delta}_i p^+\delta y_i^-}\right),$$
$$\mathcal{D}_{T2} = g^2\frac{C_F}{N_c^2-1}\int dy^- \frac{\rho}{2p^+}\langle p|\partial_\perp A^+(y^-)\partial_\perp A^+(0)|p\rangle \left(\prod_{i=1}^{n} e^{-i\bar{\Delta}_i p^+\delta y_i^-}\right). \quad (51)$$

These three coefficients $\mathcal{D}_{L1}$, $\mathcal{D}_{L2}$ and $\mathcal{D}_{D2}$ are connected to longitudinal energy loss rate $\hat{e}$, longitudinal momenta diffusion rate $\hat{e}_2$ and transverse momenta diffusion rate $\hat{q}$. Its is also worth mentioning that, while for light quark $\bar{\Delta} \sim \bar{x}_D \sim \lambda^2$, for the 'semi hard' heavy quark $\bar{\Delta} \sim \bar{x}_D - x_B\bar{y}_i \sim \lambda^{\frac{3}{2}}$. As such, the hard sector transport coefficients of a heavy-quark sample somewhat higher values of momentum fraction x than light quark transport coefficients. This supports the notion that heavy quark and light quark transport coefficients need not yield the same numerical value. The above definitions of all the hard sector transport coefficients are not truly gauge-invariant. The manifestly gauge invariant hard sector transport coefficients can only be realised with the incorporation of higher order much softer terms where $k_\perp \ll \lambda Q$. The summation over such ultra soft gluon incorporation eventually leads to the emergence of Wilson links between the the gluon field operators. This will renders the operator product gauge invariant.

We have now resummed over an arbitrary number of multiple scatterings,

$$\frac{dW_\perp^{A\mu\nu}}{d^3l_q} = \sum_{n=0}^{\infty}\frac{dW_{n\perp}^{A\mu\nu}}{d^3l_q} = W_0^{A\mu\nu}\phi(L^-, l_q^-, \vec{l}_{q\perp}), \quad (52)$$

where the final state quark distribution function is defined as $\phi(L^-, l_q^-, \vec{l}_{q\perp})$,

$$\phi \sum_{n=0}^{\infty}\frac{1}{n!}\left(\prod_{i=1}^{n}\int_0^{L^-} dY_i^-\left[-\mathcal{D}_{L1}\frac{\partial}{\partial p_i^-} + \frac{1}{2}\mathcal{D}_{L2}\frac{\partial^2}{\partial^2 p_i^-} + \frac{1}{2}\mathcal{D}_{T2}\nabla^2_{p_{i\perp}}\right]\right)\delta(l_q^- - q^- - p_0^-)\,\delta^2(\vec{l}_{q\perp}). \quad (53)$$

The heavy quark momentum distribution function, after the full multi-scattering resummation reads,

$$\phi(L^-, l_q^-, \vec{l}_{q\perp}) = \exp\left(L^-\left[\mathcal{D}_{L1}\frac{\partial}{\partial l_q^-} + \frac{1}{2}\mathcal{D}_{L2}\frac{\partial^2}{\partial^2 l_q^-} + \frac{1}{2}\mathcal{D}_{T2}\nabla^2_{l_{q\perp}}\right]\right)\delta(l_q^- - q^- - p_0^-)\,\delta^2(\vec{l}_{q\perp}), \quad (54)$$

where the derivatives over p_i have been transformed to the derivatives over l_q.

The momentum distribution function $\phi(L^-, l_q^-, \vec{l}_{q\perp})$ for the final outgoing heavy quark is the unique solution of the following diffusion equation,

$$\frac{\partial\phi}{\partial L^-} = \left[\mathcal{D}_{L1}\frac{\partial}{\partial l_q^-} + \frac{1}{2}\mathcal{D}_{L2}\frac{\partial^2}{\partial^2 l_q^-} + \frac{1}{2}\mathcal{D}_{T2}\nabla^2_{l_{q\perp}}\right]\phi(L^-, l_q^-, \vec{l}_{q\perp}). \quad (55)$$

Above mentioned differential equation describes the time evolution of the momentum distribution profile of propagating heavy quark which suffers multiple soft scatterings in the passage of its transport through a nuclear matter. The three terms in the above diffusion equation represent the contributions from longitudinal momentum change and longitudinal momentum diffusion, and the transverse momentum diffusion. The delta function initial condition: $\phi(L^- = 0, l_q^-, \vec{l}_{q\perp}) = \delta(l_q^- - q^-)\delta^2(\vec{l}_{q\perp})$, provied the following solution for the distribution fuction ϕ,

$$\phi(L^-, l_q^-, \vec{l}_{q\perp}) = \frac{1}{\sqrt{2\pi\mathcal{D}_{L2}L^-}}\exp\left[-\frac{(l_q^- - q^- + \mathcal{D}_{L1}L^-)^2}{2\mathcal{D}_{L2}L^-}\right]\quad\frac{1}{2\pi\mathcal{D}_{T2}L^-}\exp\left[\frac{-l_{q\perp}^2}{2\mathcal{D}_{T2}L^-}\right]. \quad (56)$$

One may now identify,

$$\langle l_q^-\rangle = q^- - \mathcal{D}_{L1}L^-, \quad (57)$$
$$\langle (l_q^-)^2\rangle - \langle l_q^-\rangle^2 = \mathcal{D}_{L2}L^-,$$
$$\langle l_{q\perp}^2\rangle = 2\mathcal{D}_{T2}L^-. \quad (58)$$

The coefficients D_{L1}, D_{L2} and D_{T2} are related to longitudinal drag rate $\hat{e} = dE/dt$, longitudinal straggling rate $\hat{e}_2 = d(\Delta E)^2/dt$, and the transverse momentum diffusion rate $\hat{q} = d(\Delta p_T)^2/dt$ as

$$\hat{e} = \mathcal{D}_{L1},$$
$$\hat{e}_2 = \mathcal{D}_{L2}/\sqrt{2},$$
$$\hat{q} = 2\sqrt{2}\mathcal{D}_{T2}. \quad (59)$$

A similar set of arguments may be used to simplify the (++)-projection of the hadronic tensor. At this order of approximation, we obtain the longitudinal projection of the hadronic tensor from an arbitrary number of scatterings as,

$$\frac{dW_L^{A\mu\nu}}{d^3 l_q} = \sum_{n=0}^{\infty} \frac{dW_{nL}^{A\mu\nu}}{d^3 l_q} = W_{0L}^{A\mu\nu}\phi(L^-, l_q^-, \vec{l}_{q\perp}). \quad (60)$$

V. CONCLUSIONS

The subject of heavy quark energy loss is not yet settled and requires more detailed analysis [21, 22]. In this work, the propagation of a single 'semi-hard' heavy quark in a dense nuclear medium has been studied. The formalism is an extension of the higher twist framework that includes medium induced multiple scatterings. In this formalism the higher twist corrections are magnified by the large extent of the nucleus. They are then resummed to obtain the temporal evolution for the momentum distribution of the probe. Both transverse broadening as well as the longitudinal drag and longitudinal diffusion, have been studied simultaneously.

We have focussed on the specific case of "semi-hard" quarks where the mass and momentum scale as $M, p \sim \sqrt{\lambda}Q$. In this work we have applied SCET-Glauber scaling based momentum power counting whenever required. It shows that the longitudinal momentum transfers and the transverse momentum transfers have a comparable effect on the off-shellness of the heavy-quark. This implies that longitudinal transfers, not only lead to the drag and diffusion, similar to light flavors, but will also noticeably affect the radiative loss. The calculation of this novel effect, will be carried out in a future effort. Here we focussed on whether the drag and diffusion experienced by heavy-flavors can be cast in the same form as for light flavors; this is indeed the case.

An evolution equation for the temporal development of the heavy-quark momentum distribution have been derived. All three leading transport coefficients involved are connected to the longitudinal drag, longitudinal straggling and transverse momentum diffusion coefficient. The general structure of the transport coefficients for the semi-hard heavy quarks appear to be similar to those for light quarks (or even those for a fast heavy-quark). However, a closer analysis indicates that semi-hard heavy quarks, scattering off the nucleon, sample a larger value of x than do light quarks or gluons. As such, the values of the transport coefficients for such partons may not be the same as those used for light flavors. As a corollary, the energy loss of semi-hard heavy-quarks yield a direct window into the x dependence of jet transport coefficients. The combined effect of all the three hard sector transport coefficients on the gluon bremsstrahlung spectrum off the heavy quark will be explored in a future effort.

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