Connection of g-2_{μ}, electroweak, dark matter, and collider constraints on 331 models

Chris Kelso, H. N. Long, R. Martinez, and Farinaldo S. Queiroz

Phys. Rev. D **90**, 113011 — Published 17 December 2014

DOI: 10.1103/PhysRevD.90.113011

On the Connection of $g-2_\mu$, Electroweak, Dark Matter and Collider Constraints on 331 Models

Chris Kelso[a,b,*] H.N. Long[c,†] R. Martinez[d,‡] and Farinaldo S. Queiroz[e,f§]
[a] Department of Physics and Astronomy - University of Utah - Salt Lake City, UT 84112
[b] Physics Department - University of North Florida - Jacksonville, FL 32224
[c] Institute of Physics VAST -10 Dao Tan - Ba Dinh, Hanoi, Vietnam
[d] Departamento de Fisica - Universidad Nacional de Colombia, Bogota D.C
[e] Department of Physics - University of California Santa Cruz - Santa Cruz, CA 95064
[f] Santa Cruz Institute for Particle Physics,
Santa Cruz, CA 95064

In this work we compute all contributions to the muon magnetic moment stemming from several 3-3-1 models namely, minimal 331, 331 with right handed neutrinos, 331 with heavy neutral leptons, 331 with charged exotic leptons, 331 economical and 331 with two higgs triplets. Further, we exploit the complementarity among current electroweak, dark matter and collider constraints to outline the relevant parameter space of the models capable of explaining the anomaly. Lastly, assuming that the experimental anomaly has been otherwise resolved, we derive robust 1σ bounds using the current and projected measurements.

PACS numbers: 95.35.+d, 14.60.Pq, 98.80.Cq, 12.60.Fr

I. INTRODUCTION

The muon magnetic moment ($g-2_\mu$) is one of the most precisely measured quantities in particle physics. Somewhat recently in Brookhaven, $g-2_\mu$ has been measured with great a precision reaching the level of 0.54 ppm. Since the first results were reported, a long standing discrepancy between theory and experiment of about 3.6σ has been observed, providing a hint that new physics may be around the corner. This deviation triggered a multitude of speculations about the possible origin of this mild excess (for recent reviews see Refs.[1, 2]). However, there are large theoretical uncertainties that blur the significance of this discrepancy. These uncertainties are dominated by the hadronic vacuum polarization and the hadronic contribution to the light-by-light scattering. Significant effort has been put forth to try to reduce these uncertainties [3–5]. The current deviation is $\Delta a_\mu = 295 \pm 81 \times 10^{-11}$. Out of this $\pm 81 \times 10^{-11}$ error, $\pm 51 \times 10^{-11}$ is theoretical, which is dominated by uncertainty in the lowest-order hadronic contribution ($\pm 39 \times 10^{-11}$) and in the hadronic light-by-light contribution ($\pm 26 \times 10^{-11}$) [6].

In the near future important improvements in both the theoretical and experimental situations are expected. Combining the expected progress from the theoretical side, along with the projected experimental sensitivity for the g-2 experiment at Fermilab, the precision will likely reach $\Delta a_\mu = 295 \pm 34 \times 10^{-11}$, possibly increasing the magnitude of the signal up to 5σ [6]. Hence, it is worthwhile to explore the complementarity among $g-2_\mu$, electroweak, dark matter and collider constraints in particle physics models.

In this work, we will focus our effort on electroweak extensions of the standard model known as 331 models. In these models the $SU(2)_L$ gauge group is extended to $SU(3)_L$. The motivations for considering such class of models, among others [19–21], relies on the following:
(i) They explain the number of generations: the 331 gauge symmetry in combination with QCD asymptotic freedom lead to the generation number to be three;
(ii) They have plausible dark matter candidates [7–9, 11–16];
(iii) They can accommodate the dark radiation component observed by Planck through non-thermal DM production [17]
(iv) They are generally consistent with current electroweak and collider data as we discuss further;
(v) The Peccei-Quinn symmetry, necessary to solve the strong-CP problem, follows naturally from the particle content in these models [18].

Our goal is to assess which 331 models are consistent with the current electroweak, collider and dark matter limits while being able to explain the $g-2_\mu$ anomaly and derive 1σ bounds on the particle spectrum. Previous studies have been performed in the past discussing the $g-2_\mu$ in 331 models [25, 26]. Those studies were limited to one particular model, such as the minimal 331 model and 331 model with right handed neutrinos without taking into account important electroweak, collider and dark matter constraints. In this work we will extend those studies by investigating the $g-2_\mu$ in six 331 models namely: the minimal 331 model [22], 331 model with right handed neutrinos (331 r.h.n for short) [27], 331 model with heavy neutral leptons (331LHN) [7], 331 Economical [29], 331 Minimal with two higgs triplets

* ckelso@unf.edu
† hnlong@iop.vast.ac.vn
‡ remartinezm@unal.edu.co
§ fdasilva@ucsc.edu

(RM331 for short), and the 331 with charged exotic leptons [30, 31], properly accounting for these constraints. Additionally, we derive 1σ limits based on the current and projected sensitivity for $g-2_\mu$assuming the anomaly has been otherwise resolved.

In summary our main findings are:

- *331 Minimal:* This model cannot explain $g-2_\mu$anomaly. We find a robust limit on the scale of symmetry breaking (v_χ) of 4 TeV, which can be translated into $M_{Z'} > 2.4$ TeV. As far as we know this is the strongest bound on the Z' mass in the literature. Moreover, we show that the upcoming g-2 experiment at Fermilab might be able to fiercely exclude this model.

- *331 r.h.n:* It cannot explain $g-2_\mu$ excess because it requires a rather small scale of symmetry breaking already ruled out by current collider, dark matter experiments and electroweak precision data.

- *331 LHN:*

  For heavy neutrino masses of $M_N = 1$ GeV, $v_\chi <$ 1 TeV is needed to address $g-2_\mu$. Nevertheless, current bounds prohibit this possibility. Hence, the 331LHN is excluded as a potential framework. Because the overall contribution is quite small, a projected 1σ limit of $v_\chi \gtrsim 1.5$ TeV is somewhat irrelevant compared to the current direct dark matter detection ones [9, 11]. The regime in which the heavy neutrino masses are either larger or smaller do not change our conclusions.

- *331 Economical:* Due to the large cancelation between the W' and Z' corrections the total contribution is small and requires $v_\chi < 1$ TeV to accommodate the $g-2_\mu$ excess. Such a low scale of symmetry breaking is prohibited by current data, however. In conclusion, the Economical 331 model cannot reproduce the $g-2_\mu$ reported, no meaningful current limit can be derived, but a projected one of 1.4 TeV is found.

- *RM331:* We observe that a scale of symmetry breaking of ~ 2 TeV could explain the $g-2_\mu$ excess, while being consistent with existing limits. Furthermore, a current limit of 4 TeV and projected limit of 6 TeV can be placed on the scale of symmetry breaking. Since this model, similar to the 331 minimal model, is valid up to only 5 TeV, the RM331 may be ruled out in the near future.

- *331 Exotic Leptons:*

  Regardless how massive the exotic leptons are, the total contribution to $g-2_\mu$ is negative and small. Therefore, no relevant constraint could be derived.

We have given a brief introduction to the current status of the muon anomalous magnetic moment and summarized our main findings. We now turn our attention to the corrections to $g-2_\mu$ stemming from the most popular 331 models. Throughout this work, our reasoning focuses on the leptonic sectors of such models as they are the most relevant for the $g-2_\mu$ anomaly. We also properly account for the existing electroweak and collider bounds on the other particles of the models. We provide master integrals and analytical expressions for all contributions to $g-2_\mu$ discussed in this work in the Appendix.

II. MINIMAL 331 MODEL

A. Content

The leptonic content of the minimal 331 model is comprised of three lepton triplets as follows,

$$f_L^a = (\nu^a,\ l^a,\ (l^c)^a)_L^T \sim (1,3,0), \tag{1}$$

where a runs through the three family generations. Since 331 stands for an enlarged electroweak gauge symmetry, 5 new gauge bosons are added to the SM namely, $W'^{\pm}$, $U^{\pm\pm}$ and Z'. Both $W'^{\pm}$ and $U^{\pm\pm}$ carry two units of lepton number, hence called bileptons, which interact with the SM leptons as follows [22],

$$\mathcal{L}_l^{CC} \supset -\frac{g}{2\sqrt{2}}\left[\bar{\nu}\gamma^\mu(1-\gamma_5)C\bar{l}^T W_\mu'^{-} - \bar{l}\gamma^\mu\gamma_5 C\bar{l}^T U_\mu^{--} + h.c\right], \tag{2}$$

with the respective masses,

$$M_{W'}^2 = \frac{g^2}{4}\left(v_\eta^2 + v_\chi^2 + v_\sigma^2\right), M_U^2 = \frac{g^2}{4}\left(v_\rho^2 + v_\chi^2 + 4v_\sigma^2\right), \tag{3}$$

where v_ρ, v_η, v_χ and v_σ are the vev's of the neutral scalars presented in the Eq.(6) below.

Notice that the vector current for $U^{\pm\pm}$ vanishes due to Fermi statistics. One can clearly see that both charged bosons generate contributions to $g-2_\mu$ through Fig.1(e) and Figs.1(f)-1(g), respectively. Regarding the neutral gauge boson Z', which mixes with the SM Z, we find the gauge interactions [22],

$$\mathcal{L}^{NC} \supset \bar{f}\,\gamma^\mu[g_V(f) + g_A(f)\gamma_5]\,f\,Z'_\mu. \tag{4}$$

with,

$$g_V(\mu) = \frac{g}{c_W}\frac{3\sqrt{1-4s_W^2}}{2\sqrt{3}},\ g_A(\mu) = \frac{g}{c_W}\frac{\sqrt{1-4s_W^2}}{2\sqrt{3}},$$
$$M_{Z'}^2 = \left(\frac{g^2+3g'^2}{3}\right)v_\chi^2, \tag{5}$$

where $g' = g\tan_W$. Electroweak measurements constraint this mixing angle to be quite small [28]. Note the Z' boson also contributes to the muon anomalous magnetic moment through the diagram shown in Fig.1(h). As we will see later, the magnitude of the vector and axial couplings lead to the Z' correction to $g-2_\mu$ that is

positive in this model. The Z' contribution to $g-2_\mu$ in general is proportional to $g_V^2-5g_A^2$ where g_V and g_A are the vector and axial couplings. Therefore, depending on the hypercharges assigned for the leptonic triplets, which determine g_V and g_A, the magnitude and possibly the overall sign of the Z' contribution to $g-2_\mu$ can change (see Appendix for details).

As for the scalar sector, 331 models usually advocate the presence of three scalar triplets and one sextet in order to generate masses for all fermions. In the case of the minimal 331 model those read,

$$\begin{aligned}\eta &= \left(\eta^0,\ \eta_1^+,\ \eta_2^+\right)^T,\\ \rho &= \left(\rho^+,\ \rho^0,\ \rho^{++}\right)^T,\\ \chi &= \left(\chi^-,\ \chi^{--},\ \chi^0\right)^T,\\ S &= \begin{pmatrix}\sigma_1^0 & h_2^- & h_1^+\\ h_2^- & H_1^{--} & \sigma_2^0\\ h_1^+ & \sigma_2^0 & H_1^{++}\end{pmatrix},\end{aligned} \tag{6}$$

where η^0, ρ^0, χ^0 and σ_1^0 acquire a vev v_η, v_ρ, v_χ and v_σ respectively.

The important interactions for muon magnetic moment are [23]:

$$\mathcal{L} \supset G_l\ \left[\overline{l_R}\,\nu_L\eta_1^- + \overline{l_R^c}\,\nu_L h_1^+ + \overline{l_R}\,\nu_L h_2^+ + \overline{l_R} l_L R_{\sigma_2}\right] + h.c \tag{7}$$

with [24],

$$\begin{aligned}M^2_{\eta_1^+} &= \frac{f}{\sqrt{2}}v_\chi\left(\frac{v_\rho}{v_\eta}+\frac{v_\eta}{v_\rho}\right),\\ M_{h_1^+,h_2^+} &\sim v_\chi,\\ M_{\sigma_0'} &\sim v_\chi,\end{aligned} \tag{8}$$

where $G_l = m_l\sqrt{2}/v_\eta$, with v_η being the vev of η^0, f the trilinear coupling in the scalar potential [22] and R_{σ_2} the real component field of σ_2^0 [24]. After the spontaneous symmetry breaking we find $v_\eta^2+v_\rho^2+v_\sigma^2=v^2$ where v is the SM vev. Typically v_σ, which gives rise to neutrino masses, is taken to be small, whereas v_η is assumed to be equal to v_ρ, but v_η is free to vary obeying this restriction, and vice-versa. This is important because for small values of v_η the charged scalar contribution will not be suppressed as has been previously assumed [25]. The feynmann diagrams which give rise to correction to $g-2_\mu$from these scalars are depicted in Figs.1(a)-1(b).

We have shown the relevant interactions to $g-2_\mu$ thus far, further we discuss the existing constraints on the minimal 331 model.

B. Existing Bounds

Since this model does not have a dark matter candidate, the important bounds arise from electroweak and collider data only. The muon decay $\mu \to e\nu_e\bar{\nu_\mu}$ implies $M_{W'} > 230$ GeV [32]. Measurements of flavor changing neutral current (FCNC) in meson oscillations produce a lower bound of $M_{Z'} \gtrsim 1-2$ TeV. The variation in the bound comes from the texture parametrization used in the quark mixing matrices. Additionally, electroweak bounds coming from the rare decays $B_{s,d} \to \mu^+\mu^-$ and $B_d \to K^\star(K)\mu^+\mu^-$ impose $M_{Z'} \gtrsim 1$ TeV masses [34]. Lastly, CMS Collaboration has performed Z' searches. Since no excess has been observed, a bound of 2.2 TeV has been found on Z' mass [33]. This limit can be translated into a limit on the scale of symmetry breaking of the model of $v_\chi > 3.6$ TeV. We will incorporate these bounds in our results in the next section.

III. 331 MODEL WITH RIGHT HANDED NEUTRINOS

The so called 331 model with right handed neutrinos (331 r.h.n for short) is motivated by neutrino masses. Here the neutrino masses can be easily addressed. This model is an extension of the minimal 331 model in which the third component in the leptonic triplet for a right handed neutrino is replaced as follows [27],

$$f_L^a = (\nu^a, l^a, (\nu^c)^a)_L^T \sim (1,3,-1/3), l_R^a \sim (1,1,-1). \tag{9}$$

As before five new gauge bosons are added to the SM namely, $W', X^0, X^{0\dagger}$ and Z'. Because in this model the third component in the leptonic triplet is a neutral particle, this model does not feature doubly charged bosons, it has instead neutral bosons $X^0, X^{0\dagger}$. The spontaneous symmetry breaking (SSB) induces $|M^2_{W'} - M^2_X| \leq M^2_W$ [27], and a $Z-Z'$ mixing. Since the $Z-Z'$ mixture is bounded to be very small we might consider Z and Z' as mass eigenstates. In this regime the vector and axial couplings using the notation of Eq.(4) are found to be,

$$g_V'(\mu) = \frac{g}{4c_W}\frac{(1-4s_W^2)}{\sqrt{3-4s_W^2}},\ g_A'(\mu) = -\frac{g}{4c_W\sqrt{3-4s_W^2}}, \tag{10}$$

with

$$M^2_{Z'} = \frac{g^2}{4(3-4s_w^2)}\left(4v_\chi^2 + \frac{v_\rho^2}{c_w^2} + \frac{v_\eta^2(1-2s_w^2)}{c_w^2}\right). \tag{11}$$

The Z' boson contribution to $g-2_\mu$ appears in the form of Fig.1(h). The bilepton X^0 does not contribute to $g-2_\mu$, but similar to the Minimal 331 model the singly charged boson does, through the interaction (Fig.1(e)),

$$\mathcal{L} \supset -\frac{g}{2\sqrt{2}}\left[\overline{\nu_R^c}\,\gamma^\mu(1-\gamma_5)\bar{l}\,W_\mu'^-\right]. \tag{12}$$

The mass term of the singly charged gauge boson is similar to the previous model according to Eq.(3). The

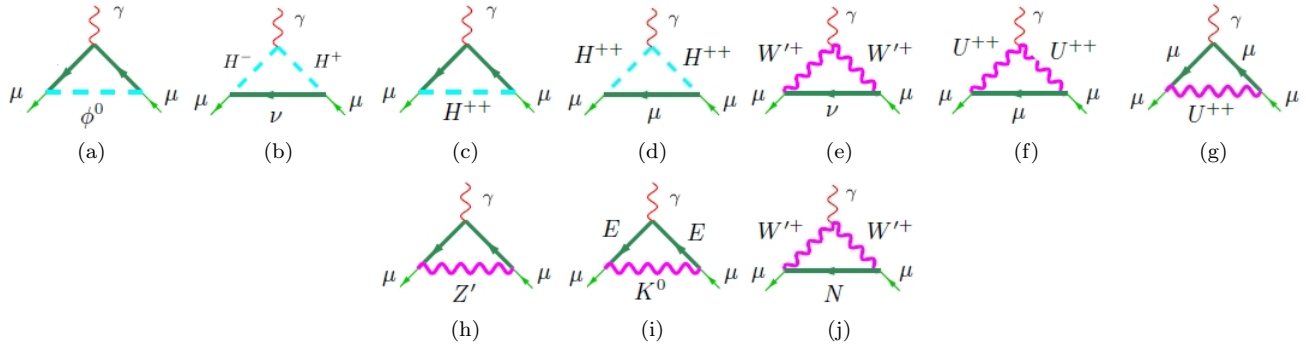

FIG. 1. Feynmann diagrams arising in 331 models studied here.

scalar sector in the 331 r.h.n is different though and it is now comprised of the following three scalar triplets,

$$\eta = \left(\eta^0,\ \eta^-,\ \eta^{0\prime}\right)^T,$$
$$\rho = \left(\rho^+,\ \rho^0,\ \rho^{\prime+}\right)^T,$$
$$\chi = \left(\chi^0,\ \chi^-,\ \chi^{\prime 0}\right)^T. \tag{13}$$

Due to a different scalar content this model has contributions to $g-2_\mu$ stemming from three scalars. Two coming from singly charged ones, with an interaction similar to Eq.(7) represented in Fig.1(b). In addition, there is a correction coming from a neutral scalar S_2, which is a combination of the real component of the ρ^0 and η^0 fields, exhibited in Fig.1(a) through the interaction,

$$\mathcal{L} \supset G_s \bar{\mu}\,\mu S_2, \tag{14}$$

where $G_s = m_\mu\sqrt{2}/(2v_\rho)$, and

With those results we have gathered all information needed for the $g-2_\mu$. We emphasize that the key differences between the 331 r.h.n model and the minimal 331 are the absence of the doubly charged and the presence of additional charged and neutral scalar contributions. Before presenting our main findings for this particular model we discuss further the existing bounds.

A. Existing Bounds

The non-observation of an excess in the dilepton search from CMS experiment has resulted in a 2.4 TeV lower bound on the Z' mass [33]. This limit can be translated into a lower bound on the scale of symmetry breaking of 7.5 TeV using Eq.(11). Electroweak data from the decays $B_{s,d} \to \mu^+\mu^-$ and $B_d \to K^\star(K)\mu^+\mu^-$ exclude Z' masses up to $\sim 1-3$ TeV [34]. Additionally, direct dark matter detection bounds coming from the underground detector LUX, have been applied to the 331 model with right-handed neutrinos to exclude a scale of symmetry breaking lower than 10 TeV [11], implying $M_{Z'} \gtrsim 4$ TeV. The latter is valid under the assumption that the complex scalar ϕ which is $\sim \eta^{0\prime}$ is a viable DM candidate. With those stringent constraints in mind we show our results concerning $g-2_\mu$.

IV. 331 MODEL WITH HEAVY NEUTRAL LEPTON

The 331 model with heavy leptons (331LHN for short) is a compelling extension of the SM because it can obey the electroweak constraints and has two viable dark matter candidates, a complex scalar and a fermion [7] in the context of the Higgs [35] and Z' portals [36] respectively. Besides, it offers a possible explanation to the dark radiation favored by current data through a sub-dominant non-thermal production of dark matter [17, 37]. Here the third component in the leptonic triplet is a heavy neutral lepton as follows,

$$f_L^a = (\nu^a, l^a, N^a)_L^T \sim (1,3,-1/3), l_R^a \sim (1,1,-1). \tag{15}$$

This model is nearly identical to the 331 model with right handed neutrinos as far the muon anomalous magnetic moment is concerned. The key differences rise from the presence of the heavy leptons represented in Fig.1(j) through the interactions,

$$\mathcal{L} \supset -\frac{g}{\sqrt{2}}\left[\overline{N_L}\,\gamma^\mu \bar{l}\,W_\mu'^-\right] - G_l\,\overline{l_R}\,N_L\eta_1^- \tag{16}$$

Notice the presence of the heavy lepton instead of the light neutrino in the previous model. In summary, the interactions that contribute to the muon anomalous magnetic moment in the 331 model with heavy leptons stem from the singly charged gauge boson Eq.(16), the Z' with the vector and axial couplings of Eq.(10), the neutral scalar S_2 via Eq.(14), singly charged h_1 through Eq.(16) and the second singly charged scalar h_2 via Eq.(7). Because of the heavy lepton, the singly charged scalars give rise to different corrections to $g-2_\mu$ as we shall see below.

A. Existing Bounds

The existing bounds on this model are very similar to the 331 r.h.n. CMS dilepton searches resulted in the lower bound $M_{Z'} \gtrsim 4$ TeV [33] in the regime where Z' boson cannot decay into heavy lepton pairs. This bound demands a scale of symmetry breaking larger than 7.5 TeV. Electroweak bounds coming from the $B_{s,d} \to \mu^+\mu^-$ and $B_d \to K^\star(K)\mu^+\mu^-$ rule out Z' masses up to ~ 3 TeV [34]. Because this model has two non-coexistent dark matter candidates, a complex scalar ($\eta^{0\prime}$) and the lightest heavy lepton (N_1 for instance), the dark matter bounds change depending on which particle is the lightest. For the scenario where the scalar is the lightest, direct dark matter detection excludes a scale of symmetry breaking lower than 10 TeV, implying $M_{Z'} \gtrsim 4$ TeV [11]. For the regime where the fermion is the DM candidate, it has been found $v_{\chi'} > 5$ TeV, i.e $M_{Z'} \gtrsim 2$ TeV [9].

V. ECONOMICAL 331 MODEL

This model refers to the 331 extension which uses the leptonic triplet of the 331 with handed neutrinos, but instead of having three scalar triplets it has only two namely [29],

$$\begin{aligned}\phi &= \left(\eta_1^+,\ \eta_2^0,\ \eta_3^+\right)^T,\\ \chi &= \left(\chi_1^0,\ \chi_2^-,\ \chi_3^0\right)^T.\end{aligned} \tag{17}$$

Here we will adopt the notation: $v_{\eta_2^0} = v\sqrt{2}, v_{\chi_1} = u/\sqrt{2}$ and $v_{\chi_3^0} = v_\chi/\sqrt{2}$, where v is the SM vev.

This model possesses a more simple scalar sector compared to the 331 with right handed neutrinos. As a result of this simplicity, corrections to $g-2_\mu$ arise from neutral and charged scalars (See Eq.(15) of [29]) similar to Eq.(14) and Eq.(7) but with the following masses after replacing v_η by v,,

$$M^2_{\eta_1^+} = \frac{\lambda_4}{2}\left(u^2+v^2+v_\chi^2\right),\ M^2_{S_2} = 2\lambda_1 v_\chi^2 \tag{18}$$

The diagrams that contribute to $g-2_\mu$rising from these scalars are exhibited in Fig.1(a)-1(b). Because the 331 Economical model has the same leptonic triplet of the 331 r.h.n , i.e, the same hypercharge configuration, the vector and axial Z' couplings are the same, but the Z' mass term turns out to be different as result of the different scalar content as follows,

$$M^2_{Z'} = \frac{g^2 c_w^2 v_\chi^2}{3-4s_w^2}. \tag{19}$$

As for the singly charged boson interaction, it is identical to the 331 r.h.n model and given in Eq.(7). In summary, the relevant contributions to $g-2_\mu$ from this model come from the charged and neutral gauge bosons.

A. Existing bounds

Data from the $B_{s,d} \to \mu^+\mu^-$ and $B_d \to K^\star(K)\mu^+\mu^-$ decays exclude Z' masses up to $\sim 1-3$ TeV depending on the parametrization in the quark mixing matrices [34]. Dilepton searches performed by CMS resulted in the lower bound $M_{Z'} \gtrsim 4$ TeV , which implies $v_\chi > 7.5$ TeV [33].

VI. MINIMAL 331 MODEL WITH TWO HIGGS TRIPLETS

The minimal 331 model with two Higgs triplets, RM331 for short, was mostly motivated by minimality due to the shortened scalar sector. This model does not have a dark matter candidate, nor does it explain the fermion masses with renormalizable Lagrangians [39]. The scalar sector of this model is comprised of two scalar triplets only namely,

$$\begin{aligned}\rho &= \left(\rho^+,\ \rho^0,\ \rho^{++}\right)^T,\\ \chi &= \left(\chi^-,\ \chi^{--},\ \chi^0\right)^T.\end{aligned} \tag{20}$$

As a result, contributions from doubly charged (Figs.1(c)-1(d)) and neutral scalars (Figs.1(a)) arise through the Lagrangian,

$$\mathcal{L} \supset \frac{m_\mu}{v_\chi}\bar{l}\, l\, S_2 + \sqrt{2}\frac{m_\mu}{v_\rho}\bar{l^c}\, l_L\, H^{--}, \tag{21}$$

with,

$$M^2_{S_2} = \lambda_2 v_\chi^2 + \frac{\lambda_3^2 v_\rho^2}{4\lambda_2},\ M^2_{H^{\pm\pm}} = \frac{\lambda_4}{2}\left(v_\chi^2+v_\rho^2\right). \tag{22}$$

This model possesses a similar hypercharge configuration, vector and axial couplings to the minimal 331 model. This means that the Z'-lepton interactions are equivalent to those described in Eq.(5), but with,

$$M_{Z'} = \frac{g^2 c_w^2 v_\chi^2}{3(1-4s_w^2)}. \tag{23}$$

The singly charged $V^\pm$ (Fig.1(e)) and doubly $U^{\pm\pm}$ (Figs.1(f)-1(g)) charged vector bosons contributions are precisely the same of the minimal 331 model, Eq.(2), changing the mass terms only accordingly,

$$M_V^2 = \frac{g v_\chi^2}{4},\ M_U^2 = \frac{g^2}{4}\left(v_\rho^2+v_\chi^2\right). \tag{24}$$

Thus, in summary, the corrections to $g-2_\mu$ stemming from this model are: neutral scalar, doubly charged scalar, doubly charged vector boson, singly charged vector boson and neutral boson.

A. Existing Limits

Currently limits based on Drell Yann production of doubly charged scalar exclude doubly charged scalars up to $\sim$ 400 GeV [38]. The muon decay $\mu \to e\nu_e\bar{\nu_\mu}$ implies $M_{W'} > 230$ GeV [32]. Flavor changing neutral current processes arising from the RM331 model are sizeable and therefore stringent constraints have been found: $v_\chi \gtrsim 1-2.7$ TeV, depending on the texture parametrization used [40]. Moreover, data from the $B_{s,d} \to \mu^+\mu^-$ and $B_d \to K^\star(K)\mu^+\mu^-$ decays rule out Z' masses up to $1-2$ TeV range [34]. In addition, the CMS Collaboration has performed Z' searches, since no excess has been observed a lower bound $M_{Z'} \gtrsim 2.2$ TeV was derived [33]. This limit can be translated into a limit on the scale of symmetry breaking of the model namely $v_\chi > 1850$ TeV.

VII. 331 MODEL WITH EXOTIC LEPTONS

A special feature of the 331 models discussed previously is the fact that one quark generation transforms in a different representation of $SU(3)_L$ compared to others, in order to satisfy the chiral anomaly cancelation condition. As a result, the Z'-quark interactions are not universal, giving rise to flavor changing neutral current processes at tree level [40]. Different 331 models can be built, in particular some are comprised of five left-handed leptonic triplets in different representations of the $SU(3)_L$ gauge group [41]. Within this context the Z'-lepton interactions are not universal and flavor changing processes can arise. We investigate these models in this section.

The leptonic sector is comprised of the following triplets,

$$\begin{aligned} f_{1L} &= \left(\nu_1, l_1^-, E_1^-\right)_L^T \sim (1,3,-2/3),\ l_1^c \sim (1,1,1) \\ f_{2,3L} &= \left(l_{2,3}^-, \nu_{2,3}, N_{2,3}\right)_L^T \sim (1,3^\star,-1/3),\ l_{2,3}^c \sim (1,1,1) \\ f_{4L} &= \left(E_2^-, N_3, N_4\right)_L^T \sim (1,3^\star,-1/3),\ E_2^c \sim (1,1,1) \\ f_{5L} &= \left(N_5, E_2^+, l_3^+\right)_L^T \sim (1,3^\star,3/3),\ E_2^c \sim (1,1,1) \end{aligned} \tag{25}$$

Using the same notation of Ref.[42], the relevant interactions for the $g-2_\mu$ are,

$$\begin{aligned} \mathcal{L} \supset\ & \frac{g'}{2\sqrt{3}s_w c_w}\bar{\mu}\gamma_\mu\left(g_V + g_A\right)\mu\, Z' \\ & -\frac{g}{\sqrt{2}}\left(\overline{N_{1L}}\,\gamma_\mu\mu_L + \bar{\mu}_L\gamma_\mu N_{4L}\right)K_\mu^+ \\ & -\frac{g}{\sqrt{2}}\left(\bar{\mu}_L\gamma_\mu E_L\right)K_\mu^0 \\ & h_1\bar{\mu}(1-\gamma_5)N\phi^+ + h_2\bar{\mu}E^-\phi^0 + h_3\bar{\mu}E_2^-\phi^0 + h.c., \end{aligned} \tag{26}$$

with,

$$\begin{aligned} g_V = \frac{-c_{2w}+2s_w^2}{2},\ g_A = \frac{c_{2w}+2s_w^2}{2}, \\ M_{Z'} = \frac{2}{9}\left(3g^2+g'^2\right)v_\chi^2, \\ M_{K^+}^2 = M_{K^0}^2 = \frac{g^2}{4}\left(2v_\chi^2+v^2\right), \\ g' = \frac{g\tan_w}{\sqrt{1-\tan_w^2/3}}, \end{aligned} \tag{27}$$

where K^+, K^0 and Z' are the gauge bosons of the model, v_χ sets the scale of the $SU(3)_L$ symmetry breaking, v is the SM vev, and ϕ^+, ϕ^0 are the heavy charged and neutral scalars evoked in the scalar triplets [43]. We have seen that corrections rising from scalar particles are suppressed by the lepton masses, so we will ignore them here. Hence the main contributions come from the gauge bosons Z' (see Fig.1(h)), K^+ (see Fig.1(e)) and K^0 (check Fig.1(i)) .

A. Existing Constraints

A study of rare decay data, $\tau \to lll, \tau \to e\gamma, \mu \to e\gamma$ and $\mu \to eee$, has placed limits on the Z' mass which range from 800 GeV-4 TeV, depending on the value of the mixing angles in the leptonic sector. This implies a lower bound $v_\chi \gtrsim 1.5-7.1$ TeV. Constraints coming from pair production of charged exotic leptons (E) at the LHC imply $M_E > 405$ GeV [44]. This can be effectively translated into a limit on the scale of symmetry breaking of the model, since it depends on the product of Yukawa coupling and v_χ and the Yukawa coupling can be arbitrarily small.

VIII. RESULTS

In this section we present our results taking into account all corrections to the muon magnetic moment stemming from all 331 models previously discussed keeping mind the current constraints. We emphasize that master integrals for computing all these contributions are given in the Appendix. In general contributions coming from scalar particles are suppressed, contributions from neutral gauge bosons can be sizeable with positive or negative sign depending on the relative magnitude of the vector and vector-axial couplings, and doubly charged gauge bosons corrections are large. We presented our numerical results in Figs.2-7. There the solid (dashed) horizontal green lines delimit the current and projected sensitive of $g-2_\mu$ experiments and the region of parameter space which a given model accommodate the reported muon magnetic moment. Below those, the solid (dashed) red lines represent the current (projected) 1σ bounds that might be placed on the models in case the muon magnetic moment is otherwise resolved. Hereunder we dis-

cuss the results shown in Figs.2-7 for the six 331 models. Our limits are summarized in Table I.

A. 331 Minimal

In Fig.2 for $v_\eta = 174$ GeV, we show the numerical results for the individual contributions to $g-2_\mu$Different values of v_η produce the same conclusions since the value of the vev only alters the contribution from the scalars which is negligible. We have not included the neutral scalar contribution because it is also suppressed by the muon mass squared. We display the corrections in terms of the scale of symmetry breaking because the particles masses have different dependencies with the scale of symmetry breaking. Therefore, if one plots the results as a function of the masses, the conclusions would be misleading, since the contributions in terms of the particle masses are not on equal footing.

The solid (dashed) green horizontal lines represent the current (projected) sensitivities to $g-2_\mu$. The solid (dashed) horizontal red lines are the current and projected 1σ bound in the case the anomaly is not resolved by any means but this model. The charged scalar correction, which is negative, has been multiplied by (-10^6) to be shown in the graph. We conclude that the doubly and singly charged vector bosons contributions are the most relevant ones and that for $v_\chi = 2$ TeV the model in principle could account for the excess. Albeit, the current LHC bounds rule out $v_\chi < 3.6$ TeV, and thus this model can be decisively excluded as an explanation of the $g-2_\mu$ anomaly.

Moreover, stringent bounds can be derived after summing up all contributions. We find that if the anomaly persists, a current (projected) 1σ limit of 4 TeV (5.8 TeV) can be placed on the scale of symmetry breaking of the model. Since this model is valid only up to 5 TeV or so, we conclude that the upcoming g-2 experiment at Fermilab will be able to undoubtedly exclude this model. We emphasize that this conclusion is irrefutable. No fine-tuning or different parameter choices can remedy this because the main contributions come from gauge bosons, whose interactions are determined by the gauge group.

B. 331 r.h.n

In Fig.3 we exhibit the individual contributions to $g-2_\mu$ coming from the 331 r.h.n model as a function of the scale of symmetry breaking. We see that the singly charged (W') and neutral (Z') gauge bosons corrections are the leading ones. We have multiplied the neutral scalar and charged scalar contributions by (10^6) and -1 respectively to depict them in the graph. Differently from the minimal 331 model, the Z' now gives a negative correction to $g-2_\mu$ due to the magnitude of the vector and axial couplings as explained previously. The Z' contribution has been multiplied by minus one to show it in the plot. We can already notice from the plot that a rather small scale of symmetry breaking is needed to explain the $g-2_\mu$ and combining all individual corrections we an overall current (projected) limit of 1 TeV (1.5) TeV in the scale of symmetry breaking. Although, current collider, dark matter experiments and electroweak precision data firmly exclude scale smaller than 7.5 TeV. Hence, the current and projected 1σ lower bounds on this model are well below the existing ones, this model is excluded as a potential candidate to explain $g-2_\mu$result in light of the current limits.

C. 331 LHN

In Fig.4 we exhibit the individual contributions as a function of the scale of symmetry breaking. Equivalently to the 331 r.h.n the singly charged and neutral gauge bosons are the most relevant corrections to $g-2_\mu$. The scalars contributions have been multiplied by 10^6 factor so we could depict them in the panel. The Z' contribution is negative and whereas the W' is positive but it strongly depends on the mass of the neutral fermion. We have adopted $M_N > 1$ GeV. The diagram in question is shown in Fig.1(j). In general the contribution stemming from W' is quite different when its mass is close to the neutral fermion mass. Since we interested in the regime which the scale of symmetry breaking is large than 1 TeV, i.e $M'_W > 330$ GeV, the results from any value of $M_N \ll 330$ GeV are similar to the 331 r.h.n model, where a low scale of symmetry breaking, less than 1TeV, is need to accommodate $g-2_\mu$ and the current (projected) limit found is 1 TeV (1.5 TeV) on the scale of symmetry breaking. However, such scale is severely excluded by dark matter, collider and electroweak bounds. In case the neutral fermion mass lies in the TeV scale the overall correction to $g-2_\mu$ is dwindled since we are suppressing the leading one (W' contribution). In summary, the model similarly to the 331 r.h.n, cannot accommodate the muon anomalous magnetic moment, while obeying the current limits.

D. 331 Economical

In Fig.5 we depict the individual contributions to $g-2_\mu$along with the current and projected sensitivities as in previous plots. We have multiplied some of the individual contributions by constants to show them in the plot, namely: neutral scalar $S_2 \times (10^6)$, charged scalar $h_1^+ \times (-10^6)$ and $Z' \times (-1)$. One can straightforwardly conclude that the W' and Z' corrections are the leading ones. It is clear from the figure that the scale of symmetry breaking (~ 800 GeV) required to reproduce the measured $g-2_\mu$is fiercely ruled out by LHC limits that prohibits scales smaller than 7.5 TeV. Anyway, we find a current (projected) limit of 1 TeV in the scale of symmetry breaking of the model. In conclusion, the Economical

331 model cannot accommodate $g-2_\mu$.

E. RM331

In Fig.6 we display the individual contributions to $g-2_\mu$in the RM331 model. We conclude that doubly charged and singly charged vector bosons are the leading ones. We we see that a scale of symmetry breaking of ~ 2 TeV can explain the $g-2_\mu$ excess. Such energy scale is consistent with the aforementioned limits, since the FCNC ones are sensitive to the parametrization scheme used in the hadronic sector. After summing up all individual corrections we find that in case the anomaly is otherwise resolved, a current lower bound of 4 TeV, and projected lower bound of 6 TeV can be placed on the scale of symmetry breaking of the model. Notice that this model is within current sensitivity of the next generation of $g-2_\mu$ experiments. Since this model, similar to the 331 minimal model, is valid up to 5 TeV only due to the Landau Pole, the RM331 might be excluded in the foreseeable future.

F. 331 Exotic Leptons

In the Fig.7 we exhibit the individual corrections from the neutral and charged gauge bosons according to Eq.(26) for exotic leptons (charged leptons) masses of $M_E = 1$ TeV. As we discussed previously, the contributions from the scalars have been ignored since they are negligible. One can easily conclude that the K^0 correction, which is negative, is the most relevant. Since all corrections are negative the model cannot accommodate the muon magnetic moment excess. Adding up all corrections, we find that the overall contribution is small and negative. However, we can still draw a bound, assuming the anomaly has been otherwise explained, because the overall contribution would still have to lie within the error bars. Hence, we find a current bound of 1 TeV and a projected one of 1.8 TeV in case the $g-2_\mu$anomaly is otherwise resolved. We emphasize that our limits are not very sensitive to the masses of the exotic leptons.

IX. CONCLUSIONS

We have computed contributions to the muon magnetic moment stemming from the main 331 models in the literature and derived bounds summarized in Table I. We exploited the complementarity among collider, dark matter and electroweak constraints to outline which models are able to address $g-2_\mu$. Moreover, we derive stringent 1σ limits on the scale of symmetry breaking of those models whenever possible.

We concluded that the *Minimal 331 model* cannot explain the $g-2_\mu$ excess. We find a robust limit on the scale of symmetry breaking of 4 TeV, which can be translated into $M_{Z'} > 2.4$ TeV. As far as we know this is the strongest bound in the Z' mass in the literature. Additionally, we show that the upcoming g-2 experiment at Fermilab will be able to undoubtedly exclude this model.

Model	$g-2_\mu$ Limit
331 Minimal	Current: $V_\chi \geq 4$ TeV
	Projected: $V_\chi \geq 5.8$ TeV
331 r.h.n	Current: $V_\chi \geq 1$ TeV
	Projected: $V_\chi \geq 1.5$ TeV
331 LHN	Current: $V_\chi \geq 1$ TeV
	Projected: $V_\chi \geq 1.5$ TeV
331 Economical	Current: $V_\chi \geq 900$ GeV
	Projected: $V_\chi \geq 1.3$ TeV
RM331	Current: $V_\chi \geq 4$ TeV
	Projected: $V_\chi \geq 5.8$ TeV
331 Exotic Leptons	Current: $V_\chi \geq 1$ TeV
	Projected: $V_\chi \geq 1.8$ TeV

TABLE I. Limits on the scale of symmetry breaking (V_χ) of the 331 models using current and projected sensitivity $g-2_\mu$ experiments.

As for the *331 r.h.n*, we find this model incapable of accommodating measured $g-2_\mu$ because it requires a scale of symmetry breaking that is already ruled out by collider and dark matter data, and place a current (projected) 1σ bound of 1 TeV (1.5 TeV) on the scale of the 331 symmetry breaking.

Regarding the *331LHN*, with $M_N = 1$ GeV, a scale of symmetry breaking much smaller than 1 TeV is needed to address the $g-2_\mu$ excess, which is already excluded by current collider and direct dark matter detection. Similarly for $M_N > 1$ GeV. A current (projected) 1σ bound of 1 TeV (1.5 TeV) on the scale of the 331 symmetry breaking was found.

Concerning the *331 Economical model*, due to the large cancelation between the W' and Z' corrections the total contribution is small and requires $v_\chi < 1$ TeV to explain the reported $g-2_\mu$. This low scale of symmetry breaking is again ruled out by current data. A current (projected) 1σ bound of 900 GeV (1.3 TeV) on the scale of the 331 symmetry breaking was derived.

As for the *RM331* model, we observed that a scale of symmetry breaking of ~ 2 TeV could explain the $g-2_\mu$ excess, while being consistent with other constraints. If instead, the anomaly is otherwise resolved, a current limit of 4 TeV, and projected of 5.8 TeV can be placed on the scale of symmetry breaking of the model. Since this model has a Landau pole at 5 TeV (similar to the Minimal 331 model), the RM331 might be entirely excluded in the next generation of experiments.

Lastly, *the 331 model with exotic leptons*, predicts a small and negative contribution to $g-2_\mu$ regardless of how massive the exotic leptons are. Thus it cannot accommodate the reported $g-2_\mu$. A current (projected)

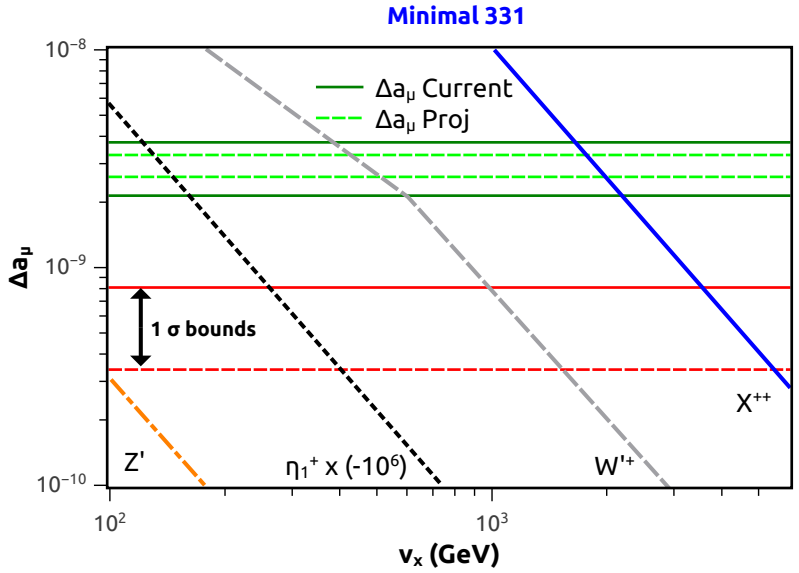

FIG. 2. Individual contributions to $g-2_\mu$. The solid (dashed) green horizontal lines represent the current (projected) sensitive to $g-2_\mu$. The solid (dashed) horizontal red lines are the current and projected 1σ bound in case the anomaly is resolved otherwise. After summing up all individual corrections we find that the scale of symmetry breaking that reproduces $g-2_\mu$ is quite small and excluded by current data. Moreover, we have derived a current (projected) 4 TeV (5.8 TeV) limit on the scale of symmetry breaking. Since this model is valid only up to 5 TeV, we conclude that the upcoming g-2 experiment at Fermilab will be able to undoubtedly exclude this model.

bound of 1 TeV (1.8 TeV) on the scale of symmetry breaking of the model was placed.

ACKNOWLEDGEMENT

The authors thank Alex Dias, Eugenio Del Nobile for useful discussions. HNL is supported by the Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2014.51. R.M thanks COLCIENCIAS for financial support. FQ is partly supported by US Department of Energy Award SC0010107 and the Brazilian National Counsel for Technological and Scientific Development (CNPq).

X. APPENDIX

In this Appendix we present master integrals for computing $g-2_\mu$ stemming from all particles discussed in this work.

Neutral Scalar

Neutral scalars in general can have scalar (g_{s1}) and pseudo-scalar (g_{p1}) couplings which shift $(g-2)_\mu$ through Fig.1(a) by

$$\Delta a_\mu(\phi) = \frac{1}{8\pi^2}\frac{m_\mu^2}{M_\phi^2}\int_0^1 dx \frac{g_{s1}^2\ P_{s1}(x) + g_{p1}^2\ P_{p1}(x)}{(1-x)(1-\lambda^2 x)+\lambda^2 x} \quad (28)$$

where $\lambda = m_\mu/M_\phi$ and,

$$P_{s1}(x) = x^2(2-x),$$
$$P_{p1}(x) = -x^3, \quad (29)$$

which gives us,

$$\Delta a_\mu(\phi) = \frac{1}{4\pi^2}\frac{m_\mu^2}{M_\phi^2}\left[g_{s1}^2\left(\ln\left(\frac{M_\phi}{m_\mu}\right)-\frac{7}{12}\right) + g_{p1}^2\left(-\ln\left(\frac{M_\phi}{m_\mu}\right)+\frac{11}{12}\right)\right] \quad (30)$$

The result in Eq.(30) is for general neutral scalars with scalar and pseudo-scalar couplings in the regime $M_\phi \gg m_\mu$. Note that neutral scalars are also bounded by LEP searches for four-lepton contact interactions. For $M_\phi > \sqrt{s}$ these bounds require $g/M_\phi < 2.5\times 10^{-4}\text{GeV}^{-1}$ [45].

Singly Charged Scalar

A general Lagrangian involving singly charged scalars with scalar (g_{s1}) and pseudo-scalar (g_{p1}) couplings which gives rise to the $g-2_\mu$ correction according to Fig.1(b),

$$\Delta a_\mu(H^+) = \frac{1}{8\pi^2}\frac{m_\mu^2}{M_{H^+}^2}\int_0^1 dx \frac{g_{s2}^2\ P_{s2}(x) + g_{p2}^2\ P_{p2}(x)}{\epsilon^2\lambda^2(1-x)(1-\epsilon^{-2}x)+x} \quad (31)$$

where

$$P_{s2}(x) = -x(1-x)(x+\epsilon)$$
$$P_{p2}(x) = -x(1-x)(x-\epsilon) \quad (32)$$

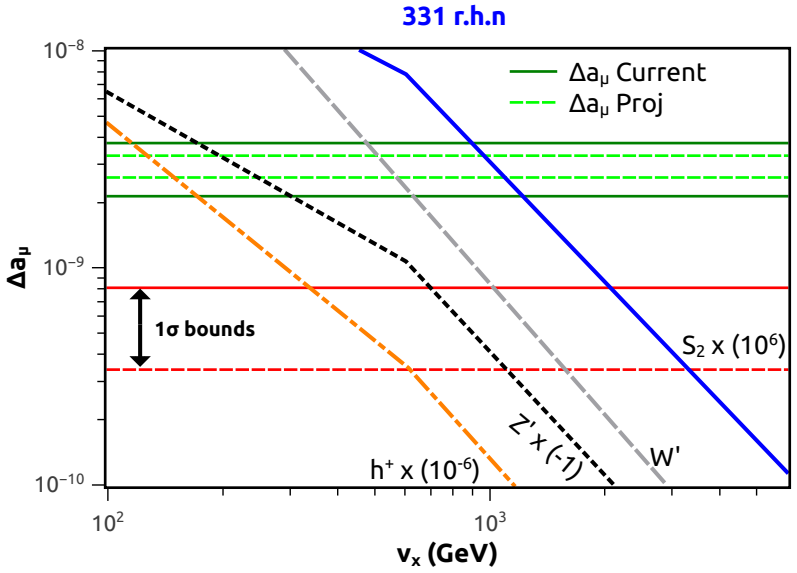

FIG. 3. Individual contributions $g-2_\mu$ in the 331 r.h.n. Adding up all contributions we conclude that this model cannot explain $g-2_\mu$ excess because it requires a scale of symmetry breaking that is already ruled out by current collider, dark matter experiments and electroweak precision data. Additionally, a current (projected) constraint of 1 TeV (1.5 TeV) might be posed.

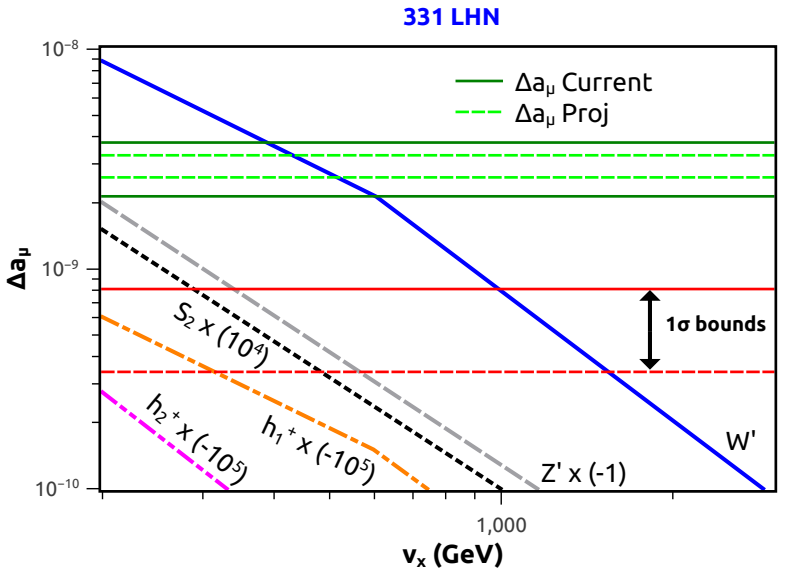

FIG. 4. Individual contributions to the muon magnetic moment in the 331 model with heavy leptons as function of the scale of symmetry breaking. With $M_N = 1$ GeV, a scale of symmetry breaking of much smaller than 1 TeV would be needed to accommodate $g-2_\mu$which is already excluded by dark matter, collider and electroweak bounds. Hence the 331LHN can not provide an explanation for the anomaly.

with $\epsilon = m_\nu/m_\mu$ and $\lambda = m_\mu/M_{H^+}$, which results in,

$$\Delta a_\mu(H^+) = \frac{1}{4\pi^2}\frac{m_\mu^2}{M_{H^+}^2}\left[g_{s2}^2\left(-\frac{m_\nu}{4m_\mu}-\frac{1}{12}\right) + g_{p2}^2\left(\frac{m_\nu}{4m_\mu}-\frac{1}{12}\right)\right] \quad (33)$$

Eq.(33) holds even if there was a charge conjugation matrix (C) as in same charged scalar contributions in the minimal 331 model presented in Eq.(7).

Doubly Charged Scalar

A doubly charged scalar contributes to $g-2_\mu$through the diagrams Fig.1(c)-1(d). From each diagram we find, respectively,

$$\Delta a_\mu(H^{\pm\pm}) = \frac{-q_H}{2\pi^2}\left(\frac{m_\mu}{M_{H^{\pm\pm}}}\right)^2\int_0^1 dx\frac{g_{s4}^2P_s(x)+g_{p4}^2P_p(x)}{\lambda^2x^2+(1-2\lambda^2)x+\lambda^2} + \frac{-q_f}{2\pi^2}\left(\frac{m_\mu}{M_{H^{\pm\pm}}}\right)^2\int_0^1 dx\frac{g_{s4}^2P_s'(x)+g_{p4}^2P_p'(x)}{\lambda^2x^2+(1-x)} \quad (34)$$

where

$$P_{s4}(x) = x^3-x \;;P_s' = 2x^2-x^3$$
$$P_{p4}(x) = x^3-2x^2+x \;;P_p' = -x^3 \quad (35)$$

and $\lambda = m_\mu/M_{H^{++}}$, $q_H = -2$ is the electric charge of the doubly charged scalar running in the loop, and $q_f = 1$ is the electric charge of the muon in the loop. The factor of four in Eq.(34) is a symmetry factor due to the presence of two identical fields in the interaction term. This expression simplifies to,

$$\Delta a_\mu(H^{++}) = \frac{-2}{3}\frac{g_{s4}^2m_\mu^2}{\pi^2M_{\phi^{\pm\pm}}^2} \quad (36)$$

when $g_{p4} = \pm g_{s4}$ and $M_{\phi^{\pm\pm}} \gg m_\mu$. In the setup where either of the above conditions fail the integral in Eq.(34) is most easily solved numerically.

Charged Lepton

Charged leptons corrects $g-2_\mu$through Fig.1(i). The contribution is given by,

$$\Delta a_\mu(E) = \frac{1}{8\pi^2}\frac{m_\mu^2}{M_{K^0}^2}\int_0^1 dx\frac{g_{v6}^2\ P_{v6}(x)+g_{a6}^2\ P_{a6}(x)}{(1-x)(1-\lambda^2x)+\epsilon^2\lambda^2x} \quad (37)$$

where

$$P_{v6}(x) = 2x(1-x)(x-2(1-\epsilon))+\lambda^2(1-\epsilon)^2x^2(1+\epsilon-x)$$
$$P_{a6}(x) = 2x^2(1+x+2\epsilon)+\lambda^2(1+\epsilon)^2x(1-x)(x-\epsilon) \quad (38)$$

with $\epsilon = M_E/m_\mu$ and $\lambda = m_\mu/M_{K^0}$. Therefore the contribution of a generic, singly charged lepton mediated by a neutral vector is found to be

$$\Delta a_\mu(E) = \frac{1}{4\pi^2}\frac{m_\mu^2}{M_{K^0}^2}\left\{g_{v6}^2\left[\frac{M_E}{m_\mu}-\frac{2}{3}\right]+g_{a6}^2\left[-\frac{M_E}{m_\mu}-\frac{2}{3}\right]\right\}, \quad (39)$$

in the $M_K^0 \gg M_E$ limit. Outside of this limit, one should solve Eq.(37) numerically, using the public Mathematica code in [45], for instance.

Neutral Vector

Contribution from a new neutral gauge boson, such as a Z', in shown in Fig.1(h) and is given by,

$$\Delta a_\mu(Z') = \frac{m_\mu^2}{8\pi^2M_Z'^2}\int_0^1 dx\frac{g_{v9}^2P_{v9}(x)+g_{a9}^2P_{a9}(x)}{(1-x)(1-\lambda^2x)+\lambda^2x}, \quad (40)$$

where $\lambda = m_\mu/M_{Z'}$ and

$$P_{v9}(x) = 2x^2(1-x)$$
$$P_{a9}(x) = 2x(1-x)\cdot(x-4)-4\lambda^2\cdot x^3. \quad (41)$$

These integrals simplify to give a contribution of

$$\Delta a_\mu(Z') = \frac{m_\mu^2}{4\pi^2M_Z'^2}\left(\frac{1}{3}g_{v9}^2-\frac{5}{3}g_{a9}^2\right) \quad (42)$$

in the limit $M_{Z'} \gg m_\mu$. This is the contribution of the Z' to the muon anomalous magnetic moment. In the regime $M_Z' > \sqrt{s}$ LEP has placed a 95% C.L upper bound of $g_{v9}/M_Z' < 2.2\times10^{-4}\text{GeV}^{-1}$ for $g_{v9} = g_{a9}$. This limit excludes the possibility of a single Z' boson to be the solution to the $g-2_\mu$ anomaly [45].

Singly Charged Vector

Their contributions to $g-2_\mu$ are depicted in Fig.1(e) and Fig.1(j) reads,

$$\Delta a_\mu(W') = \frac{1}{8\pi^2}\frac{m_\mu^2}{M_{V^+}^2}\int_0^1 dx\frac{g_{v10}^2\ P_{v10}(x)+g_{a10}^2\ P_{a10}(x)}{\epsilon^2\lambda^2(1-x)(1-\epsilon^{-2}x)+x}, \quad (43)$$

where

$$P_{v10}(x) = 2x^2(1+x-2\epsilon)+\lambda^2(1-\epsilon)^2x(1-x)(x+\epsilon)$$
$$P_{a10}(x) = 2x^2(1+x+2\epsilon)+\lambda^2(1+\epsilon)^2x(1-x)(x-\epsilon), \quad (44)$$

with $\epsilon = m_\nu/m_\mu$ and $\lambda = m_\mu/M_{W'}$. This simplifies to

$$\Delta a_\mu(W') = \frac{1}{4\pi^2}\frac{m_\mu^2}{M_{W'}^2}\left[g_{v10}^2\left(\frac{5}{6}-\frac{m_\nu}{m_\mu}\right)+g_{a10}^2\left(\frac{5}{6}+\frac{m_\nu}{m_\mu}\right)\right], \quad (45)$$

in the regime $M_{W'} \gg m_\mu$.

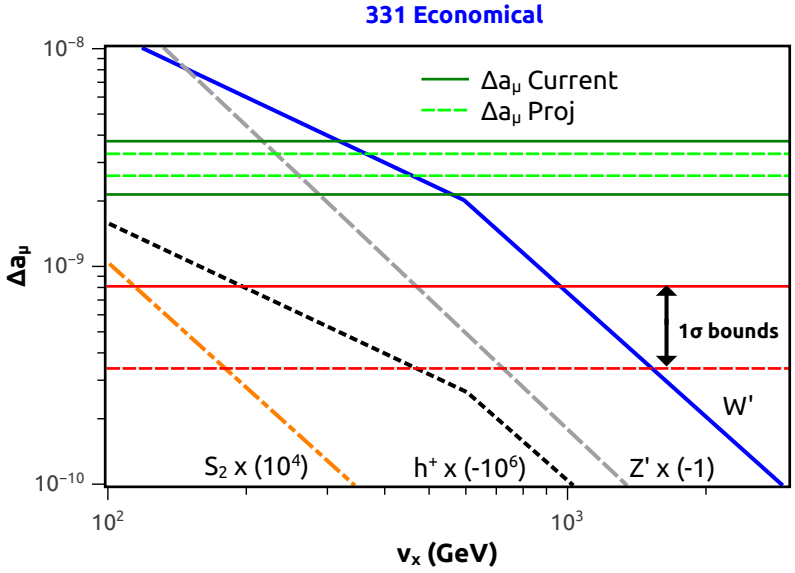

FIG. 5. Individual contributions from the 331 Economical model. We conclude from the right panel that the overall correction to $g-2_\mu$ is negative and small. Thus the model is excluded as an explanation to $g-2_\mu$. Moreover, a current (projected) limit of 900 GeV (1.3 TeV) can placed on the scale of symmetry breaking.

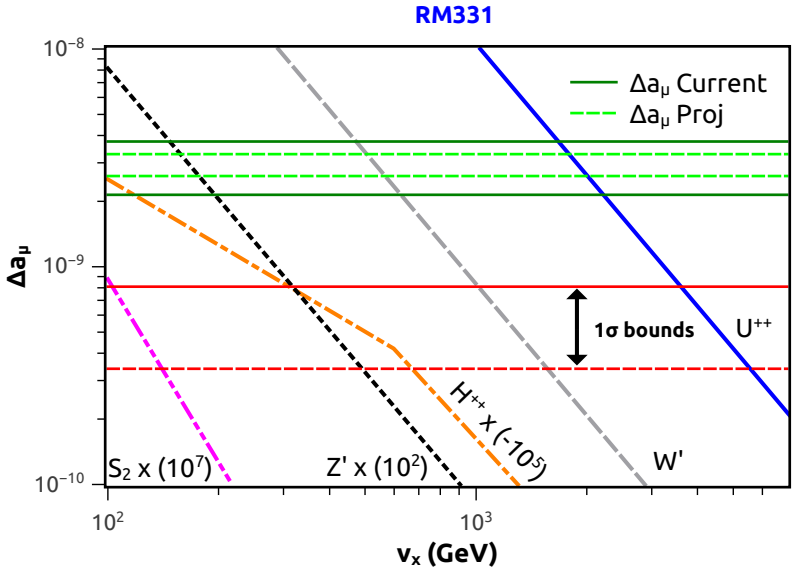

FIG. 6. Individual contributions to $g-2_\mu$in the RM331 as a function of the scale of symmetry breaking. We conclude with $v_\chi \sim 2$ TeV the model can accommodate the $g-2_\mu$ excess while being consistent with existing bounds. If the anomaly is otherwise resolved a current limit of 4 TeV will be placed, and projected of 5.8 TeV will be achieved, ruling out the entire model due to the Landau Pole at 5 TeV.

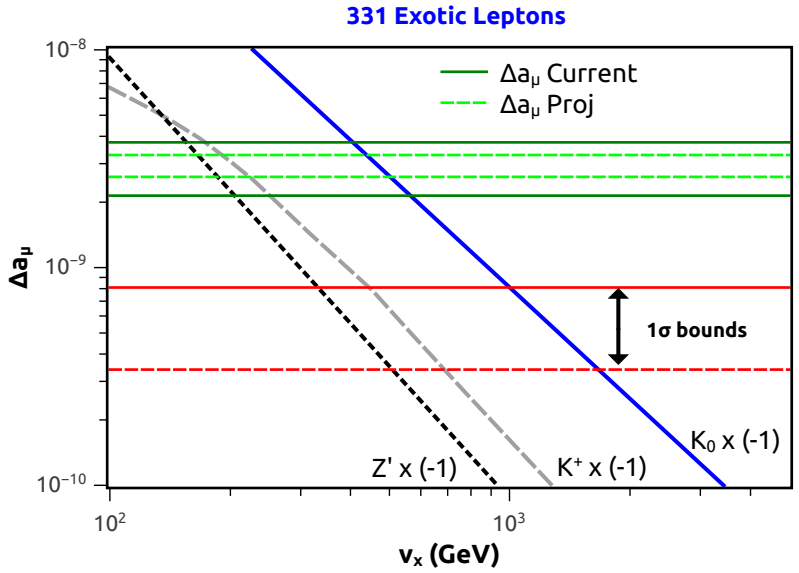

FIG. 7. Individual contributions to $g-2_\mu$in the 331 model with exotic leptons. Summing up all corrections we find that overall contribution is negative and small. We could still derive a current (projected) bound of 1 TeV (1.8 TeV) in case the $g-2_\mu$anomaly is otherwise resolved.

Doubly Charged Vector

The doubly-charged vector boson contribution to $g-2_\mu$ is exhibited in Figs.1(f)-1(g), and are given by,

$$\Delta a_\mu(U^{\pm\pm}) = \frac{1}{\pi^2}\left(\frac{m_\mu}{M_{U^{\pm\pm}}}\right)^2\int_0^1 dx\frac{g_{v11}^2P_{v11}(x)+g_{a11}^2P_{a11}(x)}{\lambda^2(1-x)^2+x} \frac{-1}{2\pi^2}\left(\frac{m_\mu}{M_{U^{\pm\pm}}}\right)^2\int_0^1 dx\frac{g_{v12}^2P'_{v12}(x)+g_{a12}^2P'_{a12}(x)}{(1-x)(1-\lambda^2x)+\lambda^2x}, \quad (46)$$

where $\lambda = m_\mu/M_{U^{\pm\pm}}$, and

$$\begin{aligned} P_{v11}(x) &= 2x^2(x-1) \\ P_{a11}(x) &= 2x^2(x+3)+4\lambda^2\cdot x(1-x)(x-1), \\ P'_{v12}(x) &= 2x(1-x)\cdot x \\ P'_{a12}(x) &= 2x(1-x)\cdot(x-4)-4\lambda^2\cdot x^3. \end{aligned} \quad (47)$$

Hence the total doubly-charged vector contribution is given by,

$$\Delta a_\mu(U^{\pm\pm}) = \frac{m_\mu^2}{\pi^2 M_{U^{\pm\pm}}^2}\left(\frac{-2}{3}g_{v12}^2+\frac{16}{3}g_{a12}^2\right) \quad (48)$$

[1] T. Blum, A. Denig, I. Logashenko, E. de Rafael, B. Lee Roberts, T. Teubner and G. Venanzoni, arXiv:1311.2198 [hep-ph].

[2] J. P. Miller, E. de Rafael and B. L. Roberts, Rept. Prog. Phys. **70**, 795 (2007) [hep-ph/0703049].

[3] C. Aubin, T. Blum, M. Golterman, S. Peris, Phys.Rev. D88 (2013) 074505, [arXiv:1307.4701];

[4] C. Aubin, T. Blum, M. Golterman, K. Maltman, S. Peris, [arXiv:1311.5504];

[5] M. Golterman, K. Maltman, S. Peris, [arXiv:1310.5928].

[6] R. M. Carey, K. R. Lynch, J. P. Miller, B. L. Roberts, W. M. Morse, Y. K. Semertzides, V. P. Druzhinin and B. I. Khazin et al., FERMILAB-PROPOSAL-0989; Andreas S. Kronfeld et al,[arxiv:1306.5009].

[7] J. K. Mizukoshi, C. A. de S.Pires, F. S. Queiroz and P. S. Rodrigues da Silva, Phys. Rev. D **83**, 065024 (2011) [arXiv:1010.4097 [hep-ph]]; M. E. Catano, R. Martinez and F. Ochoa, Phys. Rev. D **86**, 073015 (2012) [arXiv:1206.1966 [hep-ph]].

[8] J. D. Ruiz-Alvarez, C. A. de S. Pires, F. S. Queiroz, D. Restrepo and P. S. Rodrigues da Silva, Phys. Rev. D **86**, 075011 (2012) [arXiv:1206.5779 [hep-ph]].

[9] S. Profumo and F. S. Queiroz, Eur. Phys. J. **74**, 2960 (2014) [arXiv:1307.7802 [hep-ph]].

[10] D. Cogollo, A. X. Gonzalez-Morales, F. S. Queiroz and P. R. Teles, arXiv:1402.3271 [hep-ph].

[11] P. V. Dong, D. T. Huong, F. S. Queiroz and N. T. Thuy, arXiv:1405.2591 [hep-ph].

[12] P. V. Dong, N. T. K. Ngan and D. V. Soa, arXiv:1407.3839 [hep-ph].

[13] P. V. Dong, T. P. Nguyen and D. V. Soa, Phys. Rev. D **88**, no. 9, 095014 (2013) [arXiv:1308.4097 [hep-ph]].

[14] P. V. Dong, H. T. Hung and T. D. Tham, Phys. Rev. D **87**, no. 11, 115003 (2013) [arXiv:1305.0369 [hep-ph]].

[15] C. A. de S.Pires and P. S. Rodrigues da Silva, JCAP **0712**, 012 (2007) [arXiv:0710.2104 [hep-ph]].

[16] D. Fregolente and M. D. Tonasse, Phys. Lett. B **555**, 7 (2003) [hep-ph/0209119].

[17] C. Kelso, C. A. de S. Pires, S. Profumo, F. S. Queiroz and P. S. Rodrigues da Silva, Eur. Phys. J. C **74**, 2797 (2014) [arXiv:1308.6630 [hep-ph]]; F. S. Queiroz, AIP Conf. Proc. **1604**, 83 (2014) [arXiv:1310.3026 [astro-ph.CO]];

[18] P. B. Pal, Phys. Rev. D **52**, 1659 (1995) [hep-ph/9411406].

[19] A. G. Dias, P. R. D. Pinheiro, C. A. de S.Pires and P. S. Rodrigues da Silva, arXiv:1309.6644 [hep-ph]; J. G. Ferreira, C. A. de S. Pires, P. S. Rodrigues da Silva and A. Sampieri, Phys. Rev. D **88**, no. 10, 105013 (2013) [arXiv:1308.0575 [hep-ph]]; W. Caetano, C. A. de S. Pires, P. S. Rodrigues da Silva, D. Cogollo and F. S. Queiroz, Eur. Phys. J. C **73**, 2607 (2013) [arXiv:1305.7246 [hep-ph]]; A. Alves, E. Ramirez Barreto, A. G. Dias, C. A. de S. Pires, F. S. Queiroz and P. S. Rodrigues da Silva, Eur. Phys. J. C **73**, 2288 (2013) [arXiv:1207.3699 [hep-ph]];

[20] W. Caetano, D. Cogollo, C. A. de S. Pires and P. S. Rodrigues da Silva, Phys. Rev. D **86**, 055021 (2012) [arXiv:1206.5741 [hep-ph]]; A. G. Dias, C. A. de S.Pires, P. S. Rodrigues da Silva and A. Sampieri, Phys. Rev. D **86**, 035007 (2012) [arXiv:1206.2590]; A. Alves, E. Ramirez Barreto, A. G. Dias, C. A. de S. Pires, F. S. Queiroz and P. S. Rodrigues da Silva, Phys. Rev. D **84**, 115004 (2011) [arXiv:1109.0238 [hep-ph]]; A. G. Dias, C. A. de S. Pires and P. S. Rodrigues da Silva, Phys. Rev. D **82**, 035013 (2010) [arXiv:1003.3260 [hep-ph]];

[21] F. Queiroz, C. A. de S. Pires and P. S. R. da Silva, Phys. Rev. D **82**, 065018 (2010) [arXiv:1003.1270 [hep-ph]]; D. Cogollo, H. Diniz and C. A. de S. Pires, Phys. Lett. B **687**, 400 (2010) [arXiv:1002.1944 [hep-ph]]; D. Cogollo, H. Diniz and C. A. de S. Pires, arXiv:0903.0370 [hep-ph]; D. Cogollo, H. Diniz, C. A. de S. Pires and P. S. Rodrigues da Silva, Eur. Phys. J. C **58**, 455 (2008) [arXiv:0806.3087 [hep-ph]]; D. Cogollo, H. Diniz, C. A. de S. Pires and P. S. Rodrigues da Silva, Mod. Phys. Lett. A **23**, 3405 (2009) [arXiv:0709.2913 [hep-ph]].

[22] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992) [hep-ph/9206242].

[23] R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D **47**, 4158 (1993) [hep-ph/9207264].

[24] M. D. Tonasse, Phys. Lett. B **381**, 191 (1996) [hep-ph/9605230].

[25] N. A. Ky, H. N. Long and D. V. Soa, Phys. Lett. B **486**, 140 (2000) [hep-ph/0007010]; N. A. Ky and H. N. Long, hep-ph/0103247.

[26] C. Kelso, P. R. D. Pinheiro, F. S. Queiroz and W. Shepherd, Eur. Phys. J. C **74**, 2808 (2014) [arXiv:1312.0051 [hep-ph]]; C. A. De Sousa Pires and P. S. Rodrigues da Silva, Phys. Rev. D **65**, 076011 (2002) [hep-ph/0108200]; C. A. de S. Pires and P. S. Rodrigues da Silva, Phys. Rev. D **64**, 117701 (2001) [hep-ph/0103083]; C. -X. Yue, Q. -Y. Shi and T. Hua, Nucl. Phys. B **876**, 747 (2013) [arXiv:1307.5572 [hep-ph]].

[27] H. N. Long, Phys. Rev. D **54**, 4691 (1996) [hep-ph/9607439]; H. N. Long, Phys. Rev. D **53**, 437 (1996) [hep-ph/9504274].

[28] V. V. Andreev, P. Osland and A. A. Pankov, arXiv:1406.6776 [hep-ph]; M. S. Carena, A. Daleo, B. A. Dobrescu and T. M. P. Tait, Phys. Rev. D **70**, 093009 (2004) [hep-ph/0408098]; J. Alcaraz *et al.* [ALEPH and DELPHI and L3 and OPAL and LEP Electroweak Working Group Collaborations], hep-ex/0612034.

[29] W. A. Ponce, Y. Giraldo and L. A. Sanchez, Phys. Rev. D 67, 075001 (2003) [arXiv:0210026]; P. V. Dong, H. N. Long, D. T. Nhung and D. V. Soa, Phys. Rev. D **73**, 035004 (2006) [hep-ph/0601046]; J.G. Duenas, N. Gutierrez, R. Martinez, F. Ochoa. Eur.Phys.J. C60 (2009) 653; P. V. Dong and H. N. Long, Adv. High Energy Phys. **2008**, 739492 (2008) [arXiv:0804.3239 [hep-ph]]; D. Berenstein, R. Martinez, F. Ochoa and S. Pinansky, Phys. Rev. D **79**, 095005 (2009) [arXiv:0807.1126 [hep-ph]]; R. Martinez and F. Ochoa, Phys. Rev. D **90**, 015028 (2014) [arXiv:1405.4566 [hep-ph]].

[30] J. M. Cabarcas, J. Duarte and J. -A. Rodriguez, Int. J. Mod. Phys. A **29**, 1450015 (2014) [arXiv:1310.1407 [hep-ph]]; W. A. Ponce, J. B. Florez and L. A. Sanchez, Int. J. Mod. Phys. A **17**, 643 (2002) [hep-ph/0103100];

[31] W. A. Ponce, Y. Giraldo and L. A. Sanchez, [hep-ph/0201133]; J. M. Cabarcas, J. Duarte and J. -A. Rodriguez, PoS HQL **2012**, 072 (2012) [arXiv:1212.3586 [hep-ph]]; D. L. Anderson and M. Sher, Phys. Rev. D **72**, 095014 (2005) [hep-ph/0509200].

[32] D. Ng, Phys. Rev. D **49**, 4805 (1994) [hep-ph/9212284].

[33] Y. A. Coutinho, V. Salustino Guimares and A. A. Nepomuceno, Phys. Rev. D **87**, no. 11, 115014 (2013) [arXiv:1304.7907 [hep-ph]].

[34] A. J. Buras, F. De Fazio and J. Girrbach, JHEP **1402**, 112 (2014) [arXiv:1311.6729 [hep-ph], arXiv:1311.6729]; A. J. Buras, F. De Fazio and J. Girrbach-Noe, arXiv:1405.3850 [hep-ph]; A. J. Buras, F. De Fazio, J. Girrbach and M. V. Carlucci, JHEP **1302**, 023 (2013) [arXiv:1211.1237 [hep-ph]]; A. E. Carcamo Hernandez, R. Martinez and F. Ochoa, Phys. Rev. D **73**, 035007 (2006) [hep-ph/0510421]; R. Martinez and F. Ochoa, Phys. Rev. D **77**, 065012 (2008) [arXiv:0802.0309 [hep-ph]].

[35] A. Djouadi, A. Falkowski, Y. Mambrini and J. Quevillon, Eur. Phys. J. C **73**, 2455 (2013) [arXiv:1205.3169 [hep-ph]]; F. S. Queiroz and K. Sinha, Phys. Lett. B **735**, 69 (2014) [arXiv:1404.1400 [hep-ph]].

[36] A. Alves, S. Profumo and F. S. Queiroz, JHEP **1404**, 063 (2014) [arXiv:1312.5281 [hep-ph]]; G. Arcadi, Y. Mambrini, M. H. G. Tytgat and B. Zaldivar, JHEP **1403**, 134 (2014) [arXiv:1401.0221 [hep-ph]]; O. Lebedev and Y. Mambrini, arXiv:1403.4837 [hep-ph].

[37] D. Hooper, F. S. Queiroz and N. Y. Gnedin, Phys. Rev. D **85**, 063513 (2012) [arXiv:1111.6599 [astro-ph.CO]]; C. Kelso, S. Profumo and F. S. Queiroz, Phys. Rev. D **88**, no. 2, 023511 (2013) [arXiv:1304.5243 [hep-ph]]; R. Allahverdi, M. Cicoli, B. Dutta and K. Sinha, Phys. Rev. D **88**, no. 9, 095015 (2013) [arXiv:1307.5086 [hep-ph]]; F. S. Queiroz, K. Sinha and W. Wester, arXiv:1407.4110 [hep-ph]; K. Sinha, AIP Conf. Proc. **1604**, 105 (2014) [arXiv:1311.0884 [hep-ph]]; J. Hasenkamp and J. Kersten, JCAP **1308**, 024 (2013) [arXiv:1212.4160 [hep-ph]].

[38] J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012)

[39] J. G. Ferreira, Jr, P. R. D. Pinheiro, C. A. d. S. Pires and P. S. R. da Silva, Phys. Rev. D **84**, 095019 (2011) [arXiv:1109.0031 [hep-ph]].

[40] D. Cogollo, F. S. Queiroz and P. Vasconcelos, arXiv:1312.0304 [hep-ph].

[41] T. A. Nguyen, N. A. Ky and H. N. Long, Int. J. Mod. Phys. A 15, 283 (2000) [arXiv:9810273].

[42] J. M. Cabarcas, J. Duarte and J. -A. Rodriguez, Int. J. Mod. Phys. A **29**, 1450015 (2014) [arXiv:1310.1407 [hep-ph]].

[43] W. A. Ponce, J. B. Florez and L. A. Sanchez, Int. J. Mod. Phys. A **17**, 643 (2002) [hep-ph/0103100].

[44] A. Freitas, J. Lykken, S. Kell and S. Westhoff, JHEP **1405**, 145 (2014) [arXiv:1402.7065 [hep-ph]].

[45] F. S. Queiroz and W. Shepherd, Phys. Rev. D **89**, 095024 (2014) [arXiv:1403.2309 [hep-ph]].