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Current experimental data on the 125 GeV Higgs boson still allow room for large CP violation. The observables usually considered in this context are triple product asymmetries, which require an input of four visible particles after imposing momentum conservation. We point out a new class of CP violating observables in Higgs physics which require only three reconstructed momenta. They may arise if the process involves an interference of amplitudes with different intermediate particles, which provide distinct "strong phases" in the form of the Breit-Wigner widths, in addition to possible "weak phases" that arise from CP violating couplings of the Higgs in the Lagrangian. As an example, we propose a forward-backward asymmetry of the charged lepton in the three-body Higgs decay, $h \to \ell^- \ell^+ \gamma$, as a probe for CP-violating Higgs couplings to $Z\gamma$ and $\gamma\gamma$ pairs. Other processes exhibiting this type of CP-violation are also discussed.

INTRODUCTION

The observation of the 125 GeV Higgs boson at the Large Hadron Collider (LHC) [1, 2] marked the beginning of a long-term research program to look for physics beyond the Standard Model (SM) through properties of the Higgs boson. So far measurements based on the signal strength conform to SM predictions. However, some properties of the Higgs boson, in particular the tensor structure of its coupling to matter, remain relatively unconstrained by publicly available experimental data. One particularly interesting possibility is that the Higgs couplings to SM gauge bosons and/or fermions contain new sources of CP-violation (CPV). While some of these couplings may be significantly constrained by low-energy precision observables [3, 4], such constraints are not model-independent. It is therefore important to directly constrain the possibility of CP violating Higgs couplings in high-energy colliders [5–14].

There have been many works on direct measurements of CPV in Higgs physics [15–35], which all rely on constructing a CP-odd triple product asymmetries. Such an observable, however, requires presence of three linearly-independent vectors. Given that the Higgs is a scalar particle and carries no spin, momentum conservation then implies measurements of four visible momenta in order to probe CPV in the Higgs sector. One prime example is the azimuthal angle between the two decay planes of a four-body Higgs decay:

$$\cos\phi = \frac{(\vec{p}_1 \times \vec{p}_2) \cdot (\vec{p}_3 \times \vec{p}_4)}{|\vec{p}_1 \times \vec{p}_2| \ |\vec{p}_3 \times \vec{p}_4|} \ , \qquad (1)$$

which appears in channels such as $h \to 4\ell$ and $h \to \tau\tau$.

In general, CPV occurs through an interference of two amplitudes with different weak phases, that is phases which change sign under a CP transformation. If, in addition, the amplitudes also contain different strong phases, which do not change sign under CP, then one can construct simpler CPV observables. One example is the asymmetry $\mathcal{A}_{\rm CP}$ of decays into CP conjugate final states F and $\bar{F}$. Let us assume that the decay process is described by two interfering amplitudes, $\mathcal{M}_F = \mathcal{M}_1 + \mathcal{M}_2$, which can be written as $\mathcal{M}_i = |c_i| e^{i(\delta_i + \phi_i)}$, where δ_i and ϕ_i are the strong and weak phases, respectively. This then gives,

$$A_{\rm CP} = \frac{d\Gamma_F - d\Gamma_{\bar{F}}}{d\Gamma_F + d\Gamma_{\bar{F}}} \propto |c_1||c_2| \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2), \qquad (2)$$

where we see explicitly that both δ_i and ϕ_i need to be different for the asymmetry to be non-vanishing.

In flavor physics, where these types of effects have previously been studied, strong phases are often incalculable because they arise from strong interactions. There are however exceptions when strong phases come from propagation of intermediate state particles. One well-known example is time evolution of intermediate states that mix with each other, such as the B^0–$\bar{B}^0$ system. Another example that received less attention is strong phases from the propagation of weakly interacting particles with finite widths [36–40]. In this paper we point out that this latter possibility may arise in the context of decays and associated production of the Higgs boson. In this case, the weak phases may arise from couplings of the Higgs boson to the SM particles in the Lagrangian, while the strong phases could come from the finite width effects in the Breit-Wigner propagators of intermediate particles.

There are a number of specific realizations of the above scenario, with applications in both a hadron collider and a lepton collider. In this paper we focus primarily on on the process $h \to \ell^+\ell^-\gamma$. In the SM, the $\ell^+\ell^-$ pair could come from an intermediate Z boson or a photon. We allow the intermediate vector boson to be on or off shell and do not distinguish between them in our notation. This process can be used to probe the possible CP violating $h\gamma\gamma$ and $hZ\gamma$ couplings. Similarly one can consider the decay $h \to \ell^+\ell^- Z$ in which case CP vio-

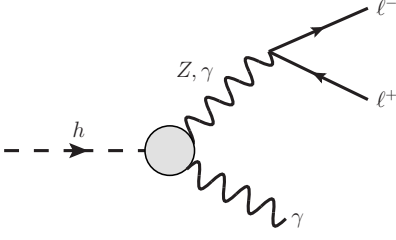FIG. 1. Feynman diagrams for the processes $h \to \ell^- \ell^+ \gamma$ where $\ell = e, \mu$.

lating $hZ\gamma$, and hZZ couplings are probed. We will also discuss $f\bar{f} \to Z/\gamma \to hV$, which is related to $h \to 2\ell + V$ by crossing symmetry, and can also be used to probe CP violating $h\gamma\gamma$, $hZ\gamma$ and hZZ couplings. For all of these cases the strong phases are provided by the widths of the Z boson propagating in the intermediate state, while the weak phases may arise from new physics Higgs couplings to matter.

CP VIOLATION IN $h \to \ell^- \ell^+ \gamma$ DECAYS

We first focus on the process $h \to \ell^- \ell^+ \gamma$ shown in Fig. 1. The couplings of the Higgs boson to $Z\gamma$ and $\gamma\gamma$ can be parametrized with the following Lagrangian,

$$\mathcal{L} \supset \frac{h}{4v}\Big(2A_2^{Z\gamma}F^{\mu\nu}Z_{\mu\nu} + 2A_3^{Z\gamma}F^{\mu\nu}\widetilde{Z}_{\mu\nu}$$
$$+ \ A_2^{\gamma\gamma}F^{\mu\nu}F_{\mu\nu} + A_3^{\gamma\gamma}F^{\mu\nu}\widetilde{F}_{\mu\nu}\Big), \qquad (3)$$

where $v = 246$ GeV, $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ and $\widetilde{V}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}V^{\rho\sigma}$. We work with effective Higgs couplings for which the SM predicts $A_1^{ZZ} = 2$ at tree level and $A_2^i \lesssim \mathcal{O}(10^{-2}-10^{-3})$ at 1-loop ($i = Z\gamma, \gamma\gamma$). The A_3^i are first induced at three loop order [41] and totally negligible. We take $A_{2,3}^i$ to be momentum independent and real as is done in [42–44]. Thus we are neglecting any potential strong phases in the effective couplings, but which in the SM are negligible [31, 45]. Since the A_2 operators are CP-even and A_3 are CP-odd, CP violation must be proportional to products of A_2^i and A_3^j in Eq. (3). In $h \to 4\ell$ we can have CP violation for $i = j$ and $i \neq j$ [35] because of the ability to form CP-odd triple products from the four visible final state momenta. As we will see, in the case of the 3-body $h \to \ell^- \ell^+ \gamma$ decay we only obtain CP violation for $i \neq j$ due to the strong phase condition discussed above, i.e. the Breit-Wigner propagators of the intermediate vector bosons of the interfering amplitudes must be distinct.

To see how CP violation arises in $h \to \ell^- \ell^+ \gamma$ decays it is instructive to analyze the process in terms of helicity amplitudes. Below we treat the leptons as massless and work in the basis where they have the spin projection $+1/2$ (R) or $-1/2$ (L) along the direction of motion of ℓ^- in the rest frame of the the $\ell^- \ell^+$

pair. We define the z-axis by the direction opposite to the motion of photon, which has the polarization tensor $\epsilon^{\pm 1} = (0, 1, \pm i, 0)/\sqrt{2}$. The angle θ_1 is then the polar angle of ℓ^- in the rest frame of $\ell^+\ell^-$. Note that for massless leptons, ℓ^+ and ℓ^- must have the same helicity $\lambda_1 = \lambda_2 \equiv \lambda$, where $\lambda = L, R$. We denote the helicity amplitudes as $\mathcal{M}(\lambda, \epsilon^{\pm 1}) \equiv \lambda_{\pm 1}(\cos\theta_1)$. In colliders we do not measure helicities, therefore we sum over λ and $\epsilon^\pm$ in the amplitude-squared.

Under **P** symmetry all helicities are flipped, while **C** exchanges particles with anti-particle (thus flipping fermion helicities), which corresponds to $\theta_1 \to \pi - \theta_1$. Thus, the CP transformation relates amplitudes with the same fermion helicity, and opposite photon helicity. Up to a convention-dependent phase, unbroken CP implies $L_{+1}(\cos\theta_1) = L_{-1}(-\cos\theta_1)$, $R_{+1}(\cos\theta_1) = R_{-1}(-\cos\theta_1)$, in which case,

$$\sum_{\text{hel.}} |\mathcal{M}|^2 = |L_{+1}(\cos\theta_1)|^2 + |L_{+1}(-\cos\theta_1)|^2$$
$$+ |R_{+1}(\cos\theta_1)|^2 + |R_{+1}(-\cos\theta_1)|^2 , \quad (4)$$

where clearly Eq. (4) is symmetric in $\cos\theta_1$. Therefore a forward-backward asymmetry in the angle θ_1 is a signal of CP violation. Similarly, unbroken C implies $L_{\pm 1}(\cos\theta_1) = R_{\pm 1}(-\cos\theta_1)$, which implies that the forward-backward asymmetry also requires C violation.

Evaluating the diagram in Fig. 1, the helicity amplitudes from the intermediate $V = Z, \gamma$ are given by

$$\lambda_{\pm 1}^V = \mp g_{V,\lambda}\frac{(A_2^{V\gamma} \pm iA_3^{V\gamma})M_1(m_h^2 - M_1^2)}{2\sqrt{2}v(M_1^2 - m_V^2 + im_V\Gamma_V)}\left(1 \mp \kappa\cos\theta_1\right),$$
$$(5)$$

where $\lambda = R, L$ ands $\kappa = +1$ for $\lambda = R$ and -1 for $\lambda = L$. We have also defined M_1 is the invariant mass of the $\ell^- \ell^+$ pair. The couplings of the vector boson to left-handed and right-handed leptons are denoted as $g_{V,L}$ and $g_{V,R}$; for the photon we have $g_{V,L} = g_{V,R} = -e$. In this form we can easily see that the conditions for CP violating asymmetry are satisfied. More specifically,

- Two different intermediate particles, Z and γ, contribute to the same amplitudes.

- $\text{Arg}(A_2^{V\gamma} + iA_3^{V\gamma})$, $V = Z, \gamma$, provide different weak phases.

- $\text{Arg}(M_1^2 - m_V^2 + im_V\Gamma_V)$, $V = Z, \gamma$, give distinct strong phases.

It should be clear by now that the forward-backward asymmetry of the ℓ^- with respect to the z-axis in the $\ell^- \ell^+$ rest frame is a CP-violating observable. We write the differential decay width as,

$$\frac{d\Gamma}{dM_1^2 d\cos\theta_1} = \left(1 + \cos^2\theta_1\right)\frac{d\Gamma_{\text{CPC}}}{dM_1^2} + \cos\theta_1\frac{d\Gamma_{\text{CPV}}}{dM_1^2}. \quad (6)$$

The first term is CP conserving and symmetric in $\cos\theta_1$, whereas the second term violates CP and gives rise to the forward-backward asymmetry. The forward-backward asymmetry can now be computed:

$$A_{\text{FB}}(M_1) = \frac{\left(\int_0^1 - \int_{-1}^0\right) d\cos\theta_1 \frac{d\Gamma}{dM_1^2 d\cos\theta_1}}{\left(\int_0^1 + \int_{-1}^0\right) d\cos\theta_1 \frac{d\Gamma}{dM_1^2 d\cos\theta_1}}$$
$$= \frac{3}{8}\frac{d\Gamma_{\text{CPV}}/dM_1^2}{d\Gamma_{\text{CPC}}/dM_1^2}. \tag{7}$$

Focusing on the CPV contribution we find,

$$\frac{d\Gamma_{CPV}}{dM_1^2} = (A_2^{Z\gamma}A_3^{\gamma\gamma} - A_2^{\gamma\gamma}A_3^{Z\gamma}) \tag{8}$$
$$\times \frac{e(g_{Z,R} - g_{Z,L})m_Z\Gamma_Z(m_h^2 - M_1^2)^3}{512\pi^3 m_h^3 v^2\left((M_1^2 - m_Z^2)^2 + m_Z^2\Gamma_Z^2\right)}.$$

The expression is non-zero only in the presence of both CP-even and CP-odd Higgs couplings. Moreover, we are only sensitive to the products of the Higgs couplings to $Z\gamma$ and $\gamma\gamma$ since this is an interference effect between Z and γ. The condition of C violation is provided by the axial coupling of the Z boson to leptons (the Higgs couplings in Eq. (3) are C-even), hence the asymmetry is proportional to $(g_{Z,R} - g_{Z,L})$. The asymmetry vanishes in the limit when Γ_Z goes to zero, as then strong phases would be absent. On the left in Fig. 2 we plot the magnitudes of the symmetric and asymmetric parts of the differential width for a choice of parameters giving rise to SM signal strengths in $\Gamma(h \to Z\gamma)$ and $\Gamma(h \to \gamma\gamma)$. The shapes of the symmetric and asymmetric parts are very similar on the Z peak. The rise of the symmetric part for $M_1 \to 0$ is due to the intermediate photon contribution. On the right in Fig. 2 we show the differential asymmetry $A_{\text{FB}}(M_1)$ for the same choice of parameters. We can also define the total integrated asymmetry,

$$\bar{A}_{\text{FB}} \equiv \frac{3\int_{M_0}^{m_h} dM_1 M_1 \frac{d\Gamma_{\text{CPV}}}{dM_1}}{8\int_{M_0}^{m_h} dM_1 M_1 \frac{d\Gamma_{\text{CPC}}}{dM_1}}, \tag{9}$$

where the cut $M_1 > M_0$ on the minimum $\ell^-\ell^+$ invariant mass is necessary to cut off the IR divergence due to the intermediate photon. As long as M_0 is not too small, an accurate estimate can be obtained in the narrow width approximation and setting $A_{2,3}^{\gamma\gamma} \to 0$ in the symmetric part. This way we get,

$$\bar{A}_{\text{FB}} \approx \frac{\Gamma_Z}{m_Z}\frac{A_2^{Z\gamma}A_3^{\gamma\gamma} - A_2^{\gamma\gamma}A_3^{Z\gamma}}{(A_2^{Z\gamma})^2 + (A_3^{Z\gamma})^2}\frac{3e(g_{Z,R} - g_{Z,L})}{2(g_{Z,R}^2 + g_{Z,L}^2)}$$
$$\approx 0.07\frac{A_2^{Z\gamma}A_3^{\gamma\gamma} - A_2^{\gamma\gamma}A_3^{Z\gamma}}{(A_2^{Z\gamma})^2 + (A_3^{Z\gamma})^2} \tag{10}$$

Clearly, if the CP-odd couplings are of the same order as the CP-even ones, then the only parametric suppression of the asymmetry is by $\Gamma_Z/m_Z \sim 3\%$. The asymmetry can be larger if $A_2^{\check{Z}\gamma}$ is much below the SM value,

although that would require a cancellation between the SM W loop and new physics contributions to $h \to Z\gamma$.

To observe an asymmetry in this channel one must compete not only with the CP conserving part of the $h \to \ell^-\ell^+\gamma$ decay, but also with the much larger irreducible $q\bar{q} \to Z\gamma$ and reducible $Z + X$ (with X faking a photon) backgrounds. We estimate the expected the significance as follows. In Ref. [46] it was estimated that after cuts $\sigma_h \sim 1.3$ fb for the CP conserving $h \to \ell^-\ell^+\gamma$ decay and $\sigma_{ib} \sim 37$ fb for the irreducible background at $\sqrt{s} = 14$ TeV LHC. We assume here that the reducible background will be of the same order as the irreducible one, thus $\sigma_b \sim 2\sigma_{ib}$. Our signal is $S \sim A_{\text{FB}}\sigma_h L$, where L is the integrated luminosity, and the background is $B \sim (\sigma_h + \sigma_b)L$. Then the significance is given by,

$$\frac{S}{\sqrt{B}} \sim \left(\frac{\bar{A}_{\text{FB}}}{0.1}\right)\sqrt{\frac{L}{3000 \text{ fb}^{-1}}}. \tag{11}$$

This suggests the high-luminosity phase of the LHC would have a chance to observe this asymmetry, especially if a matrix element method analysis similar to what has been done in [42–44] is used to boost the sensitivity significantly. This direction is currently under study.

On the other hand, a similar estimate indicates one should be able to probe $A_{\text{FB}} \sim 0.05$ in a 100 TeV pp collider with 3000 fb^{-1} even using a simpler cut-based approach akin to Ref. [46].

CP VIOLATION IN OTHER PROCESSES

We move to discussing other processes exhibiting this new class of CP violating observables. In this section we restrict to order of magnitudes estimates of the asymmetry, and briefly comment on the discovery prospects.

First, we consider the $h \to \ell^-\ell^+ Z$ decay with an on-shell Z boson. This process is very similar to the $h \to \ell^-\ell^+\gamma$ decay discussed in the previous section, except that in this case the weak phases may originate from the Higgs couplings to $Z\gamma$ and to ZZ. The former were given in Eq. (3) and we parametrize the latter as,

$$\mathcal{L} \supset \frac{h}{4v}\left(A_1^{ZZ}Z^\mu Z_\mu + A_2^{ZZ}Z^{\mu\nu}Z_{\mu\nu} + A_3^{ZZ}Z^{\mu\nu}\widetilde{Z}_{\mu\nu}\right). \tag{12}$$

The new element here is the tree-level coupling A_1^{ZZ} which is expected to be much larger than the loop induced couplings A_2^i and A_3^i. Thus, the A_1^{ZZ} squared term will dominate the symmetric CP-conserving part of the differential width, while the interference with $A_3^{Z\gamma}$ will dominate the CP violating part. Thus, the forward-backward asymmetry parametrically behaves as,

$$\bar{A}_{\text{FB}}(h \to \ell^-\ell^+ Z) \sim \frac{\Gamma_Z}{m_Z}\frac{A_3^{Z\gamma}}{A_1^{ZZ}} \lesssim 10^{-3}. \tag{13}$$
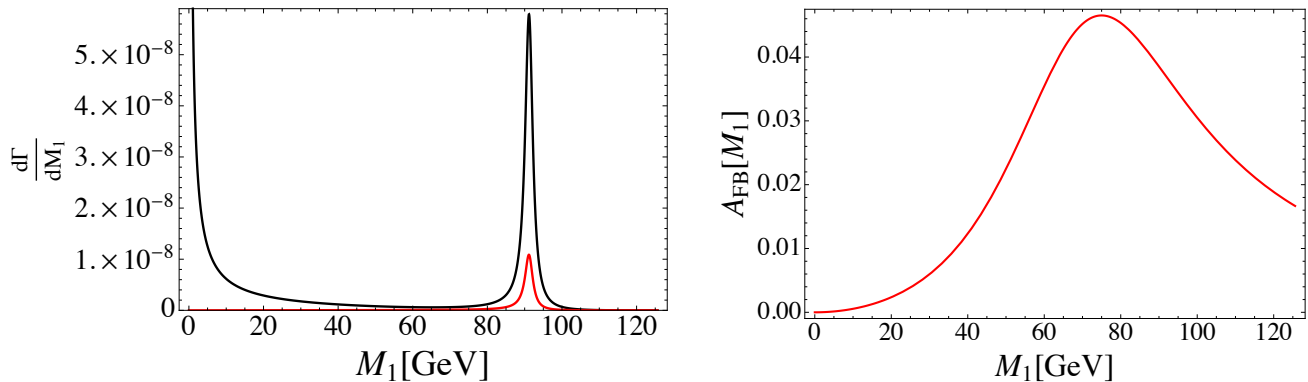
FIG. 2. **Left:** the differential decay rate $\frac{d\Gamma}{dM_1}$ for the symmetric (black) and the asymmetric part $\times 5$ (red) for $A_3^{Z\gamma} = A_{2\mathrm{SM}}^{Z\gamma}$, $A_2^{\gamma\gamma} = A_{2\mathrm{SM}}^{\gamma\gamma}$, $A_2^{Z\gamma} = A_3^{\gamma\gamma} = 0$. **Right:** For the same parameters, the dependence of the signal asymmetry on M_1.

The additional suppression by $A_3^{Z\gamma}/A_1^{ZZ} \sim 10^{-2}$ makes the asymmetry difficult to observe. Note that the closely related $h \to 4\ell$ process can also probe these tensor structures [35].

The CP violating asymmetry of the kind discussed here may also arise in 2-to-2 scattering of fermions into bosons. If one can distinguish the incoming and outgoing particle, then one possibility is to define the forward-backward asymmetry with respect to the scattering angle in the center-of-mass frame of the collision. One example is the process $e^-e^+ \to Z/\gamma \to hZ$ in an electron-positron collider. At the level of the amplitude, it is related to $h \to \ell^-\ell^+ Z$ by crossing symmetry. In this case we find,

$$\bar{A}_{\mathrm{FB}}(e^-e^+ \to hZ) \sim \frac{\Gamma_Z m_Z}{s} \frac{A_3^{Z\gamma}}{A_1^{ZZ}} \lesssim 10^{-4}. \qquad (14)$$

where $\sqrt{s}$ is the center-of-mass energy of the e^+e^- collision. We find the additional suppression factor of m_Z^2/s as compared to Eq. (13). This arises because the amplitude for producing a Higgs boson in association with a transverse Z is parametrically suppressed by $m_Z/\sqrt{s}$ compared to that with a longitudinal Z. Thus, the hZ production cross section is dominated by longitudinal Z, which does not give rise to the CP asymmetry. Due to that suppression, observing the asymmetry requires a large integrated luminosity, well beyond what is expected in the $\sqrt{s} = 250$ GeV phase of the ILC. Furthermore, the asymmetry becomes more difficult to observe as the collision energy is increased.

The same parametric dependence as in Eq. (14) applies for the process $q\bar{q} \to Z/\gamma \to hZ$ relevant for hadron colliders. The additional complication in this case is that the direction of the initial quark vs anti-quark can only be determined statistically, based on the boost of the hZ system in the laboratory frame. The asymmetry can be larger if the final state Z is replaced with a photon. For the $f\bar{f} \to Z/\gamma \to h\gamma$ process, both the symmetric and the asymmetric parts depend only on loop-induced couplings $A_{2,3}^i$. Moreover, only the transverse polarizations

of the final state vector boson are present. Assuming that the symmetric part is dominated by the intermediate Z exchange, we obtain the same parametric dependence as in the $h \to \ell^-\ell^+\gamma$ case:

$$\bar{A}_{\mathrm{FB}}(f\bar{f} \to h\gamma) \sim \frac{\Gamma_Z}{m_Z} \frac{A_2^{Z\gamma}A_3^{\gamma\gamma} - A_2^{\gamma\gamma}A_3^{Z\gamma}}{(A_2^{Z\gamma})^2 + (A_3^{Z\gamma})^2} \lesssim 10^{-1}. \qquad (15)$$

It might be interesting to look into this possibility at a 100 TeV pp collider.

All of the above examples have one common feature: CP transforms $\cos\theta \to -\cos\theta$, with θ $(\pi - \theta)$ defined by the direction of motion of a fermion f (or anti-fermion $\bar{f}$) with respect to one of the bosons in the process. This can be traced to the fact that, while f transforms to $\bar{f}$, the bosons in these processes are neutral and transform to themselves under CP (up to a helicity flip for vectors). The consequence is that the forward-backward asymmetry is a CP violating observable. The situation would be different if *both* particle pairs were CP-conjugate. For example, in the processes $f\bar{f} \to W^+W^-$ and $f\bar{f} \to f'\bar{f}'$, CP leaves θ invariant which allows a forward-backward asymmetry to arise without CP violation.

In principle, an asymmetry of the kind discussed here can also be induced by CP violating Higgs couplings to fermions in processes such as $f\bar{f} \to h \to Z\gamma$ (s-channel) interfering with $f\bar{f} \to Z\gamma$ (t-channel). In practice, however, the asymmetry is suppressed by the fermion mass and by the Higgs width, therefore it is too small to be observable.

CONCLUSIONS

In this work we proposed a new class of CP violating observables in Higgs physics without the necessity to construct triple product observables. These observables can be applied to either three-body decays or 2-to-2

scattering processes involving a Higgs boson at either a hadron or a lepton collider. They allow measurements of CP violating Higgs couplings to Z and γ gauge boson pairs as well as in principle to fermions. Given that the amount of CP violation in the SM is insufficient to generate the observed baryon asymmetry in the universe and that any observation of CP violation in the Higgs sector would be a sign of physics beyond the Standard Model, searching for these additional sources of CP violation would be of utmost importance in current and future colliders. We leave a careful study on the sensitivity and reach of this class of observables to future work.

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