A composite Higgs model with minimal fine-tuning: The large-N and weak-technicolor limit

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A Composite Higgs Model with Minimal Fine-Tuning
I. The Large-N and Weak-technicolor Limit

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Abstract

We suggest a criterion to minimize the amount of fine-tuning in a composite Higgs model. The paradigm of this type of model is the top-condensate model of Bardeen, Hill and Lindner (BHL). Although "minimally fine-tuned", this model failed to account correctly for the masses of the top quark and the 125 GeV Higgs boson. We propose a generalization of the BHL model that employs finely-tuned extended technicolor (ETC) plus technicolor (TC) interactions. The additional freedom of this model may accommodate both $m_t(173)$ and $M_H(125)$. This paper studies the large-N_{TC} and N_C limit of this model in which technicolor is weak and does not contribute to electroweak symmetry breaking. Refinements including walking-TC dynamics and a renormalization group analysis of m_t and M_H will appear in a subsequent paper. A likely generic signal of this model is enhanced production of longitudinally-polarized weak bosons, alone and in association with $H(125)$

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1. Introduction and Plan

The concept of naturalness in particle physics is over 40 years old [1, 2] and the quest for a natural theory of electroweak symmetry breaking (EWSB) has been a dominant theme for almost that long [3, 4]. This is because the *elementary* Higgs boson [5, 6, 7] introduced to trigger gauge symmetry breaking and give mass to the W and Z in the standard electroweak model [8, 9] is so very unnatural. There is no cut-off to the quadratically divergent corrections to its squared mass this side of the Planck scale. The discovery at the CERN LHC of a 125 GeV Higgs boson, $H(125)$ [10, 11], possibly the lone Higgs boson of the standard model, has left supersymmetric and composite models of a light Higgs boson as the only remaining approaches to naturalness. Both involve a new energy scale Λ — either the scale of supersymmetry breaking or the scale of the new strong dynamics binding the composite Higgs — that serves to cut off the corrections to M_H^2 at $\mathcal{O}(\Lambda^2/16\pi^2)$ or, perhaps, $\mathcal{O}(\Lambda^2/(16\pi^2)^2)$. Thus, Λ (or $\Lambda/4\pi$) must not be larger than about 1 TeV in order that the theory is natural. Generally, this is achieved by having the standard quadratic divergence in M_H^2 from the top quark (and weak bosons) canceled by contributions from partners of the top (and W, Z). The failure, so far, to find these partners at masses below 1-2 TeV[1], has put considerable stress on both supersymmetric and composite Higgs models. All such models and, in particular, composite Higgs ones — the subject of this paper — require a degree of fine-tuning of parameters that calls their "naturalness" into serious question [12, 13]

Therefore, in order to maintain the hypothesis that $H(125)$ is a fermion-antifermion composite, I will provisionally adopt a "principle of least unnaturalness": *the least unnatural description of a composite* $H(125)$ *is one that involves the smallest fine-tuning for* M_H^2 and *the fewest number of free parameters that must be fine-tuned to achieve this.*

The paradigm of this sort of light composite Higgs description is the topcolor model of Bardeen, Hill and Lindner (BHL) [14]. In their model, $q_L = (t, b)_L$ and t_R are assumed to have a new strong (presumably broken gauge) interaction at some high scale Λ, giving rise to the $SU(2) \otimes U(1)$-invariant four-fermion interaction $\mathcal{L}_{\bar{t}t}$ at energies below Λ,

$$\mathcal{L}_{\bar{t}t} = G\, \bar{q}_L^{ia}\, t_{Ra}\; \bar{t}_R^{b}\, q_{Lib}\,. \tag{1}$$

Here, the $SU(2)_{EW}$ and color-$SU(3)_C$ indices, i and a, b, are summed over; the coupling $G = \mathcal{O}(1/\Lambda^2)$. This Nambu–Jona-Lasinio (NJL) interaction produces the top-quark mass m_t *and* a $\bar{q}_L t_R$ composite scalar doublet ϕ if G satisfies

$$\frac{G N_C}{8\pi^2}\left(\Lambda^2 - m_t^2 \ln\frac{\Lambda^2}{m_t^2}\right) = 1, \text{ i.e., } G > G_c = \frac{8\pi^2}{N_C \Lambda^2}\,. \tag{2}$$

[1] https://twiki.cern.ch/twiki,bin/view/CMSPublic/PhysicsResulsB2G,
https:/twiki.cern.ch/twiki/bin/view/CMSPublic/ResultsSUS,
https:/twiki.cern.ch/twiki/bin/view/ATLASPublic/ExoticsPublicResults,
https:/twiki.cern.ch/twiki/bin/view/ATLASPublic/SupersymmetricPublicResults.

Here, Λ is also the cutoff of the momentum integral defining m_t in the NJL-bubble, or large-N_C, approximation to its gap equation, while $N_C = 3$ is the number of ordinary colors.[2] The composite scalar is a complex doublet under $SU(2)_{EW}$. It consists of three massless Goldstone bosons, eaten by W and Z, and a Higgs boson H of mass $M_H = 2m_t$.

It is clear from Eq. (2) that m_t and M_H can be nonzero but very much less than Λ if and only if G is greater than but very close to G_c. This is the fine-tuning of the BHL model, but it is the model's <u>only</u> fine-tuning. Once it is imposed, all other Λ-dependence is logarithmic. Thus, even though Λ is very large in BHL, the model exemplifies our notion of being least unnatural.

The low-energy Lagrangian describing H interactions with q_L, t_R and the EW gauge bosons is just the standard-model Lagrangian [14]. In that formulation, the negative $\mathcal{O}(\Lambda^2)$ contribution to M_H^2 and the $\ln \Lambda^2$ contribution to its quartic self-interaction are induced by the Yukawa interaction $\Gamma_t\, \bar{t}tH$, where Γ_t is obtained from the residue of the Higgs pole in the $\bar{t}t \to \bar{t}t$ amplitude in the 0^+ channel. Then, the Higgs vacuum expectation value (vev) v is determined in the usual way from the quartic scalar coupling $\lambda \sim \Gamma_t^4 N_C \ln \Lambda^2/16\pi^2$ and the negative M_H^2. The value of v is set by $M_W = \frac{1}{2}gv$, and then $m_t = \Gamma_t v/\sqrt{2}$. Thus, m_t, M_H and $M_{W,Z}$ are all closely related in the BHL model. It is the most minimal *dynamical* model of electroweak symmetry breaking.

The renormalization group equations for the Yukawa coupling Γ_t of $\bar{t}tH$, the quartic coupling λ, and the SM gauge couplings $g_{1,2,3}$ result in a significant reduction of m_t and M_H/m_t, with smaller values obtained for larger Λ. Unfortunately, even for $\Lambda = 10^{15}\,\mathrm{GeV}$, BHL obtained $m_t = 229\,\mathrm{GeV}$ and $M_H = 256\,\mathrm{GeV}$. Still, the importance of the BHL model is that it suggests a connection between the relatively large value of the top-quark mass and the lightness of the Higgs boson.

The purpose of this work is to ameliorate the fine-tuning of the BHL model and to obtain masses for the Higgs boson and top quark that are closer to their measured values. I demonstrate this in a simple model of technicolor (TC) plus *strong* extended technicolor (ETC). Technicolor with weakly-coupled extended technicolor, cannot account for the large value of m_t. But, if ETC is strong with its four-fermion coupling g_{ETC}^2/M_{ETC}^2 finely-tuned, it can produce a large m_t that is much smaller than the mass scale M_{ETC} of the ETC boson giving rise to $\mathcal{L}_{\bar{t}t}$ [15, 16]. This is similar to the BHL model, but now the relevant scale, $\Lambda \simeq M_{ETC}$, is expected to be $\mathcal{O}(10)$–$\mathcal{O}(100)\,\mathrm{TeV}$, much lower than the BHL scale.[3] Furthermore, as shown in Ref. [17], the symmetry breaking phase transition must be second order. But, then, it implies the existence of a composite complex-scalar doublet ϕ with three Goldstone bosons and a massive but light scalar that couples strongly to the top quark, *exactly as in the BHL model.* This scalar will be our candidate for $H(125)$. In our model,

[2] In the UV-complete model with Lagrangian $\mathcal{L}_{\bar{t}t}$ there is no quadratic divergence from the $HHWW$ vertex with a W-loop. Quadratic divergences involving weak-boson exchange do occur in subleading order in $1/N_C$.

[3] Relatively low masses for ETC bosons generating third-generation masses need not conflict with limits on flavor-changing neutral current interactions.

the Higgs boson is a composite of $\bar{t}t$ and $\bar{U}U$, where U is the up-component of a technifermion doublet $T = (U, D)$.

In this paper, we treat this TC-ETC model in the large-N (N_C, N_{TC}) limit. In a realistic model of this type, we expect TC to be strong enough to participate in EWSB. That is, its coupling α_{TC} is near an infrared fixed point of its β-function [18, 19] and, so, it evolves slowly [20, 21, 22, 23] and reaches the critical value α_c for chiral symmetry breaking at a scale Λ_{TC} of order several hundred GeV. This seems necessary to account for light quark and lepton masses induced by ETC bosons with M_{ETC} of many 100's of TeV [24], for then the relevant technifermion condensates at M_{ETC} are enhanced by a large anomalous dimension, $\gamma_m \simeq 1$ [25].

Including a walking α_{TC} in our analysis is a complication we will defer to a subsequent paper. The renormalization group running down from Λ of fermion and Higgs masses referred to above will also be deferred. A further simplification is that light quark and lepton masses are left out; their inclusion is not technically difficult. The phenomenology of this TC-ETC model will be developed in a third paper (see Sec. 6 for a brief foretaste).

This model has features that might allow it to account better for the Higgs and top masses than the BHL model does. First, the technifermion contribution to the composite Higgs loosens the tight connection among m_t, M_W and M_H. Second, there are now two large Yukawa couplings of the composite Higgs to fermions, Γ_t to $\bar{t}t$ and Γ_U to $\bar{U}U$. The renormalization group equations for Γ_U will involve the strong walking gauge coupling of technicolor when that is included in the model. Third — and this is a point we are uncertain about — in addition to the light scalar H induced by fine-tuning, there may be a lightest 0^{++} technihadron bound state. This state could mix with H and drive down its mass, and certainly otherwise complicate the model's phenomenology. This possibility will be considered in the third paper of this series.

In the remainder of this paper, then, we discuss the composite model in the large-N, weak-TC limit. We assume that all of EWSB comes from the single composite Higgs boson of the model, i.e., its vev is $v = 246\,\mathrm{GeV}$. Our development follows that in BHL. In Sec. 2 a model Lagrangian is presented and used to calculate the dynamical masses m_t and m_U at the scale Λ. The $\bar{q}q$ and $\bar{T}T$ scattering amplitudes are computed in Sec. 3 and their scalar and pseudoscalar (Goldstone) poles are revealed. We find that M_H (at scale Λ) generically lies between $2m_t$ and $2m_U$. The electroweak gauge boson propagators in $\mathcal{O}(g_{1,2}^2)$ and large-N approximations, including their Goldstone pole contributions, are computed in Sec. 4. A numerical study of the $2 \to 2$ scattering amplitudes in the scalar channel and the value of M_H in one fitting scheme are presented in Sec. 5. We also calculate Γ_t and the Higgs vev v from the residue of the Higgs pole in the $\bar{t}t \to \bar{t}t$ amplitude. Section 6 includes preliminary comments on the model's bound-state spectrum that may be of use to experimentalists. In particular, the possibility that weak boson production is enhanced by ρ_T and ω_T states is discussed. We summarize the large-N, weak-TC results and what remains to be done in Sec. 7.

There has been much previous work using the NJL mechanism to describe the Higgs boson, including Refs. [26, 27, 28] which preceded BHL in involving a new strong interaction of top quarks as the dynamics of EWSB. Topcolor led to the so-called top-seesaw models of Dobrescu and Hill [29] and Chivukula, et al. [30] and, more recently, Refs. [31, 32].[4] Bar-Shalom and collaborators proposed a "hybrid model" with a dynamical Higgs-like scalar plus an elementary scalar to describe $H(125)$ [33, 34]. They used an NJL Lagrangian with fourth generation quarks interacting via a topcolor interaction with scale $\Lambda \sim 1\,\text{TeV}$ to generate the dynamical scalar. Apart from the use of NJL in the bubble approximation, these models do not resemble ours, and the use of fourth generation quarks is reminiscent of the top-seesaw mechanism. Finally, as this paper was being written, there appeared one by Di Chiara, et al., who proposed a model of $H(125)$ based on TC and ETC, using a Lagrangian which is a truncated version of that introduced in Sec. 2 [35]. This model bears no further resemblance to ours; in particular, and among other things, strong, fine-tuned ETC is not employed in their paper to make the Higgs boson much lighter than the TC and ETC scales.

2. The TC-ETC Model in the Large-N Approximation.

The fermions in this model transform under electroweak $(SU(2)\otimes U(1))_{EW}$, ordinary color $SU(3)_C$ and technicolor $SU(N_{TC})$, and they are

$$\begin{aligned}
q_L &= \begin{pmatrix} t \\ b \end{pmatrix}_L \in (\mathbf{2}, \tfrac{1}{6}, \mathbf{3}, \mathbf{1})\,, \qquad t_R \in (\mathbf{1}, \tfrac{2}{3}, \mathbf{3}, \mathbf{1})\,, \quad b_R \in (\mathbf{1}, -\tfrac{1}{3}, \mathbf{3}, \mathbf{1})\,, \\
T_L &= \begin{pmatrix} U \\ D \end{pmatrix}_L \in (\mathbf{2}, 0, \mathbf{1}, \boldsymbol{R}_{TC})\,, \qquad U_R \in (\mathbf{1}, \tfrac{1}{2}, \mathbf{1}, \boldsymbol{R}_{TC})\,, \quad D_R \in (\mathbf{1}, -\tfrac{1}{2}, \mathbf{1}, \boldsymbol{R}_{TC})\,,
\end{aligned} \tag{3}$$

where $\boldsymbol{R}_{TC}$ is a complex representation of $SU(N_{TC})$. As explained above, light quarks and leptons are not dealt with in this paper. Likewise, additional technifermions are not included here.

The *hard* masses of t and U are generated by ETC interactions at a scale $\Lambda \simeq M_{ETC} = \mathcal{O}(10)$–$\mathcal{O}(100)\,\text{TeV}$. At energies below Λ, the effective interaction is taken to be a sum of terms similar to the BHL Lagrangian, $\mathcal{L}_{\bar{t}t}$, in Eq. (1):

$$\mathcal{L}_{ETC} = G_1\, \bar{q}_L^{ia}\, t_{Ra}\, \bar{t}_R^b\, q_{Lib} + G_2\, \left(\bar{q}_L^{ia}\, t_{Ra}\, \bar{U}_R^{\alpha}\, T_{Li\alpha} + \text{h.c.}\right) + G_3\, \bar{T}_L^{i\alpha}\, U_{R\alpha}\, \bar{U}_R^{\beta}\, T_{Li\beta}\,, \tag{4}$$

where the $SU(2)_{EW}$ and color-$SU(3)_C$ and $SU(N_{TC})$ indices, i and a, b, and α, β are summed over. This interaction is to be thought of as having been Fierzed from an ETC interaction involving left times right-handed currents. The $SU(3)_C$ and $SU(N_{TC})$ indices appearing here therefore cannot correspond to exchange of ordinary massless color and TC gluons. The couplings $G_{1,2,3}$ are nominally positive and of $\mathcal{O}(1/\Lambda^2)$. In this simplest form of our TC-ETC model, the D-technifermion is assumed to get no, or at least negligible, hard mass

[4] The last two papers contain a large bibliography of related work.

from ETC. Then in the neglect of EW interactions, this model has an $(SU(2)_L \otimes U(1)_R)_q \otimes (SU(2)_L \otimes U(1)_R)_T$ flavor symmetry which is explicitly broken to $SU(2) \otimes U(1)$ by the G_2 term in $\mathcal{L}_{ETC}$. *If* $\mathcal{L}_{ETC}$ generates *both* $\bar{t}t$ and $\bar{U}U$ condensates *and* $G_2 \neq 0$, this flavor symmetry is spontaneously broken to $U(1)$ and just three Goldstone bosons appear. We shall see in Sec. 6 that all three G_i are comparable and that G_2 is not weak. Therefore, there are *not* three relatively light pseudo-Goldstone bosons.

It is not difficult to add terms that generate $m_D \neq 0$, but not so easy to maintain $m_D = m_U$ in this model.[5] For example, adding

$$G_3\, \bar{T}_L^{i\alpha}\, D_{R\alpha}\;\, \bar{D}_R^{\beta}\, T_{Li\beta} \tag{5}$$

to $\mathcal{L}_{ETC}$ gives an $(SU(2)_L \otimes SU(2)_R)_T$ invariant interaction and $m_D = m_U$ only if $G_2 = 0$. But, then, there is an unacceptable triplet of very light pseudo-Goldstone bosons (see Sec. 6). Adding instead

$$G_2\,\left(\bar{q}_L^{ia}\, b_{Ra}\;\, \bar{D}_R^{\alpha}\, T_{Li\alpha} + \text{h.c.}\right) + G_3\, \bar{T}_L^{i\alpha}\, D_{R\alpha}\;\, \bar{D}_R^{\beta}\, T_{Li\beta} \tag{6}$$

generates $m_b \neq 0$ as well as $m_D \neq 0$. These masses will differ from m_t and m_U, respectively. But that does not necessarily upset the observed closeness of the ρ-parameter to one. Further analysis of such a model is beyond our scope in this paper.

Following BHL, the gap equations for m_t and m_U *renormalized at the scale* Λ are (see the Appendix) [6,7]

$$\begin{aligned} m_t &= -\tfrac{1}{2} G_1 \langle \bar{t}t \rangle - \tfrac{1}{2} G_2 \langle \bar{U}U \rangle \\ &= \frac{G_1 N_C m_t}{8\pi^2}\left(\Lambda^2 - m_t^2 \ln\frac{\Lambda^2}{m_t^2}\right) + \frac{G_2 N_{TC} m_U}{8\pi^2}\left(\Lambda^2 - m_U^2 \ln\frac{\Lambda^2}{m_U^2}\right)\,; \end{aligned} \tag{7}$$

$$\begin{aligned} m_U &= -\tfrac{1}{2} G_2 \langle \bar{t}t \rangle - \tfrac{1}{2} G_3 \langle \bar{U}U \rangle \\ &= \frac{G_2 N_C m_t}{8\pi^2}\left(\Lambda^2 - m_t^2 \ln\frac{\Lambda^2}{m_t^2}\right) + \frac{G_3 N_{TC} m_U}{8\pi^2}\left(\Lambda^2 - m_U^2 \ln\frac{\Lambda^2}{m_U^2}\right)\,. \end{aligned} \tag{8}$$

Here, N_{TC} is the dimensionality of the T-representation $\boldsymbol{R}_{TC}$. So long as $G_2 \neq 0$ — which we assume throughout this paper — the independence of the N_C and N_{TC} imply that (just multiply Eq. (7) by m_U and Eq. (8) by m_t)

$$G_2 = G_1 \frac{m_U}{m_t} = G_3 \frac{m_t}{m_U}\,. \tag{9}$$

[5] I thank Sekhar Chivukula for the conversation that led to this paragraph.

[6] As did BHL, we assume that the condensates $\langle \bar{t}\gamma_5 t\rangle = \langle \bar{U}\gamma_5 U\rangle = 0$.

[7] The gap equations approximated in Eqs. (7,8) are integrals over an ETC boson propagator with mass Λ times the mass term in the t or U propagator. In a walking-α_{TC} model, the U-mass term has both a dynamical piece, falling off roughly as $1/p$ above the technicolor scale Λ_{TC} and a hard piece $m_U(p)$ that is constant up to $p \simeq \Lambda \gg \Lambda_{TC}$. In accord with our weak-TC assumption, the dynamical piece is ignored. In any case, the hard-mass term will dominate the integral unless $m_U \ll \Lambda_{TC}^2/\Lambda$.

Then, Eqs. (7–9) imply the following generalization of the ‘fine-tuning condition in Eq. (2):

$$\frac{G_1 N_C}{8\pi^2}\left(\Lambda^2 - m_t^2 \ln\frac{\Lambda^2}{m_t^2}\right) + \frac{G_3 N_{TC}}{8\pi^2}\left(\Lambda^2 - m_U^2 \ln\frac{\Lambda^2}{m_U^2}\right) = 1\,. \tag{10}$$

The mass parameters in this weak-TC model, m_t, m_U, M_W, M_H and Λ are not independent. If m_t and m_U are nonzero, only one of the three G_i is an independent parameter.

As in BHL, Eqs. (7,8) contain this model's only quadratic divergences and, for nonzero $m_t, m_U \ll \Lambda$, its only fine-tuning of parameters. Once Eq. (10) is enforced, all other Λ-dependence is logarithmic.

3. The $2 \to 2$ Amplitudes in the Scalar and Goldstone Boson Channels

We again follow BHL to calculate the $2 \to 2$ amplitudes in the scalar and pseudoscalar channels. For the neutral scalar channel, there are three amplitudes to calculate: $\bar{t}t \to \bar{t}t$, $\bar{U}U \to \bar{U}U$ and $\bar{t}t \leftrightarrow \bar{U}U$. The effective Hamiltonian for the bubble is

$$\mathcal{H}_{0^+} = -\tfrac{1}{4}G_1 \bar{t}^a t_a\, \bar{t}^b t_b - \tfrac{1}{2}G_2 \bar{t}^a t_a\, \bar{U}^\alpha U_\alpha - \tfrac{1}{4}G_3 \bar{U}^\alpha U_\alpha\, \bar{U}^\beta U_\beta\,. \tag{11}$$

The $\bar{t}t \to \bar{t}t$ amplitude is

$$\begin{aligned}\Gamma^{\bar{t}t}_{0^+}(p) = -\tfrac{1}{2}G_1 - (-\tfrac{1}{2}G_1)^2 i \textstyle\int d^4x\, e^{ip\cdot x}\langle\Omega|T(\bar{t}^a t_a(x)\,\bar{t}^b t_b(0)|\Omega\rangle \\ -(-\tfrac{1}{2}G_2)^2 i \textstyle\int d^4x\, e^{ip\cdot x}\langle\Omega|T(\bar{t}^a U_\alpha(x)\,\bar{U}^\alpha t_a(0)|\Omega\rangle + \cdots\end{aligned} \tag{12}$$

The integrals are cut off at Λ and evaluated in the Appendix. The relation $G_2^2/G_1 = G_3$ makes this sum a geometric series,

$$\begin{aligned}\Gamma^{\bar{t}t}_{0^+}(p) \;=\; & -\tfrac{1}{2}G_1\Bigg[1 - \tfrac{G_1 N_C}{8\pi^2}\left(\Lambda^2 - m_t^2\ln\tfrac{\Lambda^2}{m_t^2}\right) - \tfrac{G_3 N_{TC}}{8\pi^2}\left(\Lambda^2 - m_U^2\ln\tfrac{\Lambda^2}{m_U^2}\right) \\ & -\frac{G_1 N_C (p^2 - 4m_t^2)}{16\pi^2}\int_0^1 dx\,\ln\left(\frac{\Lambda^2}{m_t^2 - p^2 x(1-x)}\right) \\ & -\frac{G_3 N_{TC}(p^2 - 4m_U^2)}{16\pi^2}\int_0^1 dx\,\ln\left(\frac{\Lambda^2}{m_U^2 - p^2x(1-x)}\right)\Bigg]^{-1}.\end{aligned} \tag{13}$$

By Eq.(10), the first line on the left is zero and $\Gamma^{\bar{t}t}_{0^+}$ becomes

$$\begin{aligned}\Gamma^{\bar{t}t}_{0^+}(p) \;=\; & m_t^2\Bigg[\frac{N_C m_t^2(p^2 - 4m_t^2)}{8\pi^2}\int_0^1 dx\,\ln\left(\frac{\Lambda^2}{m_t^2 - p^2x(1-x)}\right) \\ & +\frac{N_{TC} m_U^2 (p^2 - 4m_U^2)}{8\pi^2}\int_0^1 dx\,\ln\left(\frac{\Lambda^2}{m_U^2 - p^2x(1-x)}\right)\Bigg]^{-1}.\end{aligned} \tag{14}$$

The scalar-channel amplitudes for $\bar{U}U \to \bar{U}U$ and $\bar{t}t \leftrightarrow \bar{U}U$ are $\Gamma_{0^+}^{\bar{U}U} = (m_U/m_t)^2\Gamma_{0^+}^{\bar{t}t}$ and $\Gamma_{0^+}^{\bar{t}t\leftrightarrow\bar{U}U} = (m_U/m_t)\Gamma_{0^+}^{\bar{t}t}$. Then the sum of the four $2 \to 2$ amplitudes in the neutral scalar channel is

$$\begin{aligned}\Gamma_{0^+}(p) \quad &= \quad (m_t + m_U)^2\left[\frac{N_C m_t^2(p^2 - 4m_t^2)}{8\pi^2}\int_0^1 dx\, \ln\left(\frac{\Lambda^2}{m_t^2 - p^2x(1-x)}\right)\right. \\ &\left. +\frac{N_{TC} m_U^2(p^2 - 4m_U^2)}{8\pi^2}\int_0^1 dx\, \ln\left(\frac{\Lambda^2}{m_U^2 - p^2x(1-x)}\right)\right]^{-1} . \end{aligned} \tag{15}$$

The scalar amplitude has a pole at $p^2 = M_H^2$, the solution of

$$\begin{aligned}&N_C m_t^2(M_H^2 - 4m_t^2)\int_0^1 dx\, \ln\left(\frac{\Lambda^2}{m_t^2 - M_H^2x(1-x)}\right) \\ &+N_{TC} m_U^2(M_H^2 - 4m_U^2)\int_0^1 dx\, \ln\left(\frac{\Lambda^2}{m_U^2 - M_H^2x(1-x)}\right) = 0\,. \end{aligned} \tag{16}$$

This result, the Higgs mass at scale Λ in the large-N approximation, will be modified by renormalization-group running from Λ down to the H-pole. The BHL mass, $M_H(\Lambda) = 2m_t$, is obtained by setting $m_U = 0$ in Eq. (16). A very good approximation to the solution to Eq. (16) is

$$M_H = 2\sqrt{\frac{N_C m_t^4 + N_{TC} m_U^4}{N_C m_t^2 + N_{TC} m_U^2}} \tag{17}$$

The effective Hamiltonian for the neutral and charged Goldstone poles in $2 \to 2$ scattering is

$$\begin{aligned}\mathcal{H}_{0^-} \quad &= \quad \tfrac{1}{4}G_1\bar{t}^a\gamma_5 t_a\,\bar{t}^b\gamma_5 t_b + \tfrac{1}{2}G_2\bar{t}^a\gamma_5 t_a\,\bar{U}^\alpha\gamma_5 U_\alpha + \tfrac{1}{4}G_3\bar{U}^\alpha\gamma_5 U_\alpha\,\bar{U}^\beta\gamma_5 U_\beta \\ &- G_1\,\bar{b}_L^a\,t_{Ra}\,\bar{t}_R^b\,b_{Lb} - G_2\,\left(\bar{b}_L^a\,t_{Ra}\,\bar{U}_R^\alpha\,D_{L\alpha} + \text{h.c.}\right) - G_3\,\bar{D}_L^\alpha\,U_{R\alpha}\,\bar{U}_R^\beta\,D_{L\beta}\,. \end{aligned} \tag{18}$$

The $\bar{t}t \to \bar{t}t$ amplitude is (note the i's in $i\gamma_5$, left out in BHL):

$$\Gamma_{0^-}^{\bar{t}t}(p) = -\tfrac{1}{2}G_1 - (\tfrac{1}{2}G_1)^2 i\int d^4x\, e^{ip\cdot x}\langle\Omega|T(\bar{t}^a i\gamma_5 t_a(x)\,\bar{t}^b i\gamma_5 t_b(0)|\Omega\rangle + \cdots\,. \tag{19}$$

Proceeding as in the scalar case. the sum of the $2 \to 2$ amplitudes in the neutral channel is

$$\begin{aligned}\Gamma_{0^-}^0(p) \quad &= \quad \frac{8\pi^2(m_t + m_U)^2}{p^2}\left[N_C m_t^2\int_0^1 dx\, \ln\left(\frac{\Lambda^2}{m_t^2 - p^2x(1-x)}\right)\right. \\ &\left. +N_{TC} m_U^2\int_0^1 dx\, \ln\left(\frac{\Lambda^2}{m_U^2 - p^2x(1-x)}\right)\right]^{-1} . \end{aligned} \tag{20}$$

The corresponding amplitude in the charged $t\bar{b} \to t\bar{b}$ channel is

$$\Gamma_{0^-}^{t\bar{b}}(p) = -\tfrac{1}{4}G_1 - (G_1)^2 i\int d^4x\, e^{ip\cdot x}\langle\Omega|T(\bar{t}_R^a b_{La}(x)\,\bar{b}_L^b t_{Rb}(0)|\Omega\rangle + \cdots\,. \tag{21}$$

Including the other channels, these sum up to

$$\Gamma^{\pm}_{0^-}(p) = \frac{2\pi^2(m_t+m_U)^2}{p^2}\Bigg[N_C m_t^2\int_0^1 dx\, x\ln\left(\frac{\Lambda^2}{m_t^2 x - p^2 x(1-x)}\right) + N_{TC} m_U^2\int_0^1 dx\, x\ln\left(\frac{\Lambda^2}{m_U^2 x - p^2 x(1-x)}\right)\Bigg]^{-1}. \qquad (22)$$

The manipulations [14] used to obtain these results are given in the Appendix.

As noted above, there may be an isotriplet of pseudo-Goldstone bosons that acquire mass from the G_2-interaction. This is discussed briefly in Sec. 6 and considered in more detail in a later paper.

4. The Electroweak Gauge Boson Propagators

In this section we compute the EW propagators in the NJL-bubble approximation, neglecting EW gauge-boson radiative corrections. As in BHL, the EW fields are rescaled to bring the gauge coupling into their kinetic terms, i.e., $(1/4g^2)F^2_{\mu\nu}$. The $(SU(2)\otimes U(1))_{EW}$ currents are

$$\begin{aligned} j^A_\mu &= \bar q_L\gamma_\mu\frac{\tau_A}{2}q_L + \bar T_L\gamma_\mu\frac{\tau_A}{2}T_L \qquad (A=1,2,3)\,;\\ j^0_\mu &= \tfrac{1}{6}\left(\bar q_L\gamma_\mu q_L + \bar q_R\gamma_\mu q_R\right) + \bar q_R\gamma_\mu\frac{\tau_3}{2}q_R + \bar T_R\gamma_\mu\frac{\tau_3}{2}T_R\,. \end{aligned} \qquad (23)$$

The inverse W-propagator is

$$\begin{aligned} \frac{1}{g_2^2}(D^{\pm}(p))^{-1}_{\mu\nu} &= \frac{1}{g_2^2}(p_\mu p_\nu - p^2 g_{\mu\nu}) + \frac{i}{2}\int d^4x\, e^{ip\cdot x}\langle\Omega|T(j^{(1+i2)}_{L\mu}(x)\, j^{(1-i2)}_{L\nu}(0))|\Omega\rangle\\ &= (p_\mu p_\nu - p^2 g_{\mu\nu})\left(\frac{1}{g^2_{2W}(p^2)} - \frac{f^2_W(p^2)}{p^2}\right). \end{aligned} \qquad (24)$$

In the second line of Eq. (24), $g^{-2}_{2W}(p^2)$ is computed from the bare inverse W-propagator plus the one-loop correlator $\Pi^{\pm}_{\mu\nu}(p)$ of a pair of charged weak currents. It is given in the TC-ETC model by (see the Appendix for details)

$$\begin{aligned} g^{-2}_{2W}(p^2) &= g_2^{-2} + \frac{1}{16\pi^2}\int_0^1 dx\, 2x(1-x)\Bigg[N_C\ln\left(\frac{\Lambda^2}{m_t^2 x - p^2 x(1-x)}\right)\\ &+ N_{TC}\ln\left(\frac{\Lambda^2}{m_U^2 x - p^2 x(1-x)}\right)\Bigg]. \end{aligned} \qquad (25)$$

The contribution to the W-propagator from the massless pole in $\Gamma^{\pm}_{0^-}$ in Eq. (22) is

$$\begin{aligned} f^2_W(p^2) &= \frac{1}{16\pi^2}\int_0^1 dx\, x\Bigg[N_C\, m_t^2\ln\left(\frac{\Lambda^2}{m_t^2 x - p^2 x(1-x)}\right)\\ &+ N_{TC}\, m_U^2\ln\left(\frac{\Lambda^2}{m_U^2 x - p^2 x(1-x)}\right)\Bigg]. \end{aligned} \qquad (26)$$

A comment is in order here: The fermion masses in the one-loop-EW and fermion-bubble-sum contributions to the weak-boson propagators must be the hard masses generated by $\mathcal{L}_{ETC}$. It is these masses that satisfy the gap Eqs. (7,8), and those relations are used to remove the Λ^2-dependence from the bubble sums. Furthermore, the masses in the $m^2 g_{\mu\nu}$ part of the EW loop must be the same as those in the $m^2 p_\mu p_\nu/p^2$ terms coming from the bubble sum in order that Ward-Takahashi (WT) identities are maintained and the propagators are transverse. Therefore, in accord with the model defined by $\mathcal{L}_{ETC}$, $m_b = M_D = 0$ in $g^2_{2W}(p^2)$ and $f^2_W(p^2)$. A more complete treatment of the propagators will include the dynamical strong-TC contributions to the Goldstone poles and, to satisfy the WT identities, the fermion masses. These do not have $\ln\Lambda^2$ dependence as they are cut-off by TC dynamics (mainly) at scale Λ_{TC}. This more complicated analysis is deferred to a later paper. The upshot of all this is that in the weak-TC limit all of EWSB comes from $\mathcal{L}_{ETC}$ and, since there is just one complex Higgs doublet,

$$f^2_W(0) = 1/(4\sqrt{2}G_F) = (123\,\mathrm{GeV})^2 \;\cong M^2_W/g^2_{2W}(0), . \tag{27}$$

The inverse neutral propagator matrix is

$$\begin{aligned}\frac{1}{g_i g_j}(D^0(p))^{-1}_{\mu\nu} &= \begin{pmatrix} 1/g_2^2 & 0 \\ 0 & 1/g_1^2 \end{pmatrix}(p_\mu p_\nu - p^2 g_{\mu\nu}) \\ &+ \; i\int d^4x\, e^{ip\cdot x}\langle\Omega| \begin{pmatrix} T(j^3_\mu(x) j^3_\nu(0)) & T(j^3_\mu(x) j^0_\nu(0)) \\ T(j^0_\mu(x) j^3_\nu(0)) & T(j^0_\mu(x) j^0_\nu(0)) \end{pmatrix} |\Omega\rangle\,. \end{aligned} \tag{28}$$

As for the W-propagator, this is calculated from the bare inverse propagators, the one-loop neutral-current correlators, and the large-N bubble sums for $\Gamma^0_{0^-}$. This gives

$$\frac{1}{g_i g_j}(D^0(p))^{-1}_{\mu\nu} = (p_\mu p_\nu - p^2 g_{\mu\nu}) \begin{pmatrix} 1/g^2_{2Z}(p^2) & 0 \\ 0 & 1/g^2_{1Z}(p^2) \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \frac{f^2_Z(p^2)}{p^2}\,, \tag{29}$$

where $g^2_{2Z}(p^2)$, $g^2_{1Z}(p^2)$, and $f^2_Z(p^2)$b have been defined to give a massless photon pole in the diagonalized neutral propagator. Reading off f^2_Z from the massless pole term in the

$\langle j^3_\mu j^0_\nu \rangle$-correlator, we obtain (see the Appendix)

$$\begin{aligned} f_Z^2(p^2) &= \frac{1}{32\pi^2}\int_0^1 dx\left[N_C\, m_t^2 \ln\!\left(\frac{\Lambda^2}{m_t^2 - p^2x(1-x)}\right) + N_{TC}\, m_U^2 \ln\!\left(\frac{\Lambda^2}{m_U^2 - p^2x(1-x)}\right)\right] \\ &+ \frac{N_C p^2}{16\pi^2}\int_0^1 dx\, \tfrac{1}{3}x(1-x)\ln\!\left(\frac{-p^2x(1-x)}{m_t^2 - p^2x(1-x)}\right); \qquad (30) \end{aligned}$$

$$\begin{aligned} g_{2Z}^{-2}(p^2) &= g_2^{-2} + \frac{1}{16\pi^2}\int_0^1 dx\, x(1-x)\Bigg\{N_C\left[\tfrac{4}{3}\ln\!\left(\frac{\Lambda^2}{m_t^2 - p^2x(1-x)}\right) + \tfrac{2}{3}\ln\!\left(\frac{\Lambda^2}{-p^2x(1-x)}\right)\right] \\ &+ N_{TC}\left[\ln\!\left(\frac{\Lambda^2}{m_U^2 - p^2x(1-x)}\right) + \ln\!\left(\frac{\Lambda^2}{-p^2x(1-x)}\right)\right]\Bigg\}; \qquad (31) \end{aligned}$$

$$\begin{aligned} g_{1Z}^{-2}(p^2) &= g_1^{-2} + \frac{1}{16\pi^2}\int_0^1 dx\, x(1-x)\Bigg\{N_C\left[\tfrac{20}{9}\ln\!\left(\frac{\Lambda^2}{m_t^2 - p^2x(1-x)}\right) + \tfrac{2}{9}\ln\!\left(\frac{\Lambda^2}{-p^2x(1-x)}\right)\right] \\ &+ N_{TC}\left[\ln\!\left(\frac{\Lambda^2}{m_U^2 - p^2x(1-x)}\right) + \ln\!\left(\frac{\Lambda^2}{-p^2x(1-x)}\right)\right]\Bigg\}. \qquad (32) \end{aligned}$$

The $(-p^2x(1-x))$ arguments of the logarithms come from a b or D-fermion loop with $m_b = m_D = 0$.

The ρ-parameter,

$$\rho \cong \frac{f_W^2(0)}{f_Z^2(0)} = \frac{(32\pi^2)^{-1}\left[N_C\, m_t^2\left(\ln(\Lambda^2/m_t^2) + \frac{1}{2}\right) + N_{TC}\, m_U^2\left(\ln(\Lambda^2/m_U^2) + \frac{1}{2}\right)\right]}{(32\pi^2)^{-1}\left[N_C\, m_t^2 \ln(\Lambda^2/m_t^2) + N_{TC}\, m_U^2\ln(\Lambda^2/m_U^2)\right]}\,, \qquad (33)$$

the running of the EW gauge couplings, and the W and Z pole masses, solutions of

$$\begin{aligned} M_W^2 &= g_{2W}^2(M_W^2) f_W^2(M_W)\,, \\ M_Z^2 &= (g_{1Z}^2(M_Z^2) + g_{2Z}^2(M_Z^2)) f_Z^2(M_Z)\,, \qquad (34) \end{aligned}$$

will be discussed in the numerical calculations next, in Sec. 5.

There has been much discussion over the years of the constraint on technicolor theories from the S-parameter [36, 37, 38, 39]. However, as emphasized in Refs. [40, 41], all of these calculations of S assume that TC dynamics is QCD-like, with asymptotic freedom setting in rather quickly above Λ_{TC}. But TC dynamics cannot be QCD-like. As noted earlier, α_{TC} must be a walking gauge coupling to avoid unwanted large flavor-changing neutral current interactions, and this invalidates the assumptions made to calculate S. Calculating S in the strong dynamics of walking technicolor is now the object of a number of groups using lattice gauge theoretic techniques; see, e.g., Refs. [42, 43, 44].

5. Numerical Calculations in the Large-N Approximation

Here we present outcomes of the large-N, weak-TC results of Secs. 3 and 4 for a simple numerical scheme. In this scheme we fix Λ and then calculate $m_t(\Lambda)$ using one-loop QCD

Λ	m_t	m_U	M_H	Γ_t	$v = \sqrt{2}m_t/\Gamma_t$
20 TeV	134 GeV	167 GeV	330 GeV	0.783	242 GeV
500 TeV	118 GeV	126 GeV	250 GeV	0.685	244 GeV
Λ	ρ	g_1	g_2	M_W(pole)	M_Z(pole)
20 TeV	1.0520	0.3941	0.7187	80.8	91.0
500 TeV	1.0301	0.4230	0.7714	80.6	93.0

Table 1: The Higgs mass, ρ-parameter and the W, Z-pole masses calculated for ETC scales $\Lambda = 20$ and $500\,\text{TeV}$. The calculation scheme adopted is described in the text.

running with six flavors from Λ down to $2m_t$ and with five flavors down to $m_t = 173\,\text{GeV}$ [45]. Fixing $f_W(0) = (4\sqrt{2}G_F)^{-1/2} \cong 123\,\text{GeV}$ in Eq. (33) then determines $N_{TC}m_U^2$. Note that, in leading-log approximation, this same combination, $N_{TC}m_U^2$, appears in the formula for M_H^2, Eq.(16). Finally, we choose $N_{TC} = 15$ (which corresponds to an $SU(6)$ TC gauge group with $T = (U, D)$ in the antisymmetric second-rank tensor representation) and calculate $M_H(\Lambda)$. As a check on our calculation, we obtain the Higgs vev v from $m_t(\Lambda) = \Gamma_t(\Lambda)v/\sqrt{2}$, where

$$\left(\Gamma_t(\Lambda)/\sqrt{2}\right)^2 = \lim_{p^2 \to M_H^2} (p^2 - M_H^2)\Gamma_{0^+}^{\bar{t}t}(p)\,, \tag{35}$$

and compare the result to the input $2f_W(0) = 246\,\text{GeV}$.

We consider two cases, $\Lambda = 20\,\text{TeV}$ and $\Lambda = 500\,\text{TeV}$. There is no obvious reason not to have such a high scale for generating m_t since the only price is more fine tuning of G_1 in Eq. (4). In a more complete TC-ETC model, such a large ETC mass for the third generation may also suffice to produce masses for the lighter quarks while suppressing their flavor-changing neutral current interactions. This would eliminate the need for a "tumbled" spectrum of ETC masses. The difference in the two cases we consider is greater than one might have anticipated given the merely logarithmic dependence on Λ of the $2 \to 2$ amplitudes calculated in Sec. 3.

The results are in Table 1. The EW couplings $g_{1,2}$ used in Eqs. (25,31,32) to calculate the W and Z pole-masses were determined by requiring that $g_{1,2Z}^{-2}$ at $p = M_Z = 91.18\,\text{GeV}$ give $\sin^2\theta_W(M_Z) = g_1^2/(g_1^2 + g_2^2) = 0.23116$ and $\alpha(M_Z) = 1/128$ [45]. All the results in the lower half of the table are very good except for a slightly high Z-pole mass in the second case.

It is worth recalling that the Higgs mass will be renormalized from Λ down to the electroweak scale. In BHL [14], $M_H(\Lambda = 10^{15}\,\text{GeV}) = 330\,\text{GeV}$ decreased to $256\,\text{GeV}$. We expect our values of M_H to decrease as well but, of course, we cannot guess by how much — especially since the effect of a walking TC coupling has to be included in the running of m_U. Another thing is that, since we are assuming there is only one Higgs boson, its coupling to the top quark *at the top mass* will be $v/(\sqrt{2}m_t) \cong 1$, so that the ggH coupling of the Higgs to QCD gluons will have its standard-model strength and the production rates

$\sigma(gg \to \gamma\gamma,\, ZZ^*)$ will be as in the standard model.

6. Preliminary Remarks on the Model's Phenomenology

The low-lying bound-state spectrum of this model depends on the magnitude of the ETC couplings G_i and the TC gauge coupling α_{TC}.[8] The chiral-flavor symmetry of our model is $(SU(2)_L \otimes U(1)_R)_q \otimes (SU(2)_L \otimes U(1)_R)_T$ explicitly broken to $SU(2) \otimes U(1)$ by G_2.

Suppose first, then, that $G_2 = 0$. There are three possibilities: (1) G_1 and G_3 are supercritical, i.e., $G_1 > 8\pi^2/N_C\Lambda^2$, $G_3 > 8\pi^2/N_{TC}\Lambda^2$ and m_t, $m_U \neq 0$ are solutions of the gap Eqs. (7,8); (2) G_3 is supercritical ($m_U \neq 0$), but G_1 is not ($m_t = 0$); and (3) vice-versa. Possibilities (2) and (3) are excluded because (2) m_t cannot be zero and (3) TC would likely generate a dynamical mass for U and D, spontaneously breaking their chiral symmetry, now $SU(2)_L \otimes SU(2)_R$, giving rise to an additional triplet of massless Goldstone bosons. They could acquire mass only from the EW gauge interactions and they would be very light [24], hence excluded (e.g., by production of the charged pair in e^+e^- annihilation). In possibility (1), we would have two light composite scalar doublet bound states, hence two light Higgs bosons and two massless Goldstone triplets. One combination of the Goldstone triplets would be eaten by the W and Z, but the orthogonal triplet is again very light and excluded.

Once $G_2 \neq 0$, its magnitude is fixed by Eqs. (9, 10). Both m_t and m_U are nonzero. Now, both terms in Eq. (10) must be less than one. We saw in Sec. 5 that a very small m_U/m_t or m_t/m_U is unlikely to be compatible with $f_W(0) \cong 123\,\text{GeV}$.[9] Thus, G_2 is not much different from G_1 and/or G_3 and it cooperates with them to make the model just barely critical, producing $0 < m_t,\, m_U \ll \Lambda$ — our model's fine-tuning.

What does this mean for the spectrum of relatively low-lying bound states? So long as TC is present and confining, we expect isovector ρ_T and isoscalar ω_T which are $\bar{T}T$ states. It is not clear how heavy the lightest ρ_T and ω_T are. If their binding is due mainly to TC, we would guess their masses are in the range 500 GeV to 2 TeV. If the strong ETC interactions $\mathcal{L}_{ETC}$ contribute to their mass *other than through the hard mass* m_U*,* they might be much heavier. Because the hard technifermion mass is an $I = 1$ operator, the neutral and charged ρ_T and the ω_T should all have nearly the same mass. It is also possible that the mass-eigenstate vectors are ideally mixed $\bar{U}U$ and $\bar{D}D$ states.

Assuming they are lighter than $\sim 2\,\text{TeV}$, the ρ_T and ω_T will be produced at the LHC at observable rates by the Drell-Yan process, $\bar{q}q \to \gamma, Z, W \to \rho_T$ or ω_T [46] and, if they are heavy enough, via weak vector boson fusion (see Delgado, Grojean, Maina and Rosenfeld in Ref. [47]).

[8]Presumably, once its effects are included, α_{TC} becomes strong enough to confine technicolor at a distance scale of $\mathcal{O}(1\,\text{TeV}^{-1})$.

[9]If we relax this condition on $f_W(0)$, then there must be at least two Higgs doublets and, therefore, two Higgs bosons in a TC-ETC model. This is an interesting complication that we do not pursue in this paper.

How do ρ_T and ω_T decay? There may be a triplet of lightest "pseudoscalars", induced by the criticality of $\mathcal{L}_{ETC}$ and by TC. This triplet would be an admixture of $\bar{q}t$ and $\bar{T}U$ states that is orthogonal to the three Goldstone bosons eaten by $W^\pm$ and Z^0. They are not light pseudo-Goldstone bosons, for they get a large mass from the near-critical G_2 interaction. In fact, there is no obvious reason that they are much lighter than ρ_T and ω_T. Thus, we expect the vectors' dominant decay modes to involve *longitudinally-polarized* weak bosons, the erstwhile "pions" absorbed in the Higgs mechanism, alone and possibly in association with $H(125)$:

$$\rho_T^{\pm,0} \rightarrow W_L^\pm Z_L,\ W_L^+ W_L^- \quad \text{and} \quad W_L^\pm H,\ Z_L H\,; \tag{36}$$

$$\omega_T \rightarrow W_L^+ W_L^- Z_L \quad \text{and} \quad Z_L H\,. \tag{37}$$

These decays are strong (TC) interactions. Thus, heavier ρ_T and ω_T are unlikely to be narrow resonances. In that case, the presence of the ρ_T and ω_T will be signaled by increases in the rates of the above processes at higher invariant masses.

7. Summary and Plans

In this paper we presented a simple model of a light composite Higgs boson. It is inspired by the top-condensate model of Bardeen, Hill and Lindner [14] and, in our view, its paradigmatic position as a dynamical model embodying our notion of "least unnaturalness". Our model combines technicolor with strong extended technicolor to jointly account for electroweak symmetry breaking, the light Higgs boson discovered at the LHC, and the top quark. The strong ETC interaction with finely-tuned couplings is essential for these to occur at energies much less than the ETC scale Λ. This mechanism was anticipated in Ref. [17].

Our simple model employs one technifermion doublet $T = (U, D)$ interacting with the third generation quarks $q = (t, b)$ via three ETC interactions with strengths $G_1, G_2, G_3 = \mathcal{O}(1/\Lambda^2)$, where $\Lambda \sim 10$–$500\,\text{TeV}$. These interactions were treated in the NJL approximation of large (N_C, N_{TC}). While the TC interaction of T is expected to be an important part of the model, it is also a significant complication. We neglected TC in this paper.

The solution of the model in this large-N, weak-TC limit then closely followed BHL: The gap equations in Sec. 2 for the hard masses m_t and m_U are quadratically divergent, and requiring $m_t, m_U \ll \Lambda$ is a fine tuning of a part in $\mathcal{O}(\Lambda^2/m^2)$. These gap equations also imply the relation $G_2 = G_2(m_U/m_t) = G_3(m_t/m_U)$ among the model's ETC couplings. This relation was essential for turning the complicated NJL bubble sums for the $2 \rightarrow 2$ scattering amplitudes in Sec. 3 into simple geometric series. As in BHL, all Λ^2-dependence in these amplitudes was removed by applying the condition (10) for nontrivial solutions to the gap equations. In the scalar channel, the Higgs boson pole occurs at $M_H^2 \cong 4(N_C m_t^4 + N_{TC} m_U^4)/(N_C m_t^2 + N_{TC} m_U^2)$. This is the model's only Higgs boson (necessarily, since G_2 is not weak), so its vev is $v = 246\,\text{GeV}$. There are three Goldstone boson channels. Their massless poles disappear from the physical spectrum, producing the the massive W and Z-boson poles in their propagators (Sec. 4). Integrals used in Secs. 3 and 4 are in the Appendix.

In Sec. 5 we carried out a simple numerical analysis of our model by (1) fixing Λ and then $m_t(\Lambda)$ so that $m_t = 173\,\mathrm{GeV}$ at the weak scale and (2) determining $N_{TC} m_U^2(\Lambda)$ from the residue of the Goldstone pole in the W-propagator, $f_W(0) = 123\,\mathrm{GeV}$. Fixing N_{TC} then determined $M_H(\Lambda)$. The results are in Table 1 for the two choices $\Lambda = 20\,\mathrm{TeV}$ and $500\,\mathrm{TeV}$. As in BHL, these values of the Higgs mass are expected to decrease when run down to the weak scale. However, the effect of TC on the running is unknown and, like the inclusion of TC dynamics, is deferred to the next paper. The ρ-parameter and the W and Z-pole masses were also calculated and in quite good agreement with experiment. Of course, more elaborate numerical schemes are possible, e.g., a "best fit" to m_t and M_H with fixed values of Λ and the Higgs vev $v = 2f_W(0) = 246\,\mathrm{GeV}$, or even a scan over Λ for a best fit to m_t and M_H.

Finally, in Sec. 6 we speculated briefly on the model's phenomenology. We noted that the model with $G_2 = 0$ is excluded because it has a triplet of nearly massless pseudo-Goldstone bosons; the charged ones would have been discovered decades ago in e^+e^- annihilation. Further, the constraint $f_W(0) = 123\,\mathrm{GeV}$ implies that m_t and m_U are likely to be comparable and this, in turn, implies that all three G_i are comparable and *nearly* critical, i.e., nearly large enough to induce nonzero m_t, m_U by themselves. Given this, it is difficult to see what this model's phenomenology is because it will be controlled by two strong interactions, TC and ETC, with very different energy scales. One possibility that suggested itself deals with the model's lowest lying spin-one, isovector and isoscalar $\bar{T}T$ states. If their binding is determined by TC, not ETC, dynamics, the masses of these ρ_T and ω_T should be $\frac{1}{2}$–$2\,\mathrm{TeV}$ and possibly within reach of the LHC. Their spin-zero π_T partners are not pseudo-Goldstone bosons and, so, are likely to be as heavy as they are. Then, the principal observational modes of the vectors are their *strong* decays to longitudinally polarized weak bosons, either in diboson and triboson combinations or in association with $H(125)$. Beyond this, understanding the phenomenology of this model, or any model like it, requires a much better understanding of its dynamics. This and the phenomenology are the subjects of planned papers.

Appendix: Integrals used in the text

The calculations presented here come from Ref. [14] and C. T. Hill (private communication). Momentum integrals are in Minkowski space until Wick-rotated and then cut off at momentum Λ.

Sections 2 and 3:

The fermion condensates at scale Λ in Eq. (7):

$$\begin{aligned}
\langle \bar{t}t \rangle_\Lambda &= \sum_a \langle \bar{t}^a(0) t_a(0) \rangle_\Lambda = -iN_C (\mathrm{Tr} S_t(0))_\Lambda \\
&\equiv -4iN_C \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{m_t}{k^2 - m_t^2} = -\frac{N_C m_t}{4\pi^2} \int_0^{\Lambda^2} dk^2 \frac{k^2}{k^2 + m_t^2} \\
&\cong -\frac{N_C m_t}{4\pi^2} \left[\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2) \right] , \qquad (38)
\end{aligned}$$

for $\Lambda^2 \gg m_t^2$.
The scalar $t\bar{t} \to t\bar{t}$ integral in Eq. (12):

$$\begin{aligned}
& i \int d^4x \, e^{ip\cdot x} \langle \Omega | T(\bar{t}^a t_a(x) \, \bar{t}^b t_b(0)) | \Omega \rangle \\
&= 4iN_C \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{k\cdot(k+p) + m_t^2}{((k+p)^2 - m_t^2)(k^2 - m_t^2)} \\
&= 2iN_C \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{((k+p)^2 - m_t^2) + (k^2 - m_t^2) - (p^2 - 4m_t^2)}{((k+p)^2 - m_t^2)(k^2 - m_t^2)} \\
&\cong \frac{N_C}{4\pi^2} \left[\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2) \right] + \frac{N_C (p^2 - 4m_t^2)}{8\pi^2} \int_0^1 dx \ln\left(\frac{\Lambda^2}{m_t^2 - p^2 x(1-x)} \right) . \qquad (39)
\end{aligned}$$

The Goldstone boson $t\bar{t} \to t\bar{t}$ integral in Eq. (19):

$$\begin{aligned}
& i \int d^4x \, e^{ip\cdot x} \langle \Omega | T(\bar{t}^a i\gamma_5 t_a(x) \, \bar{t}^b i\gamma_5 t_b(0)) | \Omega \rangle \\
&= 4iN_C \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{k\cdot(k+p) - m_t^2}{((k+p)^2 - m_t^2)(k^2 - m_t^2)} \\
&= 2iN_C \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{((k+p)^2 - m_t^2) + (k^2 - m_t^2) - p^2}{((k+p)^2 - m_t^2)(k^2 - m_t^2)} \\
&\cong \frac{N_C}{4\pi^2} \left[\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2) \right] + \frac{N_C p^2}{8\pi^2} \int_0^1 dx \ln\left(\frac{\Lambda^2}{m_t^2 - p^2 x(1-x)} \right) . \qquad (40)
\end{aligned}$$

The Goldstone boson $t\bar{b} \to t\bar{b}$ integral in Eq. (22):

$$
\begin{aligned}
& i \int d^4x \, e^{ip\cdot x} \langle \Omega | T(\bar{t}^a_R b_{La}(x) \, \bar{b}^b_L t_{Rb}(0)) | \Omega \rangle \\
& = 2iN_C \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{k\cdot(k+p)}{(k+p)^2(k^2-m_t^2)} = 2iN_C \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{(k+p)^2 - k\cdot p - p^2}{(k+p)^2(k^2-m_t^2)} \\
& \cong \frac{N_C}{8\pi^2}\left[\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2)\right] + \frac{N_C p^2}{8\pi^2} \int_0^1 dx \, x \int_0^{\Lambda^2} \frac{k^2}{k^2 + m_t^2 x - p^2 x(1-x)} \\
& \cong \frac{N_C}{8\pi^2}\left[\Lambda^2 - m_t^2 \ln(\Lambda^2/m_t^2)\right] + \frac{N_C p^2}{8\pi^2} \int_0^1 dx \, x \ln\left(\frac{\Lambda^2}{m_t^2 x - p^2 x(1-x)}\right) . \qquad (41)
\end{aligned}
$$

Section 4:

There are two types of terms in this calculation, the correlator of two weak currents and the sum of large-N bubbles inserted into this correlator. The first is the standard one-loop correction to the weak polarization tensors $\Pi_{\mu\nu}(p)$, while the second produces the Goldstone pole in this one-loop term. We calculate the terms here for the product of two charged tb currents. For the simple one-loop correlator of two *conserved* currents, we use dimensional regularization to avoid a spurious quadratic divergence. With $d = 4 - \epsilon$ and $m_b = 0$, we have for the $t\bar{b} \to t\bar{b}$ contribution:

$$
\begin{aligned}
\Pi^\pm_{\mu\nu}(p) &= \frac{i}{g_2^2}\left(\frac{g_2}{\sqrt{2}}\right)^2 \int d^4x \, e^{ip\cdot x} \langle \Omega | T(\bar{t}^a_L \gamma_\mu b_{La}(x) \, \bar{b}^b_L \gamma_\nu t_{Lb}(0) | \Omega \rangle \\
&= \frac{dN_C \Gamma(2-d/2)}{4(4\pi)^{d/2}} \int_0^1 dx \, x \left[\frac{2(p_\mu p_\nu - p^2 g_{\mu\nu})(1-x) + m_t^2 g_{\mu\nu}}{(m_t^2 x - p^2 x(1-x))^{(2-d/2)}}\right] . \qquad (42)
\end{aligned}
$$

Using

$$
\frac{d\,\Gamma(2-d/2)(\Delta^2)^{(d/2-2)}}{4(4\pi)^{d/2}} = \frac{2}{16\pi^2}\left[\epsilon^{-1} - \tfrac{1}{2}\gamma + \tfrac{1}{2}\ln 4\pi - \tfrac{1}{4} - \tfrac{1}{2}\ln\Delta^2 + \mathcal{O}(\epsilon)\right] \longrightarrow \frac{1}{16\pi^2}\ln(\Lambda^2/\Delta^2) , \qquad (43)
$$

we get for the sum of the quark and technifermion loops:

$$
\begin{aligned}
\Pi^\pm_{\mu\nu}(p) &= \frac{N_C}{16\pi^2} \int_0^1 dx \, x \left[2(p_\mu p_\nu - p^2 g_{\mu\nu})(1-x) + m_t^2 g_{\mu\nu}\right] \ln\left(\frac{\Lambda^2}{m_t^2 x - p^2 x(1-x)}\right) \\
&+ \; (N_C, m_t \to N_{TC}, m_U) . \qquad (44)
\end{aligned}
$$

The charged Goldstone-boson pole contribution to $\Pi^{\pm}_{\mu\nu}$ is

$$\begin{aligned}
\Pi^{\pm}_{\mu\nu, GB}(p) &= \frac{i}{g_2^2}\left(\frac{g_2}{\sqrt{2}}\right)^2 \int d^4x d^4y\, e^{ip\cdot(x-y)} \langle\Omega| T\left[j^{(1+i2)}_{L\mu}(x)\,(i\mathcal{L}_{ETC}(0)+\cdots)\, j^{(1-i2)}_{L\nu}(0)\right]|\Omega\rangle \\
&= \frac{2p_\mu p_\nu}{(16\pi^2)^2}\Bigg\{\left[\int_0^1 dx\, x N_C m_t \ln\left(\frac{\Lambda^2}{m_t^2 x - p^2 x(1-x)}\right)\right]^2 \left(-4\Gamma^{t\bar{b}}_{0^-}(p)\right) \\
&+ (t\bar{b} \leftrightarrow U\bar{D}) + (U\bar{d} \to U\bar{D}) \text{ terms}\Bigg\} \qquad (45) \\
&= -\frac{p_\mu p_\nu}{16\pi^2 p^2}\int_0^1 dx\, x\left[N_C m_t^2 \ln\left(\frac{\Lambda^2}{m_t^2 x - p^2 x(1-x)}\right) + (N_C, m_t \to N_{TC}, m_U)\right].
\end{aligned}$$

The factor of $-4\Gamma^{t\bar{b}}_{0^-}$ comes from the first term on the right in Eq. (21), which indicates that $G_1 + \cdots$ sums to this. These GB-pole terms combine with the $m^2 g_{\mu\nu}$-terms in Eq. (44) to make a transverse massless-GB pole term. Then, with Eq. (24), $g^{-2}_{2W}(p^2)$ and $f^2_W(p^2)$ are easily read off, and are given in Eqs. (25,26).

The calculations for the neutral EW propagator matrix are similar, if more tedious. The important thing there is to arrange the terms so that there is a massless photon pole.

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