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Are Scalar and Tensor Deviations Related in Modified Gravity?

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Modified gravity theories on cosmic scales have three key deviations from general relativity. They can cause cosmic acceleration without a physical, highly negative pressure fluid, can cause a gravitational slip between the two metric potentials, and can cause gravitational waves to propagate differently, e.g. with a speed different from the speed of light. We examine whether the deviations in the metric potentials as observable through modified Poisson equations for scalar density perturbations are related to or independent from deviations in the tensor gravitational waves. We show analytically they are independent instantaneously in covariant Galileon gravity – e.g. at some time one of them can have the general relativity value while the other deviates – though related globally – if one deviates over a finite period, the other at some point shows a deviation. We present expressions for the early time and late time de Sitter limits, and numerically illustrate their full evolution. This in(ter)dependence of the scalar and tensor deviations highlights complementarity between cosmic structure surveys and future gravitational wave measurements.

I. INTRODUCTION

Extensions to general relativity have become a subject of intense interest in the last decade, due to both observational evidence for acceleration of the cosmic expansion and intriguing new theoretical work. For general relativity to explain cosmic acceleration it requires a physical energy-momentum component that violates the strong energy condition, for example a cosmological constant or scalar field with highly negative pressure. These explanations have difficulties with fine tuning and naturalness, so an attractive alternative has been to consider altering the structure of the gravitational action itself, e.g. through scalar-tensor theories, higher dimensional gravity, or massive gravitons.

Inventing a sound, consistent gravity theory is no easy task, and furthermore the theory must satisfy observational cosmology constraints such as an early universe behavior similar to general relativity, with radiation and matter domination, a deviation near the present to explain cosmic acceleration, and growth of large scale massive structures not too dissimilar from in general relativity. Indeed the specific expansion and growth histories, and their comparison, provide one of the key signatures of modified gravity. The evolution of linear density perturbations, and their gravitational lensing of light, can be phrased in terms of, respectively, nonrelativistic and relativistic modified Poisson equations where Newton's constant becomes two distinct functions of space and time. These effective gravitational strengths can not only differ from Newton's constant, but from each other; this divergence is called the gravitational slip.

Observationally there are no constraints yet on cosmological gravitational waves, but many modified theories predict that their propagation will differ from general relativity; in particular, as we focus on here, the sound speed of these tensor perturbations may deviate from the speed of light. Recently, [1] illustrated a relation between the gravitational slip and the gravitational wave propagation in several modified gravity theories: a deviation in one led to a deviation in the other.

Here we consider the generality of such a relation. If it arose from the intrinsic structure of the gravity theory, it would be a powerful tool for detecting some modifications and predicting others. We work within Horndeski gravity, the most general single scalar field gravity that obeys second order field equations, and in particular Galileon gravity plus an extension with disformal couplings.

In Sec. II we review the relevant equations of motion that lead to the modified Poisson equations defining the gravitational strength functions and their slip, as well as the sound speed of gravitational waves. Section III examines these expression analytically, in the early time limit and for the asymptotic de Sitter late time attractor of cosmic acceleration. We display the full evolution for various cases in Sec. IV and discuss the results in Sec. V.

II. MODIFICATIONS BEYOND GENERAL RELATIVITY

The metric for a spatially flat Friedmann-Robertson-Walker cosmology linearly perturbed by scalar density modes is

$$ds^2 = -(1+2\psi)dt^2 + a^2(1-2\phi)\delta_{ij}dx^i dx^j \ , \qquad (1)$$

in Newtonian gauge, where a is the cosmic expansion factor, and ψ and ϕ are the metric potentials. We will be particularly interested in whether they are equal, as in general relativity (we assume no fluid anisotropic stress, which can generate a difference between them). The gravitational slip is defined as

$$\eta = \frac{\phi}{\psi} \, , \qquad (2)$$

and we will explore the conditions under which it deviates from unity in a modified gravity theory.

The metric potentials can be probed observationally by the growth of linear density perturbations and the gravitationally lensing they induce. For subhorizon scales we write the modified Poisson equations in the standard quasistatic approximation (see Appendix A and [2] for discussion of the quasistatic approximation in unusual cases) as

$$\nabla^2\phi = 4\pi a^2 G_{\rm eff}^{(\phi)}\rho_m\,\delta_m \quad (3)$$
$$\nabla^2\psi = 4\pi a^2 G_{\rm eff}^{(\psi)}\rho_m\,\delta_m \quad (4)$$
$$\nabla^2(\psi+\phi) = 8\pi a^2 G_{\rm eff}^{(\psi+\phi)}\rho_m\,\delta_m\,. \quad (5)$$

Here the $G_{\rm eff}$ are the modified Newton's constants giving the gravitational strength, ρ_m is the matter density, and $\delta_m = \delta\rho_m/\rho_m$ is the density perturbation. The second equation is central to the motion of nonrelativistic matter and hence the growth of massive structures, while the third is central to the motion of relativistic particles, and hence the propagation of light [3]. Note that $G_{\rm eff}^{(\psi+\phi)} = G_{\rm eff}^{(\phi)} + G_{\rm eff}^{(\psi)}$. The gravitational slip is then

$$\eta = \frac{G_{\rm eff}^{(\phi)}}{G_{\rm eff}^{(\psi)}}\,. \quad (6)$$

We now specialize to covariant Galileon gravity [4–6], a subset of the general Horndeski theory [2, 7–9], except that we will also allow a derivative coupling term related to the increasingly investigated disformal field theories [10–13]. Such a theory has a number of useful properties, such as involving only second order equations of motion and having a shift symmetry (as well as softly broken Galilean symmetry).

The action following the notation of [14] is

$$S = \int d^4x\,\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{c_2}{2}\,(\partial\pi)^2 - \frac{c_3}{M^3}(\partial\pi)^2\Box\pi \right.$$
$$\left. - \frac{c_4}{2}\mathcal{L}_4 - \frac{c_5}{2}\mathcal{L}_5 - \frac{M_{\rm Pl}}{M^3}c_{\rm G}G^{\mu\nu}\,\partial_\mu\pi\partial_\nu\pi - \mathcal{L}_m\right]\,, \quad (7)$$

where π is the Galileon scalar field, the c_i are constant coefficients, $\mathcal{L}_m$ is the fluid Lagrangian, and

$$\mathcal{L}_4 = (\nabla_\mu\pi)(\nabla^\mu\pi)\left[2(\Box\pi)^2 - 2\pi_{;\mu\nu}\pi^{;\mu\nu} \right.$$
$$\left. - R(\nabla_\mu\pi)(\nabla^\mu\pi)/2\right]/M^6 \quad (8)$$
$$\mathcal{L}_5 = (\nabla_\mu\pi)(\nabla^\mu\pi)\left[(\Box\pi)^3 - 3(\Box\pi)\pi_{;\mu\nu}\pi^{;\mu\nu} \right.$$
$$\left. + 2\pi_{;\mu}{}^{;\nu}\pi_{;\nu}{}^{;\rho}\pi_{;\rho}{}^{;\mu} - 6\pi_{;\mu}\pi^{;\mu\nu}\pi^{;\rho}G_{\nu\rho}\right]/M^9\,. \quad (9)$$

The equations of motion can be written as coupled first order differential equations for the Hubble expansion $H = d\ln a/d(H_0 t)$ and field evolution $x = d(\pi/M_{\rm Pl})/d\ln a$, as in [14]. The gravitational slip becomes

$$\eta = \frac{\kappa_4\kappa_6 - \kappa_5\kappa_1}{2(\kappa_3\kappa_6 - \kappa_1^2)}\,, \quad (10)$$

where the κ_i involve sums of terms depending on c_i, H, and x (see Appendix B for the explicit expressions). When all $c_i = 0$, i.e. general relativity, then all κ_i vanish except $\kappa_3 = -1$ and $\kappa_4 = -2$, so in this case indeed $\eta = 1$.

The expression for the gravitational wave speed c_T for Horndeski gravity appears in Eq. (28), with Eqs. (17) and (20), of [15]. Note that for the Galileon gravity we consider, the relevant Horndeski functions

$$G_4 = \frac{1}{2} + \frac{c_4}{4}H^4x^4 \quad (11)$$
$$G_5 = \frac{-3c_5}{4}H^4x^4 + c_G\frac{\pi}{M_{\rm Pl}}\,. \quad (12)$$

The c_G term comes from the derivative coupling to the Einstein tensor, $c_G G^{\mu\nu}\nabla_\mu\pi\nabla_\nu\pi$, we have included in the Galileon action. This disformal coupling also allows us to study other gravity theories of interest such as purely kinetic coupled gravity [16] and some aspects of Fab 4 [17] and Fab 5 gravity [18].

The tensor sound speed is then given by

$$c_T^2 = \frac{2\kappa_3}{\kappa_4} \quad (13)$$
$$= \frac{1 + \frac{c_4}{2}H^4x^4 + 3c_5H^6x^5\left(\frac{H'}{H} + \frac{x'}{x}\right) - c_G H^2x^2}{1 - \frac{3c_4}{2}H^4x^4 + 3c_5H^6x^5 + c_G H^2x^2}\,,$$

where prime denotes $d/d\ln a$. Note that c_2 and c_3 do not explicitly enter; this is an important point.

If only c_2 and c_3 are nonzero, then κ_1 vanishes, $\kappa_4 = 2\kappa_3$, and then from Eq. (10) we have $\eta = 1$. This is also apparent from the full equation for $\phi - \psi$ in Eq. (C1) of [14]. This raises the conjecture that there are some conditions under which the gravitational wave speed and gravitational slip are closely connected, as investigated by [1].

However, we see that c_4, c_5, and c_G can all contribute to both c_T and η; if any of them are nonzero then there can be deviations from general relativity in these observables. It is not obvious that these terms enter these two quantities in the same way, though, and so the potential exists for a deviation signature to be evident in only one of them. That is, we want to see if we can have $\eta = 1$ but $c_T \neq 1$, or $c_T = 1$ but $\eta \neq 1$, or if indeed a deviation in one forces a deviation in the other.

III. EARLY AND LATE TIME LIMITS

In the early radiation and matter dominated eras we generally want modified gravity deviations from general relativity to be small, to preserve excellent agreement with primordial nucleosynthesis and cosmic microwave background observations. For the Galileon case, [14] calculated that at early times $G_{\rm eff}^{(\phi)} = 1 + \mathcal{O}(\Omega_\pi)$, where Ω_π is the fraction of critical density contributed by the modified gravity scalar field. We can carry out a similar computation for $G_{\rm eff}^{(\psi)}$ and find that also $G_{\rm eff}^{(\psi)} = 1 + \mathcal{O}(\Omega_\pi)$; for example if the c_5 term dominates the Galileon Lagrangian as expected at early times then $G_{\rm eff}^{(\psi)} = 1 + (759/224)\Omega_\pi$ during matter domination.

Thus $\eta_{\rm early}=1+\mathcal{O}(\Omega_\pi)$, and we can similarly calculate that $c^2_{T,\rm early}=1+\mathcal{O}(\Omega_\pi)$. For example, in matter domination with the leading c_5 term we have

$$\eta_{\rm early}=1+\frac{111}{32}\Omega_\pi \tag{14}$$

$$c^2_{T,\rm early}=1+\frac{15}{56}\Omega_\pi\ . \tag{15}$$

Thus at early times a deviation signature in one, while small, does imply a deviation in the other, since they both arise from the effective dark energy density (since only one term in the Lagrangian dominates).

At later times, however, multiple Lagrangian terms can be comparable and this connection can be broken. We show the evolution of η and c_T in the next section, but first we demonstrate analytically the breakdown of the connection between the scalar and tensor deviations for late time cosmology, i.e. when the effective dark energy is non-negligible, and in particular when it completely dominates in the de Sitter limit.

Let us define

$$e=c_4H^4x^4\ ,\quad f=c_5H^6x^5\ ,\quad a=c_GH^2x^2\,. \tag{16}$$

We can write

$$c_T^2-1=\frac{2e-2a-3f\left(1-\frac{H'}{H}-\frac{x'}{x}\right)}{1-\frac{3}{2}e+a+3f} \tag{17}$$

$$\kappa_1\,x=-(c_T^2-1)\left(1-\frac{3}{2}e+a+3f\right)-6e\left(\frac{H'}{H}+\frac{x'}{x}\right) +2a\left(\frac{H'}{H}+\frac{x'}{x}\right)+3f\left(\frac{4H'}{H}+\frac{3x'}{x}\right) \tag{18}$$

$$\kappa_4=2\kappa_3+2(c_T^2-1)\left(1-\frac{3}{2}e+a+3f\right)\ . \tag{19}$$

From Eq. (10) we see that

$$\eta-1\propto(\kappa_4-2\kappa_3)\kappa_6+\kappa_1(2\kappa_1-\kappa_5)\ , \tag{20}$$

so to achieve vanishing slip at some moment the vanishing deviation c_T^2-1 is insufficient – one must also have the conditions $H'=0=x'$ (or very small κ_1 as in the early universe when it is proportional to Ω_π, or an instant of cancellation in the evolution of the various terms from the Lagrangian).

The conditions $H'=0=x'$ are the fixed points for the equations of motion, corresponding to de Sitter cosmology. In fact, in the de Sitter limit $\eta_{\rm dS}=1$ regardless of the value of c_T^2, as noted in [14]. (We can see this here by substituting the algebraic constraints Eqs. (67)–(68) from [14] into Eq. (20), resulting in its right hand side vanishing.) But in general, except for these two exceptions – de Sitter late time limit and negligible effective dark energy early time limit – $c_T^2=1$ at some moment does not imply $\eta=1$ then. Nor is the converse true: $\eta=1$ at some moment does not imply $c_T^2=1$ then, except at early times (not even in the de Sitter limit, as we consider below). Basically, the two equations of motion give two constraints, and fixing the present dark energy density would give a third, but this is only three equations for five quantities ($c_{2\ldots5}$, c_G) and so η is not completely determined without further assumptions about the values of the c_i; in particular nothing forces $\eta=1$. So $c_T^2=1$ does not guarantee $\eta=1$.

(One might note that tensor deviations can occur in other ways than through changing the gravitational wave speed, i.e. through a graviton mass term, a transverse traceless source term, or a running of the Planck mass [1]. These potentially add more parameters, which further underdetermines the system. However, in theories where these extra effects are independent from the tensor sound speed [19] and if they are fixed to zero deviation from general relativity, these extra conditions might be sufficient to close the system and guarantee $\eta=1$. This is what occurs in the Horndeski case considered by [1]; we discuss it further in Appendix A. However, this does not help much since by Eq. (20) this forces a constraint on the expansion history H, and does not solve the following counterexample.)

Now consider the converse: does $\eta=1$ imply $c_T^2=1$? This obviates the issue of tensor mode propagation depending on more than the gravitational wave sound speed. If lack of deviations from general relativity in the scalars must imply lack of deviations in the tensor modes, then $c_T^2=1$ is a necessary condition.

One cannot obtain a de Sitter state with only one of the c_i's nonzero. Since we require nonzero c_4, c_5, or c_G to have $c_T^2\neq1$, for simplicity we could look at pairs involving at least one of them. Indeed we then find that these cases violate $c_T^2=1$. However, although this shows that nothing in the structure of the theory requires scalar mode deviations (or lack thereof) to guarantee tensor mode deviations (or lack thereof), these "pair" cases do generally have an instability in the scalar sound speed, $c_s^2<0$, or sometimes a ghost (see Fig. 5 in [14]) and so one might prefer a purely healthy theory. Therefore we take the simple illustrations of some triplet cases.

Recall that $\eta_{\rm dS}=1$. It is convenient to define E, F, A as the values that e, f, a from Eqs. (16) take at the de Sitter fixed point. We then find

$$c^2_{T,\rm dS}=\frac{1+\frac{1}{2}E}{1-\frac{3}{2}E}\quad\text{for }\{c_2,c_3,c_4\} \tag{21}$$

$$c^2_{T,\rm dS}=\frac{1}{1+3F}\quad\text{for }\{c_2,c_3,c_5\} \tag{22}$$

$$c^2_{T,\rm dS}=\frac{1-A}{1+A}\quad\text{for }\{c_2,c_3,c_G\}\ . \tag{23}$$

This shows that any of c_4, c_5, c_G can give deviations in the tensor sector while keeping the scalar slip as general relativity. The last case is particularly interesting since the derivative coupling shows up in purely kinetic gravity [16], Fab 4 gravity [17], and Fab 5 gravity [18]. Indeed Eq. (23) appears in Eq. (B23) of [18], using the opposite sign convention for c_G. (Recall that c_2 and c_3 do not contribute explicitly to c_T^2 and so are somewhat irrelevant.)

In the next section we exhibit numerical solutions where all terms are present, at arbitrary times. Finally we consider the global situation where one sector looks like general relativity at all times.

IV. EVOLUTION OF DEVIATIONS

We solve the coupled equations of motion $x'(H, x)$, $H'(H, x)$ (e.g. see Eqs. 8, 9 of [14]) from standard early matter dominated initial conditions to the future de Sitter attractor, and use the results in Eqs. (10) and (13). In Fig. 1 we show the evolution of the slip and gravitational wave sound speed for the uncoupled Galileon gravity case of Fig. 10 from [14]. Here $c_{2...5}$ are nonzero and the present dark energy density is $\Omega_{\pi,0} = 0.72$. We clearly see that $\eta = 1$ does not imply $c_T^2 = 1$ during the late time de Sitter state, or in the recent past. Similarly this verifies that $c_T^2 = 1$ does not imply $\eta = 1$. Thus the lack of scalar deviations at some moment does not guarantee the lack of tensor deviations, or the converse. The derivative coupled Galileon case, also as in Fig. 10 of [14], is shown in Fig. 2 and again demonstrates that no firm relation exists between slip and gravitational wave speed. These cases are free of ghosts or instabilities.

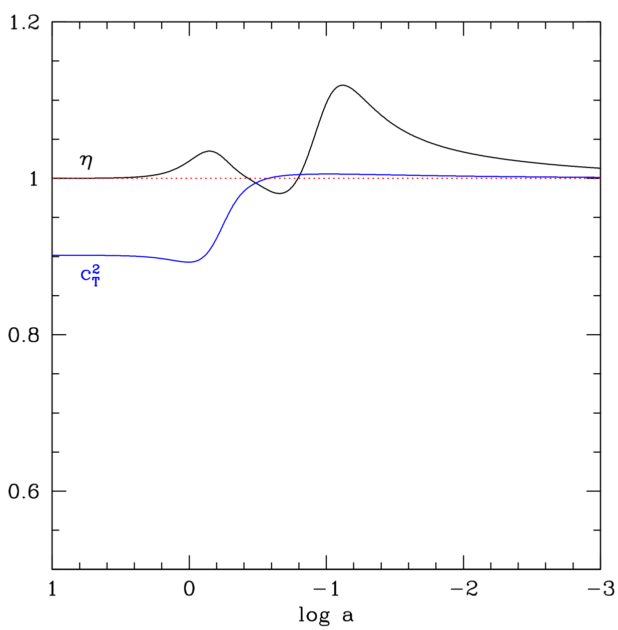

FIG. 1. The evolution of the gravitational slip η and gravitational wave sound speed squared c_T^2 is plotted vs the log of the expansion factor for an uncoupled Galileon gravity model. At late times the slip goes to unity, the general relativity value, but c_T^2 does not.

Note that while $\eta_{\rm dS} = 1$, this does not mean $G_{\rm eff} = G_N$. Indeed from Fig. 10 of [14] we see that $G_{\rm eff,dS}$ can be substantially greater than unity: $G_{\rm eff}/G_N \approx 33$ for the uncoupled case and 6.5 for the coupled case. This high-

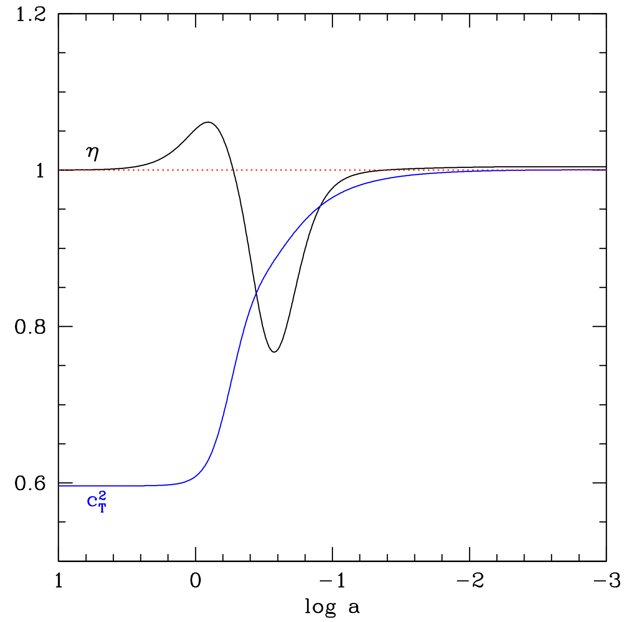

FIG. 2. As Fig. 1 but for a derivatively coupled Galileon gravity model. Again, at late times the slip goes to unity, the general relativity value, but c_T^2 does not.

lights that η is not the only ingredient in the modified Poisson equations; observationally we also care about the absolute gravitational strength, e.g. $G_{\rm eff}^{(\psi)}$ entering growth of structure or $(1+\eta)\, G_{\rm eff}^{(\psi)}$ entering gravitational lensing. Both growth and lensing must be measured to extract η. (See [20] for details on how other cosmological and astrophysical quantities enter the observations, and which combinations can be determined.)

To demonstrate that $G_{\rm eff} \neq G_N$ is not responsible for $c_T^2 \neq 1$, despite $\eta = 1$, we plot in Fig. 3 an uncoupled case from Fig. 6 of [14] where $G_{\rm eff,dS} = G_N$. Although $\eta_{\rm dS} = 1$ and $G_{\rm eff,dS}/G_N = 1$, still $c_T^2 \neq 1$; while the scalar sector looks like general relativity with a cosmological constant, the tensor sector has a clear deviation.

That conclusion holds for an instantaneous measurement. Let us now consider the global case, where $\eta = 1$ for all time, and its implications for the tensor sector. (Note from Eq. 17 that the only way $c_T^2 = 1$ for all time is for $c_4 = c_5 = c_G = 0$, i.e. general relativity must hold.) Suppose $\eta = 1$ for all time. (We leave aside the early universe where $\eta - 1 \sim \Omega_\pi \ll 1$.) Then from Eq. (20) we see that its right hand side is a polynomial of H and x. For this to vanish at all times, the coefficients of the polynomial (hence the c_i) must be zero, so we have general relativity.

Thus global nondeviation in slip leads to global nondeviation in c_T^2 (and all other gravitational wave propagation characteristics) for our Galileon case. However, if $\eta \neq 1$ for some finite period then c_T^2 will deviate during some (possibly different) period, and if $c_T^2 \neq 1$ for some finite period then η will deviate during some (possibly

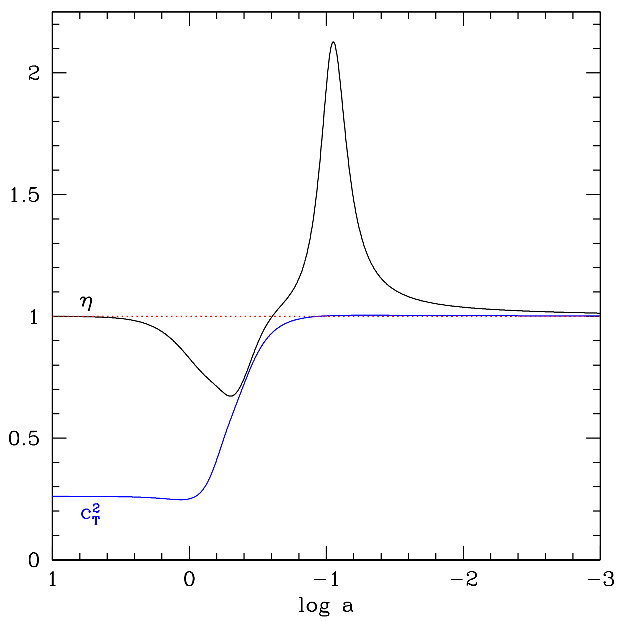

FIG. 3. As Fig. 1 but for an uncoupled Galileon gravity model where the gravitational strength $G_{\rm eff}$ returns to general relativity at late times. Although at late times the slip also goes to the general relativity value of unity, c_T^2 does not.

different) period.

V. CONCLUSIONS

Modified gravity is an active area of exploration on both the theoretical and observational fronts. It has the potential to give rise to cosmic acceleration, a disconnect between the expansion history and structure growth history, deviations from Newton's constant in the gravitational strengths entering the modified Poisson equations (changing cosmic growth and gravitational lensing), and deviations from general relativity in gravitational wave propagation. Detecting these deviations would be revolutionary, and if there were a connection between the deviations then this might lead to insights about the modified gravity theory.

We explored whether such a connection necessarily exists between the gravitational slip of the scalar metric potentials and the tensor modes, as recently suggested by [1]. The answer is no and yes. Working within covariant Galileon gravity, and its extension to derivative coupling to the Einstein tensor (as appears in other modified gravity theories), we show both analytically and numerically that one can have zero deviation in slip at some moment and still have a gravitational wave speed different from the speed of light, hence a separate signature of breaking general relativity. In particular, during the de Sitter attractor of cosmic acceleration or even the incompletely matter dominated era, this holds. Similarly, one can have a gravitational wave speed equaling the speed of light at some moment, but a deviation in the gravitational slip. Only when the effective dark energy density makes a negligible contribution, $\Omega_\pi \ll 1$, does lack of a deviation in one sector necessitate simultaneous general relativistic behavior in the other. Globally in time, however, if one deviation ever occurs, the other will deviate at some time; and if one deviation never occurs, the other will never deviate.

Taken together, these conditions imply that observations of cosmic large scale structure, its growth, evolution, and gravitational lensing effects, are complementary to future measurements of gravitational waves. Gravitational wave observations thus have a definite role to play in understanding the nature of cosmic acceleration and gravity.

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Appendix A: Relation to Gravitational Property Functions

In [19] they describe the gravitational sector in terms of four "property functions" α_i: α_K describing the kinetic properties, α_B the braiding of the kinetic and metric terms, α_M the running of the Planck mass, and α_T the tensor gravitational wave speed. In Galileon gravity, the gravitational wave propagation equation is affected only by their speed and the Planck mass running. The latter vanishes in the de Sitter limit, however, so we have focused here on the gravitational wave speed; our conclusions do not alter when considering the other terms.

In terms of our notation, we can write

$$\alpha_T = c_T^2 - 1 = \frac{2\kappa_3}{\kappa_4} - 1 \tag{A1}$$

$$\alpha_B = \frac{2\kappa_5 x}{\kappa_4} \tag{A2}$$

$$\alpha_K = \frac{4\kappa_2 x^2}{\kappa_4} \tag{A3}$$

$$M_*^2 = \frac{-\kappa_4}{2} \tag{A4}$$

$$\alpha_M = \frac{d\ln M_*^2}{d\ln a} = \frac{\kappa_4'}{\kappa_4}\ . \tag{A5}$$

The gravitational slip involves two terms contributing to the anisotropic stress, from the scalar sector and from the gravitational waves. This leaves open the possibility that the terms can cancel under special circumstances, and this allows $\eta = 1$ in the de Sitter limit despite c_T^2 deviating from 1 (and hence $\alpha_T \neq 0$). Conversely, when $c_T^2 = 1$ at some moment (say in the recent past), the slip can still deviate from general relativity because of a contribution proportional to α_M.

Finally, note that [2] and [19] caution that the quasistatic approximation involves not only the Hubble scale but the braiding scale. Basically, if the coefficients of the spatial derivative terms (Laplacians) in the equations for the metric potentials become small, then we can no longer neglect the time derivative terms. The condition for continued validity of the quasistatic approximation is

$$\alpha_B \, \frac{k}{aH} \gg 1 \ . \tag{A6}$$

We have verified numerically that for the cases used in our Figures, today $\alpha_B \sim \mathcal{O}(1)$ and $\alpha_B > 0.01$ for $a > 10^{-3}$, and so the quasistatic approximation holds on sufficiently subhorizon observational scales.

Appendix B: Galileon Functions

In Eq. (10) we use quantities κ_i for notational simplicity. Here we exhibit them for covariant Galileon gravity allowing derivative coupling, following [14].

$$\kappa_1 = -6c_4 H^3 x^2 \left(H'x + Hx' + \frac{Hx}{3} \right) + 2c_{\rm G} \left(HH'x + H^2 x' + H^2 x \right) + c_5 H^5 x^3 \left(12Hx' + 15H'x + 3Hx \right) \tag{B1}$$

$$\kappa_2 = -\frac{c_2}{2} + 6c_3 H^2 x + 3c_{\rm G} H^2 - 27 c_4 H^4 x^2 + 30 c_5 H^6 x^3 \tag{B2}$$

$$\kappa_3 = -1 - \frac{c_4}{2} H^4 x^4 + c_{\rm G} H^2 x^2 - 3c_5 H^5 x^4 \left(Hx' + H'x \right) \tag{B3}$$

$$\kappa_4 = -2 + 3c_4 H^4 x^4 - 2c_{\rm G} H^2 x^2 - 6c_5 H^6 x^5 \tag{B4}$$

$$\kappa_5 = 2c_3 H^2 x^2 - 12 c_4 H^4 x^3 + 4c_{\rm G} H^2 x + 15 c_5 H^6 x^4 \tag{B5}$$

$$\kappa_6 = \frac{c_2}{2} - 2c_3 \left(H^2 x' + HH'x + 2H^2 x \right) + c_4 \left(12 H^4 x x' + 18 H^3 x^2 H' + 13 H^4 x^2 \right) - c_{\rm G} \left(2HH' + 3H^2 \right) - c_5 \left(18 H^6 x^2 x' + 30 H^5 x^3 H' + 12 H^6 x^3 \right) \ . \tag{B6}$$

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