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Supernova Bounds on Weinberg's Goldstone Bosons

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Recently, Weinberg proposed a scenario where Goldstone bosons may be masquerading as fractional cosmic neutrinos. We calculate the energy loss rates through the emission of these Goldstone bosons in a post-collapse supernova core. Invoking the well established emissivity bound from the Supernova 1987A observations and simulations, we find that nuclear bremsstrahlung processes can notably impose a bound on the Goldstone boson coupling to the Standard Model Higgs, g, dependent on the mass of the associated radial field, m_r. We apply the supernova emissivity bound at typical core conditions: a density of $\rho = 3 \cdot 10^{14}$ g/cm^3 and a temperature $T = 30$ MeV. Even in the conservative limit where m_r is large enough compared with the Goldstone boson energies attainable at this temperature, our bound $|g| \lesssim 0.011\,(m_r/500\text{ MeV})^2$ is very competitive to those derived from current and projected sensitivities of collider experiments.

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I. INTRODUCTION

The cosmic microwave background (CMB) radiation, if combined with other observational data, can be used to constrain the effective number of light neutrino species. The WMAP9 data combined with eCMB, BAO, and H_0 measurements has inferred $N_\nu = 3.55^{+0.49}_{-0.48}$ at 68% CL [1]. Latest Planck data combined with WP, highL, BAO, and H_0 measurements gives $N_\nu = 3.52^{+0.48}_{-0.45}$ at 95% CL [2]. Most recently, with the inclusion of the B-mode polarization data by the BICEP2 experiment [3], evidence for an extra weakly-interacting light species becomes favorable, with $N_\nu \simeq 4$ (see e.g. Ref. [4]). These bounds are consistent with that from the big bang nucleosynthesis (BBN) $N_\nu = 3.71^{+0.47}_{-0.45}$ (see e.g. Ref. [5]). On the other hand, the standard scenario with three active, massless neutrinos predicts $N_\nu = 3.046$ at the CMB epoch [6].

Recently, Weinberg [7] has investigated whether Goldstone bosons can be masquerading as fractional cosmic neutrinos. The motivation is that they would be massless or nearly massless, and their characteristic derivative coupling would make them very weakly-interacting at sufficiently low temperatures. The most crucial criterion is that those Goldstone bosons have to decouple from the thermal bath early enough so that their temperature is lower than that of the neutrinos. A simple extended Higgs sector in the Standard Model (SM) has been proposed to realize this idea such that the Goldstone bosons contribute significantly to the effective number of light species. The thermal history of these Goldstone bosons depends crucially on their coupling to the Standard Model Higgs field and the mass of the radial field. An upper bound on the coupling constant can be quickly derived using the limit on the invisible decay width of the SM Higgs. In this work we will examine the viability of this scenario by considering the cooling of a post-collapse supernova core, such as the Supernova 1987A.

II. WEINBERG'S MODEL

Let us first briefly summarize Weinberg's model [7] following the convention of Ref. [8]. Consider the simplest possible broken continuous symmetry, a global $U(1)$ symmetry associated with the conservation of some quantum number W. A single complex scalar field $S(x)$ is introduced for breaking this symmetry spontaneously. With this field added to the SM, the Lagrangian is

$$\begin{aligned}\mathcal{L} = (\partial_\mu S^\dagger)(\partial^\mu S) + \mu^2\, S^\dagger S - \lambda\,(S^\dagger S)^2 \\ - g\,(S^\dagger S)(\Phi^\dagger \Phi) + \mathcal{L}_{\rm SM}\,, \end{aligned} \quad (1)$$

where Φ is the SM Higgs doublet, μ^2, g, and λ are real constants, and $\mathcal{L}_{\rm SM}$ is the usual SM Lagrangian. One separates a massless Goldstone boson field $\alpha(x)$ and a massive radial field $r(x)$ in $S(x)$ by defining

$$S(x) = \frac{1}{\sqrt{2}}\,(\langle r\rangle + r(x))\, e^{2\imath\alpha(x)}\,, \quad (2)$$

where the fields $r(x)$ and $\alpha(x)$ are real. In the unitary gauge, one sets $\Phi^{\rm T} = (0\,,\, \langle\varphi\rangle + \varphi(x))/\sqrt{2}$, where $\varphi(x)$ is the physical Higgs field. The Lagrangian in Eq. (1) thus becomes

$$\begin{aligned}\mathcal{L} = \frac{1}{2}(\partial_\mu r)(\partial^\mu r) + \frac{1}{2}\frac{(\langle r\rangle + r)^2}{\langle r\rangle^2}(\partial_\mu\alpha)(\partial^\mu\alpha) \\ + \frac{\mu^2}{2}(\langle r\rangle + r)^2 - \frac{\lambda}{4}(\langle r\rangle + r)^4 \\ - \frac{g}{4}(\langle r\rangle + r)^2(\langle\varphi\rangle + \varphi)^2 + \mathcal{L}_{\rm SM}\,. \end{aligned} \quad (3)$$

In Eq. (3), we have replaced $\alpha(x) \to \alpha(x)/(2\langle r\rangle)$ in order to achieve a canonical kinetic term for the $\alpha(x)$ field. In this model, the interaction of the Goldstone bosons with the SM particles arises entirely from a mixing of the radial boson with the Higgs boson via the mixing angle

$$\tan 2\theta = \frac{2g\,\langle\varphi\rangle\,\langle r\rangle}{m_\varphi^2 - m_r^2}\,. \tag{4}$$

The φ-r mixing allows the SM Higgs boson to decay into a pair of the Goldstone bosons with the decay width

$$\Gamma_{\varphi\to 2\alpha} = \frac{g^2\,\langle\varphi\rangle^2\,m_\varphi^3}{32\pi\,(m_\varphi^2 - m_r^2)^2}\,. \tag{5}$$

For $\langle\varphi\rangle = 247$ GeV, $m_\varphi = 125$ GeV, and assuming $m_r \ll m_\varphi$, one obtains a constraint of $|g| \lesssim 0.018$. In Ref. [8] it is pointed out that by including the $\varphi \to rr$ channel, the constraint can be improved to $|g| \lesssim 0.011$. Further collider signatures of this model have been investigated therein and in Ref. [9]. In the future, the International Linear Collider (ILC) may constrain the branching ratio of Higgs invisible decays to $< 0.4 - 0.9\%$ [10], improving the collider bound on $|g|$ by a factor of $5-7$.

From the mixing term $-g\,\langle\varphi\rangle\,\langle r\rangle\,\varphi r$ and the interaction term $(1/\langle r\rangle)\,r\,\partial_\mu\alpha\,\partial^\mu\alpha$ in the Lagrangian (Eq. (3)) as well as the SM Higgs-fermion coupling $-m_f\varphi\bar{f}f/\langle\varphi\rangle$, an effective interaction between the Goldstone bosons and any SM fermion f,

$$+g\,m_f\,\bar{f}f\,\varphi r\,\partial_\mu\alpha\,\partial^\mu\alpha\,, \tag{6}$$

is produced. In the early universe, the Goldstone bosons remain in thermal equilibrium via the processes $\alpha\alpha \leftrightarrow \bar{f}f$, where f are SM fermions in the thermal bath. If the Goldstone bosons freeze out before the muon annihilation occurs, they contribute about 0.39 to the effective number of neutrino types in the era before recombination. Weinberg has made an order-of-magnitude estimate

$$\frac{g^2 m_\mu^7 M_{\rm Pl}}{m_\varphi^4 m_r^4} \approx 3\,, \tag{7}$$

which shows that for $g = 0.005$ the Goldstone bosons decouples at muon annihilation for $m_r \approx 500$ MeV (see also Ref. [11]). While a more accurate calculation is underway [12], in this work we will use $m_r = 500$ MeV as a benchmark.

III. SUPERNOVA COOLING DUE TO GOLDSTONE BOSON EMISSION FROM PAIR ANNIHILATION PROCESSES

Now we turn to supernova cooling. The observed duration of neutrino burst events from Supernova 1987A in several detectors confirmed the standard picture of neutrino cooling of post-collapse supernova. In the second phase of neutrino emission, a light particle which interact even more weakly than neutrinos could lead to more efficient energy loss and shorten the neutrino burst duration. Demanding that the novel cooling agent X should not have affected the total cooling time significantly, an upper bound on their emissivity can be derived [13, 14]

$$\epsilon_X \equiv \frac{Q_X}{\rho} \lesssim 10^{19}\,{\rm erg}\cdot{\rm g}^{-1}\cdot{\rm s}^{-1} = 7.324\cdot 10^{-27}\,{\rm GeV}\,, \tag{8}$$

where Q_X is the energy loss rate. This bound, dubbed the "Raffelt criterion", is to be applied at typical core conditions, i.e. a density $\rho = 3\cdot 10^{14}$ g/cm^3 and a temperature $T = 30$ MeV. It has been used exhaustively in the literature to constrain the properties of exotic particles, notably the axions [15–17], right-handed neutrinos [15], Kaluza-Klein gravitons [18, 19], and unparticles [20, 21] etc. Among all, the authors of Ref. [19] have performed self-consistent simulations of the early, neutrino-emitting phase of a proto-neutron star including energy losses due to the Kaluza-Klein gravitons in large extra dimension scenarios. From their subsequent probabilistic analyses they inferred bounds on the radii of the extra dimensions for the cases of 2 and 3 extra dimensions. They found excellent agreement between their simulation results and those obtained by using the Raffelt criterion.

Stellar energy loss due to Goldstone boson pair emission had been considered for the Compton-like process [22]. Here, from their effective interaction with the SM fermions (Eq. (6)), the Goldstone bosons can be produced in electron-positron pair annihilation $e^+e^- \to \alpha\alpha$, in photon scattering $\gamma\gamma \to \alpha\alpha$ and in nuclear bremsstrahlung processes $NN \to NN\alpha\alpha$. The number densities of neutron, proton, electron, and electron neutrino in the supernova core are determined by the baryon density n_B, charge neutrality and β-equilibrium conditions. The chemical potential of each particle at $T = 30$ MeV are $\mu_n = 971$ MeV, $\mu_p = 923$ MeV, $\mu_e = 200$ MeV, and $\mu_{\nu_e} = 152$ MeV, respectively, for a fixed lepton fraction $Y_L = 0.3$. The degeneracy parameter for the neutron is $\eta_n \equiv (\mu_n - m_n)/T \approx 1.05$ in this case, corresponding to neither strongly non-degenerate nor degenerate case. On the other hand, the electrons are highly degenerate.

i) For the $e^+(p_1)\,e^-(p_2) \to \alpha(q_1)\,\alpha(q_2)$ process, the amplitude squared, summed over the initial spins, is

$$\sum_{\rm spins} |\mathcal{M}_{e^+e^-\to\alpha\alpha}|^2 = \frac{16\,g^2\,m_e^2\,(q_1\cdot q_2)^2\,\left[(p_1\cdot p_2) - m_e^2\right]}{(s-m_\varphi^2)^2\,(s-m_r^2)^2}\,, \tag{9}$$

where $s = (p_1+p_2)^2 = (q_1+q_2)^2$ is the center-of-mass (cm) energy squared. Denote the energies of the $e^\pm$ and the Goldstone boson pairs by E_1, E_2, ω_1, and ω_2, respectively. The energy loss rate due to this process is

$$\begin{aligned} Q_{e^+e^-\to\alpha\alpha} &= \frac{1}{2!}\int\prod_{j=1}^{2}\frac{d^3\vec{q}_j}{(2\pi)^3 2\omega_j}\int\prod_{i=1}^{2}\frac{2\,d^3\vec{p}_i}{(2\pi)^3\,2E_i} \\ &\times \frac{1}{4}\sum_{\rm spins}|\mathcal{M}_{e^+e^-\to\alpha\alpha}|^2(2\pi)^4\delta^4(p_1+p_2-q_1-q_2) \\ &\times f_1 f_2\,(\omega_1+\omega_2)\,, \end{aligned} \tag{10}$$

where $f_1(\vec{p}_1) = (e^{(E_1+\mu_e)/T}+1)^{-1}$ and $f_2(\vec{p}_2) = (e^{(E_2-\mu_e)/T}+1)^{-1}$ are the distribution functions for the positron and the electron, respectively. A symmetry factor of 1/2! is included for the identical particles in the final state. In the large m_r limit, the r field propagator can be expanded in powers of (s/m_r^2). In this work we use only the leading term in the expansion, as in Ref. [7]. The results we will present should thus be regarded as conservative estimates, since all higher terms contribute positively to the energy loss rate. Performing the $d^3\vec{q}_1 d^3\vec{q}_2$ integral analytically, we obtain

$$\int \frac{d^3\vec{q}_1}{\omega_1}\frac{d^3\vec{q}_2}{\omega_2}\frac{(q_1\cdot q_2)^2}{m_r^4}\,\delta^4(p_1+p_2-q_1-q_2) = \frac{\pi}{2}\frac{(p_1+p_2)^4}{m_r^4}\,, \tag{11}$$

analogous to the Lenard's Identity for the $e^+e^- \to \nu\bar{\nu}$ process [23]. Then following Ref. [24], we define these two dimensionless functions

$$\begin{aligned} U_k &\equiv \frac{1}{\pi^2}\int_0^\infty \frac{|\vec{p}_1|^2 d|\vec{p}_1|}{T^3}\left(\frac{E_1}{T}\right)^k f_1(\vec{p}_1)\,, \\ \Phi_k &\equiv \frac{1}{\pi^2}\int_0^\infty \frac{|\vec{p}_2|^2 d|\vec{p}_2|}{T^3}\left(\frac{E_2}{T}\right)^k f_2(\vec{p}_2)\,. \end{aligned} \tag{12}$$

The energy loss rate can then be expressed as

$$Q_{e^+e^-\to\alpha\alpha} = \frac{T^{11}}{16\pi}\left(\frac{g^2 m_e^2}{m_r^4 m_\varphi^4}\right)\sum C_{ij}(U_i\,\Phi_j + \Phi_i\,U_j)\,, \tag{13}$$

where the sum runs over $\{i,j\}$ pairs, with $C_{23} = 2$, $C_{12} = 1/3$, $C_{03} = -1$, $C_{01} = C_{-12} = -1/3$, and $C_{-10} = -2/3$. Evaluating the U_k, Φ_k functions numerically for the typical supernova core condition $\rho = 3\cdot 10^{14}$ g/cm^3, $T = 30$ MeV and $\mu_e = 200$ MeV, we find the emissivity due to the process $e^+e^- \to \alpha\alpha$ is

$$\epsilon_{e^+e^-\to\alpha\alpha} = 1.73\cdot 10^{-28}\ \text{GeV}\,g^2\left(\frac{m_r}{500\ \text{MeV}}\right)^{-4}. \tag{14}$$

One sees that for m_r around 500 MeV, even with $g \approx 0.018$ saturating the collider bound, contribution from Goldstone boson emission to supernova cooling is far from competing with that from neutrino emission.

ii) The energy loss rate for the photon scattering process can be calculated similarly. The amplitude squared for the process $\gamma(p_1)\,\gamma(p_2) \to \alpha(q_1)\,\alpha(q_2)$ is

$$\begin{aligned} |\mathcal{M}_{\gamma\gamma\to\alpha\alpha}|^2 = \left(\frac{\alpha}{4\pi}\right)^2 \frac{16\,G_F}{\sqrt{2}}\,|F|^2\,(q_1\cdot q_2)^2 \\ \times \frac{g^2\,\langle\varphi\rangle^2}{(s-m_\varphi^2)^2}\,\frac{(p_1\cdot p_2)^2}{(s-m_r^2)^2}\,, \end{aligned} \tag{15}$$

and the resulting energy loss rate in the large m_r limiting case is

$$Q_{\gamma\gamma\to\alpha\alpha} = \left(\frac{1}{2!}\right)^2 \frac{1819.8}{5\sqrt{2}\pi}\left(\frac{\alpha}{2\pi}\right)^2 G_F\,|F|^2\,\frac{g^2\,\langle\varphi\rangle^2}{m_\varphi^4\,m_r^4}\,T^{13}\,. \tag{16}$$

Here, α and G_F are the fine-structure constant and the Fermi constant, respectively, and the symmetry factor $(1/2!)^2$ is included for identical particles in the initial and in the final state. The form factor F enters through the amplitude for the SM Higgs decay to two photons (see e.g. Ref. [25, 26]), in this case a function of the cm energy $\sqrt{s}$ in the photon collision. The cm energies attainable at the typical temperature in the post-collapse supernova core correspond to the mass of the light (sub-GeV) Higgs boson studied in Ref. [27, 28]. For simplicity, we use a constant value of $|F|^2 = 4$ to approximate the result of Ref. [28], and find that the emissivity is

$$\epsilon_{\gamma\gamma\to\alpha\alpha} \sim \frac{6.32\cdot 10^{-29}\ \text{GeV}}{(\rho/3\cdot 10^{14}\ \text{g/cm}^3)}\,\frac{g^2}{\left(\frac{m_r}{500\ \text{MeV}}\right)^4}\left(\frac{T}{30\ \text{MeV}}\right)^{13}, \tag{17}$$

even smaller than that from the electron-positron annihilation process.

IV. SUPERNOVA COOLING DUE TO GOLDSTONE BOSON EMISSION FROM NUCLEAR BREMSSTRAHLUNG PROCESSES

Now we turn to evaluate the energy loss rate due to the nuclear bremsstrahlung process

$$\begin{aligned} Q_{NN\to NN\alpha\alpha} = \frac{\mathcal{S}}{2!}\int\prod_{j=1}^{2}\frac{d^3\vec{q}_j}{(2\pi)^3\,2\omega_j}\int\prod_{i=1}^{4}\frac{d^3\vec{p}_i}{(2\pi)^3\,2E_i} \\ \times\sum_{\text{spins}}|\mathcal{M}_{NN\to NN\alpha\alpha}|^2\,f_1 f_2(1-f_3)(1-f_4)\,(\omega_1+\omega_2) \\ \times(2\pi)^4\delta^4(p_1+p_2-p_3-p_4-q_1-q_2)\,, \end{aligned} \tag{18}$$

where $p_{1,2}$ are the four-momenta of the initial-state nucleons, and $p_{3,4}$ those of the final-state nucleons $N = p, n$. For nn or pp interactions, the symmetry factor for identical particles is $\mathcal{S} = \frac{1}{4}$, whereas for np interactions it is 1. The amplitude squared $|\mathcal{M}_{nn\to nn\alpha\alpha}|^2$ is summed over initial and final nucleon spins but without being averaged. In the non-relativistic limit, the occupation numbers are given by the normalized Maxwell-Boltzmann distribution $f(\vec{p}) = (n_B/2)\,(2\pi/m_N T)^{3/2}\,e^{-\vec{p}^2/2m_N T}$.

To calculate the scattering amplitude, first we need to obtain the effective coupling of the Goldstone bosons to the nucleons through the Higgs. We follow the Shifman-Vainshtein-Zakharov (SVZ) approach [29, 30] to evaluate the matrix element $\left\langle N|\sum_q m_q\,\bar{q}q + \sum_Q m_Q\,\bar{Q}Q|N\right\rangle$, with q, Q denoting the light and the heavy quarks, respectively. Using the SVZ heavy quark expansion

$$\sum_Q m_Q\,\bar{Q}Q \to -\frac{2}{3}\frac{\alpha_s}{8\pi}n_h\,G^a_{\mu\nu}G^{a\,\mu\nu}\,, \tag{19}$$

in the $m_q \to 0$ limit we obtain the effective Lagrangian for the interaction of Weinberg's Goldstone bosons with the nucleons

$$\mathcal{L}_{\text{eff}} = \frac{2}{27}\,n_h\,g\,\frac{m_N}{m_r^2\,m_\varphi^2}\,\partial_\mu\alpha\,\partial^\mu\alpha\,\bar{\psi}_N\psi_N\,, \tag{20}$$

with n_h the number of heavy quarks. From this we define an effective coupling $g_N \equiv (2/27)\, n_h\, g$, to be used in the following calculation.

Armed with this knowledge, we follow the prescription given in Ref. [31] to calculate the amplitude for the nuclear bremsstrahlung process. In the one-pion exchange (OPE) approximation, there are four direct and four exchange diagrams, corresponding to the Goldstone boson pairs being emitted by any one of the nucleons. In total there are 64 diagrams to calculate, which can be grouped into 8 categories. Denote the 4-momenta of the exchanged pions by $k_a \equiv p_2 - p_4$ (in the direct diagrams) and $l_a \equiv p_2 - p_3$ (in the exchange diagrams), respectively. In young supernova cores, $k_a^2 \simeq -|\vec{k}|^2$, $l_a^2 \simeq -|\vec{l}|^2$, and $|\vec{k}|^2, |\vec{l}|^2 \sim 3 m_N T$. Again we work in the conservative large m_r limit, using only the leading term in the (s/m_r^2) expansion of the r field propagator. Summing all diagrams from the 8 categories and expanding in powers of (T/m_N), we find the amplitude squared for the nuclear bremsstrahlung process $nn \to nn\alpha\alpha$ to be

$$\sum_{\text{spins}} |\mathcal{M}_{nn\to nn\alpha\alpha}|^2 \approx (2!)^2 \left(\frac{g_N\, m_N}{m_r^2\, m_\varphi^2}\right)^2 \left(\frac{2 m_N f}{m_\pi}\right)^4 (q_1 \cdot q_2)^2 \, \frac{(2q^2)^2\, m_N^2}{(2p\cdot q)^4} \times 256 \left\{ \frac{|\vec{k}|^4}{(|\vec{k}|^2 + m_\pi^2)^2} + \frac{|\vec{l}|^4}{(|\vec{l}|^2 + m_\pi^2)^2} + \frac{|\vec{k}|^2 |\vec{l}|^2 - 2|\vec{k}\cdot\vec{l}|^2}{(|\vec{k}|^2 + m_\pi^2)(|\vec{l}|^2 + m_\pi^2)} + \ldots \right\}, \tag{21}$$

with $q = q_1 + q_2$. Here, $\alpha_\pi \equiv (2 m_N f/m_\pi)^2/(4\pi) \approx 15$ with $f \approx 1$ being the pion-nucleon "fine-structure" constant. The $(2!)^2$ factor arises from the Wick contraction of the two Goldstone bosons in the final state. Considering only the leading terms in the (T/m_N) expansion of the amplitude squared and neglecting the pion mass m_π in the curly brackets, the phase space integral in Eq. (18) can be performed analytically as for the axion or neutrino emission cases [32]. We estimate the energy loss rate due to $nn \to nn\alpha\alpha$ in the non-degenerate (ND) case to be

$$Q^{\rm ND}_{nn\to nn\alpha\alpha} \simeq \frac{1056\sqrt{\pi}}{(2\pi)^6} \left(3 - \frac{2\beta}{3}\right) n_B^2 \times \left(\frac{g_N\, m_N}{m_r^2 m_\varphi^2}\right)^2 \left(\frac{2 m_N f}{m_\pi}\right)^4 \frac{T^{9.5}}{m_N^{4.5}}\,. \tag{22}$$

The β term arises from the averaging of the $(\vec{k}\cdot\vec{l})$ term over the nucleon scattering angle and we find that $\beta = 2.0938$. In the large m_r limiting case, the very strong temperature dependence arises from the presence of the $(q_1 \cdot q_2)^2/m_r^4$ term in the amplitude squared because of the $\partial_\mu \alpha\, \partial^\mu \alpha \bar{f} f$ type coupling [33] in Eq. (6). In comparison, in the ND limit the temperature dependence of the energy loss rate is $T^{3.5}$ and $T^{5.5}$ for the axion and the neutrino emission cases, respectively [31, 32]. In large extra dimension scenarios with 2 and 3 extra dimensions, the temperatuer dependence of the Kaluza-Klein graviton emissivity is $T^{5.42}$ and $T^{6.5}$, respectively [19]. We compare the emissivity due to the Goldstone bosons

$$\epsilon^{\rm ND}_{nn\to nn\alpha\alpha} = \frac{Q^{\rm ND}_{nn\to nn\alpha\alpha}}{\rho} \simeq \frac{6.65\cdot 10^{-22}\ {\rm GeV}}{(\rho/3\cdot 10^{14}\ {\rm g/cm^3})}\, g_N^2 \left(\frac{m_r}{500\ {\rm MeV}}\right)^{-4} \left(\frac{T}{30\ {\rm MeV}}\right)^{9.5}, \tag{23}$$

with the emissivity bound in Eq. (8), which should be applied at $\rho = 3\cdot 10^{14}$ g/cm^3 and $T = 30$ MeV [14]. We obtain a constraint of

$$g_N^2 \left(\frac{m_r}{500\ {\rm MeV}}\right)^{-4} \lesssim 1.1\cdot 10^{-5}\,, \tag{24}$$

on the coupling of Weinberg's Goldstone bosons to nucleons through the Higgs. This implies for the coupling constant (cf. Eq. (1)) to the Higgs that

$$|g| \lesssim 0.011 \left(\frac{m_r}{500\ {\rm MeV}}\right)^2, \tag{25}$$

from the relation $g_N = (2/27)\, n_h\, g$, with the number of heavy quark flavours $n_h = 4$. One sees that the supernova bound is competitive and complementary to the collider bound $g \lesssim 0.018\,(0.011)$, which is insensitive to the m_r value. We have checked the pion mass effects on the energy loss rate by keeping the m_π^2 in the denominators in Eq. (21) and performing the phase space integrals using the Monte Carlo routine VEGAS [34]. We find that the reduction is 12% at $T = 30$ MeV and only 5% at $T = 80$ MeV, milder than that in the axion emission case. It remains to estimate the emissivity for more general cases, i.e. for smaller m_r values, and including the higher-order terms in the (T/m_N) expansion of the amplitude squared (Eq. (21)), to find the modifications of this bound. Besides using the OPE approximation, one may also estimate the emissivity due to nuclear bremsstrahlung processes in a model-independent way following Refs. [18, 35]. In this approach, the emissivity is related to the measured nucleon-nucleon total cross section by taking the soft radiation $(\omega_1 + \omega_2 \to 0)$ limit.

Eq. (23) imparts the impression that our supernova bound on the Goldstone boson coupling is very sensitive to the supernova core temperature. For example, if we assume that the temperature at supernova core is $T = 20$ MeV, our bound in Eq. (25) would be 6.86 times weaker. The authors of Ref. [19] did not present the results for more than 3 large extra dimensions, otherwise we would know whether one can still apply the Raffelt criterion at $T = 30$ MeV in the case of emissivities with stronger T-dependence. It is appropriate to perform a simulation of the early phase of a proto-neutron star including energy losses due to Goldstone boson emission, this is however beyond the scope of this work.

V. SUMMARY AND OUTLOOK

In conclusion, we have determined the allowed range for the coupling constant g in dependence of m_r, the mass of the radial field $r(x)$ in Weinberg's extended Higgs model, in which new Goldstone bosons may be masquerading as fractional cosmic neutrinos. In the conservative large m_r limit, we have estimated the energy loss rates in post-collapse supernova cores due to Goldstone boson emission in different channels including the e^+e^- annihilation, photon scattering and nuclear bremsstrahlung processes. We present our main result in Eq. (25), obtained by confronting our estimate for the nuclear bremsstrahlung processes with the well established emissivity bound from the Supernova 1987A observations and simulations, known as the "Raffelt criterion". We applied the Raffelt criterion at typical core conditions: a density of $\rho = 3 \cdot 10^{14}$ g/cm^3 and a temperature $T = 30$ MeV, and discussed the validity in our case. We found that even in the conservative limit where m_r is large enough compared with the Goldstone boson energies attainable at this temperature, our bound is highly competitive to that derived from collider experiments. In the future, if the ILC can indeed improve the collider bound to $|g| < 0.0015$, Weinberg's estimate (Eq. (7)) would require $m_r < 274$ MeV in order that the Goldstone bosons contribute 0.39 to N_ν. In this case our bound is at least as good as $|g| < 0.0033$, still competitive. Technical details, investigation of more general cases, as well as other astrophysical constraints will be presented in a following work [12].

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[1] G. Hinshaw *et al.* [WMAP Collaboration], arXiv:1212.5226 [astro-ph.CO].
[2] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1303.5076 [astro-ph.CO].
[3] P. A. R. Ade *et al.* [BICEP2 Collaboration], arXiv:1403.3985 [astro-ph.CO].
[4] E. Giusarma, E. Di Valentino, M. Lattanzi, A. Melchiorri and O. Mena, arXiv:1403.4852 [astro-ph.CO].
[5] G. Steigman, Adv. High Energy Phys. **2012**, 268321 (2012) [arXiv:1208.0032 [hep-ph]].
[6] G. Mangano, G. Miele, S. Pastor and M. Peloso, Phys. Lett. B **534**, 8 (2002) [astro-ph/0111408].
[7] S. Weinberg, Phys. Rev. Lett. **110**, 241301 (2013) [arXiv:1305.1971 [astro-ph.CO]].
[8] K. Cheung, W. -Y. Keung and T. -C. Yuan, Phys. Rev. D **89** (2014) 015007 [arXiv:1308.4235 [hep-ph]].
[9] L. A. Anchordoqui, L. H. M. Da Silva, P. B. Denton, H. Goldberg, T. C. Paul, B. J. Vlcek and T. J. Weiler, arXiv:1312.2547 [hep-ph].
[10] P. Bechtle, S. Heinemeyer, O. Stl, T. Stefaniak and G. Weiglein, arXiv:1403.1582 [hep-ph].
[11] C. Garcia-Cely, A. Ibarra and E. Molinaro, JCAP **1311** (2013) 061 [arXiv:1310.6256 [hep-ph]].
[12] W.-Y. Keung, K.-W. Ng, H. Tu and T.-C. Yuan, in preparation.
[13] G. G. Raffelt, Phys. Rept. **198**, 1 (1990).
[14] G. G. Raffelt, Lect. Notes Phys. **741**, 51 (2008) [hep-ph/0611350].
[15] G. Raffelt and D. Seckel, Phys. Rev. Lett. **60** (1988) 1793.
[16] M. S. Turner, Phys. Rev. Lett. **60** (1988) 1797.
[17] R. Mayle, J. R. Wilson, J. R. Ellis, K. A. Olive, D. N. Schramm and G. Steigman, Phys. Lett. B **203** (1988) 188.
[18] C. Hanhart, D. R. Phillips, S. Reddy and M. J. Savage, Nucl. Phys. B **595** (2001) 335 [nucl-th/0007016].
[19] C. Hanhart, J. A. Pons, D. R. Phillips and S. Reddy, Phys. Lett. B **509** (2001) 1 [astro-ph/0102063].
[20] S. Hannestad, G. Raffelt and Y. Y. Y. Wong, Phys. Rev. D **76** (2007) 121701 [arXiv:0708.1404 [hep-ph]].
[21] A. Freitas and D. Wyler, JHEP **0712** (2007) 033 [arXiv:0708.4339 [hep-ph]].
[22] D. Chang and W. -Y. Keung, Phys. Lett. B **217** (1989) 238.
[23] A. Lenard, Phys. Rev. **90** (1953) 968.
[24] D. G. Yakovlev, A. D. Kaminker, O. Y. Gnedin and P. Haensel, Phys. Rept. **354** (2001) 1 [astro-ph/0012122].
[25] W. J. Marciano, C. Zhang and S. Willenbrock, Phys. Rev. D **85**, 013002 (2012) [arXiv:1109.5304 [hep-ph]].
[26] J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, Front. Phys. **80** (2000) 1.
[27] J. R. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B **106** (1976) 292.
[28] H. Leutwyler and M. A. Shifman, Phys. Lett. B **221** (1989) 384.
[29] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147** (1979) 385.
[30] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147** (1979) 448.
[31] R. P. Brinkmann and M. S. Turner, Phys. Rev. D **38**, 2338 (1988).
[32] G. Raffelt and D. Seckel, Phys. Rev. D **52** (1995) 1780 [astro-ph/9312019].
[33] We note that the temperature exponent of a value 9.5 coincides with the corresponding calculation [21] of the

scalar unparticle bremsstrahlung with an unparticle dimension $d_{\mathcal{U}} = 4$ since the operator $\partial_\mu \alpha\, \partial^\mu \alpha$ is of dimension 4. We thank A. Freitas for his nice communication.

[34] G. P. Lepage, J. Comput. Phys. **27** (1978) 192.

[35] C. Hanhart, D. R. Phillips and S. Reddy, Phys. Lett. B **499** (2001) 9 [astro-ph/0003445].