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Double logarithms of Q^2/m_c^2 that appear in the cross section for $e^+e^- \to J/\psi + \eta_c$ at next-to-leading order (NLO) in the strong coupling α_s account for the bulk of the NLO correction at B-factory energies. (Here, Q^2 is the square of the center-of-momentum energy, and m_c is the charm-quark mass.) We analyze the double logarithms that appear in the contribution of each of NLO Feynman diagram, and we find that the double logarithms arise from both the Sudakov and the endpoint regions of the loop integration. The Sudakov double logarithms cancel in the sum over all diagrams. We show that the endpoint region of integration can be interpreted as a pinch-singular region in which a spectator fermion line becomes soft or soft and collinear to a produced meson. This interpretation may be important in establishing factorization theorems for helicity-flip processes, such as $e^+e^- \to J/\psi + \eta_c$, and in resumming logarithms of Q^2/m_c^2 to all orders in α_s.

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I. INTRODUCTION

Measurements of the exclusive double-charmonium production cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ by the Belle [1, 2] and *BABAR* [3] collaborations have stimulated a good deal of theoretical activity. Calculations within the nonrelativistic QCD (NRQCD) factorization formalism [4] at leading order (LO) in α_s and v [5, 6] yield values for the production cross section that lie almost an order of magnitude below the measured values. Here, α_s is the strong coupling and v is the velocity of the charm quark (c) or charm antiquark ($\bar{c}$) in the charmonium rest frame.

Calculations at next-to-leading order (NLO) in α_s [7, 8] and NLO in v^2 [9–11] seem to resolve this discrepancy. However, the large NLO corrections raise issues about the convergence of the α_s and v expansions. In the case of the v expansion, the large NLO corrections arise from several sources, each of which contributes a modest correction that is consistent with a convergent expansion [9–11]. In the case of the α_s expansion, the NLO contribution produces a correction of about 100%. It has been pointed out that the bulk of the NLO correction at B-factory energies arises from double logarithms of Q^2/m_c^2, where Q^2 is the square of the e^+e^--center-of-momentum energy and m_c is the charm-quark mass [12]. This suggests that one might gain control of the α_s expansion by resumming the large double (and single) logarithms of Q^2/m_c^2.

The standard tool for the resummation of logarithms in exclusive processes is the light-cone formalism [13, 14]. However, the evolution of light-cone distributions of mesons produces only single logarithms and, so, does not control the double logarithms that appear in NLO amplitude for $e^+e^- \to J/\psi + \eta_c$. It has been suggested in Ref. [12] that the double logarithms could be related to endpoint singularities in the light-cone hard-scattering kernels.[1] Our analysis confirms this conjecture.

In this paper we identify the loop-momentum regions that give rise to the singularities in the double logarithms of Q^2/m_c^2 that appear in the limit $m_c \to 0$. (An abbreviated discussion of these double logarithms and the associated singularities was given in Ref. [15].) We find that the singularities that are associated with the large double logarithms arise from both the Sudakov region and the endpoint region of the loop-momentum integration. The Sudakov logarithms cancel in the sum over Feynman diagrams.[2] Our results for the double logarithms agree diagram by diagram with those that were obtained in the complete NLO calculations in Refs. [7, 8]. A new insight that follows from our analysis is that the endpoint singularity corresponds to a pinch-singular region of the loop-momentum integration in which a spectator-quark line becomes either soft or soft and collinear. (Our nomenclature is that the active-quark line is the quark line to which the virtual photon attaches and that the spectator-quark line is the other quark line.) This

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[1] Such singularities are discussed in Refs. [13, 14].
[2] The cancellation of Sudakov logarithms for color-singlet mesons was noted in Refs. [13, 14].

pinch singularity has a (logarithmically) divergent power count by virtue of the fact that the process $e^+e^- \to J/\psi + \eta_c$ proceeds through a helicity flip. The insight that the endpoint singularity corresponds to a soft-quark pinch-singular region could have important implications for the all-orders factorization of the infrared singularities for this process and for the resummation of the associated logarithms.[3]

The remainder of this paper is organized as follows. In Sec. II, we outline our strategy for computing the double logarithms and analyzing the associated singularities. We compute the double logarithms and identify the loop-momentum regions that give rise to the associated singularities in Sec. III. In this section, we compute both the Sudakov and the endpoint double-logarithmic contribution for each NLO Feynman diagram. In Sec. IV, we give a general argument for the cancellation of the Sudakov double logarithms that is based on a soft-collinear approximation and diagrammatic Ward identities. Sec. V contains a general analysis of the endpoint region, in which we show that the endpoint singularities can yield logarithmic divergences, but not power divergences. We summarize our results in Sec. VI.

II. STRATEGY OF THE COMPUTATION

According to the NRQCD factorization formalism [4], the amplitude for the process $e^+e^- \to J/\psi + \eta_c$ can be written as a sum of products of short-distance coefficients (SDCs) with NRQCD long-distance matrix elements (LDMEs). The double logarithms of Q^2/m_c^2 that are the focus of this paper appear in the single SDC that arises in the NRQCD factorization expression for the amplitude at LO in v.[4] This SDC can be obtained perturbatively by comparing the full-QCD amplitude $i\mathcal{A}[e^+e^- \to c\bar{c}_1(^3S_1) + c\bar{c}_1(^1S_0)]$ with the NRQCD expression for the amplitude. (Here the subscripts 1 indicate that the $c\bar{c}$ pairs are in color-singlet states.) Because the NRQCD LDMEs for the $c\bar{c}$ states are insensitive to momentum scales of order m_c or larger, the double logarithms of Q^2/m_c^2 in the SDC are contained entirely in the full-QCD amplitude. Therefore, in our calculation, we focus on the full-QCD amplitude $i\mathcal{A}[e^+e^- \to c\bar{c}_1(^3S_1) + c\bar{c}_1(^1S_0)]$.

The process $e^+e^- \to c\bar{c}_1(^3S_1) + c\bar{c}_1(^1S_0)$ consists of the process $e^+e^- \to \gamma^*$, followed by the process $\gamma^* \to c\bar{c}_1(^3S_1) + c\bar{c}_1(^1S_0)$. Because the process $e^+e^- \to \gamma^*$ does not receive QCD corrections in relative order $\alpha^0\alpha_s$, we need to consider only the amplitude $i\mathcal{A}[\gamma^* \to c\bar{c}_1(^3S_1) + c\bar{c}_1(^1S_0)]$ in order to compute the NLO corrections relative to the LO amplitude. (Here, α is the electromagnetic coupling constant.) In the remainder of this paper, we will denote the amplitude $i\mathcal{A}[\gamma^* \to c\bar{c}_1(^3S_1) + c\bar{c}_1(^1S_0)]$ by $i\mathcal{A}$.

The process $e^+e^- \to J/\psi + \eta_c$ does not satisfy quark-helicity conservation. In order to produce a helicity flip, the amplitude $i\mathcal{A}$ must contain at least one numerator factor m_c. It follows that the amplitude is suppressed by a factor of m_c/Q relative to a helicity-conserving amplitude. In our calculation, we retain all numerator terms that are proportional to powers of m_c, except as noted. We also keep m_c nonzero in denominators in order to regulate singularities.

Our strategy in analyzing the double logarithms in $i\mathcal{A}$ is to examine the double-logarithmic singularities that appear in the limit $m_c \to 0$. In characterizing these singularities, we ignore factors m_c from propagator numerators and spinor normalizations. From a general Landau analysis of pinch singularities [19–23], we expect singularities to arise from regions of loop momentum in which an internal line is soft and/or collinear to an external line. If m_c is nonzero, then there is no pinch when internal quark lines are soft or collinear or when internal gluon lines are collinear. However, there can still be a pinch when an internal gluon line is soft. We regulate these soft singularities by using dimensional regularization in $d = 4 - 2\epsilon$ dimensions. The soft singularities then produce single poles in ϵ. The amplitudes that we consider are ultraviolet (UV) finite. However, after we reduce tensor integrals to scalar integrals, individual terms can contain UV divergences, which we also regulate dimensionally and which also produce single poles in ϵ. These UV divergences cancel in the sum of contributions from each Feynman diagram.

Although we work in $d = 4 - 2\epsilon$ dimensions, we evaluate Dirac traces and numerator algebra in four dimensions. That simplification does not affect the calculation of the double logarithms of m_c because terms of order ϵ in the numerator can contribute only in conjunction with a soft pole or a UV pole. The coefficient of a soft pole can contain only a single (collinear) logarithm of m_c, while the coefficient of a UV pole cannot contain any logarithm of m_c. Following this procedure for the numerator algebra, we reproduce all of the double logarithms of m_c that appear in the exact NLO calculation of Refs. [7, 8].

We will see that the double logarithmic singularities that appear in the limit $m_c \to 0$ arise from two sources. One

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[3] The existing proof of factorization for exclusive double-quarkonium production in Refs. [16, 17] applies only to processes that do not flip the quark helicity, and, so, is not relevant to the process $e^+e^- \to J/\psi + \eta_c$.

[4] Double logarithms of Q^2/m_c^2 were also found in the order-$\alpha_s v^2$ correction to the amplitude [18].

source is a region of loop momentum in which the momentum of a gluon becomes both soft and collinear. This is the "Sudakov" region. A second source is a region of loop momentum in which one gluon carries away almost all of the momentum of a spectator quark and the other gluon carries away almost all of the momentum of a spectator antiquark. This is the "endpoint" region. As we will show, the endpoint region can also be characterized as a momentum region in which a spectator-quark line becomes either soft or soft and collinear. In a general analysis of pinch singularities, such a momentum configuration produces a pinch. Furthermore, as is evident from our calculation, in order m_c/Q, the pinch has the correct power counting to produce a (logarithmic) singularity. In contrast, in order $(m_c/Q)^0$, this same pinch does not have the correct power counting to produce a singularity. Consequently, endpoint momentum configurations do not play a role in non-helicity-flip processes at the leading non-trivial order in m_c/Q.[5]

III. CALCULATION OF THE DOUBLE LOGARITHMS

In this section, we evaluate the double logarithms that appear in the NLO QCD corrections to the amplitude $\gamma^* \to J/\psi + \eta_c$, and we identify the momentum regions that are associated with the double-logarithmic singularities in the limit $m_c \to 0$.

A. Kinematics, conventions, and nomenclature

First, we describe the kinematics, conventions, and nomenclature that we use in calculating the double logarithms and throughout this paper. We work in the Feynman gauge. We use the light-cone momentum coordinates $k = [k^+, k^-, \boldsymbol{k}_\perp] = [(k^0 + k^3)/\sqrt{2}, (k^0 - k^3)/\sqrt{2}, \boldsymbol{k}_\perp]$ and work in the e^+e^--center-of-momentum frame. Because our calculation is at LO in v, we set the relative momentum of the c and $\bar{c}$ in each charmonium equal to zero. Then, the momenta of the c and $\bar{c}$ in the J/ψ are both $p = [(\sqrt{P^2 + m_c^2} + P)/\sqrt{2}, (\sqrt{P^2 + m_c^2} - P)/\sqrt{2}, \mathbf{0}_\perp]$, and the momenta of the c and $\bar{c}$ in the η_c are both $\bar{p} = [(\sqrt{P^2 + m_c^2} - P)/\sqrt{2}, (\sqrt{P^2 + m_c^2} + P)/\sqrt{2}, \mathbf{0}_\perp]$, where P is the magnitude of the 3-momentum of any of the c's or $\bar{c}$'s. The momentum of the virtual photon is $2(p + \bar{p})$, which implies that $Q^2 = 16(P^2 + m_c^2)$. Note that $p^+ = \bar{p}^- \sim Q$ and $p^- = \bar{p}^+ \sim m_c^2/Q$.

If a momentum k has light-cone components whose orders of magnitude are $P\lambda[1, (\eta^+)^2, \eta^+]$, then we say that k is soft if $\lambda \ll 1$, and we say that k is collinear to plus if $\eta^+ \ll 1$. If k has light-cone components whose orders of magnitude are $P\lambda[(\eta^-)^2, 1, \eta^-]$, then we say that k is soft if $\lambda \ll 1$, and we say that k is collinear to minus if $\eta^- \ll 1$. Hence, p is collinear to plus and $\bar{p}$ is collinear to minus in the limit $m_c^2/P^2 \to 0$.

The amplitudes that correspond to each diagram contain spin and color projectors that put the $Q\bar{Q}$ pairs into states of definite spin and color [26]. When the relative momentum of the c and $\bar{c}$ in each charmonium is zero, the spin-singlet $\otimes$ color-singlet and spin-triplet $\otimes$ color-singlet projectors are given by

$$\Pi_1(\bar{p}, \bar{p}) = -\frac{1}{2\sqrt{2}m_c}\gamma^5(\not{\bar{p}} + m_c) \otimes \frac{\mathbf{1}}{\sqrt{N_c}} = -\frac{1}{2\sqrt{2}m_c}(-\not{\bar{p}} + m_c)\gamma^5 \otimes \frac{\mathbf{1}}{\sqrt{N_c}}, \tag{1a}$$

$$\Pi_3(p, p, \lambda) = -\frac{1}{2\sqrt{2}m_c}\not{\epsilon}^*(\lambda)(\not{p} + m_c) \otimes \frac{\mathbf{1}}{\sqrt{N_c}} = -\frac{1}{2\sqrt{2}m_c}(-\not{p} + m_c)\not{\epsilon}^*(\lambda) \otimes \frac{\mathbf{1}}{\sqrt{N_c}}, \tag{1b}$$

where $\epsilon^*(\lambda)$ is the polarization vector for the $c\bar{c}$ pair in the spin-triplet state, $N_c = 3$ is the number of colors, $\mathbf{1}$ is the unit color matrix, and we use nonrelativistic normalization for the spinors.

B. Evaluation of the LO diagrams

In Fig. 1, we show the diagrams at LO in α_s for the process $\gamma^* \to J/\psi + \eta_c$. A straightforward computation of the contribution to the amplitude from these diagrams yields

$$i\mathcal{A}^\mu_{\rm LO} = \frac{-i256\pi\alpha_s C_F}{m_c Q^4}\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu(\lambda)p_\alpha\bar{p}_\beta, \tag{2}$$

where $C_F = (N_c^2 - 1)/(2N_c)$ and $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric tensor in 4 dimensions, for which we use the convention $\epsilon_{0123} = +1$. We have suppressed the factor $-i$ times the charm-quark charge that is associated with the electromagnetic vertex.

[5] The suppression of endpoint singularities by inverse powers of the large momentum transfer was noted in Refs. [13, 14, 24, 25].

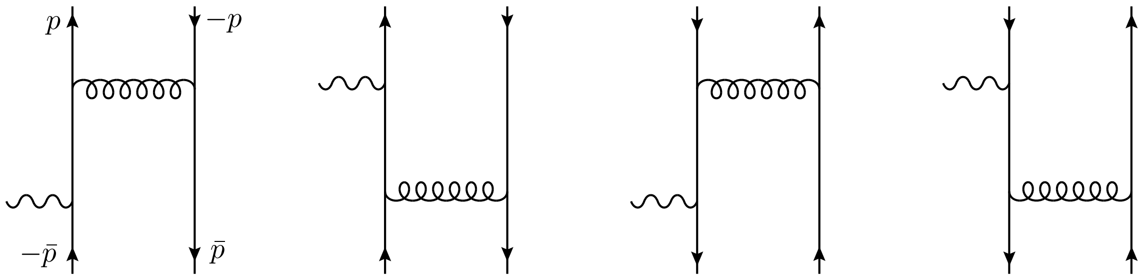

FIG. 1: Feynman diagrams for the process $\gamma^* \to J/\psi + \eta_c$ at LO in α_s. The upper $c\bar{c}$ pair corresponds to the J/ψ, and the lower $c\bar{c}$ pair corresponds to the η_c.

C. Evaluation of the double logarithms in the NLO diagrams

Now we calculate the double logarithms that arise from the Feynman diagrams that contribute to the NLO QCD corrections to the amplitude. As we will explain in Secs. III D and III E, only certain diagrams can potentially yield double logarithms of Q^2/m_c^2. These diagrams are shown in Fig. 2.

1. *Diagram of figure 2(a)*

In order to illustrate our methods, we discuss in some detail the diagram that is shown in Fig. 2(a). The associated amplitude is

$$\begin{aligned} i\mathcal{A}^{(\mathrm{a})\mu}_{\mathrm{NLO}} = & \int_k \frac{1}{N_c}\mathrm{Tr}\Big[\Pi_3(p,p,\lambda)(-ig_s\gamma^\alpha T^a)\frac{i(\not{k}+2\not{p}+\not{\bar{p}}+m_c)}{(k+2p+\bar{p})^2-m_c^2+i\varepsilon}\gamma^\mu\frac{i(\not{k}-\not{\bar{p}}+m_c)}{(k-\bar{p})^2-m_c^2+i\varepsilon} \\ & \times(-ig_s\gamma^\beta T^b)\Pi_1(\bar{p},\bar{p})(-ig_s\gamma_\alpha T^a)\frac{i(-\not{k}-\not{p}+m_c)}{(k+p)^2-m_c^2+i\varepsilon}(-ig_s\gamma_\beta T^b)\Big] \\ & \times\frac{-i}{k^2+i\varepsilon}\frac{-i}{(k+p+\bar{p})^2+i\varepsilon}, \end{aligned} \tag{3}$$

where $g_s=\sqrt{4\pi\alpha_s}$, T^a is the generator of the fundamental (triplet) representation of SU(3) with adjoint-representation color index $a=1,\cdots,N_c^2-1$, and the trace is over the gamma and color matrices. The symbol $\int_k$ is defined by

$$\int_k \equiv \mu^{2\epsilon}\int\frac{d^dk}{(4\pi)^d}, \tag{4}$$

where μ is the dimensional-regularization scale. As we have mentioned, because the process $e^+e^-\to J/\psi+\eta_c$ does not satisfy quark-helicity conservation, we retain factors of m_c in the numerator. Factors of m_c can come from the numerators of the quark propagators or from the numerators of the spin projectors.

As a first step in reducing the tensor integrals in Eq. (3) to scalar integrals, we decompose the loop momentum k as follows:

$$k=\frac{k\cdot(p+\bar{p})}{(p+\bar{p})^2}(p+\bar{p})+\frac{k\cdot(p-\bar{p})}{(p-\bar{p})^2}(p-\bar{p})+k_\perp, \tag{5}$$

which is valid to all orders in m_c^2/Q^2. Here, $k_\perp=[0^+,0^-,\boldsymbol{k}_\perp]$. Because the propagator denominators and the numerator factors $k\cdot p$ and $k\cdot\bar{p}$ are independent of the angles of $\boldsymbol{k}_\perp$, we can carry out the average over the angles of $\boldsymbol{k}_\perp$ easily, setting terms linear or cubic in $k_\perp$ to zero and making the replacement $k_\perp^\mu k_\perp^\nu\to-(1/2)g_\perp^{\mu\nu}\boldsymbol{k}_\perp^2$ for terms that are quadratic in $k_\perp$, where $-g_\perp^{\mu\nu}$ can be expressed as

$$-g_\perp^{\mu\nu}=-g^{\mu\nu}+\frac{(p+\bar{p})^\mu(p+\bar{p})^\nu}{(p+\bar{p})^2}+\frac{(p-\bar{p})^\mu(p-\bar{p})^\nu}{(p-\bar{p})^2}. \tag{6}$$

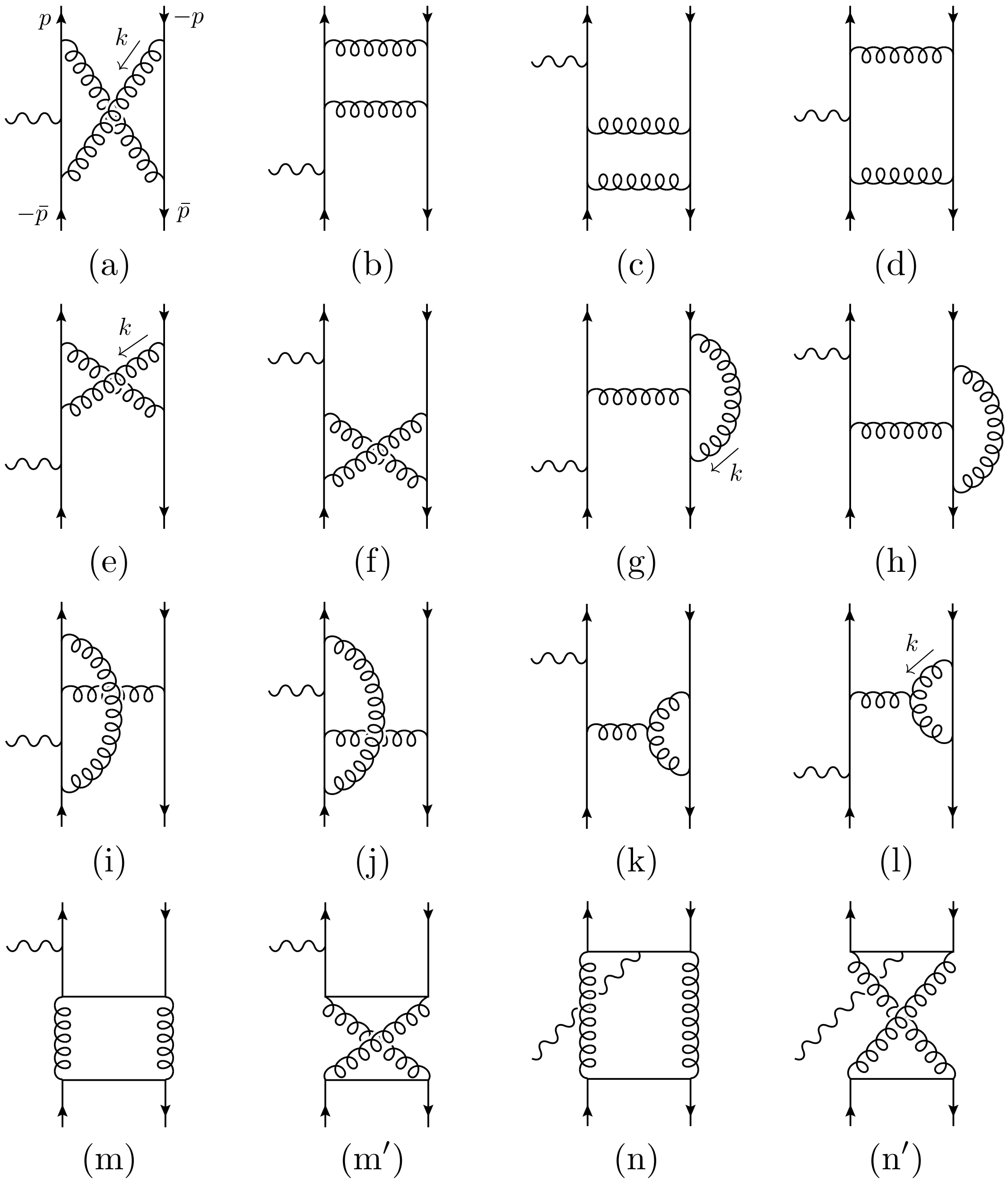

FIG. 2: One-loop diagrams for the process $e^+e^- \to J/\psi + \eta_c$ that potentially contain double logarithms in Q^2/m_c^2. The upper $c\bar{c}$ pair corresponds to the J/ψ, and the lower $c\bar{c}$ pair corresponds to the η_c. We do not show diagrams that are charge conjugates of the diagrams in the figure.

After carrying out the trace over the gamma and color matrices, we find that $i\mathcal{A}^{(\mathrm{a})\mu}_{\mathrm{NLO}}$ can be written as

$$\begin{aligned} i\mathcal{A}^{(\mathrm{a})\mu}_{\mathrm{NLO}} &= \frac{2g_s^4}{m_c} C_F \left(C_F - \frac{C_A}{2} \right) \epsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu(\lambda) p_\alpha \bar{p}_\beta \int_k \frac{1}{\mathcal{D}} \bigg[3k^2 + 4k\cdot p - 2k\cdot\bar{p} - 4p\cdot\bar{p} + \boldsymbol{k}_\perp^2 \\ &\quad + \frac{4(p\cdot\bar{p}k\cdot p - m_c^2 k\cdot\bar{p})}{(p+\bar{p})^2(p-\bar{p})^2} (k^2 + 4k\cdot p + 2k\cdot\bar{p}) \bigg], \end{aligned} \tag{7}$$

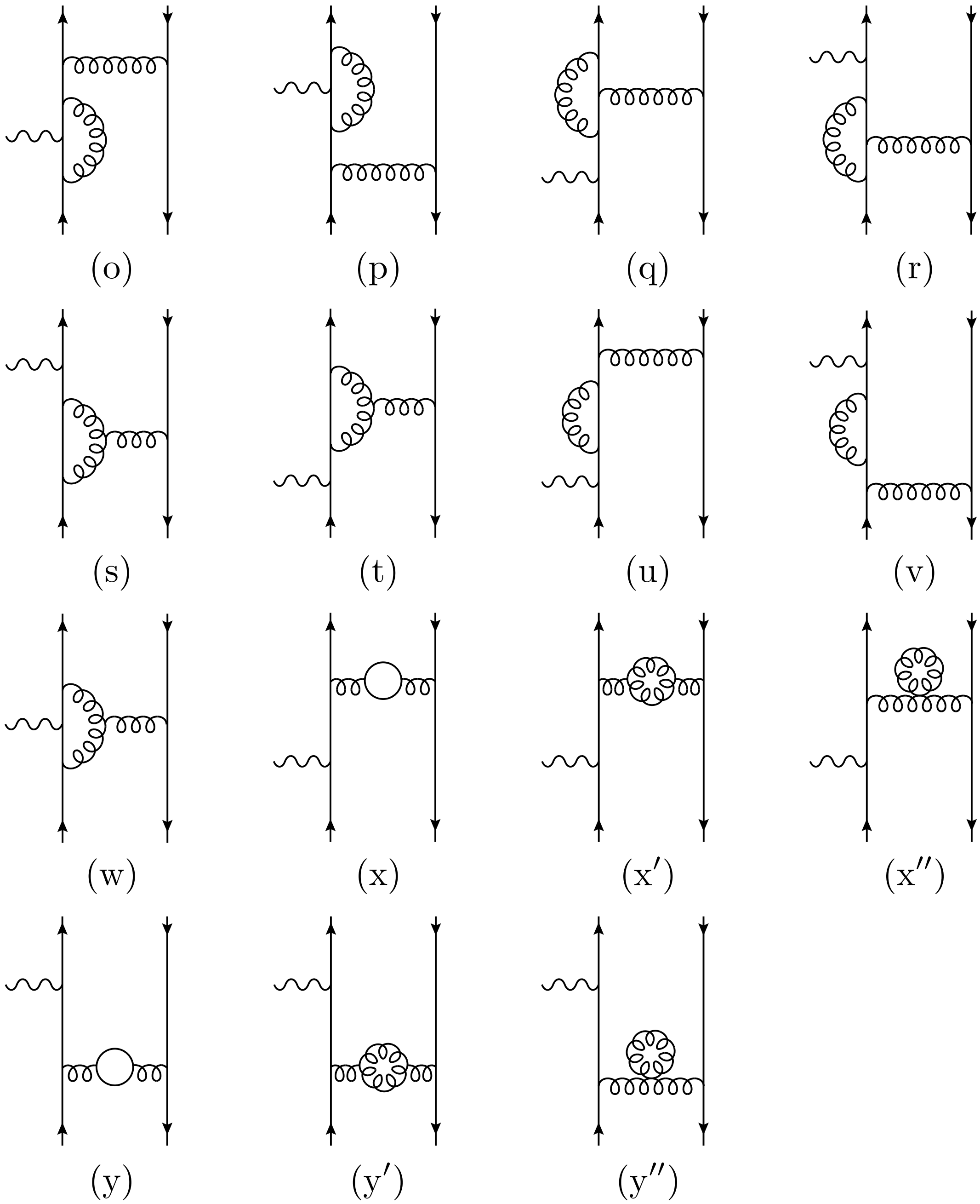

FIG. 3: One-loop diagrams for the process $e^+e^- \to J/\psi + \eta_c$ that do not contain the double logarithms in Q^2/m_c^2. The upper $c\bar{c}$ pair corresponds to the J/ψ, and the lower $c\bar{c}$ pair corresponds to the η_c. We do not show diagrams that are charge conjugates of the diagrams in the figure.

where $C_A = N_c$, $\mathcal{D} = D_0 D_1 D_2 D_3 D_4$, and

$$D_0 = k^2 + i\varepsilon, \tag{8a}$$
$$D_1 = (k - \bar{p})^2 - m_c^2 + i\varepsilon, \tag{8b}$$
$$D_2 = (k + p)^2 - m_c^2 + i\varepsilon, \tag{8c}$$
$$D_3 = (k + p + \bar{p})^2 + i\varepsilon, \tag{8d}$$
$$D_4 = (k + 2p + \bar{p})^2 - m_c^2 + i\varepsilon. \tag{8e}$$

We can then use the identity

$$\boldsymbol{k}_\perp^2=\frac{[k\cdot(p+\bar{p})]^2}{(p+\bar{p})^2}+\frac{[k\cdot(p-\bar{p})]^2}{(p-\bar{p})^2}-k^2,\tag{9}$$

to eliminate $\boldsymbol{k}_\perp^2$. Writing the numerator in terms of the D_i's, we obtain the following scalar integrals:

$$\begin{aligned}i\mathcal{A}_{\rm NLO}^{(\rm a)\mu}=&\frac{2g_s^4}{m_c}C_F\left(C_F-\frac{C_A}{2}\right)\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^*(\lambda)p_\alpha\bar{p}_\beta\int_k\frac{1}{\mathcal{D}}\Bigg[\frac{D_0D_3}{(p-\bar{p})^2}-\frac{3D_2D_3}{(p-\bar{p})^2}+\frac{D_0D_2}{(p+\bar{p})^2}\\&-\frac{2p\cdot\bar{p}}{(p+\bar{p})^4}(D_0D_1+D_3D_4)+\frac{8m_c^2p\cdot\bar{p}D_1D_3}{(p-\bar{p})^2(p+\bar{p})^4}+\frac{6(p\cdot\bar{p})^2+21m_c^2p\cdot\bar{p}+7m_c^4}{(p-\bar{p})^2(p+\bar{p})^4}D_2D_4\\&-\frac{4[m_c^4+(p\cdot\bar{p})^2]D_0D_4}{(p-\bar{p})^2(p+\bar{p})^4}-\frac{2(p\cdot\bar{p})^2-5m_c^2p\cdot\bar{p}+m_c^4}{(p-\bar{p})^2(p+\bar{p})^4}(D_1D_2-2D_1D_4)\\&+\left(6-\frac{2m_c^2(9m_c^2+p\cdot\bar{p})}{(p-\bar{p})^2(p+\bar{p})^2}\right)D_2\Bigg],\end{aligned}\tag{10}$$

where we have made use of the identities

$$2D_0-D_1-2D_3+D_4=0,\tag{11a}$$

$$D_1-2D_2+D_4=4(p\cdot\bar{p}+m_c^2).\tag{11b}$$

The scalar integrals can be evaluated by using standard methods. If we ignore all integrals that do not produce the double logarithms in Q^2/m_c^2, we obtain

$$\begin{aligned}i\mathcal{A}_{\rm NLO}^{(\rm a)\mu}\approx\ &i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\left(C_F-\frac{C_A}{2}\right)\\&\times\Bigg\{\int_k\frac{1}{(k^2+i\varepsilon)[(k+p)^2-m_c^2+i\varepsilon][(k-\bar{p})^2-m_c^2+i\varepsilon]}\\&+\int_k\frac{1}{[(k+p+\bar{p})^2+i\varepsilon][(k+2p+\bar{p})^2-m_c^2+i\varepsilon][(k+p)^2-m_c^2+i\varepsilon]}\\&+\int_k\frac{1}{(k^2+i\varepsilon)[(k+p)^2-m_c^2+i\varepsilon][(k+p+\bar{p})^2+i\varepsilon]}\Bigg\},\end{aligned}\tag{12}$$

where the LO amplitude is given in Eq. (2).

2. *The scalar integral $\mathcal{S}$*

Let us consider the first scalar integral in Eq. (12), which we denote by $\mathcal{S}$:

$$\begin{aligned}\mathcal{S}\equiv&\int_k\frac{1}{(k^2+i\varepsilon)[(k+p)^2-m_c^2+i\varepsilon][(k-\bar{p})^2-m_c^2+i\varepsilon]}\\=&\int_{\boldsymbol{k}_\perp}\int\frac{dk^+}{2\pi}\int\frac{dk^-}{2\pi}\frac{1}{(2k^+k^--\boldsymbol{k}_\perp^2+i\varepsilon)(2k^+k^--\boldsymbol{k}_\perp^2+2k^+p^-+2k^-p^++i\varepsilon)}\\&\times\frac{1}{2k^+k^--\boldsymbol{k}_\perp^2-2k^+\bar{p}^--2k^-\bar{p}^++i\varepsilon},\end{aligned}\tag{13}$$

where

$$\int_{\boldsymbol{k}_\perp}\equiv\mu^{2\epsilon}\int\frac{d^{d-2}\boldsymbol{k}_\perp}{(2\pi)^{d-2}}.\tag{14}$$

We remind the reader that, because we work in the center-of-momentum frame, $p^+=\bar{p}^-\sim Q$ and $p^-=\bar{p}^+\sim m_c^2/Q$.

As we have explained, we can understand the origin of the double logarithms of Q^2/m_c^2 in $\mathcal{S}$ by analyzing the pinch singularities in $\mathcal{S}$. From a general Landau analysis [19–23], we know that the only possible pinch singularities are those that correspond to k soft, collinear-to-plus, and collinear-to-minus [27]. We now verify by direct examination of $\mathcal{S}$ that those pinch singularities are present.

In the k^- complex plane, the integrand in Eq. (13) has three poles, which are located at

$$k^- = \frac{\boldsymbol{k}_\perp^2 - i\varepsilon}{2k^+}, \tag{15a}$$

$$k^- = \frac{\boldsymbol{k}_\perp^2 - 2k^+p^- - i\varepsilon}{2(k^+ + p^+)}, \tag{15b}$$

$$k^- = \frac{\boldsymbol{k}_\perp^2 + 2k^+\bar{p}^- - i\varepsilon}{2(k^+ - \bar{p}^+)}. \tag{15c}$$

In the k^+ complex plane, the integrand in Eq. (13) has three poles, which are located at

$$k^+ = \frac{\boldsymbol{k}_\perp^2 - i\varepsilon}{2k^-}, \tag{16a}$$

$$k^+ = \frac{\boldsymbol{k}_\perp^2 - 2k^-p^+ - i\varepsilon}{2(k^- + p^-)}, \tag{16b}$$

$$k^+ = \frac{\boldsymbol{k}_\perp^2 + 2k^-\bar{p}^+ - i\varepsilon}{2(k^- - \bar{p}^-)}. \tag{16c}$$

In the $k_\perp^i$ complex plane, the integrand in Eq. (13) has poles at

$$k_\perp^i = \pm\sqrt{2k^+k^- - (k_\perp^j)^2 + i\varepsilon}, \tag{17a}$$

$$k_\perp^i = \pm\sqrt{2(k^+k^- + k^+p^- + k^-p^+) - (k_\perp^j)^2 + i\varepsilon}, \tag{17b}$$

$$k_\perp^i = \pm\sqrt{2(k^+k^- - k^+\bar{p}^- - k^-\bar{p}^+) - (k_\perp^j)^2 + i\varepsilon}, \tag{17c}$$

where $(k_\perp^j)^2$ is summed over $j \neq i$ in the dimensionally regulated expression.

Consider first the case $m_c \neq 0$. The poles in k^- pinch the k^- contour at $k^- = 0$ when k^+ and $\boldsymbol{k}_\perp$ go to zero in a fixed ratio. The combinations of poles that provide a pinch depend on the sign of k^+. Similarly, the k^+ contour is pinched at $k^+ = 0$ when k^- and $\boldsymbol{k}_\perp$ go to zero in a fixed ratio. The $k_\perp^i$ contour is also pinched at $k_\perp^i = 0$ when k^+, k^-, and $k_\perp^j$ go to zero. These pinches correspond to the soft singularity at $k = 0$.[6]

Now consider the case $m_c = 0$. Again there are pinches corresponding to a soft singularity: the k^- contour is pinched at $k^- = 0$ when k^+ and $\boldsymbol{k}_\perp$ go to zero in a fixed ratio; the k^+ contour is pinched at $k^+ = 0$ when k^- and $\boldsymbol{k}_\perp$ go to zero in a fixed ratio; and the $k_\perp^i$ contour is pinched at $k_\perp^i = 0$ when k^+, k^-, and $k_\perp^j$ go to zero. (The combinations of poles that provide the pinches for given signs of k^+ and k^- are different in the massless case than in the massive case.)

In the case $m_c = 0$, there are additional pinches that correspond to collinear singularities. The first and second k^- poles pinch the k^- contour at $k^- = 0$ when $-p^+ < k^+ < 0$ and $\boldsymbol{k}_\perp$ goes to zero such that $\boldsymbol{k}_\perp^2/k^+ = 0$ and $\boldsymbol{k}_\perp^2/(k^+ + p^+) = 0$. The $k_\perp^i$ contour is also pinched at $k_\perp^i = 0$ when k^- and $k_\perp^j$ go to zero. These pinches correspond to a collinear-to-plus singularity. Note that the pinches in the k^- and $k_\perp^i$ contours occur even when k^+ is arbitrarily close to 0 or to $-p^+$. There is no pinch in the k^+ contour, but this is consistent with a collinear-to-plus momentum configuration, in which k^+ takes on any value in the range $-p^+ < k^+ < 0$. When $m_c = 0$, there are also pinches that correspond to a collinear-to-minus singularity: the first and third k^+ poles pinch the k^+ contour at $k^+ = 0$ when $0 < k^- < \bar{p}^-$ and $\boldsymbol{k}_\perp$ goes to zero such that $\boldsymbol{k}_\perp^2/k^- = 0$ and $\boldsymbol{k}_\perp^2/(k^- - \bar{p}^-) = 0$; the $k_\perp^i$ contour is pinched at $k_\perp^i = 0$ when k^+ and $k_\perp^j$ go to zero.

We conclude that there are pinches in $\mathcal{S}$ that correspond to soft, collinear-to-plus, and collinear-to-minus singularities, as expected. These pinches correspond to Sudakov logarithms of m_c. In particular, the double logarithms of m_c result from an overlap between the soft and collinear pinches [27].

---

[6] It is possible that some, but not all, of the components of k can have pinched contours of integration. In this case, it may be possible to avoid the singular region by deforming the unpinched contour. For example, the second and third denominators in Eq. (13) pinch the k^- contour at $k^- = -p^- + \bar{p}^-$ and pinch the k^+ contour at $k^+ = -p^+ + \bar{p}^+$. However, it is easy to see that, for these values of k^- and k^+, the $k_\perp^i$ contour is not pinched at $k_\perp^i = 0$, even if $k_\perp^j = 0$. It can also be seen that, for these values of k^- and k^+, the first denominator in Eq. (13) is off shell by order $p^+\bar{p}^- \sim Q^2$, and so is inconsistent with the scaling for a soft singularity.

$\mathcal{S}$ can be evaluated easily by combining denominators with Feynman parameters. The result is

$$\mathcal{S} = \frac{i}{4\pi^2 Q^2}\frac{p^+ + p^-}{p^+ - p^-}\Bigg[\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{4\pi\mu^2 e^{-\gamma_{\rm E}}}{m_c^2}\right)\left(\log\frac{p^-}{p^+} + i\pi\right) +2\,\mathrm{Sp}\left(1-\frac{p^-}{p^+}\right) + \frac{1}{2}\log^2\frac{p^-}{p^+} - \pi^2 + i\pi\log\frac{p^+p^-}{(p^+-p^-)^2} + O(\epsilon)\Bigg], \tag{18}$$

where $\gamma_{\rm E}$ is the Euler-Mascheroni constant, and $\mathrm{Sp}(z)$ is the Spence function:

$$\mathrm{Sp}(z) = -\int_0^z du \frac{\log(1-u)}{u}, \tag{19}$$

for any complex number $z \notin [1,\infty)$. The subscript IR on ϵ indicates that the pole is associated with the infrared divergence. If we retain only the terms that are singular in ϵ and the double logarithms, then we have

$$\mathcal{S} \approx \frac{i}{4\pi^2 Q^2}\Bigg[\left(-\frac{1}{\epsilon_{\rm IR}} + \log\frac{m_c^2}{\mu^2}\right)\log\frac{Q^2}{m_c^2} + \frac{1}{2}\log^2\frac{Q^2}{m_c^2}\Bigg]. \tag{20}$$

The single logarithm of Q^2/m_c^2 corresponds, in the limit $m_c \to 0$, to the collinear-to-plus and collinear-to-minus singularities. The double logarithm of Q^2/m_c^2 corresponds, in the limit $m_c \to 0$, to the overlap of the soft singularity and the collinear-to-plus and collinear-to-minus singularities. The single logarithm of m_c^2/μ^2 corresponds to the soft singularity.

In the second integral in Eq. (12), we can write $k' = k + p + \bar{p}$, where $k + p + \bar{p}$ is the momentum of the gluon that connects the quark line with momentum p and the quark line with momentum $\bar{p}$. Then, this integral becomes

$$\int_k \frac{1}{[(k+p+\bar{p})^2 + i\varepsilon][(k+2p+\bar{p})^2 - m_c^2 + i\varepsilon][(k+p)^2 - m_c^2 + i\varepsilon]} = \int_{k'} \frac{1}{(k'^2 + i\varepsilon)[(k'+p)^2 - m_c^2 + i\varepsilon][(k'-\bar{p})^2 - m_c^2 + i\varepsilon]}, \tag{21}$$

which is identical to $\mathcal{S}$.

3. The scalar integral $\mathcal{E}$

Now we consider the third integral in Eq. (12), which we denote by $\mathcal{E}$:

$$\mathcal{E} \equiv \int_k \frac{1}{(k^2 + i\varepsilon)[(k+p)^2 - m_c^2 + i\varepsilon][(k+p+\bar{p})^2 + i\varepsilon]}. \tag{22}$$

If we change the loop momentum to the spectator-quark momentum $\ell = -k - p$, then $\mathcal{E}$ can be written as

$$\mathcal{E} = \int_\ell \frac{1}{(\ell^2 - m_c^2 + i\varepsilon)[(\ell+p)^2 + i\varepsilon][(\ell-\bar{p})^2 + i\varepsilon]}. \tag{23}$$

Except for the m_c^2 terms in the denominator factors, this expression is identical to the one for $\mathcal{S}$ in Eq. (13). Hence, in the limit $m_c \to 0$, $\mathcal{E}$ will develop singularities when ℓ, the spectator-quark momentum, becomes soft and/or collinear-to-plus or collinear-to-minus. In $\mathcal{S}$, the collinear singularities were regulated by m_c, while the soft singularities required an additional regulator, which we took to be dimensional. In contrast, as can be seen by inspection, both the soft and collinear singularities in $\mathcal{E}$ are regulated by m_c.

When ℓ is soft, the gluon with momentum k carries away all of the collinear-to-plus momentum of the spectator quark with momentum $-p$, and the gluon with momentum $k+p+\bar{p}$ carries away all of the collinear-to-minus momentum of the spectator quark with momentum $\bar{p}$. Therefore, the region of loop momentum in which ℓ is soft corresponds to the endpoint region. From our analysis of $\mathcal{S}$, we know that the double logarithms arise from contributions in which ℓ is simultaneously soft and collinear. Therefore, the double logarithms in $\mathcal{E}$ are endpoint double logarithms.

$\mathcal{E}$ can be evaluated straightforwardly by using Feynman parameters to combine denominators. The result is

$$\mathcal{E} = \frac{i}{4\pi^2 Q^2}\frac{p^+ + p^-}{p^+ - p^-}\left[\frac{1}{2}\log^2\frac{p^-}{p^+} + 2\,\mathrm{Sp}\left(-\frac{p^-}{p^+}\right) + \frac{\pi^2}{6} + i\pi\log\frac{p^-}{p^+} + O(\epsilon)\right]. \tag{24}$$

If we expand the result in powers of m^2/Q^2 and retain only the double logarithms in Q^2/m_c^2, then we obtain

$$\mathcal{E} \approx \frac{i}{8\pi^2 Q^2}\log^2\frac{Q^2}{m_c^2}. \tag{25}$$

TABLE I: Endpoint and Sudakov double logarithms that arise from each diagram in Fig. 2 in units of $i\mathcal{A}_{\rm LO} \times (-i\alpha_s \pi Q^2)/2$.

Diagram	Endpoint double logarithm	Sudakov double logarithm
(a)	$(C_F - \frac{1}{2}C_A)\mathcal{E}$	$2(C_F - \frac{1}{2}C_A)\mathcal{S}$
(b)	$C_F\mathcal{E}$	0
(c)	$2\,C_F\mathcal{E}$	0
(d)	$\frac{1}{2}\,C_F\mathcal{E}$	0
(e)	$(C_F - \frac{1}{2}C_A)\mathcal{E}$	$(C_F - \frac{1}{2}C_A)\mathcal{S}$
(f)	0	$(C_F - \frac{1}{2}C_A)\mathcal{S}$
(g)	0	$-(C_F - \frac{1}{2}C_A)\mathcal{S}$
(h)	0	$-(C_F - \frac{1}{2}C_A)\mathcal{S}$
(i)	0	$-(C_F - \frac{1}{2}C_A)\mathcal{S}$
(j)	0	$-(C_F - \frac{1}{2}C_A)\mathcal{S}$
(k) – (n′)	0	0

4. Summary of the double logarithms

Direct evaluation of the remaining diagrams in Fig. 2 shows that the double logarithms in each diagram arise solely from the two scalar integrals $\mathcal{S}$ and $\mathcal{E}$. A summary of the double logarithms from each diagram is given in Table I. The authors of Ref. [12] have also computed these double logarithms, and our results agree diagram-by-diagram with their results [28].

We note that the Sudakov double logarithms cancel in the sum over all diagrams, and, so, the double logarithm in the NLO correction to the amplitude is given entirely by the sum of the endpoint double logarithms. Multiplying by a factor of two to take into account the charge conjugates of the diagrams in Fig. 2, we obtain for the double logarithm in the NLO correction

$$\begin{aligned} i\mathcal{A}^\mu_{\rm NLO} &\approx i\mathcal{A}^\mu_{\rm LO}(-i\alpha_s\pi Q^2)\left[\frac{7}{2}C_F + (2C_F - C_A)\right]\mathcal{E} \\ &\approx i\mathcal{A}^\mu_{\rm LO}\,\frac{7N_c^2-11}{32N_c}\,\frac{\alpha_s}{\pi}\log^2\frac{Q^2}{m_c^2}, \end{aligned} \tag{26}$$

in agreement with the results in Refs. [8, 12].

D. Identifying the Sudakov double logarithms

Now, let us discuss a streamlined method for identifying the Sudakov double logarithms. The discussion in Sec. III C shows that the Sudakov double logarithms arise from the scalar integral $\mathcal{S}$ and come from the region of integration in which a gluon, which we label with the momentum k, is simultaneously soft and collinear. In the soft approximation, we can simplify the amplitude numerators. For example, we can write a quark-gluon vertex and the surrounding propagator and spin-projector numerators as

$$\not\epsilon^*(\not p + m_c)\gamma^\mu(\not p - \not k + m_c) \approx 2p^\mu \not\epsilon^*(\not p + m_c), \tag{27}$$

where μ is the polarization of the gluon, we have used $\not p^2 = m_c^2$, and we have dropped the term proportional to k. By making similar manipulations, we find that, in the soft approximation, we can always replace the quark-gluon vertex and an adjacent propagator numerator with $\pm 2p^\mu$ or $\pm 2\bar{p}^\mu$, where the + (−) sign applies when the soft gluon is attached to the quark (antiquark) line. Therefore, a Sudakov double logarithm can only arise if the gluon with momentum k attaches at one end to a collinear-to-plus-quark line (containing momentum p) and at the other end to a collinear-to-minus-quark line (containing momentum $\bar{p}$). Otherwise, the contribution is subleading in Q because the numerator acquires a factor of $p^\mu p_\mu = m_c^2$ or $\bar{p}^\mu \bar{p}_\mu = m_c^2$. Using this reasoning, we can see that diagrams (b), (c), and (d) cannot contribute Sudakov double logarithms when the outer gluon is soft.

If a soft gluon enters a propagator that is off shell by order Q^2, then the result is also power suppressed. For this reason, diagrams (b), (c), and (d), cannot contribute Sudakov double logarithms when the inner gluon is soft, and diagrams (o)–(w) cannot contribute Sudakov double logarithms when the gluon that does not connect to a spectator line is soft. For the same reason, the diagrams (x), (x′), (x″), (y), (y′), and (y″) cannot contribute Sudakov double logarithms when the vacuum-polarization gluon is soft.

Diagrams (k) and (l) contain only gluons that are connected to propagators that are off shell by order Q^2 and, so, do not contribute Sudakov double logarithms.

We conclude that only diagrams (a), (e)–(j), and (m)–(n′) can contribute Sudakov double logarithms, in accordance with Table I.

E. Identifying the endpoint double logarithms

We can also streamline the method for identifying the endpoint double logarithms. The endpoint double logarithms arise when a spectator-quark propagator carries a momentum that is soft and collinear. This can happen only when a gluon carries away almost all of the spectator momentum p, which arises from the J/ψ, and a second gluon carries away almost all of the spectator momentum $\bar{p}$, which arises from the η_c. That possibility exists only for the diagrams of Figs. 2(a)–(f), (k), and (l). In each of these diagrams, we can reproduce the endpoint double logarithm that is given in Table I by using a soft approximation for the momentum of the spectator quark ℓ. Specifically, we neglect ℓ^2, $\ell\cdot p$, and $\ell\cdot\bar{p}$ in comparison with $p\cdot\bar{p}$ in both numerators and denominators. In principle, for purposes of extracting endpoint logarithms, we could neglect ℓ^2 in comparison with $\ell\cdot p$ and $\ell\cdot\bar{p}$ in denominators. However, we retain ℓ^2 in denominators in order to maintain the UV finiteness of the integrals. We also neglect m_c^2 in comparison with $p\cdot\bar{p}$.

The diagram of Fig. 2(a) gives

$$\begin{aligned} i\mathcal{A}_{\rm NLO}^{({\rm a})\mu} \approx\; & i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\left(C_F-\frac{C_A}{2}\right)\int_\ell \frac{4p\cdot\bar{p}(p\cdot\bar{p}-\ell\cdot\bar{p}-\ell^2)+4(\ell\cdot p)^2-2\ell^2\,\ell\cdot p}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]} \\ & \times\frac{1}{[(\ell-p-\bar{p})^2-m_c^2+i\varepsilon][(\ell+p+\bar{p})^2-m_c^2+i\varepsilon]}, \end{aligned} \tag{28}$$

where, in the numerator, we have ignored terms of order m_c^2/Q^2 or higher. Then, the soft approximation gives, up to corrections of order m_c^2/Q^2,

$$\begin{aligned} i\mathcal{A}_{\rm NLO}^{({\rm a})\mu,\,{\rm soft}} &= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\left(C_F-\frac{C_A}{2}\right)\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]} \\ &= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\left(C_F-\frac{C_A}{2}\right)\mathcal{E}. \end{aligned} \tag{29}$$

The diagram of Fig. 2(b) gives

$$\begin{aligned} i\mathcal{A}_{\rm NLO}^{({\rm b})\mu} \approx\; & i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,C_F \\ & \times\int_\ell \frac{4\ell\cdot p+2\ell\cdot p\,\ell\cdot\bar{p}/p\cdot\bar{p}}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon][(\ell+2p)^2-m_c^2+i\varepsilon]}, \end{aligned} \tag{30}$$

where we have neglected corrections of order m_c^2/Q^2. Applying the soft approximation, we obtain

$$\begin{aligned} i\mathcal{A}_{\rm NLO}^{({\rm b})\mu,\,{\rm soft}} =\; & i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,C_F\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]} \\ & \times\frac{4\ell\cdot p}{\ell^2+4\ell\cdot p+3m_c^2+i\varepsilon}. \end{aligned} \tag{31}$$

Since ℓ^μ in this integral is cut off at a scale of order m_c, the numerator factor $4\ell\cdot p$ cancels the last denominator factor, up to corrections of order m_c^2/Q^2. Then, we have

$$\begin{aligned} i\mathcal{A}_{\rm NLO}^{({\rm b})\mu,\,{\rm soft}} &\approx i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,C_F\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]} \\ &= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,C_F\mathcal{E}. \end{aligned} \tag{32}$$

The diagram of Fig. 2(c) gives a contribution that is similar to the one from the diagram of Fig. 2(b):

$$
\begin{aligned}
i\mathcal{A}_{\rm NLO}^{({\rm c})\mu,\,{\rm soft}} &= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,2C_F\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]}\\
&\quad\times\frac{-4\ell\cdot\bar{p}}{\ell^2-4\ell\cdot\bar{p}+3m_c^2+i\varepsilon}\\
&\approx i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,2C_F\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]}\\
&= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,2C_F\mathcal{E}.
\end{aligned}
\tag{33}
$$

The diagram in Fig. 2(d) yields

$$
\begin{aligned}
i\mathcal{A}_{\rm NLO}^{({\rm d})\mu,\,{\rm soft}} &= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,\frac{C_F}{2}\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]}\\
&\quad\times\frac{8p\cdot\bar{p}\,\ell^2-16\ell\cdot p\,\ell\cdot\bar{p}}{(\ell^2+4\ell\cdot p+3m_c^2+i\varepsilon)(\ell^2-4\ell\cdot\bar{p}+3m_c^2+i\varepsilon)}.
\end{aligned}
\tag{34}
$$

In the limit $m_c\to 0$, the first term in the numerator gives the integral

$$
\int_\ell \frac{1}{(\ell^2+2\ell\cdot p+i\varepsilon)(\ell^2-2\ell\cdot\bar{p}+i\varepsilon)(\ell^2+4\ell\cdot p+i\varepsilon)(\ell^2-4\ell\cdot\bar{p}+i\varepsilon)}.
\tag{35}
$$

This integral has a logarithmically divergent soft power count and a logarithmically divergent collinear power count. However, in the soft region, in which we can neglect ℓ^2 in comparison with $\ell\cdot p$ or $\ell\cdot\bar{p}$, there is no pinch in either the ℓ^+ or ℓ^- contour of integration. Therefore this integral does not give an endpoint double logarithm. Retaining the second term in the numerator in Eq. (34), we obtain, up to corrections of order m_c^2/Q^2,

$$
\begin{aligned}
i\mathcal{A}_{\rm NLO}^{({\rm d})\mu,\,{\rm soft}} &\approx i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,\frac{C_F}{2}\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]}\\
&= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\,\frac{C_F}{2}\mathcal{E}.
\end{aligned}
\tag{36}
$$

The contribution of the diagram in Fig. 2(e) can be evaluated in a similar manner:

$$
\begin{aligned}
i\mathcal{A}_{\rm NLO}^{({\rm e})\mu,\,{\rm soft}} &\approx i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\left(C_F-\frac{C_A}{2}\right)\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]}\\
&= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\left(C_F-\frac{C_A}{2}\right)\mathcal{E}.
\end{aligned}
\tag{37}
$$

The contribution of the diagram in Fig. 2(f) is, up to corrections of order m_c^2/Q^2,

$$
\begin{aligned}
i\mathcal{A}_{\rm NLO}^{({\rm f})\mu,\,{\rm soft}} &= i\mathcal{A}_{\rm LO}^{\mu}\frac{-i\alpha_s\pi Q^2}{2}\left(C_F-\frac{C_A}{2}\right)\int_\ell \frac{1}{(\ell^2-m_c^2+i\varepsilon)[(\ell+p)^2+i\varepsilon][(\ell-\bar{p})^2+i\varepsilon]}\\
&\quad\times\frac{2\ell\cdot p-2\ell\cdot\bar{p}}{2p\cdot\bar{p}}.
\end{aligned}
\tag{38}
$$

Since all of the numerator terms are proportional to ℓ, this integral does not give a divergent soft power count in the limit $m_c\to 0$. Hence, it does not contribute an endpoint double logarithm.

In the same manner, the diagrams in Figs. 2(k) and (l) lead to integrals that do not give divergent soft power counts and, therefore, do not contribute endpoint double logarithms.

IV. GENERAL ANALYSIS OF THE SUDAKOV DOUBLE LOGARITHMS

As we have mentioned, the singularities that appear in the Sudakov double logarithms of Q^2/m_c^2 as $m_c\to 0$ arise from a region in which the momentum of a gluon is simultaneously soft and collinear. Consequently, we can organize these singularities by making use of soft and/or collinear approximations for the amplitudes.

Consider, for example, Figs. 2(a) and (g), for the situation in which the gluon with momentum k is collinear to plus. Then, the upper quark-gluon vertex and the propagator and spin-projector numerator factors surrounding it can be written as

$$(-\not{p} - \not{k} + m_c)\gamma^\nu(-\not{p} + m_c)\not{\epsilon}^* \approx -2(p+k)^\nu(-\not{p} + m_c)\not{\epsilon}^* + m_c\gamma^\nu\not{k}\not{\epsilon}^*, \tag{39}$$

where ν is the polarization index of the gluon, we have used $\not{k}\not{p} \propto \not{p}^2 = m_c^2$, and we have dropped terms of order m_c^2. In the case of massless quarks, one obtains the collinear-to-plus approximation by retaining only the first term on the right side of Eq. (39). Since that term is proportional to k^ν for k collinear to p, one can use graphical Ward identities to simplify the amplitude. In the case of nonzero quark masses, the second term in Eq. (39) generally spoils this approach. However, if k is soft in comparison with p, as well as collinear, then we can drop the second term on the right side of Eq. (39), and the standard collinear-to-plus approximation holds. Since the current in Eq. (39) now lies in the + light-cone direction, up to terms of order m_c^2, we can make a collinear-to-plus approximation in the gluon propagator [29–31], by making the replacement in the gluon polarization tensor

$$g_{\mu\nu} \to \frac{k_\mu \bar{n}_\nu}{k\cdot\bar{n} - i\varepsilon}, \tag{40}$$

where $\bar{n}$ is a unit vector in the − light-cone direction, and the index ν corresponds to the upper attachment of the gluon to the quark line with momentum $-p$. The sign of $i\varepsilon$ is fixed by the sign in the μ-side fermion propagator in the original Feynman diagram. (We always choose k to flow out of collinear-to-plus lines and into collinear-to-minus lines.) This soft-collinear-to-plus approximation is valid unless the μ attachment of the gluon is to a line that is also collinear to plus. Hence, the approximation always holds, in the soft-collinear limit, for the diagrams that produce Sudakov logarithms because the invariant Q^2 in the logarithm can appear only if the soft-collinear gluon connects a line carrying momentum p to a line carrying momentum $\bar{p}$.

In a similar fashion, if k is collinear to minus, then we can make the replacement

$$g_{\mu\nu} \to \frac{k_\mu n_\nu}{k\cdot n + i\varepsilon}, \tag{41}$$

where n is a unit vector in the + light-cone direction.

The replacement (40) can also be regarded as a soft approximation [32, 33] to the μ attachment of the gluon. For example, in the case of the diagram of Fig. 2(a), the lower vertex of the gluon with momentum k and the surrounding propagator and spin-projector factors can be written in the soft approximation as

$$(-\not{\bar{p}} + \not{k} + m_c)\gamma^\mu(-\not{\bar{p}} + m_c)\gamma^5 \approx -2\bar{p}^\mu(-\not{\bar{p}} + m_c)\gamma^5, \tag{42}$$

where we have neglected k relative to $\bar{p}$. Then, the replacement (40) is also valid by virtue of the fact that, in the limit $m_c \to 0$, the current at lower vertex of the gluon lies in the − light-cone direction.

The soft approximation applies only when the current on the μ side of the gluon is in the − light-cone direction. In contrast, the soft-collinear-to-plus approximation applies whenever the current on the μ side of the gluon is different from the plus direction, and so the collinear approximation can be more versatile than the soft approximation.

For the diagram of Fig. 2(a) we can apply the soft-collinear-to-plus approximation (40) to the lower vertex of the gluon with momentum k and then make use of the graphical Ward identity (Feynman identity)

$$\not{k} = (-\not{\bar{p}} + \not{k} - m_c) - (-\not{\bar{p}} - m_c). \tag{43}$$

The result, including the adjacent propagator and the factor $(-\not{\bar{p}} + m_c)\gamma^5$ from the spin projector is

$$\frac{1}{-\not{\bar{p}} + \not{k} - m_c + i\varepsilon}\frac{\not{k}\bar{n}_\nu}{k\cdot\bar{n} - i\varepsilon}(-\not{\bar{p}} + m_c)\gamma^5 = \frac{\bar{n}_\nu}{k\cdot\bar{n} - i\varepsilon}(-\not{\bar{p}} + m_c)\gamma^5. \tag{44}$$

Similarly, for the diagram of Fig. 2(g), we can apply the soft-collinear-to-plus approximation (40) to the lower vertex of the gluon with momentum k and make use of the Feynman identity to obtain

$$\gamma^5(\not{\bar{p}} + m_c)\frac{\not{k}\bar{n}_\nu}{k\cdot\bar{n} - i\varepsilon}\frac{1}{\not{\bar{p}} - \not{k} - m_c + i\varepsilon} = -\frac{\bar{n}_\nu}{k\cdot\bar{n} - i\varepsilon}\gamma^5(\not{\bar{p}} + m_c). \tag{45}$$

Now, in the soft approximation, we can neglect k in comparison with p and $\bar{p}$ in quark propagator numerators, and we can neglect all of the invariants involving k in comparison with $p\cdot\bar{p}$ in propagator denominators. Therefore, in the diagram of Fig. 2(a), we can neglect k in the gluon propagator that has momentum $p + \bar{p} + k$ and in the quark propagator that has momentum $2p + \bar{p} + k$. Then, in the soft-collinear-to-plus approximation, the contributions of the diagrams of Figs. 2(a) and (g) have exactly the same propagator and vertex factors, except for a relative minus sign. Furthermore, because the outgoing $c\bar{c}$ pairs are in color-singlet states, the diagrams of Figs. 2(a) and (g) have the same color factors. Therefore, the contributions of these diagrams cancel when the gluon with momentum k is soft and collinear to plus.

This type of pair-wise cancellation occurs for all of the soft-collinear-to-plus contributions from the diagrams of Figs. 2(a), (e), (f), (g), (h), (i), and (j), including contributions from the diagram of Fig. 2(a) in which the gluon with momentum $p + \bar{p} + k$ is soft and collinear to plus. Similar cancellations occur for the diagrams of Figs. 2(a), (e), (f), (g), (h), (i), and (j), when a gluon has soft-collinear-to-minus momentum.

Now consider the diagrams of Figs. 2(m) and (m′). In each diagram, we apply the soft-collinear-to-plus approximation to the lower vertex of the gluon whose upper vertex connects to the spectator-quark. The resulting expressions for the two diagrams differ only by a minus sign and a color factor. Let T^a be the color matrix that is associated with the lower vertex of the gluon to which soft-collinear-to-plus approximation is applied and let T^b be the color matrix that is associated with the lower vertex of the other gluon. Then, the diagram of Fig. 2(m) contributes T^aT^b and the diagram of Fig. 2(m′) contributes $-T^bT^a$ to the color factor on the lower quark line. These contributions combine to give $[T^a, T^b] = if^{abc}T^c$, which has zero overlap with the color-singlet lower meson. (Here, f^{abc} is the structure constant of SU(3).) In each diagram, we can also apply the soft-collinear-to-minus approximation the upper vertex of the gluon whose lower vertex connects to the spectator-quark line. Again, we obtain a vanishing contribution. Similar arguments show that the soft-collinear-to-plus and soft-collinear-to-minus contributions from the diagrams of Figs. 2(n) and (n′) cancel.

These arguments go through in the same fashion for the charge-conjugate diagrams. We conclude that, in the sum of all diagrams, Sudakov double logarithms cancel, verifying the results of our explicit calculation.

We note that, after the application of the soft-collinear-to-plus approximation and the Feynman identity, the soft-collinear-to-plus contributions take the forms that are shown in Fig. 4. Here, the double line is an "eikonal" line, with gluon-eikonal vertex $\bar{n}_\nu$ and eikonal propagator $-i/(k \cdot \bar{n} - i\varepsilon)$, which arise from the replacement (40). (There is no propagator between the attachment of the eikonal line to the quark line and the adjacent vertex on the quark line.) For example, the contribution of Fig. 4(a) arises from the contribution of Fig. 2(a) in which the gluon with momentum k is soft and collinear to plus, and the contribution of Fig. 4(b) arises from the contribution of Fig. 2(g). Now, the pair-wise cancellation of these contributions is obvious from the diagrams in Fig. 4. All of the soft-collinear-to-plus (minus) contributions are associated entirely with the upper (lower) meson and have the form of contributions to the meson distribution amplitude. We define the hard subdiagram to be the subdiagram in which all propagators are off shell by order Q^2. Then, as is depicted in Figs. 4(i)–(l), the soft-collinear contributions have been factored from the hard subdiagram.

In arriving at the forms in Fig. 4, we have made use of the fact that a soft momentum can be neglected in the hard subdiagram and the fact that both of the mesons are color-singlet states. We can arrive at the forms in Fig. 4 without making these assumptions as follows. As we have already mentioned, the replacement (40) applies whenever the μ vertex of the soft-collinear-to-plus gluon attaches to a line that is not collinear to plus. Therefore, we can apply the replacement (40) to diagrams in which the soft-collinear gluon enters the hard subdiagram. These diagrams give a vanishing contribution in the soft limit. Nevertheless, we can include them formally. Then, we can apply the diagrammatic Ward identities to arrive at the forms in Fig. 4 directly. For example, the contributions from diagrams of Figs. 2(a), (e), and (l) in which the gluon with momentum k is soft and collinear to plus combine to give the contribution of the diagram in Fig. 4(a). Discussions of the required non-Abelian diagrammatic Ward identities can be found, for example, in Refs. [29, 31].

The factorized forms in Figs. 4(i)–(l) rely on the soft-collinear approximation and the diagrammatic Ward identities. The cancellations of the Sudakov logarithms then follow from the color-singlet natures of the mesons. Given the general natures of these arguments, we expect them to hold to all orders in perturbation theory. For the case of massless quarks, all-orders arguments for the cancellation of Sudakov logarithms in exclusive meson amplitudes have been given in Refs. [34–38]. The all-orders cancellation of soft divergences in exclusive meson amplitudes can also be established for the case of massless quarks in the context of soft-collinear effective theory (SCET) by making use of a field redefinition [39].

V. GENERAL ANALYSIS OF THE ENDPOINT REGION

As we have mentioned, the singularities that appear as $m_c \to 0$ in the endpoint double logarithms of Q^2/m_c^2 arise from the region of loop integration in which the momentum ℓ of the internal spectator-quark line is simultaneously soft and collinear. Hence, in order to analyze those singularities, we need to consider only diagrams that can give rise to such a momentum configuration. A necessary condition is that both gluons attach to the spectator line. The

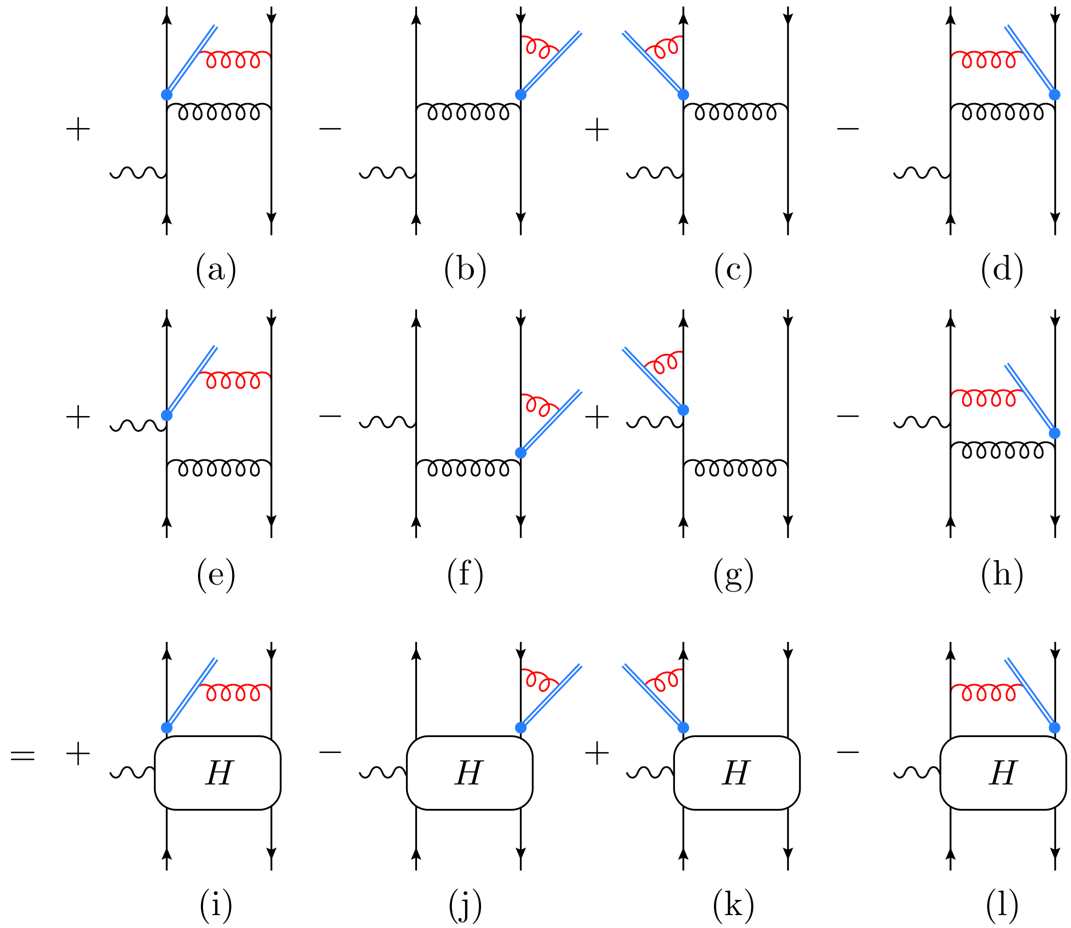

FIG. 4: Contributions that arise from regions of integration in which a gluon momentum is soft and collinear. The double lines are "eikonal" lines, whose Feynman rules are described in the text. H denotes the hard subdiagram. We have not shown diagrams that can be obtained by charge conjugation or diagrams that can be obtained by interchanging the upper and lower mesons.

diagrams satisfying this condition are shown in Figs. 2(a)–(f), (k) and (l).

At leading power in m_c/Q, power counting arguments [31] show that there are no logarithmic singularities as $m_c \to 0$ that arise from the region $\ell \to 0$. Therefore, the endpoint double logarithms appear only in contributions in which there is at least one numerator factor m_c, *i.e.*, a helicity flip.

We now argue that, if a contribution is to produce a double logarithm, then it must contain exactly one numerator factor m_c. First, we note that the contribution must contain an odd number of numerator factors of m_c in order to produce the helicity flip that is required by the process $e^+e^- \to J/\psi + \eta_c$. If there are three or more numerator factors of m_c, then, either the contribution is suppressed by powers of m_c/Q, or the integration produces two or more inverse powers of m_c. In the latter case, the inverse powers of m_c must arise from either the soft singularity or the collinear singularity. The singularity that produces the inverse power of m_c cannot produce a logarithm. Therefore, such contributions contain, at most, a single logarithm of m_c.[7] In principle, contributions that contain a single numerator factor of m_c can diverge as inverse powers of m_c in the limit $m_c \to 0$, but, as we will see, such contributions vanish when the numerator trace is taken.

In the diagrams of Figs. 2(a), (e), (f), (k), and (l), the momenta of the propagators on the active-quark lines contain both p and $\bar{p}$. Since $p \cdot \bar{p} \sim P^2 \sim Q^2$, we can ignore ℓ in the denominators of those propagators. In the limit $m_c \to 0$, the two gluon-propagator denominators and the spectator-quark-propagator denominator produce factors

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[7] Detailed-power counting arguments show that the soft singularity can be, at most, quadratically divergent, while the collinear singularities can be, at most, linearly divergent. This situation occurs only in the case of the diagram of Fig. 2(d). Hence, a single logarithm can arise only if the soft singularity produces two inverse powers of m_c and the collinear singularity produces a logarithm.

$1/(\ell^2+2p\cdot\ell+i\varepsilon)$, $1/(\ell^2-2\bar{p}\cdot\ell+i\varepsilon)$, and $1/(\ell^2+i\varepsilon)$, respectively, where we have dropped the m_c^2 terms in the propagator denominators. Hence, if we are to obtain a logarithmically divergent soft power count (λ^{-4}), then we cannot have any numerator factors of ℓ. This implies that we must retain the factor m_c in the numerator of the spectator-quark propagator and that we can drop m_c elsewhere in the numerator.

In the diagram of Fig. 2(b), the denominator of the outermost active-quark propagator produces a factor $1/(\ell^2+4p\cdot\ell+i\varepsilon)$ in the limit $m_c\to 0$. Hence, taking into account the two gluon-propagator denominators and spectator-quark-propagator denominator, we see that the propagator denominators, by themselves, produce a linearly divergent soft power count and a linearly divergent collinear-to-plus power count. However, as we now show, the numerator factors reduce both of these power counts to logarithmic ones. First, we rewrite the numerator factors that are associated with the outermost gluon and the spin projector for the J/ψ as

$$\gamma_\mu(\not{p}-m_c)\not{\epsilon}^*\gamma^\mu = 2m_c\not{\epsilon}^*, \tag{46}$$

where we have used the fact that $p\cdot\epsilon^*=0$. Now, because this factor contains the numerator power of m_c, the numerator of the spectator-quark propagator must contribute a factor $\not{\ell}$. Furthermore, if ℓ is proportional to p, then, the upper active and spectator propagator combine with the expression (46) to give the structure $\not{p}\not{\epsilon}^*\not{p}=-\not{\epsilon}^*p^2=-\not{\epsilon}^*m_c^2$. That is, the numerator vanishes, up to terms of order m_c^2. This implies that, in the trace over the gamma matrices, ℓ must appear in the combination $\ell\cdot p$. This numerator factor reduces both the soft and collinear power counts to logarithmic ones.

In the case of the diagram of Fig. 2(c), the denominator of the outermost active-quark propagator contributes a factor $1/(\ell^2-4\bar{p}\cdot\ell+i\varepsilon)$. Hence, taking into account the two gluon-propagator denominators and spectator-quark-propagator denominator, we see that the propagator denominators, by themselves, produce linearly divergent soft and collinear-to-minus power counts. We rewrite the numerator factors that are associated with the outermost gluon and the spin projector for the η_c as

$$\gamma_\mu(-\not{\bar{p}}-m_c)\gamma^5\gamma^\mu = (-2\not{\bar{p}}+4m_c)\gamma^5. \tag{47}$$

Now, it is easy to see that there must be a numerator factor of ℓ from either the outermost active-quark propagator or the spectator-quark propagator. Otherwise, there will be two factors of $\not{\bar{p}}$ that are either adjacent or separated by γ^5, resulting in an expression that vanishes, up to terms of order m_c^2. Furthermore, if ℓ is proportional to $\bar{p}$, then the numerator vanishes, up to terms of order m_c^2, because we again have a situation in which two factors of $\not{\bar{p}}$ are either adjacent or separated by γ^5. Therefore, ℓ must appear in the combination $\ell\cdot\bar{p}$ in the trace over gamma matrices. The factor $\ell\cdot\bar{p}$ reduces both the soft and collinear-to-minus power counts to logarithmic ones.

In the case of diagram of Fig. 2(d), the denominators of the active-quark propagators produce factors $1/(\ell^2+4p\cdot\ell+i\varepsilon)$ and $1/(\ell^2-4\bar{p}\cdot\ell+i\varepsilon)$ in the limit $m_c\to 0$. Hence, taking into account the two gluon-propagator denominators and spectator-quark-propagator denominator, we see that the propagator denominators, by themselves, produce a quadratically divergent soft power count and linearly divergent collinear-to-plus and collinear-to-minus power counts. One can apply the arguments that were used for the numerators of the diagrams of Figs. 2(b) and (c) separately to each of the gluons in the diagram of Fig. 2(d). The conclusion is that the numerator contains two factors of $\not{\ell}$ and that the numerator vanishes, up to terms of order m_c^2, if ℓ is proportional to p or to $\bar{p}$. Therefore, the trace contains a factor $\ell\cdot p\,\ell\cdot\bar{p}$ or a factor ℓ^2. Either factor reduces the soft and collinear power counts to logarithmic ones. (In fact, there is no collinear pinch if the numerator factor is ℓ^2.)

We note that the appearance of singularities that arise from the regions in which ℓ is soft or soft-collinear relies on the presence of a numerator factor m_c. Consider, for example, the process in which a virtual photon produces a spin-zero, S-wave meson (η_c) and a spin-zero, P-wave meson (h_c). This process proceeds without a helicity flip. Therefore, at LO in m_c/Q, we can set $m_c=0$ everywhere in the numerator. Then, for each of the contributions of the diagrams of Figs. 2(a)–(f), (k), and (l), there must be a numerator factor of ℓ from the spectator-quark propagator. In addition, by making use of the identity (47) with $m_c=0$, we can see that the contributions of the diagrams of Figs. 2(b) and (c) contain an additional factor of ℓ from the outermost active-quark propagator and that the contribution of the diagram of Fig. 2(d) contains two additional factors of ℓ from the two active-quark propagators. Otherwise, these contributions would vanish because there would be two factors of $\not{p}$ or two factors of $\not{\bar{p}}$ that are adjacent or are separated by γ^5. These numerator factors of ℓ are sufficient to eliminate the soft singularity in the contributions of each of the diagrams of Figs. 2(a)–(f), (k), and (l). We have verified this analysis by carrying out explicit calculations of the contributions of each diagram. Arguments that are similar to the preceding one apply to situations in which one or both mesons are spin-one states. The conclusion is that, at leading order in m_c/Q, for processes that do not involve a helicity flip, there are no singularities that arise from the region in which the spectator quark carries a soft or a soft-collinear momentum (endpoint singularities). This conclusion is in agreement with explicit calculations for helicity non-flip processes [12, 40, 41] and with general analyses of leading pinch singularities [31].

VI. SUMMARY

In this paper we have analyzed double logarithms of Q^2/m_c^2 that appear in the NLO QCD corrections to the process $e^+e^- \to J/\psi + \eta_c$. We have identified the origins of these double logarithms by examining, in the limit $m_c \to 0$, the pinch singularities in the contours of integration in the amplitudes. We have found that the double logarithms are of two types: Sudakov double logarithms and endpoint double logarithms. The Sudakov double logarithms are characterized by singularities in the limit $m_c \to 0$ that arise from a momentum region in which the momentum of a gluon is both soft and collinear to one of the outgoing mesons. The endpoint double logarithms are characterized by singularities in the limit $m_c \to 0$ that arise from a momentum region in which one gluon carries away almost all of the momentum of a spectator quark and the other gluon carries away almost all of the momentum of a spectator antiquark. We have carried out an explicit calculation that shows that the Sudakov and endpoint double logarithms account for all of the double logarithms that arise from each Feynman diagram.

When one sums over the contributions of all of the diagrams, the Sudakov double logarithms cancel. We have shown that this cancellation can be understood by means of a general argument that is based on a soft-collinear approximation and graphical Ward identities.

We have found that the endpoint singular region can be interpreted as a pinch-singular region in which the momentum of the spectator-quark line is both soft and collinear. Such a pinch-singular region is allowed by a general Landau analysis. However, in the case of processes that proceed without a helicity flip, it does not give rise to a singular power count at leading order in m_c/Q. In the case of processes that proceed only through a helicity flip, such as $e^-e^+ \to J/\psi + \eta_c$, the endpoint singular region can give rise to a singular power, owing to the presence of one or more factors of m_c in numerators of Feynman amplitudes at the leading nontrivial order in m_c/Q. We have given a general analysis of the power counting in the endpoint singular region at NLO in α_s. That analysis shows that the endpoint singularities can produce logarithms, but not inverse powers, of m_c. It is apparent that the endpoint region can give rise to single logarithms of Q^2/m_c^2, as well as double logarithms. The single logarithms correspond to a region of integration in which the momentum of a spectator-quark line is soft, but not collinear.

Finally, we remark that the insight that the endpoint singular region is a pinch-singular region in which a spectator-quark line is either soft or soft-collinear might allow one to make progress organizing endpoint singularities to all orders in perturbation theory. We note that SCET, as it is presently formulated [42], does not include modes in which quarks are soft, and so it would be necessary to augment SCET in order to apply it to the endpoint region. The all-orders organization of endpoint singularities, in the context of SCET or in the context of traditional diagrammatic approaches, would be a key ingredient in the resummation of the endpoint logarithms.

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