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Phys. Rev. D **90**, 035016 — Published 18 August 2014

DOI: [10.1103/PhysRevD.90.035016](https://doi.org/10.1103/PhysRevD.90.035016)

# Top Partners and Higgs Production

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(Dated: June 19, 2014)

The Higgs boson is produced at the LHC through gluon fusion at roughly the Standard Model rate. New colored fermions, which can contribute to  $gg \rightarrow h$ , must have vector-like interactions in order not to be in conflict with the experimentally measured rate. We examine the size of the corrections to single and double Higgs production from heavy vector-like fermions in  $SU(2)_L$  singlets and doublets and search for regions of parameter space where double Higgs production is enhanced relative to the Standard Model prediction. We compare production rates and distributions for double Higgs production from gluon fusion using an exact calculation, the low energy theorem (LET), where the top quark and the heavy vector-like fermions are taken to be infinitely massive, and an effective theory (EFT) where top mass effects are included exactly and the effects of the heavy fermions are included to  $\mathcal{O}\left(\frac{1}{M_X^2}\right)$ . Unlike the LET, the EFT gives an extremely accurate description of the kinematic distributions for double Higgs production.

PACS numbers:

## I. INTRODUCTION

Having discovered a particle with the generic properties of the Standard Model Higgs boson, the next important step is to determine what, if any, deviations from the standard picture are allowed by the data. The observed production and decay modes of the Higgs boson are within  $\sim 20\%$  of the expectation for a weakly coupled Higgs particle[1, 2] and so the possibilities for new physics in the Higgs sector are highly constrained[3]. A convenient framework to examine possible new high scale physics is the language of effective field theories, where the theory is constructed to reduce to the Standard Model at the electroweak scale, but new interactions are allowed at higher scales. We study an extension of the Standard Model where there are new massive quarks which are allowed to interact with the Standard Model particles, and thus potentially modify Higgs production and decay rates. Heavy fermions occur in many Beyond the Standard Model (BSM) scenarios, in particular Little Higgs models [4–7] and composite Higgs models[8–12] in which the Higgs is strongly interacting at high scales. We consider models with both charge  $\frac{2}{3}$  and  $-\frac{1}{3}$  heavy quarks, but where there are no additional Higgs bosons beyond the Standard Model  $SU(2)_L$  doublet.

New heavy colored fermions which couple to the Higgs boson cannot occur in chiral multiplets since they would give large contributions to the rate for Higgs production from gluon fusion[13, 14]. A single  $SU(2)_L$  heavy quark doublet with corresponding right-handed heavy quark singlets would increase the gluon fusion Higgs production rate by a factor  $\sim 9$ , which is definitively excluded. Vector-like quarks, on the other hand, decouple at high energy and can be accommodated both by precision electroweak data, and by Higgs production measurements. Models with a single multiplet of new vector-like fermions have been studied extensively in the context of single and double Higgs production from gluon fusion[4, 15–22]. The rates for both single and double Higgs production in this class of models are close to those of the Standard Model and the gluon fusion processes are insensitive to the top partner masses and couplings. This general feature is a result of the structure of the quark mass matrix and can be proven using the Higgs low energy theorems (LETs)[12, 23, 24].

We study more complicated models with several multiplets of vector-like quarks in both  $SU(2)_L$  doublet and singlet representations[25], which are allowed to mix with the Standard Model quarks and with each other. Higgs production from gluon fusion can be significantly altered from the Standard Model prediction when this mixing is allowed[18, 21, 24]. We explore the possibility of having the double Higgs production rate be strongly enhanced or suppressed relative to the Standard Model, while keeping the single Higgs rate close to that of the Standard Model. Models with multiple representations of vector-like fermions have also been considered in the context of flavor, where they have been used to generate a hierarchy of masses for the Standard Model fermions[26, 27].

Effective field theory (EFT) techniques can be used to integrate out the effects of heavy fermions. Low energy physics is then described by an effective Lagrangian,

$$L_{eff} = L_{SM} + \sum_i \frac{f_i O_i}{\Lambda^2} + \dots, \quad (1)$$

where  $O_i$  are the dimension-6 operators corresponding to new physics at the scale  $\Lambda$ . These operators have been catalogued under various assumptions[11, 28, 29] and in this paper, we consider only those operators affecting the gluon fusion production of Higgs bosons. We calculate the contributions to the  $f_i$  obtained by integrating out heavy vector-like quarks in  $SU(2)_L$  singlet and doublet representations, using the equations of motion. The new physics arising from the heavy vector-like quarks yields corrections to the Standard Model  $SU(2)_L \times U(1)$  gauge couplings and to the Yukawa couplings of the light fermions.

For arbitrary fermion mass matrices, we compute both single and double Higgs production from gluon fusion. As a by product of our calculation, we compare rates found by diagonalizing the mass matrices exactly, from the effective theory of Eq. 1 which contains terms of  $\mathcal{O}\left(\frac{m_t^2}{\Lambda^2}\right)$ , and from the low energy theorems, where  $m_t \rightarrow \infty$  along with the new vector-like quarks, in order to establish the numerical accuracy of the various approximations.

Section II contains a brief description of the class of models studied here. A description of single and double Higgs production using the LET description and the EFT with top and bottom quark mass effects included is given in Section III. Analytic results in an example with small mixing between the Standard Model  $3^{rd}$  generation quarks and the heavy quarks are given in Section IV in order to give an intuitive understanding of the new physics resulting from integrating out the heavy vector-like fermions, while Section V summarizes limits from precision electroweak measurements. Our major results are contained in Section VI, where total rates and distributions for double Higgs production are given in the full theory, the LET, and the EFT. Finally, some conclusions are in Section VII.

## II. THE MODEL

We consider models where in addition to the Standard Model field content, there are two vector-like  $SU(2)_L$  singlets,  $U$  and  $D$ , and one vector-like  $SU(2)_L$  doublet,  $Q$ , with hypercharges  $Y = 4/3, -2/3$ , and  $1/3$ , respectively. We only allow mixing between the new fermions and the  $3^{rd}$  generation Standard Model quarks since the interactions of the two light generations of quarks are highly constrained. The Standard Model  $3^{rd}$  generation fermions are

$$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, t_R, b_R, \quad (2)$$

and the heavy vector-like fermions are,

$$Q = \begin{pmatrix} T \\ B \end{pmatrix}, U, D, \quad (3)$$

where the left- and right- handed components have identical transformation properties under  $SU(2)_L \times U(1)$ , allowing for Dirac mass terms. Finally, the Higgs doublet takes its usual form in unitary gauge after electroweak symmetry breaking,

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}, \quad (4)$$

where  $v = 246$  GeV is the Higgs vev, and  $h$  is the Higgs boson. The Standard Model Lagrangian involving the third generation fermions and the Higgs boson is,

$$L_{SM} = i\bar{q}_L \not{D} q_L + i\bar{t}_R \not{D} t_R + i\bar{b}_R \not{D} b_R - \left( \lambda_t \bar{q}_L \tilde{H} t_R + \lambda_b \bar{q}_L H b_R + h.c. \right) + |D_\mu H|^2 - V(H), \quad (5)$$

where  $\tilde{H} \equiv i\sigma_2 H^*$ ,  $V(H)$  is the Higgs potential,  $D_\mu = (\partial_\mu - i\frac{g}{2}T \cdot W_\mu - i\frac{g'}{2}YB_\mu - ig_s t \cdot G_\mu)$ ,  $T^a = \sigma^a$  for  $SU(2)_L$  doublets,  $T^a = 0$  for  $SU(2)_L$  singlets,  $\sigma^a$  are the Pauli matrices, for the quarks  $t$  are the  $SU(3)_C$  fundamental representation matrices, for the Higgs  $t = 0$ , and  $Q = (Y + T^3)/2$  is the electric charge operator. The classical equations of motion corresponding to Eq. 5 are [28],

$$\begin{aligned} i\not{D} q_L &= \lambda_t \tilde{H} t_R + \lambda_b H b_R \\ i\not{D} t_R &= \lambda_t \tilde{H}^\dagger q_L \\ i\not{D} b_R &= \lambda_b H^\dagger q_L. \end{aligned} \quad (6)$$

The most general Lagrangian coupling the third generation quarks and the new fermions is  $L_{NP}$ ,

$$\begin{aligned}
L_{NP} &\equiv L'_M + L'_{KE} + L'_Y \\
L'_M &= -M\bar{Q}Q - M_U\bar{U}U - M_D\bar{D}D \\
L'_{KE} &= \bar{Q}(i\not{D})Q + \bar{U}(i\not{D})U + \bar{D}(i\not{D})D \\
L'_Y &= -\left\{ \lambda_1\bar{Q}_L\tilde{H}U_R + \lambda_2\bar{Q}_LHD_R + \lambda_3\bar{Q}_R\tilde{H}U_L + M_4\bar{q}_LQ_R + M_5\bar{U}_Lt_R + M_6\bar{D}_Lb_R \right. \\
&\quad \left. + \lambda_7\bar{q}_L\tilde{H}U_R + \lambda_8\bar{q}_LHD_R + \lambda_9\bar{Q}_L\tilde{H}t_R + \lambda_{10}\bar{Q}_LHb_R + \lambda_{11}\bar{Q}_RHD_L + h.c. \right\}.
\end{aligned} \tag{7}$$

The much studied cases where the Standard Model top quark mixes with only a singlet or doublet vector-like fermion[15–17, 19, 30–34] can be obtained from this study, as can the composite model case where the Standard Model quarks do not couple to the Higgs doublet ( $\lambda_t = \lambda_b = \lambda_7 = \lambda_9 = \lambda_{10} = 0$ ). We will consider various mass hierarchies in the following sections.

The mass and Yukawa interactions can be written as

$$-L_{Y'} = \bar{\chi}_L^t M^{(t)}(h) \chi_R^t + \bar{\chi}_L^b M^{(b)}(h) \chi_R^b + h.c., \tag{8}$$

where  $\chi_{L,R}^t \equiv (t, T, U)_{L,R}$ ,  $\chi_{L,R}^b \equiv (b, B, D)$ , and the Higgs-dependent fermion mass matrices are

$$M^{(t)}(h) = \begin{pmatrix} \lambda_t(\frac{h+v}{\sqrt{2}}) & M_4 & \lambda_7(\frac{h+v}{\sqrt{2}}) \\ \lambda_9(\frac{h+v}{\sqrt{2}}) & M & \lambda_1(\frac{h+v}{\sqrt{2}}) \\ M_5 & \lambda_3(\frac{h+v}{\sqrt{2}}) & M_U \end{pmatrix}, \quad M^{(b)}(h) = \begin{pmatrix} \lambda_b(\frac{h+v}{\sqrt{2}}) & M_4 & \lambda_8(\frac{h+v}{\sqrt{2}}) \\ \lambda_{10}(\frac{h+v}{\sqrt{2}}) & M & \lambda_2(\frac{h+v}{\sqrt{2}}) \\ M_6 & \lambda_{11}(\frac{h+v}{\sqrt{2}}) & M_D \end{pmatrix}, \tag{9}$$

where typically  $\lambda_i \sim \mathcal{O}(1)$ . The mass eigenstate fields,  $\psi^t \equiv (T_1, T_2, T_3)$  and  $\psi^b \equiv (B_1, B_2, B_3)$ , are found by means of bi-unitary transformations,

$$\begin{aligned}
-L_{Y'} &= \bar{\chi}_L^t (V_L^{t\dagger} V_L^t) M^{(t)}(h) (V_R^{t\dagger} V_R^t) \chi_R^t + \bar{\chi}_L^b (V_L^{b\dagger} V_L^b) M^{(b)}(h) (V_R^{b\dagger} V_R^b) \chi_R^b + h.c. \\
&= \bar{\psi}_L^t M_{diag}^t \psi_R^t + \bar{\psi}_L^b M_{diag}^b \psi_R^b + \bar{\psi}_L^t \mathcal{Y}^t \psi_R^t h + \bar{\psi}_L^b \mathcal{Y}^b \psi_R^b h + h.c.,
\end{aligned} \tag{10}$$

and  $(T_1, B_1)$  are the Standard Model 3<sup>rd</sup> generation quarks. The diagonal mass matrices can be written,

$$\begin{aligned}
M_{diag}^t &= V_L^t M^{(t)}(0) V_R^{t\dagger} \\
(M_{diag}^t)^2 &= V_L^t M^{(t)}(0) M^{(t)}(0)^\dagger V_L^{t\dagger} \\
&= V_R^t M^{(t)}(0)^\dagger M^{(t)}(0) V_R^{t\dagger},
\end{aligned} \tag{11}$$

where we have set  $h = 0$ , the Yukawa matrix is

$$\mathcal{Y}^t h = V_L^t \left( M^{(t)}(h) - M^{(t)}(0) \right) V_R^{t\dagger} \tag{12}$$

and similarly in the  $b$  sector.

The couplings to the  $W$  contain both left- and right- handed contributions,

$$\begin{aligned}
L_W &= \frac{g}{\sqrt{2}} \left( \bar{\chi}_R^{t,2} \gamma_\mu \chi_R^{b,2} + \sum_{i=1,2} \bar{\chi}_L^{t,i} \gamma_\mu \chi_L^{b,i} \right) W^{+\mu} + h.c. \\
&= \frac{g}{\sqrt{2}} \sum_{j,k=1,2,3} \left( \bar{\psi}_L^{t,j} (U_L)_{jk} \gamma_\mu \psi_L^{b,k} + \bar{\psi}_R^{t,j} (U_R)_{jk} \gamma_\mu \psi_R^{b,k} \right) W^{+\mu} + h.c.,
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
(U_L)_{jk} &= \sum_{i=1,2} (V_L^t)_{ji} (V_L^{b\dagger})_{ik} \\
(U_R)_{jk} &= (V_R^t)_{j2} (V_R^{b\dagger})_{2k}
\end{aligned} \tag{14}$$

Finally, the couplings to the  $Z$  are,

$$\begin{aligned}
L_Z = & \frac{g}{2c_W} \sum_{j,k=1,2,3} \left\{ \overline{\psi}_{L,j}^t (X_L^t)_{jk} \gamma_\mu \psi_{L,k}^t + \overline{\psi}_{R,j}^t (X_R^t)_{jk} \gamma_\mu \psi_{R,k}^t \right. \\
& \left. - \overline{\psi}_{L,j}^b (X_L^b)_{jk} \gamma_\mu \psi_{L,k}^b - \overline{\psi}_{R,j}^b (X_R^b)_{jk} \gamma_\mu \psi_{R,k}^b \right\} Z^\mu \\
& - \frac{g}{2c_W} (2s_W^2) J_{EM}^\mu Z_\mu,
\end{aligned} \tag{15}$$

where  $s_W = \sin \theta_W$ ,  $c_W = \cos \theta_W$ ,  $\theta_W$  is the weak mixing angle,

$$\begin{aligned}
(X_L^t)_{jk} &= \sum_{i=1,2} (V_L^t)_{ji} (V_L^{t\dagger})_{ik} \\
(X_R^t)_{jk} &= (V_R^t)_{j2} (V_R^{t\dagger})_{2k} \\
(X_L^b)_{jk} &= \sum_{i=1,2} (V_L^b)_{ji} (V_L^{b\dagger})_{ik} \\
(X_R^b)_{jk} &= (V_R^b)_{j2} (V_R^{b\dagger})_{2k}
\end{aligned} \tag{16}$$

and  $J_{EM}^\mu$  is the usual electromagnetic current,

$$J_{EM}^\mu = Q_t \left[ \overline{\psi}_L^t \gamma^\mu \psi_L^t + \overline{\psi}_R^t \gamma^\mu \psi_R^t \right] + Q_b \left[ \overline{\psi}_L^b \gamma^\mu \psi_L^b + \overline{\psi}_R^b \gamma^\mu \psi_R^b \right]. \tag{17}$$

The  $Z$  couplings contain flavor non-diagonal contributions due to the off diagonal terms in  $X_{L,R}^{t,b}$ . It is straightforward to apply the results of Eqs. 13 and 15 to find the gauge boson couplings in a specific model.

### III. EFFECTIVE THEORY RESULTS

In this section, we consider single and double Higgs production from gluon fusion in the general model described in the previous section. We begin with the results using the LET, in which the top quark and all top partners are taken infinitely massive. We next include the top quark and bottom quark masses exactly and compute to  $\mathcal{O}(\frac{1}{M_X^2})$ , where  $M_X$  is a generic heavy vector fermion mass. These results (EFT) are then matched to an effective Lagrangian to determine the coefficients of the dimension-6 operators. We are interested in comparing the numerical accuracy of the two approximations with the exact calculations for the gluon fusion rates.

#### A. Effective Theory From Low Energy Theorems

The low energy theorems can be used to integrate out the effect of the charge  $\frac{2}{3}$  massive particles, including the top quark. In the limit in which fermion masses ( $M_{T_1}, M_{T_2}, M_{T_3}$ ) are much heavier than the Higgs mass, the  $hgg$  coupling can be found from the low energy effective interaction of a colored Dirac fermion with the gluon field strength[35],

$$L_{hgg}^{(t)} = \frac{\alpha_s}{24\pi} h \left( \frac{\partial}{\partial h} \ln \left[ \det(M^{(t)}(h)^\dagger M^{(t)}(h)) \right] \right)_{h=0} G^{A,\mu\nu} G_{\mu\nu}^A, \tag{18}$$

where  $G_{\mu\nu}^A$  is the gluon field-strength tensor. With no approximation on the relative size of the parameters in  $M^{(t)}$ , the LET gives for the contributions from the top sector alone,

$$L_{hgg}^{(t)} = \frac{\alpha_s}{12\pi} \frac{h}{v} \left[ 1 + 2\lambda_3 v^2 \left( \frac{\lambda_1 \lambda_t - \lambda_7 \lambda_9}{X} \right) \right] G^{A,\mu\nu} G_{\mu\nu}^A, \tag{19}$$

where,

$$X \equiv -\frac{v}{2\sqrt{2}} \det M^{(t)}(0) = v^2 \lambda_3 (\lambda_1 \lambda_t - \lambda_7 \lambda_9) + 2 \left[ -\lambda_1 M_4 M_5 + M M_5 \lambda_7 + M_U M_4 \lambda_9 - \lambda_t M M_U \right]. \tag{20}$$

Having non-zero  $\lambda_3$ , the coupling between the doublet and singlet vector-like quarks and Higgs boson, is critical for achieving a result which is different from the LET for the Standard Model:

$$L_{h^2 gg}^{SM} = \frac{\alpha_s}{12\pi} \left[ \frac{h}{v} - \frac{h^2}{2v^2} + \dots \right] G^{A,\mu\nu} G_{\mu\nu}^A. \quad (21)$$

This can be understood by noting that when the mass matrix factorizes,

$$\det \left( M^{(t)}(h) \right) = F \left( \frac{h}{v} \right) G(\lambda_i, M_X, m_t), \quad (22)$$

the LET has no dependence on the heavy mass scales and Yukawa couplings as in Eq. 21 [12, 23, 24]. In the limit  $\lambda_3 \rightarrow 0$ , we have:

$$\det M^{(t)}(h) \Big|_{\lambda_3=0} = -\frac{h+v}{2\sqrt{2}} X \Big|_{\lambda_3=0}. \quad (23)$$

and the LET reduces to the Standard Model result.

In the limit  $M, M_U \gg M_5, M_4, v$  and all the Yukawa couplings  $\lambda_i$  are  $\mathcal{O}(1)$ ,

$$L_{hgg}^{(t)} \rightarrow \frac{\alpha_s}{12\pi} \frac{h}{v} \left[ 1 - \lambda_3 v^2 \left( \frac{\lambda_1 \lambda_t - \lambda_7 \lambda_9}{M M_U \lambda_t} \right) \right] G^{\mu\nu, A} G_{\mu\nu}^A. \quad (24)$$

If, motivated by composite models[12], we assume that there are no couplings of the Standard Model quarks to the Higgs, then  $\lambda_t = \lambda_b = \lambda_7 = \lambda_9 = \lambda_{10} = 0$ , and with no assumption about the relative sizes of the remaining terms,

$$L_{hgg}^{(t)} \rightarrow \frac{\alpha_s}{12\pi} \frac{h}{v} G^{A,\mu\nu} G_{\mu\nu}^A, \quad (25)$$

and the Standard Model result is recovered. Similarly to above, in this limit the determinant of the mass matrix factorizes.

Double Higgs production can also be found using the LET [15, 18, 35],

$$L_{hhgg}^{(t)} = \frac{\alpha_s}{48\pi} h^2 \left( \frac{\partial^2}{\partial h^2} \ln \left[ \det M^{(t)}(h)^\dagger M^{(t)}(h) \right] \right)_{h=0} G^{A,\mu\nu} G_{\mu\nu}^A, \quad (26)$$

and we obtain,

$$L_{hhgg}^{(t)} = -\frac{\alpha_s}{24\pi} \frac{h^2}{v^2} \left\{ 1 - 2\lambda_3 v^2 (\lambda_1 \lambda_t - \lambda_7 \lambda_9) \left[ \frac{1}{X} - \frac{2\lambda_3 v^2 (\lambda_1 \lambda_t - \lambda_7 \lambda_9)}{X^2} \right] \right\} G^{A,\mu\nu} G_{\mu\nu}^A. \quad (27)$$

In the limit  $M, M_U \gg M_4, M_5, v$ ,

$$L_{hhgg}^{(t)} = -\frac{\alpha_s}{24\pi} \frac{h^2}{v^2} \left\{ 1 + \lambda_3 v^2 \left( \frac{\lambda_1 \lambda_t - \lambda_7 \lambda_9}{M M_U \lambda_t} \right) \right\} G^{A,\mu\nu} G_{\mu\nu}^A. \quad (28)$$

Since the  $b$  quark is not a heavy fermion, the effective  $ggh$  Lagrangian in the charge  $-\frac{1}{3}$  sector requires more care and the LET cannot be naively applied. In the next section, we formally integrate out the heavy  $T_2, T_3$  and  $B_2, B_3$  fields, while retaining all mass dependence from the light Standard Model-like quarks,  $T_1, B_1$ .

## B. Effective Theory with Top and Bottom Quark Masses

The effects of finite top and bottom quark masses can be included by using the classical equations of motion to integrate out the heavy fields  $T_2, T_3, B_2$  and  $B_3$ [27, 28, 30, 31, 36]. We assume that  $M, M_U$  and  $M_D$  are of similar

magnitude and are much larger than  $v$ , that the Yukawa couplings,  $\lambda_i$ , are of  $\mathcal{O}(1)$ , and expand to  $\mathcal{O}(1/M_X^2)$ ,

$$\begin{aligned}
U_L &= \left( -\frac{\lambda_7}{M_U} + \frac{\lambda_1 M_4}{M M_U} - \frac{\lambda_t M_5}{M_U^2} \right) (\tilde{H}^\dagger q_L) + \mathcal{O}\left(\frac{1}{M_X^3}\right) \\
U_R &= \left[ -\frac{M_5}{M_U} + \left( \frac{\lambda_3 \lambda_9}{M M_U} - \frac{\lambda_t \lambda_7}{M_U^2} \right) (H^\dagger H) \right] t_R - \frac{\lambda_7}{M_U^2} i(D_\mu \tilde{H})^\dagger \gamma^\mu q_L + \mathcal{O}\left(\frac{1}{M_X^3}\right) \\
D_L &= \left( -\frac{\lambda_8}{M_D} + \frac{\lambda_2 M_4}{M M_D} - \frac{\lambda_b M_6}{M_D^2} \right) (H^\dagger q_L) + \mathcal{O}\left(\frac{1}{M_X^3}\right) \\
D_R &= \left[ -\frac{M_6}{M_D} + \left( \frac{\lambda_{10} \lambda_{11}}{M M_D} - \frac{\lambda_b \lambda_8}{M_D^2} \right) (H^\dagger H) \right] b_R - \frac{\lambda_8}{M_D^2} i(D_\mu H)^\dagger \gamma^\mu q_L + \mathcal{O}\left(\frac{1}{M_X^3}\right) \\
Q_L &= \left[ -\frac{M_4}{M} + \left( \frac{\lambda_8 \lambda_{11}}{M M_D} - \frac{\lambda_b \lambda_{10}}{M^2} \right) (H H^\dagger) + \left( \frac{\lambda_3 \lambda_7}{M M_U} - \frac{\lambda_t \lambda_9}{M^2} \right) \tilde{H} \tilde{H}^\dagger \right] q_L \\
&\quad - \frac{\lambda_9}{M^2} (i \not{D} \tilde{H}) t_R - \frac{\lambda_{10}}{M^2} (i \not{D} H) b_R + \mathcal{O}\left(\frac{1}{M_X^3}\right) \\
Q_R &= \left( -\frac{\lambda_9}{M} + \frac{\lambda_1 M_5}{M M_U} - \frac{\lambda_t M_4}{M^2} \right) \tilde{H} t_R \\
&\quad + \left( -\frac{\lambda_{10}}{M} + \frac{\lambda_2 M_6}{M M_D} - \frac{\lambda_b M_4}{M^2} \right) H b_R + \mathcal{O}\left(\frac{1}{M_X^3}\right).
\end{aligned} \tag{29}$$

Substituting Eq. 29 into  $L'_M + L'_Y$ ,

$$\begin{aligned}
L'_M + L'_Y &\equiv L_{eff}^{(a)} \\
&\rightarrow - \left\{ \left( -\frac{M_5 \lambda_7}{M_U} - \frac{M_4 \lambda_9}{M} + \frac{\lambda_1 M_4 M_5}{M M_U} + \frac{\lambda_3 \lambda_7 \lambda_9}{M M_U} (H^\dagger H) \right) \bar{q}_L \tilde{H} t_R \right. \\
&\quad + \left( -\frac{M_6 \lambda_8}{M_D} - \frac{M_4 \lambda_{10}}{M} + \frac{\lambda_2 M_4 M_6}{M M_D} + \frac{\lambda_8 \lambda_{10} \lambda_{11}}{M M_D} (H^\dagger H) \right) \bar{q}_L H b_R \\
&\quad \left. + h.c. \right\} + \delta L_{eff}^h + \mathcal{O}\left(\frac{1}{M_X^3}\right),
\end{aligned} \tag{30}$$

where  $\delta L_{eff}^h$  collects the contributions from the terms in Eq. 29 containing derivatives of the Higgs field[31],

$$\begin{aligned}
\delta L_{eff}^h &= \left\{ \frac{1}{4} \left( \frac{\lambda_7^2}{M_U^2} - \frac{\lambda_8^2}{M_D^2} \right) (H^\dagger i D_\mu H) (\bar{q}_L \gamma^\mu q_L) \right. \\
&\quad - \frac{1}{4} \left( \frac{\lambda_7^2}{M_U^2} + \frac{\lambda_8^2}{M_D^2} \right) (H^\dagger \sigma^a i D_\mu H) (\bar{q}_L \gamma^\mu \sigma^a q_L) \\
&\quad - \frac{\lambda_9^2}{2 M^2} \left[ (H^\dagger i D_\mu H) (\bar{t}_R \gamma^\mu t_R) \right] + \frac{\lambda_{10}^2}{2 M^2} \left[ (H^\dagger i D_\mu H) (\bar{b}_R \gamma^\mu b_R) \right] \\
&\quad \left. + \frac{\lambda_9 \lambda_{10}}{M^2} \left[ (\tilde{H}^\dagger i D_\mu H) (\bar{t}_R \gamma^\mu b_R) \right] \right\} + h.c. + \mathcal{O}\left(\frac{1}{M_X^3}\right).
\end{aligned} \tag{31}$$

Eq. 31 corresponds to  $\Delta L_{F_1}$  of Refs. [29, 37].

Similarly, substituting Eq. 29 into the kinetic energy terms of  $L'_{KE}$ ,

$$\begin{aligned}
L'_{KE} &\equiv L_{eff}^{(b)} \\
&\rightarrow \frac{1}{2} \left\{ \lambda_t \bar{q}_L \tilde{H} t_R \left( \frac{M_4^2}{M^2} + \frac{M_5^2}{M_U^2} + (H^\dagger H) \left[ \frac{\lambda_7^2}{M_U^2} + \frac{\lambda_9^2}{M^2} \right] \right) \right. \\
&\quad + \lambda_b \bar{q}_L H b_R \left( \frac{M_4^2}{M^2} + \frac{M_6^2}{M_D^2} + (H^\dagger H) \left[ \frac{\lambda_8^2}{M_D^2} + \frac{\lambda_{10}^2}{M^2} \right] \right) \\
&\quad \left. + h.c. + \mathcal{O}\left(\frac{1}{M_X^3}\right) \right\}.
\end{aligned} \tag{32}$$

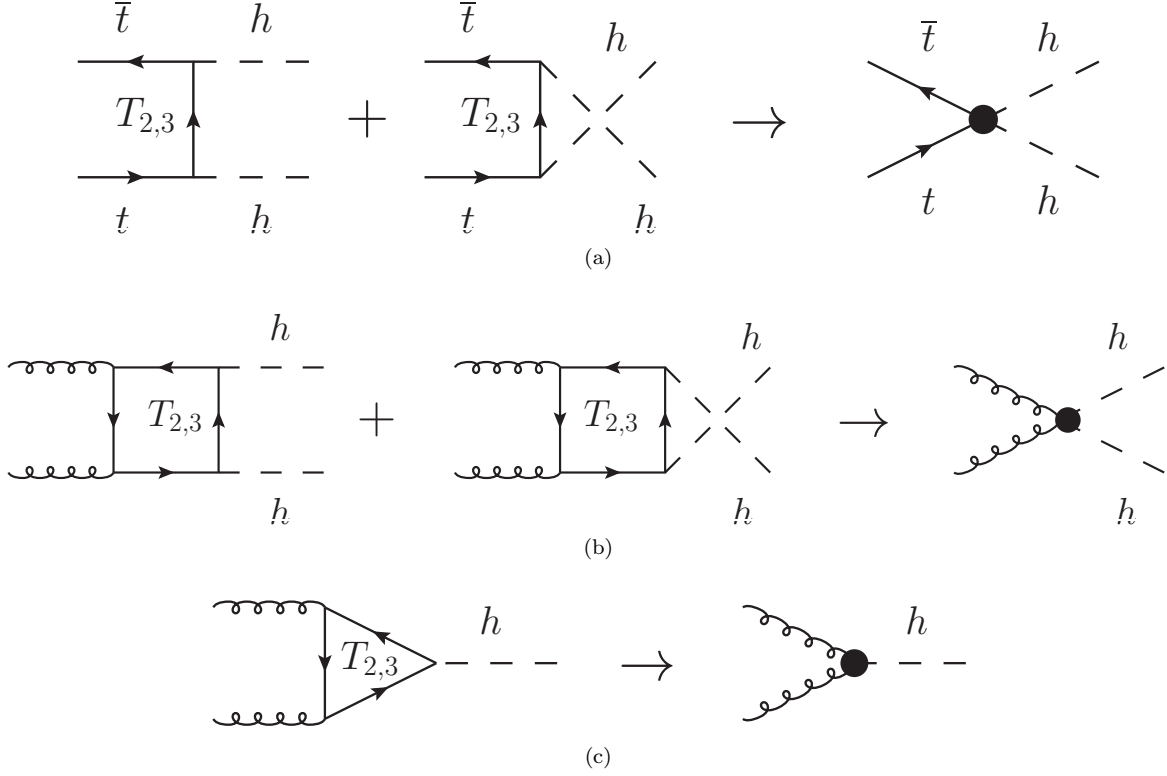


FIG. 1: Representative diagrams corresponding to integrating out heavy fields and generating the (a)  $\bar{t}thh$ , (b)  $G^{A,\mu\nu}G_{\mu\nu}^Ah^2$ , and (c)  $G^{A,\mu\nu}G_{\mu\nu}^Ah$  operators in Eq. 33.

The effective low energy Lagrangian after electroweak symmetry breaking contains only Standard Model fields, but non-Standard Model coefficients and operators have been generated by integrating out the heavy fields. The procedure of integrating out by the equations of motion occurs at tree level. However, at loop level, integrating out heavy colored particles will generate operators of the form  $G^{A,\mu\nu}G_{\mu\nu}^Ah^2$  and  $G^{A,\mu\nu}G_{\mu\nu}^Ah$ , which need to be included in the effective Lagrangian:

$$\begin{aligned}
L_{eff} &= L_{SM} + L_{eff}^{(a)} + L_{eff}^{(b)} + \delta L_{eff}^h + \frac{c_g \alpha_s}{12\pi v} G^{A,\mu\nu} G_{\mu\nu}^A h - \frac{c_{gg} \alpha_s}{24\pi v^2} G^{A,\mu\nu} G_{\mu\nu}^A h^2 \\
&= i\bar{q}_L \not{D} q_L + i\bar{t}_R \not{D} t_R + i\bar{b}_R \not{D} b_R + |D_\mu H|^2 - V(H) - m_t \bar{t}t - Y_t \bar{t}th + c_{2h}^{(t)} \bar{t}th^2 \\
&\quad - m_b \bar{b}b - Y_b \bar{b}bh + c_{2h}^{(b)} \bar{b}bh^2 + \frac{c_g \alpha_s}{12\pi v} G^{A,\mu\nu} G_{\mu\nu}^A h - \frac{c_{gg} \alpha_s}{24\pi v^2} G^{A,\mu\nu} G_{\mu\nu}^A h^2 \\
&\quad + \frac{g}{\sqrt{2}} \left\{ \left[ \delta g_L \bar{t}_L \gamma^\mu b_L + \delta g_R \bar{t}_R \gamma^\mu b_R \right] W_\mu^+ + h.c. \right\} \\
&\quad + \frac{g}{c_W} \left\{ \bar{t}_L \gamma_\mu t_L \delta Z_L^t + \bar{t}_R \gamma_\mu t_R \delta Z_R^t + \bar{b}_L \gamma_\mu b_L \delta Z_L^b + \bar{b}_R \gamma_\mu b_R \delta Z_R^b \right\} Z^\mu \\
&\quad + \delta L_{eff}^{h'} + \mathcal{O}\left(\frac{1}{M_X^3}\right).
\end{aligned} \tag{33}$$

The non-Standard Model like gauge boson coupling in lines 4 and 5 in the above equation originate from  $\delta L_{eff}^h$ , and  $\delta L_{eff}^{h'}$  is defined to be  $\delta L_{eff}^h$  with these terms removed. In Fig. 1 we show representative diagrams illustrating the generation of the (a)  $\bar{t}th^2$ , (b)  $G^{A,\mu\nu}G_{\mu\nu}^Ah^2$ , and (c)  $G^{A,\mu\nu}G_{\mu\nu}^Ah$  effective operators. To  $\mathcal{O}\left(\frac{1}{M_X^2}\right)$ , the Yukawa



couplings are shifted from their Standard Model values,

$$\begin{aligned}
\sqrt{2}Y_t &= \lambda_t \left\{ 1 - \left[ \frac{M_4^2}{2M^2} + \frac{M_5^2}{2M_U^2} + \frac{3v^2}{4} \left( \frac{\lambda_7^2}{M_U^2} + \frac{\lambda_9^2}{M^2} \right) \right] \right\} \\
&\quad - \frac{M_5\lambda_7}{M_U} - \frac{M_4\lambda_9}{M} + \frac{\lambda_1 M_4 M_5}{M M_U} + \frac{3v^2}{2M M_U} \lambda_3 \lambda_7 \lambda_9 \\
&= \sqrt{2} \frac{m_t}{v} + \frac{v^2}{M M_U} \lambda_3 \lambda_7 \lambda_9 - \lambda_t \frac{v^2}{2} \left( \frac{\lambda_7^2}{M_U^2} + \frac{\lambda_9^2}{M^2} \right) \\
\frac{Y_t v}{m_t} &\equiv 1 + \delta_t \\
\sqrt{2}Y_b &= \lambda_b \left\{ 1 - \left[ \frac{M_4^2}{2M^2} + \frac{M_6^2}{2M_D^2} + \frac{3v^2}{4} \left( \frac{\lambda_8^2}{M_D^2} + \frac{\lambda_{10}^2}{M^2} \right) \right] \right\} \\
&\quad - \frac{M_6\lambda_8}{M_D} - \frac{M_4\lambda_{10}}{M} + \frac{\lambda_2 M_4 M_6}{M M_D} + \frac{3v^2}{2M M_D} \lambda_8 \lambda_{10} \lambda_{11} \\
&= \sqrt{2} \frac{m_b}{v} + \frac{v^2}{M M_D} \lambda_8 \lambda_{10} \lambda_{11} - \lambda_b \frac{v^2}{2} \left( \frac{\lambda_8^2}{M_D^2} + \frac{\lambda_{10}^2}{M^2} \right) \\
\frac{Y_b v}{m_b} &\equiv 1 + \delta_b.
\end{aligned} \tag{34}$$

We see that  $Y_t$  and  $Y_b$  are no longer proportional to  $m_t = M_{T_1}$  and  $m_b = M_{B_1}$ . Non-Standard Model couplings of the fermions to Higgs pairs are also generated, as are Higgs-gluon effective couplings,

$$\begin{aligned}
\frac{c_{2h}^{(t)}}{v} &= \frac{3}{2\sqrt{2}} \left\{ -\frac{\lambda_3 \lambda_7 \lambda_9}{M M_U} + \frac{1}{2} \lambda_t \left( \frac{\lambda_7^2}{M_U^2} + \frac{\lambda_9^2}{M^2} \right) \right\} \\
&= -\frac{3}{2v^2} \left( Y_t - \frac{m_t}{v} \right) \\
&= -\frac{3}{2} \frac{m_t \delta_t}{v^3} \\
\frac{c_{2h}^{(b)}}{v} &= \frac{3}{2\sqrt{2}} \left\{ -\frac{\lambda_8 \lambda_{10} \lambda_{11}}{M M_D} + \frac{1}{2} \lambda_b \left( \frac{\lambda_8^2}{M_D^2} + \frac{\lambda_{10}^2}{M^2} \right) \right\} \\
&= -\frac{3}{2} \frac{m_b \delta_b}{v^3} \\
c_g &= v^2 \left[ -\frac{\lambda_1 \lambda_3}{M M_U} - \frac{\lambda_2 \lambda_{11}}{M M_D} + \frac{1}{2} \left( \frac{\lambda_7^2}{M_U^2} + \frac{\lambda_8^2}{M_D^2} + \frac{\lambda_9^2 + \lambda_{10}^2}{M^2} \right) \right] \\
&= -c_{gg}.
\end{aligned} \tag{35}$$

The top and bottom quark couplings to  $ggh$  and  $gghh$  are not included in  $c_g$  and  $c_{gg}$ , but can be calculated at one-loop using the effective interactions of Eq. 33. The effective Lagrangian depends on only 3 new parameters:  $c_g$ ,  $\delta_t$ , and  $\delta_b$ , along with the physical masses,  $m_t = M_{T_1}$  and  $m_b = M_{B_1}$ , and  $v$ . It is important to note, that within the context of this model, the coefficients of the effective Lagrangian cannot all be independently varied. This feature can also arise in composite Higgs models [21].

The non-Standard Model couplings to the  $W$  and  $Z$  are given to  $\mathcal{O}(1/M_X^2)$  by,

$$\begin{aligned}
\delta g_L &= -\frac{v^2}{4} \left( \frac{\lambda_7^2}{M_U^2} + \frac{\lambda_8^2}{M_D^2} \right) & \delta g_R &= \frac{v^2 \lambda_9 \lambda_{10}}{2M^2} \\
\delta Z_L^t &= -\frac{v^2 \lambda_7^2}{4M_U^2} & \delta Z_R^t &= \frac{v^2 \lambda_9^2}{4M^2} \\
\delta Z_L^b &= \frac{v^2 \lambda_8^2}{4M_D^2} & \delta Z_R^b &= -\frac{v^2 \lambda_{10}^2}{4M^2}
\end{aligned} \tag{36}$$

## IV. UNDERSTANDING THE FULL THEORY

### A. Hierarchy 1

In order to understand some general features of the mass matrices we consider a hierarchy where the mixing angles are small,

$$\theta \sim \frac{\lambda_i v}{M_4} \sim \frac{\lambda_i v}{M_5} \sim \frac{M_4}{M} \sim \frac{M_5}{M} \quad \text{and} \quad \theta^2 \sim \frac{\lambda_i v}{M}. \quad (37)$$

This maintains the hierarchy,  $\lambda_i v \ll M_4, M_5 \ll M, M_U, M_D$ , keeping the off-diagonal elements of the mass matrices small. In this limit the matrices which diagonalize the top quark mass matrix can be written as

$$V_L^t = \begin{pmatrix} 1 - \frac{1}{2}\theta_L^{D^2} & -\theta_L^D & -\theta_L^{S^2} \\ \theta_L^D & 1 - \frac{1}{2}\theta_L^{D^2} & \theta_L^{H^2} \\ \theta_L^{S^2} & -\theta_L^{H^2} & 1 \end{pmatrix} \\ V_R^t = \begin{pmatrix} 1 - \theta_R^{S^2} & -\theta_R^{D^2} & -\theta_R^S \\ \theta_R^{D^2} & 1 & -\theta_R^{H^2} \\ \theta_R^S & \theta_R^{H^2} & 1 - \frac{\theta_R^{S^2}}{2} \end{pmatrix}, \quad (38)$$

where the matrices of Eq. 38 are unitary to  $\mathcal{O}(\theta^3)$ .<sup>1</sup> The angles  $\theta^D$  ( $\theta^S$ ) can be thought of as the doublet (singlet) vector fermion-mixing with the Standard Model-like top quark, and  $\theta^H$  as the doublet-singlet vector fermion mixing. All angles are assumed to scale as Eq. 37.

In the small angle limit of Eq. 38, we can then solve for the parameters of the Lagrangian:

$$\mathcal{M}^{(t)}(0) = \begin{pmatrix} \lambda_t \frac{v}{\sqrt{2}} & M_4 & \lambda_7 \frac{v}{\sqrt{2}} \\ \lambda_9 \frac{v}{\sqrt{2}} & M & \lambda_1 \frac{v}{\sqrt{2}} \\ M_5 & \lambda_3 \frac{v}{\sqrt{2}} & M_U \end{pmatrix} \\ = \begin{pmatrix} M_{T_1} \left(1 - \frac{\theta_L^{D^2}}{2} - \frac{\theta_R^{S^2}}{2}\right) & M_{T_2} \theta_L^D - M_{T_1} \theta_R^{D^2} & M_{T_3} \theta_L^{S^2} - M_{T_1} \theta_R^S \\ M_{T_2} \theta_R^{D^2} - M_{T_1} \theta_L^D & M_{T_2} \left(1 - \frac{\theta_L^{D^2}}{2}\right) & -M_{T_3} \theta_L^{H^2} - M_{T_2} \theta_R^{H^2} + M_{T_1} \theta_L^D \theta_R^S \\ M_{T_3} \theta_R^S - M_{T_1} \theta_L^{S^2} & M_{T_2} \theta_L^{H^2} + M_{T_3} \theta_R^{H^2} & M_{T_3} \left(1 - \frac{\theta_R^{S^2}}{2}\right) \end{pmatrix}$$

As can be seen, according to the  $\theta$  scaling behaviour, this obeys the structure that we want ( $\lambda_i v \ll \lambda_{4,5} \ll M$ ). In the fermion mass-eigenstate basis,

$$L \sim - \sum_{i,j=1,2,3} \bar{\psi}_i \mathcal{Y}_{ij}^t \psi_j^t h, \quad (39)$$

the small angle approximation to the charge  $\frac{2}{3}$  Yukawa interactions is,

$$v \times \mathcal{Y}^t = \begin{pmatrix} M_{T_1} & M_{T_1} \theta_R^{D^2} & M_{T_3} \theta_L^{S^2} \\ M_{T_2} \theta_R^{D^2} & 0 & -M_{T_3} \theta_L^{H^2} - M_{T_2} \theta_R^{H^2} \\ M_{T_1} \theta_L^{S^2} & M_{T_2} \theta_L^{H^2} + M_{T_3} \theta_R^{H^2} & 0 \end{pmatrix}. \quad (40)$$

The mass matrix in the  $b$  quark sector can be parameterized in an identical fashion to the above discussion.

The  $W$  interactions defined in Eqs. 13 and 14, in the small angle approximation of Eq. 38, are

$$U_L = \begin{pmatrix} 1 - \frac{1}{2}(\theta_L^{D^b} - \theta_L^{D^t})^2 & \theta_L^{D^b} - \theta_L^{D^t} & \theta_L^{S^{b^2}} \\ \theta_L^{D^t} - \theta_L^{D^b} & 1 - \frac{1}{2}(\theta_L^{D^b} - \theta_L^{D^t})^2 & -\theta_L^{H^{b^2}} \\ \theta_L^{S^{t^2}} & -\theta_L^{H^{t^2}} & 0 \end{pmatrix} \quad (41)$$

$$U_R = \begin{pmatrix} 0 & -\theta_R^{D^{t^2}} & 0 \\ -\theta_R^{D^{b^2}} & 1 & \theta_R^{H^{b^2}} \\ 0 & \theta_R^{H^{t^2}} & 0 \end{pmatrix}, \quad (42)$$

---

<sup>1</sup> Note that the hierarchy determines the leading behavior of the  $\theta$  expansion of the mixing matrices. Higher orders of this expansion are determined by unitarity.

where we have added the superscripts  $b, t$  to indicate mixing in the bottom and top sectors, respectively. The  $Z$ -fermion interactions defined in Eqs. 15 and 16, in the small angle approximation are

$$\begin{aligned} X_L^t &= \begin{pmatrix} 1 & 0 & \theta_L^{St^2} \\ 0 & 1 & -\theta_L^{Ht^2} \\ \theta_L^{St^2} & -\theta_L^{Ht^2} & 0 \end{pmatrix} \\ X_R^t &= \begin{pmatrix} 0 & -\theta_R^{Dt^2} & 0 \\ -\theta_R^{Dt^2} & 1 & \theta_R^{Ht^2} \\ 0 & \theta_R^{Ht^2} & 0 \end{pmatrix}. \end{aligned} \quad (43)$$

The results for the bottom sector can be found by the replacement  $t \rightarrow b$ .

Comparing to the EFT of Eq. 33 in the small angle approximation described above,

$$\begin{aligned} Y_t &= \frac{M_{T_1}}{v} \\ Y_b &= \frac{M_{B_1}}{v} \\ c_{2h}^{(t)} &= c_{2h}^{(b)} = 0 \\ c_g &= -c_{gg} = 0. \end{aligned} \quad (44)$$

In the EFT, this hierarchy reduces to the Standard Model and so does not produce large deviations in Higgs production rates.

## B. Hierarchy 2

Hierarchy 1 appears to give small  $\lambda_7, \lambda_9, \lambda_3, \lambda_8, \lambda_{10}, \lambda_{11}$ , which are the parameters that give deviations from the Standard Model. We now describe a different hierarchy with  $M_{4,5} \ll \lambda_i v \ll M$ :

$$\theta \sim \frac{M_{4,5}}{\lambda_i v} \sim \frac{\lambda_i v}{M} \quad \text{and} \quad \theta^2 \sim \frac{M_{4,5}}{M}. \quad (45)$$

The diagonalization matrices can be parameterized in both the  $t$  sector as<sup>2</sup>,

$$\begin{aligned} V_L^t &= \begin{pmatrix} 1 - \frac{1}{2}\theta_L^{S^2} & -\theta_L^{D^2} & -\theta_L^S \\ \theta_L^{D^2} + \theta_L^H \theta_L^S & 1 - \frac{1}{2}\theta_L^{H^2} & \theta_L^H \\ \theta_L^S & -\theta_L^H & 1 - \frac{1}{2}(\theta_L^{S^2} + \theta_L^{H^2}) \end{pmatrix} \\ V_R^t &= \begin{pmatrix} 1 - \frac{1}{2}\theta_R^{D^2} & -\theta_R^D & -\theta_R^{S^2} \\ \theta_R^D & 1 - \frac{1}{2}(\theta_R^{D^2} + \theta_R^{H^2}) & -\theta_R^H \\ \theta_R^D \theta_R^H + \theta_R^{S^2} & \theta_R^H & 1 - \frac{1}{2}\theta_R^{H^2} \end{pmatrix}. \end{aligned} \quad (46)$$

---

<sup>2</sup> We omit the superscript  $t$  and  $b$  on the mixing angles where it is obvious.

The parameters of the original top mass matrix,  $M^{(t)}(0)$  from Eq. 9, can be solved for to  $\mathcal{O}(\theta^2)$ ,

$$\begin{aligned}
\lambda_t \frac{v}{\sqrt{2}} &= M_{T_1} \left( 1 - \frac{\theta_L^{S^2}}{2} - \frac{\theta_R^{D^2}}{2} \right) \\
M_4 &= M_{T_2} \left( \theta_L^{D^2} + \theta_L^S \theta_L^H \right) + M_{T_3} \theta_L^S \theta_R^H - M_{T_1} \theta_R^D \\
\lambda_7 \frac{v}{\sqrt{2}} &= M_{T_3} \theta_L^S - M_{T_1} \theta_R^{S^2} \\
\lambda_9 \frac{v}{\sqrt{2}} &= M_{T_2} \theta_R^D - M_{T_1} \theta_L^{D^2} \\
M &= M_{T_2} \left[ 1 - \frac{1}{2} \left( \theta_L^{H^2} + \theta_R^{D^2} + \theta_R^{H^2} \right) \right] - M_{T_3} \theta_L^H \theta_R^H \\
\lambda_1 \frac{v}{\sqrt{2}} &= -M_{T_3} \theta_L^H - M_{T_2} \theta_R^H \\
M_5 &= M_{T_3} (\theta_R^D \theta_R^H + \theta_R^{S^2}) + M_{T_2} \theta_R^D \theta_L^H - M_{T_1} \theta_L^S \\
\lambda_3 \frac{v}{\sqrt{2}} &= M_{T_3} \theta_R^H + M_{T_2} \theta_L^H + M_{T_1} \theta_L^S \theta_R^D \\
M_U &= M_{T_3} \left[ 1 - \frac{1}{2} \left( \theta_L^{S^2} + \theta_L^{H^2} + \theta_R^{H^2} \right) \right] - M_{T_2} \theta_L^H \theta_R^H
\end{aligned} \tag{47}$$

Finally, the Higgs couplings to the charge  $\frac{2}{3}$  fermions can be written as in Eq. 39,

$$\begin{aligned}
v \times \mathcal{Y}^t = & \begin{pmatrix} M_{T_1} \left( 1 - \theta_L^{S^2} - \theta_R^{D^2} \right) & M_{T_1} \theta_R^D - M_{T_2} \theta_L^H \theta_L^S - 2M_{T_3} \theta_L^S \theta_R^H & M_{T_1} \theta_R^D \theta_R^H + M_{T_3} \theta_L^S \\ M_{T_1} \theta_L^H \theta_L^S + M_{T_2} \theta_R^D & M_{T_2} \left( \theta_L^{H^2} + \theta_R^{D^2} + \theta_R^{H^2} \right) + 2M_{T_3} \theta_L^H \theta_R^H & -M_{T_3} \theta_L^H - M_{T_2} \theta_R^H \\ M_{T_1} \theta_L^S - 2M_{T_2} \theta_L^H \theta_R^D - M_{T_3} \theta_R^D \theta_R^H & 2M_{T_1} \theta_L^S \theta_R^D + M_{T_2} \theta_L^H + M_{T_3} \theta_R^H & M_{T_3} \left( \theta_L^{H^2} + \theta_L^{S^2} + \theta_R^{H^2} \right) + 2M_{T_2} \theta_L^H \theta_R^H \end{pmatrix}
\end{aligned} \tag{48}$$

Again, the  $b$  sector mass matrix and mixing can be parameterized in a similar fashion to the above.

Comparing to the EFT in Eq. 33 (counting  $M_{T_1}/M_{T_{2,3}} \sim M_{B_1}/M_{B_{2,3}} \sim \theta$ ), this hierarchy yields small deviations from the Standard Model,

$$\begin{aligned}
Y_t &= \frac{M_{T_1}}{v} \left( 1 - \theta_R^{Dt^2} - \theta_L^{St^2} \right) \\
Y_b &= \frac{M_{B_1}}{v} \left( 1 - \theta_R^{Db^2} - \theta_L^{Sb^2} \right) \\
c_{2h}^{(t)} &= \frac{3M_{T_1}}{2v^2} \left( \theta_R^{Dt^2} + \theta_L^{St^2} \right) \\
c_{2h}^{(b)} &= \frac{3M_{B_1}}{2v^2} \left( \theta_R^{Db^2} + \theta_L^{Sb^2} \right) \\
c_g = -c_{gg} &= (2\theta_L^{Ht^2} + \theta_L^{St^2}) + (2\theta_R^{Ht^2} + \theta_R^{Dt^2}) + 2 \frac{M_{T_2}^2 + M_{T_3}^2}{M_{T_2} M_{T_3}} \theta_L^{Ht} \theta_R^{Ht} \\
&\quad + (2\theta_L^{Hb^2} + \theta_L^{Sb^2}) + (2\theta_R^{Hb^2} + \theta_R^{Db^2}) + 2 \frac{M_{B_2}^2 + M_{B_3}^2}{M_{B_2} M_{B_3}} \theta_L^{Hb} \theta_R^{Hb}.
\end{aligned} \tag{49}$$

Again, the superscripts  $t, b$  indicate mixing angles in the top and bottom sectors, respectively. Now we want to match onto the LET, i.e, integrate out the top quark ( $T_1$ ), along with the heavier fermions  $T_2, T_3, B_2$ , and  $B_3$ . The effective Higgs-gluon interactions are

$$\mathcal{L}_{\text{LET}} = \frac{\alpha_s}{12\pi} \left[ \left( 1 + c_g^{\text{LET}} \right) \frac{h}{v} - \frac{1 + c_{gg}^{\text{LET}}}{2} \frac{h^2}{v^2} \right] G^{A,\mu\nu} G_{\mu\nu}^A. \tag{50}$$

To obtain  $c^{\text{LET}}$  we use the full LET for the top quark sector in Eqs. 24 and 28, then add in the effect of integrating out the heavy bottom quark partners, that is, the heavy down-type quark contributions to  $c_g$  and  $c_{gg}$  in Eq. 35. This

yields to  $\mathcal{O}(\theta^2)$ ,

$$\begin{aligned} c_g^{LET} &= -c_{gg}^{LET} \\ &= 2 \left( \theta_L^{Hb^2} + \theta_L^{Ht^2} + \theta_R^{Hb^2} + \theta_R^{Ht^2} \right) + \theta_L^{Sb^2} + \theta_R^{Db^2} + 2 \frac{M_{T_2}^2 + M_{T_3}^2}{M_{T_2} M_{T_3}} \theta_L^{Ht} \theta_R^{Ht} + 2 \frac{M_{B_2}^2 + M_{B_3}^2}{M_{B_2} M_{B_3}} \theta_L^{Hb} \theta_R^{Hb}. \end{aligned} \quad (51)$$

For degenerate heavy fermions,  $c_g$  is positive definite and so the contribution to double Higgs production from  $c_{gg}$  always decreases the rate. Additionally, to increase the double Higgs contribution from  $c_{gg}$ ,  $\theta_L^H$  and  $\theta_R^H$  should have opposite signs.

The mixing matrices for the  $W$  interactions are (Eqs. 13 and 14).

$$\begin{aligned} U_L &= \begin{pmatrix} 1 - \frac{1}{2} (\theta_L^{Sb^2} + \theta_L^{St^2}) & \theta_L^{Db^2} - \theta_L^{Dt^2} + \theta_L^{Hb} \theta_L^{Sb} & \theta_L^{Sb} \\ \theta_L^{Dt^2} - \theta_L^{Db^2} + \theta_L^{Ht} \theta_L^{St} & 1 - \frac{1}{2} (\theta_L^{Hb^2} + \theta_L^{Ht^2}) & -\theta_L^{Hb} \\ \theta_L^{St} & -\theta_L^{Ht} & \theta_L^{Hb} \theta_L^{Ht} + \theta_L^{Sb} \theta_L^{St} \end{pmatrix} \\ U_R &= \begin{pmatrix} \theta_R^{Db} \theta_R^{Dt} & -\theta_R^{Dt} & -\theta_R^{Hb} \theta_R^{Dt} \\ -\theta_R^{Db} & 1 - \frac{1}{2} (\theta_R^{Db^2} + \theta_R^{Hb^2} + \theta_R^{Dt^2} + \theta_R^{Ht^2}) & \theta_R^{Hb} \\ -\theta_R^{Db} \theta_R^{Ht} & \theta_R^{Ht} & \theta_R^{Hb} \theta_R^{Ht} \end{pmatrix} \end{aligned} \quad (52)$$

The mixing matrices for  $Z$  interactions (Eqs. 15 and 16) in the charge  $\frac{2}{3}$  sector are,

$$\begin{aligned} X_L^t &= \begin{pmatrix} 1 - \theta_L^{St^2} & \theta_L^{Ht} \theta_L^{St} & \theta_L^{St} \\ \theta_L^{Ht} \theta_L^{St} & 1 - \theta_L^{Ht^2} & -\theta_L^{Ht} \\ \theta_L^{St} & -\theta_L^{Ht} & \theta_L^{Ht^2} + \theta_L^{St^2} \end{pmatrix} \\ X_R^t &= \begin{pmatrix} \theta_R^{Dt^2} & -\theta_R^{Dt} & -\theta_R^{Ht^2} \\ -\theta_R^{Dt} & 1 - \theta_R^{Dt^2} - \theta_R^{Ht^2} & \theta_R^{Ht} \\ -\theta_R^{Dt} \theta_R^{Ht} & \theta_R^{Ht} & \theta_R^{Ht^2} \end{pmatrix} \end{aligned} \quad (53)$$

The  $Z$  couplings in the bottom sector are found from Eq. 53 with the replacement  $t \rightarrow b$ .

## V. LIMITS FROM PRECISION MEASUREMENTS

New heavy quarks which couple to the Standard Model gauge bosons are restricted by the oblique parameters [38]. In addition, the couplings of charge  $-\frac{1}{3}$  quarks are significantly limited by the measurements of  $Z \rightarrow b\bar{b}$ . These limits typically require small mixing parameters. .

General formulas for the contributions of the fermion sector to  $\Delta S$  and  $\Delta T$  are given in Appendix A. It is useful to consider several special cases here. For the case with only a top partner singlet ( $T_3$ ) with a mass  $M_{T_3} \gg M_{T_1}$ , the only non-zero entries of the left-handed mixing matrices are,

$$\begin{aligned} V_{L,11}^t &= V_{L,33}^t = c_L \\ V_{L,31}^t &= -V_{L,13}^t = -s_L \\ V_{L,11}^b &= 1, \end{aligned} \quad (54)$$

while  $V_R^{t,b}$  can be set to the unit matrix,  $c_L \equiv \cos \theta_L$ ,  $s_L \equiv \sin \theta_L$ , and

$$\tan(2\theta_L) = \frac{\sqrt{2}v}{M_U} \left( \frac{\lambda_7}{1 - (\lambda_7^2 + \lambda_t^2) \frac{v^2}{2M_U^2}} \right) \quad (55)$$

The result for large top partner masses is (after subtracting the Standard Model top and bottom contributions),

$$\begin{aligned} \left[ \Delta T \right]_{top \text{ singlet}} &= \frac{N_c}{16\pi s_W^2 M_W^2} s_L^2 \left( -(1 + c_L^2) M_{T_1}^2 - 2c_L^2 M_{T_1}^2 \ln \left( \frac{M_{T_1}^2}{M_{T_3}^2} \right) + s_L^2 M_{T_3}^2 \right) \\ \left[ \Delta S \right]_{top \text{ singlet}} &= -\frac{N_c}{18\pi} s_L^2 \left( 5c_L^2 + (1 - 3c_L^2) \ln \left( \frac{M_{T_1}^2}{M_{T_3}^2} \right) \right), \end{aligned} \quad (56)$$

where  $N_C = 3$ , in agreement with Ref. [15], which found that fits to the oblique parameters require  $s_L \lesssim 0.16$  for  $M_{T_3} \sim 1$  TeV at 95% confidence level. For fixed values of the Yukawa couplings,  $\lambda_i$ , the mixing angle scales for large  $M_{T_3}$  as,

$$s_L \sim \frac{v\lambda_i}{M_{T_3}} \quad (57)$$

and the contributions to the oblique parameters from the top partner decouple,

$$\left[ \Delta T \right]_{top \text{ singlet}} \sim \left[ \Delta S \right]_{top \text{ singlet}} \sim \frac{\lambda_i^2 v^2}{M_{T_3}^2}. \quad (58)$$

The limit on the angle  $s_L$  in the above example arises because of the mixing with the Standard Model top quark. Ref. [15] contains an example where there is a heavy vector-like  $SU(2)_L$  doublet,  $Q$ , along with vector-like charge  $\frac{2}{3}$  and  $-\frac{1}{3}$  quarks,  $U$  and  $D$ , which are not allowed to mix with the Standard Model fermions. This corresponds to  $M_4 = M_5 = M_6 = \lambda_7 = \lambda_8 = \lambda_9 = \lambda_{10} = 0$  in Eq. 7. In this case, limits from the oblique parameters require that the heavy fermions be approximately degenerate,  $M_{T_2} \simeq M_{T_3} \simeq M_{B_2} \simeq M_{B_3}$ , while one combination of mixing angles is unconstrained.

Limits can be also obtained from  $Z$  decays to  $b\bar{b}$  by comparing the experimental result[39] for  $R_b$  with the recent Standard Model calculation[40],

$$\begin{aligned} R_b &\equiv \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow b\bar{b})_{SM}} \\ R_b^{exp} &= 0.21629 \pm 0.00066 \\ R_b^{SM} &= 0.2154940. \end{aligned} \quad (59)$$

$R_b$  can be related to the anomalous couplings of the  $b$  quark to the  $Z$  given in Eq. 33

$$\frac{R_b^{exp}}{R_b^{SM}} = 1 - 3.57\delta g_L^b + 0.65g_R^b. \quad (60)$$

From Eqs. 36 and 60, we extract the 95% confidence level bound,

$$\left( \frac{M}{\lambda_8} \right)^2 \left( \frac{1}{1 + 0.224(1 - \frac{\lambda_{10}M}{\lambda_8 M_D})^2} \right) \gtrsim (2 \text{ TeV})^2. \quad (61)$$

The following discussion focuses on Hierarchy 2 of Section IV, although it can be shown that the conclusions are quite generic. We start by counting the degrees of freedom. Naively, there are 6 masses,

$$M_{T_1}, M_{T_2}, M_{T_3}, M_{B_1}, M_{B_2}, M_{B_3} \quad (62)$$

and 12 angles,

$$\theta_{L,R}^{St}, \theta_{L,R}^{Dt}, \theta_{L,R}^{Ht}, \theta_{L,R}^{Sb}, \theta_{L,R}^{Db}, \theta_{L,R}^{Hb}. \quad (63)$$

However,  $M_4$  and  $M$  are the same in the top and bottom sectors, leaving a total of 16 independent parameters. Considering Eqs. 44 and 53, we see that if we forbid mixing between particles with different quantum numbers then FCNCs involving the  $Z$  are eliminated. That is,  $\theta_L^{St}$  mixes a component of the Standard Model  $SU(2)_L$  doublet with an  $SU(2)_L$  singlet, and  $\theta_R^{Dt}$  mixes a Standard Model  $SU(2)_L$  singlet with a component of a vector fermion doublet. We set these angles to zero to avoid restrictions from deviations in the  $3^{rd}$  generation quark neutral current couplings, in particular  $Z \rightarrow b\bar{b}$ :

$$\theta_L^{St} = \theta_R^{Dt} = \theta_L^{Sb} = \theta_R^{Db} = 0. \quad (64)$$

The angles  $\theta_{L,R}^{Ht}$  and  $\theta_{L,R}^{Hb}$  are left nonzero, since from Eq. 51 we see that these are intimately tied to deviations from Standard Model Higgs production rates. The  $Z$  couplings to the top quark and heavy up-type vector quarks are then

$$\begin{aligned} X_L^t &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \theta_L^{Ht^2} & -\theta_L^{Ht} \\ 0 & -\theta_L^{Ht} & \theta_L^{Ht^2} + \theta_L^{St^2} \end{pmatrix} \\ X_R^t &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - \theta_R^{Ht^2} & \theta_R^{Ht} \\ 0 & \theta_R^{Ht} & \theta_R^{Ht^2} \end{pmatrix}, \end{aligned} \quad (65)$$

and the  $t$  and  $b$  quarks have Standard Model-like neutral current couplings.

The  $W$ -mixing matrices in Hierarchy 2 are,

$$U_L = \begin{pmatrix} 1 & \theta_L^{D_{b^2}} - \theta_L^{D_{t^2}} & 0 \\ \theta_L^{D_{t^2}} - \theta_L^{D_{b^2}} & 1 - \frac{1}{2}(\theta_L^{H_{b^2}} + \theta_L^{H_{t^2}}) & -\theta_L^{H_{b^2}} \\ 0 & -\theta_L^{H_{t^2}} & \theta_L^{H_{b^2}}\theta_L^{H_{t^2}} \end{pmatrix} \quad (66)$$

$$U_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 - \frac{1}{2}(\theta_R^{H_{b^2}} + \theta_R^{H_{t^2}}) & \theta_R^{H_{b^2}} \\ 0 & \theta_R^{H_{t^2}} & \theta_R^{H_{b^2}}\theta_R^{H_{t^2}} \end{pmatrix}.$$

$U_R$  only depends on  $\theta_{L,R}^{H_{t^2}}$  and  $\theta_{L,R}^{H_{b^2}}$ , the mixing angles between the heavy vector fermions, while  $U_L$  still depends on the mixing between the heavy states with the Standard Model. Forcing the heavy-light mixing to be isospin conserving,  $\theta_L^{D_{b^2}} = \theta_L^{D_{t^2}}$ ,  $U_L$  becomes

$$U_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \frac{1}{2}(\theta_L^{H_{b^2}} + \theta_L^{H_{t^2}}) & -\theta_L^{H_{b^2}} \\ 0 & -\theta_L^{H_{t^2}} & \theta_L^{H_{b^2}}\theta_L^{H_{t^2}} \end{pmatrix} \quad (67)$$

and there are no gauge boson currents mixing the Standard Model top and bottom quarks with the new vector fermions.

To summarize, taking into consideration electroweak precision observables, it is reasonable to impose the constraints:

$$\theta_L^{S_{t^2}} = \theta_R^{D_{t^2}} = \theta_L^{S_{b^2}} = \theta_R^{D_{b^2}} = 0, \quad \theta_L^{D_{b^2}} = \theta_L^{D_{t^2}}. \quad (68)$$

Under this assumption, the non-zero mixing angles are,

$$\theta_R^{S_{t^2}}, \theta_L^{D_{t^2}}, \theta_{L,R}^{H_{t^2}}, \theta_R^{S_{b^2}}, \theta_{L,R}^{H_{b^2}}, \quad (69)$$

There are 2 constraints from  $M_4$  and  $M$ ,

$$\begin{aligned} M_4 &= M_{T_2}\theta_L^{D_{t^2}^2} = M_{B_2}\theta_L^{D_{b^2}^2} \\ M &= M_{T_2}\left(1 - \frac{1}{2}\theta_L^{H_{t^2}^2} - \frac{1}{2}\theta_R^{H_{t^2}^2}\right) - M_{T_3}\theta_L^{H_{t^2}}\theta_R^{H_{t^2}} \\ &= M_{B_2}\left(1 - \frac{1}{2}\theta_L^{H_{b^2}^2} - \frac{1}{2}\theta_R^{H_{b^2}^2}\right) - M_{B_3}\theta_L^{H_{b^2}}\theta_R^{H_{b^2}}. \end{aligned} \quad (70)$$

So,  $\theta_L^{D_{t^2}} = \theta_L^{D_{b^2}}$  is only consistent if  $M_{T_2} = M_{B_2}$ , which fully eliminates isospin violation in the mixing between the new heavy states and the  $3^{rd}$  generation quarks. To make things simpler, we can also assume  $M_{T_3} = M_{B_3}$ , and then Eq. 70 is satisfied when  $\theta_L^{H_{t^2}} = \theta_L^{H_{b^2}}$  and  $\theta_R^{H_{t^2}} = \theta_R^{H_{b^2}}$ . (There are other possible solutions not requiring  $M_{T_3} = M_{B_3}$ , but for simplicity we focus on this limit.)

Now we only have a few remaining degrees of freedom: 4 masses (2 of which are known)

$$M_{T_1}, M_{B_1}, M_{T_2} = M_{B_2}, M_{T_3} = M_{B_3} \quad (71)$$

and five angles,

$$\theta_R^{S_{t^2}}, \theta_R^{S_{b^2}}, \theta_L^{D_{t^2}} = \theta_L^{D_{b^2}}, \theta_L^{H_{t^2}} = \theta_L^{H_{b^2}}, \theta_R^{H_{t^2}} = \theta_R^{H_{b^2}}. \quad (72)$$

At lowest order these angles are unconstrained by  $Z \rightarrow b\bar{b}$  and the oblique parameters only constrain the mixing among the heavy quarks. These constraints can be found in Ref. [15]. Although this result can be shown generically without assuming that  $\theta_L^D$  and  $\theta_R^S$  are small, these angles will manifest themselves in the CKM matrix when considering mixing among the first three generations [20]. We therefore continue with the small angle approximation.

## VI. RESULTS FOR HIGGS PRODUCTION

In this section, we compare the accuracy of the low energy theorem (LET) with the effective Lagrangian obtained by including the top and bottom quark mass effects (EFT), Eq. 33, as well as with predictions obtained using the

full theory. We have two goals: the first is to understand the numerical limitations of the approximations to the full theory. Our second goal is to search for a regime where single Higgs production from gluon fusion occurs at approximately the Standard Model rate, while double Higgs production is significantly altered. Again, we focus on Hierarchy 2 of Section IV B, since Hierarchy 1 (Section IV A) does not lead to significant deviations from the Standard Model (Eq. 44).

We normalize the predictions to the Standard Model rates,

$$\begin{aligned} R_h &\equiv \frac{\sigma(gg \rightarrow h)}{\sigma(gg \rightarrow h)_{SM}} \\ R_{hh} &= \frac{\sigma(gg \rightarrow hh)}{\sigma(gg \rightarrow hh)_{SM}}. \end{aligned} \quad (73)$$

The low energy theorems of Eqs. 19 and 27, including only the up-type quarks, predict to  $\mathcal{O}(\delta_{LET})$ ,

$$\begin{aligned} R_h &\sim 1 + 2\delta_{LET} \\ R_{hh} &\sim 1 + 2\delta_{LET} - \frac{4\delta_{LET}}{F_0^{SM}(M_{T_1} \rightarrow \infty)}, \end{aligned} \quad (74)$$

and

$$F_0^{SM}(M_{T_1} \rightarrow \infty) \equiv 1 - \frac{3M_h^2}{s - M_h^2}, \quad (75)$$

where  $\delta_{LET} = 2\lambda_3 v^2(\lambda_1 \lambda_t - \lambda_7 \lambda_9)/X$  is given in Eq. 19 and  $F_0$  is defined in Eqs. 79, 83, 84. In the effective field theory language of Eq. 33,  $\delta_{LET} = c_g$ . The presence of the  $\lambda_3$  coupling does indeed allow single Higgs production to differ from the Standard Model prediction. However, once  $R_h$  is measured to be approximately 1, the deviations of  $R_{hh}$  from 1 are restricted to be small. Thus in order for the double Higgs rate to be different from the Standard Model prediction, we need a region of parameter space where the low energy theorem is not valid.

The rate for single Higgs production in the effective theory including all top and bottom quark mass effects (EFT), but integrating out the heavy vector-like fermions to  $\mathcal{O}\left(\frac{1}{M_X^2}\right)$  and assuming  $\delta_b, \delta_t$  and  $c_g$  are small, is given by,

$$\begin{aligned} R_h &\rightarrow \frac{|(1 + \delta_t)F_{1/2}(\tau_{T_1}) + (1 + \delta_b)F_{1/2}(\tau_{B_1}) + c_g F_{1/2}^\infty|^2}{|F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1})|^2} \\ &\sim 1 + 2 \left[ \frac{\delta_t |F_{1/2}(\tau_{T_1})|^2 + \delta_b |F_{1/2}(\tau_{B_1})|^2 + (\delta_t + \delta_b) \text{Re}\left(F_{1/2}(\tau_{T_1})F_{1/2}^*(\tau_{B_1})\right)}{|F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1})|^2} \right] \\ &\quad + 2 \left[ \frac{c_g F_{1/2}^\infty \text{Re}\left(F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1})\right)}{|F_{1/2}(\tau_{T_1}) + F_{1/2}(\tau_{B_1})|^2} \right], \end{aligned} \quad (76)$$

where  $\tau_i \equiv 4M_i^2/M_h^2$ ,

$$\begin{aligned} F_{1/2}(\tau) &= -2\tau[1 + (1 - \tau)f(\tau)] \\ f(\tau) &= \begin{cases} \left[\sin^{-1}\left(\frac{1}{\sqrt{\tau}}\right)\right]^2 & \text{if } \tau \geq 1 \\ -\frac{1}{4}\left[\ln\left(\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}}\right) - i\pi\right]^2 & \text{if } \tau < 1 \end{cases}, \end{aligned} \quad (77)$$

and  $F_{1/2}^\infty = -\frac{4}{3}$  in the  $M_{T_1} \rightarrow \infty$  limit of  $F_{1/2}(\tau_{T_1})$ . Neglecting the  $b$  contribution and noting that  $F_{1/2}(\tau_{T_1})$  is well approximated by  $F_{1/2}^\infty$ ,

$$R_h \sim 1 + 2(\delta_t + c_g). \quad (78)$$

The  $c_g$  contribution is in agreement with the LET result of Eq. 74.



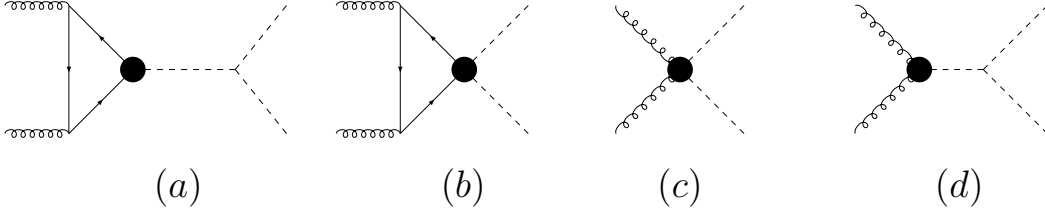


FIG. 2: Non-box contributions to the spin-0 component of  $gg \rightarrow hh$ . The dark circles represent the non-Standard Model contributions, while the solid lines are either  $t$ - or  $b$  quarks.

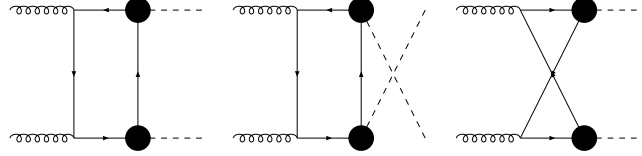


FIG. 3: Box contributions to  $gg \rightarrow hh$ . The dark circles represent the non-Standard Model contributions, while the solid lines are either  $t$ - or  $b$  quarks. The crossed diagrams from the initial state are not shown.

Double Higgs production can be analyzed in a similar fashion. The diagrams shown in Fig. 2 contribute only to the spin-0 projection, while the box diagrams shown in Fig 3 have both spin-0 and spin-2 components. The amplitude for  $g^{A,\mu}(p_1)g^{B,\nu}(p_2) \rightarrow h(p_3)h(p_4)$  is

$$A_{AB}^{\mu\nu} = \frac{\alpha_s}{3\pi v^2} \delta_{AB} \sum_i \left[ P_1^{\mu\nu}(p_1, p_2) F_0^i(s, t, u, M_j) + P_2^{\mu\nu}(p_1, p_2, p_3) F_2^i(s, t, u, M_j) \right], \quad (79)$$

where the sum is over the diagrams,  $M_j$  denotes all relevant quark masses,  $P_1$  and  $P_2$  are the orthogonal projectors onto the spin-0 and spin-2 states respectively,

$$\begin{aligned} P_1^{\mu\nu}(p_1, p_2) &= p_1 \cdot p_2 g^{\mu\nu} - p_1^\nu p_2^\mu, \\ P_2^{\mu\nu}(p_1, p_2, p_3) &= p_1 \cdot p_2 g^{\mu\nu} + \frac{1}{p_T^2} (M_h^2 p_1^\nu p_2^\mu - 2p_1 \cdot p_3 p_2^\mu p_3^\nu - 2p_2 \cdot p_3 p_1^\nu p_3^\mu + s p_3^\mu p_3^\nu), \end{aligned} \quad (80)$$

$s, t$ , and  $u$  are the partonic Mandelstam variables,

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_2 - p_3)^2, \quad (81)$$

and  $p_T$  is the transverse momentum of the Higgs particle,

$$p_T^2 = \frac{ut - M_h^4}{s}. \quad (82)$$

The individual contributions from the diagrams of Figs. 2 and 3 to  $\mathcal{O}(\frac{1}{M_X^2})$  are:

$$\begin{aligned} F_0^{(a)} &= \frac{9M_h^2}{4(s - M_h^2)} \left[ (1 + \delta_t) F_{1/2} \left( \frac{4M_{T_1}^2}{s} \right) + (1 + \delta_b) F_{1/2} \left( \frac{4M_{B_1}^2}{s} \right) \right] \\ F_0^{(b)} &= \frac{9}{4} \delta_t F_{1/2} \left( \frac{4M_{T_1}^2}{s} \right) + \frac{9}{4} \delta_b F_{1/2} \left( \frac{4M_{B_1}^2}{s} \right) \\ F_0^{(c)} &= -c_{gg} \\ F_0^{(d)} &= -c_g \frac{3M_h^2}{s - M_h^2} \\ F_0^{(box)} &= (1 + 2\delta_t) F_0^{(box, SM)}(s, t, u, M_{T_1}) + (1 + 2\delta_b) F_0^{(box, SM)}(s, t, u, M_{B_1}) \end{aligned} \quad (83)$$

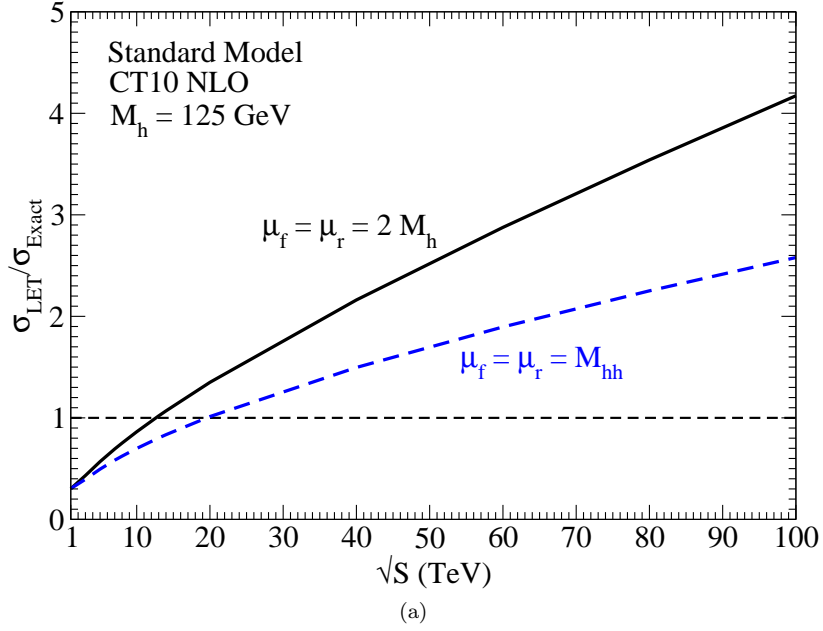


FIG. 4: Standard Model rate for  $pp \rightarrow hh$  from gluon fusion using the LET of Eq. 74 normalized to the exact cross section. This plot uses CT10NLO PDFs.

where  $F_0^{box,SM}(s, t, u, M_{T_1}) \rightarrow 1$  for  $M_{T_1} \rightarrow \infty$  and  $F_0^{(box,SM)}(s, t, u, M_j)$  contains the 6 box diagrams with a fermion of mass  $M_j$  in the loop. Analytic results can be found in Refs. [41, 42]<sup>3</sup>. In the effective theory, the spin-0 contribution is,

$$\begin{aligned} F_0 &= F_0^{(a)} + F_0^{(b)} + F_0^{(c)} + F_0^{(b)} + F_0^{(box)} \\ &\rightarrow \left[ 1 - \delta_t - c_{gg} \right] - \frac{3M_h^2}{s - M_h^2} \left[ 1 + \delta_t + c_g \right] \end{aligned} \quad (84)$$

where the  $2^{nd}$  line is found in the limit  $M_{T_1}^2 \gg s$  and neglects the  $b$  contribution. Taking  $c_{gg} = -c_g$ ,

$$F_0 \rightarrow \left[ 1 + \delta_t + c_g \right] F_0^{SM}(M_{T_1} \rightarrow \infty) - 2(c_g + \delta_t). \quad (85)$$

The  $c_g$  contribution is in agreement with the LET result of Eq. 74, while the  $\delta_t$  contribution is no longer proportion to the Standard Model result.

The LET prediction for the total cross section for double Higgs production in the Standard Model normalized to the exact result is given in Fig. 4 as a function of center-of-mass energy. At  $\sqrt{S} = 13$  TeV, the LET is a reasonable approximation to the total rate, while at higher energies the deviation from the exact result becomes large. We show this for two choices of factorization and renormalization scales,  $\mu_f = \mu_r = 2M_h$  (solid) and  $\mu_f = \mu_r = M_{hh}$  (dashed). The size of the deviation between the LET and exact calculation is very sensitive to the scale choices.

The divergence of the LET from the exact result can be understood by examining the partonic cross section for  $gg \rightarrow hh$  shown in Fig. 5. For partonic sub-energies above around 1 TeV, the LET and the exact results increasingly differ. The LET contains terms  $\sim \frac{M_{hh}^2}{M_{T_1}^2}$ , which are not present in the exact result.

The first hierarchy of small angles of Section IV reduces to the Standard Model, so we do not expect to gain insight from examining this limit. The second hierarchy, (Section IV B), however, is more interesting. In Figs. 6 and 7 we show the total cross sections for  $gg \rightarrow hh$  at  $\sqrt{S} = 13$  TeV and 100 TeV as a function of the lightest top partner mass,  $M_{T_2}$ , for a specific choice of small angles using the parameterization of Eq. 46. The LET significantly overestimates the rate at  $\sqrt{S} = 100$  TeV, but is a reasonable approximation at  $\sqrt{S} = 13$  TeV. The EFT, which contains the top and

<sup>3</sup> Our normalization is  $\frac{3}{4}$  times that of Ref. [41] for the boxes.

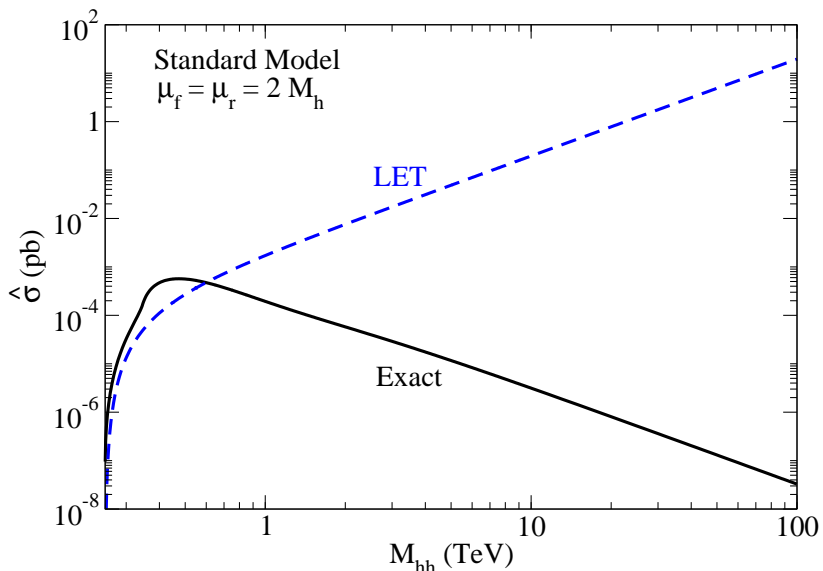


FIG. 5: Standard model partonic cross section for  $gg \rightarrow hh$ .

bottom quark contributions exactly, agrees within a few percent with the exact calculation. From Eqs. 49 and 51, we see that the EFT and LET depend on differences between the heavy vector-like quark masses and not the overall mass scale. This result is confirmed in Figs. 6 and 7, which show all the results are insensitive to the heavy quark mass scale.

It is well known that the LET does not accurately reproduce distributions for double Higgs production[15, 18, 43]. For a choice of small angles and heavy quark masses, we show the invariant mass distribution of the Higgs bosons,  $\frac{d\sigma}{dM_{hh}}$ , in Figs. 8, 9, 10, and 11 at the LHC with  $\sqrt{S} = 13$  and 100 TeV. We include the Standard Model distributions for comparison. The LET does a poor job of reproducing the exact distributions, both in the Standard Model and in the top partner model. The curves labelled “SM” and “Full Theory” contain the exact one-loop calculations for the Standard Model and top partner model respectively, while the curve labelled “Top EFT” is the top partner model calculation using the results of Eq. 33. The EFT reproduces the exact calculation quite accurately. We show this for two parameter points to illustrate the robustness of this conclusion. Both points reproduce the Standard Model single Higgs production rate to within  $\sim 10\%$ . In a given model, therefore, the EFT can be used not only for the total rate, but also for distributions. The distributions in the top partner model are quite similar to the Standard Model. Scanning over small angles, we were not able to find an example with a large deviation from the Standard Model.

## VII. CONCLUSIONS

We considered a scenario with both  $SU(2)_L$  singlet and doublet vector-like fermions. Such a scenario could in principle have large deviations from the Standard Model predictions for single and double Higgs production. However, we were unable to find parameters consistent with electroweak precision measurements and the single Higgs production rate which gave a significant deviation from the Standard Model prediction for double Higgs production.

We constructed two versions of an effective theory. The well known low energy theorem (LET) treats all fermions as infinitely massive. The total cross section for Higgs pair production is well approximated by the LET at  $\sqrt{S} = 13$  TeV, but increasingly differs at higher energies. The LET cannot reproduce the invariant mass distribution of the  $hh$  pairs. In order to include top quark mass effects, we derived an effective Lagrangian (EFT) containing only light fermions, but with non-Standard Model coefficients, which we computed to  $\mathcal{O}(\frac{1}{M_{\tilde{\chi}}^2})$ . The EFT obtains accurate results for both total and differential double Higgs rates. Our results can be used to reliably compute the leading effects of models with heavy vector-like fermions.

An important result is the observation that the coefficients of the effective Lagrangian of Eq. 33 are not free parameters, but are related to each other in any consistent model. Despite the proliferation of Yukawa couplings in Eq. 9, a consistent treatment yields an effective Lagrangian which depends on only 3 parameters,  $\delta_b$ ,  $\delta_t$ , and  $c_g$ . This is similar to the case in composite Higgs models where deviations in Yukawa couplings and new effective operators relevant for double Higgs production are tightly correlated [21]. Hence, we expect the EFT used to study Higgs

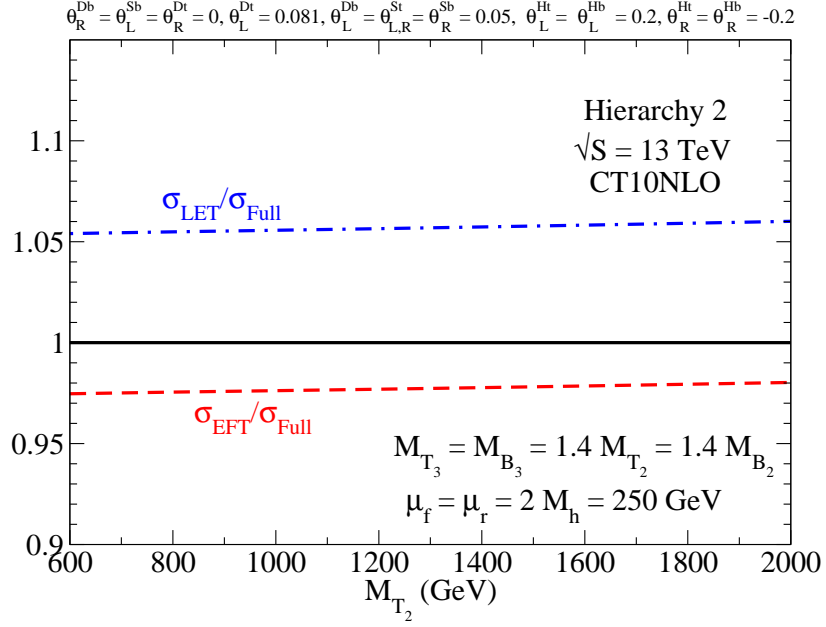


FIG. 6: Total cross section for  $pp \rightarrow hh$  for a choice of small angles using the hierarchy of Section IV B. The EFT and LET results are normalized to the exact one-loop calculation.

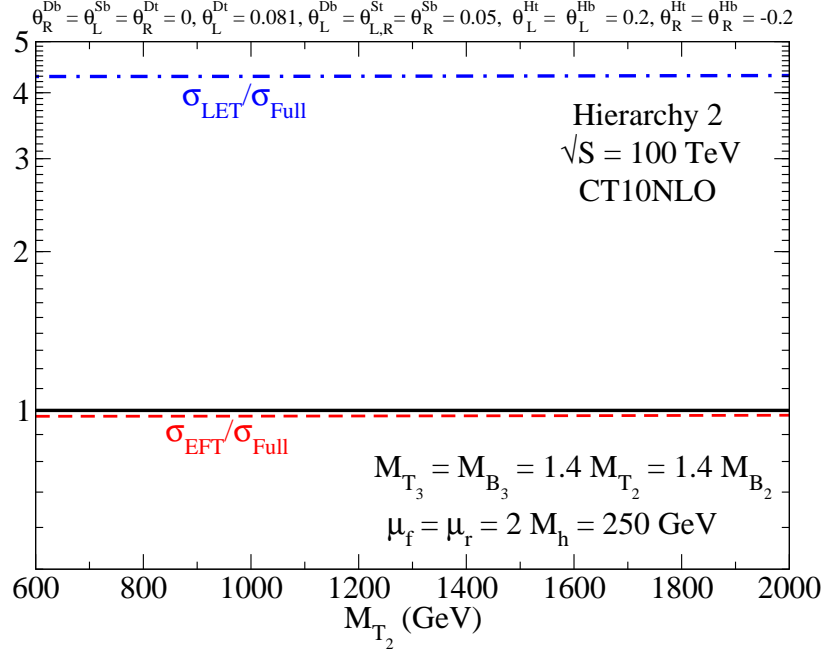


FIG. 7: Same as Fig. 6, except  $\sqrt{S} = 100 \text{ TeV}$ .

production in composite Higgs models to be a very good approximation to a complete calculation.

#### Acknowledgements

This work is supported by the U.S. Department of Energy under grant No. DE-AC02-98CH10886.

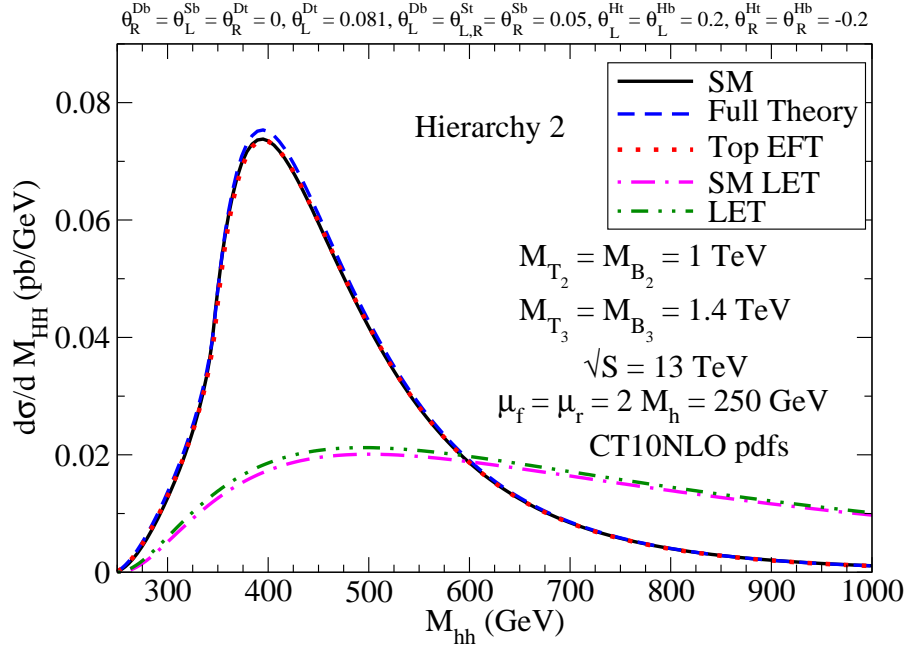


FIG. 8: Invariant mass distributions for  $pp \rightarrow hh$  at the LHC. The SM and SM LET curves represent the exact Standard Model calculation, along with the LET limit. The curves labelled Full Theory, Top EFT, and LET are the top partner model in the small angle hierarchy of Section IV B, using the exact one-loop calculation, the EFT of Eq. 83, and the LET of Eq. 74.

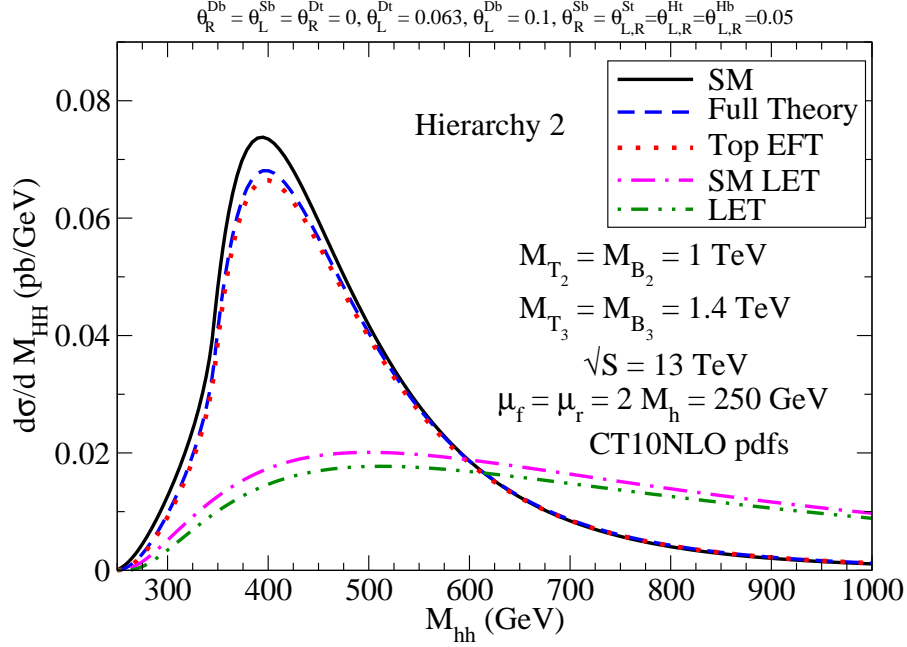
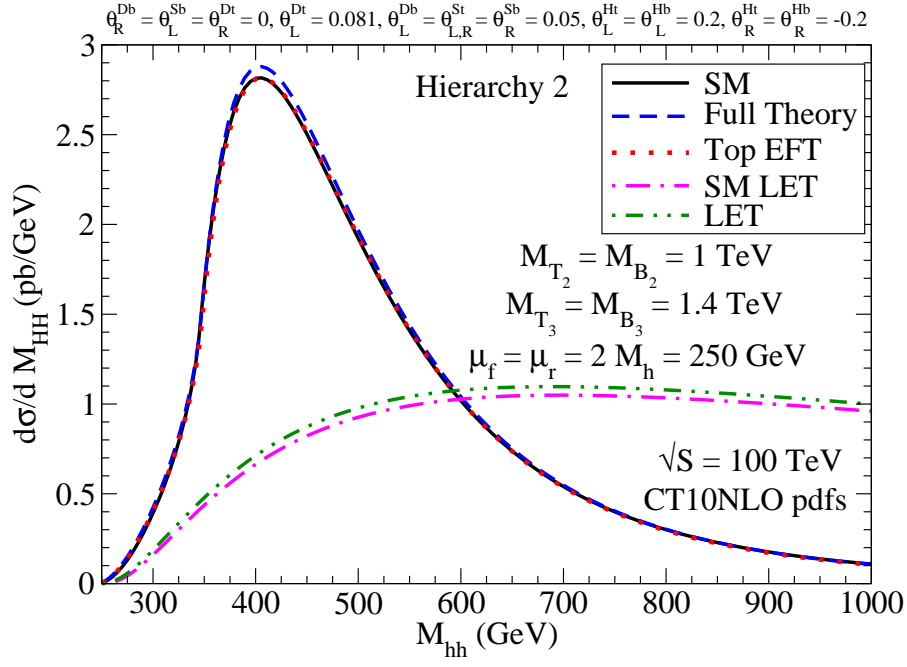
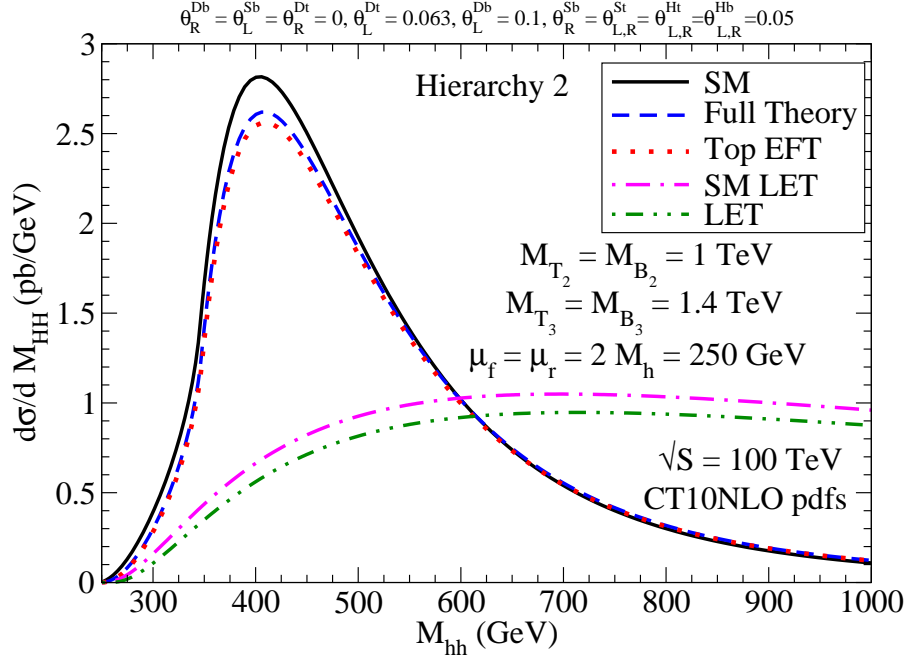


FIG. 9: Same as Fig. 8 with a different parameter point.

FIG. 10: Same as Fig. 8 with  $\sqrt{S} = 100 \text{ TeV}$ .FIG. 11: Same as Fig. 9 with  $\sqrt{S} = 100 \text{ TeV}$ .

### Appendix A: Oblique Parameters

The limits on the parameters of the fermion sector arising from contributions to gauge boson 2-point functions can be studied using the  $S, T$  and  $U$  functions following the notation of Peskin and Takeuchi[38],

$$\begin{aligned}\alpha S &= \left( \frac{4s_W^2 c_W^2}{M_Z^2} \right) \left\{ \Pi_{ZZ}(M_Z^2) - \Pi_{ZZ}(0) - \Pi_{\gamma\gamma}(M_Z^2) - \frac{c_W^2 - s_W^2}{c_W s_W} \Pi_{\gamma Z}(M_Z^2) \right\} \\ \alpha T &= \left( \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} \right).\end{aligned}\tag{86}$$

In terms of the mixing angles and the mass eigenstates of the full theory, the contributions from heavy quarks, including the Standard Model top and bottom quarks, to  $\Delta T$  and  $\Delta S$  are[32, 44],<sup>4</sup>

$$\begin{aligned}\Delta T &= \frac{N_c}{16\pi s_W^2 M_W^2} \left\{ \Sigma_{i,j=1,2,3} \left[ \left( |U_{L,ij}|^2 + |U_{R,ij}|^2 \right) \theta_+(M_{T_i}, M_{B_j}) + 2U_{L,ij} U_{R,ij}^\dagger \theta_-(M_{T_i}, M_{B_j}) \right] \right. \\ &\quad - \Sigma_{i<j=1,2,3} \left[ \left( |X_{L,ij}^t|^2 + |X_{R,ij}^t|^2 \right) \theta_+(M_{T_i}, M_{T_j}) + 2X_{L,ij}^t X_{R,ij}^{t\dagger} \theta_-(M_{T_i}, M_{T_j}) \right] \\ &\quad \left. - \Sigma_{i<j=1,2,3} \left[ \left( |X_{L,ij}^b|^2 + |X_{R,ij}^b|^2 \right) \theta_+(M_{B_i}, M_{B_j}) + 2X_{L,ij}^b X_{R,ij}^{b\dagger} \theta_-(M_{B_i}, M_{B_j}) \right] \right\} \\ \Delta S &= \frac{N_c}{2\pi M_Z^2} \left\{ \Sigma_{i,j=1,2,3} \left[ \left( |U_{L,ij}|^2 + |U_{R,ij}|^2 \right) \psi_+(M_{T_i}, M_{B_j}) + 2U_{L,ij} U_{R,ij}^\dagger \psi_-(M_{T_i}, M_{B_j}) \right] \right. \\ &\quad - \Sigma_{i<j=1,2,3} \left[ \left( |X_{L,ij}^t|^2 + |X_{R,ij}^t|^2 \right) \chi_+(M_{T_i}, M_{T_j}) + 2X_{L,ij}^t X_{R,ij}^{t\dagger} \chi_-(M_{T_i}, M_{T_j}) \right] \\ &\quad \left. - \Sigma_{i<j=1,2,3} \left[ \left( |X_{L,ij}^b|^2 + |X_{R,ij}^b|^2 \right) \chi_+(M_{B_i}, M_{B_j}) + 2X_{L,ij}^b X_{R,ij}^{b\dagger} \chi_-(M_{B_i}, M_{B_j}) \right] \right\},\end{aligned}\tag{87}$$

where the functions  $\theta_\pm, \chi_\pm$  are defined below and  $N_c = 3$ .

$$\begin{aligned}\theta_+(m_1, m_2) &= m_1^2 + m_2^2 - \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \left( \frac{m_1^2}{m_2^2} \right) \\ \theta_-(m_1, m_2) &= 2m_1 m_2 \left[ \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \ln \left( \frac{m_1^2}{m_2^2} \right) - 2 \right] \\ \theta_+(m, m) &= 0 \\ \theta_-(m, m) &= 0\end{aligned}\tag{88}$$

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<sup>4</sup> We assume all entries in the mixing matrices are real.

and

$$\begin{aligned}
\psi_+(m_1, m_2) &= \frac{22m_1^2 + 14m_2^2}{9} - \frac{M_Z^2}{9} \log\left(\frac{m_1^2}{m_2^2}\right) + \frac{11m_1^2 + M_Z^2}{18} f(m_1, m_1) \\
&\quad + \frac{7m_1^2 - M_Z^2}{18} f(m_2, m_2) \\
\psi_-(m_1, m_2) &= - \mid (m_1 m_2 \mid \left[ 4 + \frac{1}{2} \left( f(m_1, m_1) + f(m_2, m_2) \right) \right] \\
\chi_+(m_1, m_2) &= \frac{m_1^2 + m_2^2}{2} - \frac{(m_1^2 - m_2^2)^2}{3M_Z^2} + \left[ \frac{(m_1^2 - m_2^2)^3}{6M_Z^4} - \left( \frac{M_Z^2}{2} \right) \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \right] \ln\left(\frac{m_1^2}{m_2^2}\right) \\
&\quad + \frac{m_1^2 - M_Z^2}{6} f(m_1, m_2) + \frac{m_2^2 - M_Z^2}{6} f(m_2, m_2) \\
&\quad + \left[ \frac{M_Z^2}{3} - \frac{m_1^2 + m_2^2}{6} - \frac{(m_1^2 - m_2^2)^2}{6M_Z^2} \right] f(m_1, m_2) \\
\chi_-(m_1, m_2) &= - \mid m_1 m_2 \mid \left[ 2 + \left( \frac{m_1^2 - m_2^2}{M_Z^2} - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \right) \ln\left(\frac{m_1^2}{m_2^2}\right) \right. \\
&\quad \left. + \frac{1}{2} \left( f(m_1, m_1) + f(m_2, m_2) \right) - f(m_1, m_2) \right] \\
\chi_+(m, m) &= 0 \\
\chi_-(m, m) &= 0
\end{aligned} \tag{89}$$

and

$$\begin{aligned}
f(m_1, m_2) &= - \left( 2 \frac{\sqrt{\Delta}}{M_Z} \right) \left[ \arctan\left(\frac{m_1^2 - m_2^2 + M_Z^2}{M_Z \sqrt{\Delta}}\right) - \arctan\left(\frac{m_1^2 - m_2^2 - M_Z^2}{M_Z \sqrt{\Delta}}\right) \right] \quad \text{if } \Delta > 0 \\
&= 0 \quad \text{if } \Delta = 0 \\
&= \frac{1}{M_Z} \sqrt{-\Delta} \ln\left(\frac{m_1^2 + m_2^2 - M_Z^2 + M_Z \sqrt{-\Delta}}{m_1^2 + m_2^2 - M_Z^2 - M_Z \sqrt{-\Delta}}\right) \quad \text{if } \Delta < 0 \\
\Delta &= -M_Z^2 - \frac{m_1^4 + m_2^4}{M_Z^2} + 2m_1^2 + 2m_2^2 + \frac{2m_1^2 m_2^2}{M_Z^2} \\
&= -M_Z^2 \left( 1 - \frac{m_1^2 + m_2^2}{M_Z^2} \right)^2 + \frac{4m_1^2 m_2^2}{M_Z^2}
\end{aligned} \tag{90}$$



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