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Supergravity as a Special Case of Local Nilpotent Fermionic Symmetry

Hitoshi Nishino[1] and Subhash Rajpoot[2]

Department of Physics & Astronomy
California State University
1250 Bellflower Boulevard
Long Beach, CA 90840

Abstract

We start with a four-dimensional (4D) system only with local nilpotent fermionic symmetry, and show that massive $N = 1$ supergravity is realized as a special case. Our field content in 4D is $(e_\mu{}^m, \psi_\mu, \omega_\mu{}^{rs}, \chi)$, where $\psi_\mu{}^I$ is a vector-spinor in the Majorana representation in four-dimenisons, while χ is a compensator Majorana spinor, and $\omega_\mu{}^{rs}$ is the Lorentz connection in the first-order formalism. Applying a similar method to 10D, we start with the field content $(e_\mu{}^m, \psi_\mu, \omega_\mu{}^{rs}, A_{\mu\nu\rho}, B_{\mu\nu}, \lambda, \varphi, \chi)$ with nilpotent fermionic symmetry, and show that the conventional massive type-IIA supergravity comes out as a special case of our system. These explicit results indicate that the most known massive supergravity theories are just special cases of more fundamental systems with nilpotent fermionic symmetry. Our nilpotent fermionic charge N_α satisfying $\{N_\alpha, N_\beta\} = 0$ resembles the BRST charge $Q_{\rm B}$ in topological field theory with the 'twisting of supersymmetry'. If we interpret our charge N_α as twisted supersymmetry, it becomes clear how our system evades the Haag-Lopuszański-Sohnius theorem for the uniqueness of supergravity.

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[1] E-Mail: hnishino@csulb.edu

[2] E-Mail: rajpoot@csulb.edu

1. Introduction

In our recent paper [1], we have demonstrated that some supersymmetric integrable systems in lower dimensions ($D \leq 3$) can be generated by a system in dimensions $D = 2+2$ only with local nilpotent fermionic symmetry. In such a system, a vector-spinor plays the role of the gauge field for local nilpotent fermionic symmetry.

In a subsequent paper [2], we have also presented a self-dual Yang-Mills (SDYM) system with a vector-spinor gauging local nilpotent symmetry generates in $D = 2+2$ contains the usual supersymmetric SDYM system as special exact solutions. Our mechanism is shown to work also in dimensions $D = 8+0$ and $D = 7+0$ for generalized self-dualities with reduced holonomies $SO(7)$ and G_2, respectively. In other words, we have shown that supersymmetric systems are realized as the sub-systems of larger systems only with local nilpotent fermionic symmetry.

In supergravity theory [3][4][5][6], it was shown long time ago that the gravitino field equation satisfies the so-called consistency condition [4], so that supergravity appears to be the 'unique' gauge theory for interacting vector-spinors. To elucidate this, we consider a system of the field content (A_μ, ψ_μ^i) in four-dimensions (4D), where $i = 1, 2$ for the **2** representation of $SO(2)$, and suppose there is a minimal coupling between the $SO(2)$ gauge field A_μ and the Majorana vector-spinor ${\psi_\mu}^i$ in the **2** of $SO(2)$. Consider the lagrangian[3)]

$$\mathcal{L}_1 \equiv -\tfrac{1}{4}(F_{\mu\nu})^2 - \tfrac{1}{2}(\overline{\psi}_\mu{}^i \gamma^{\mu\nu\rho} D_\nu {\psi_\rho}^i) \tag{1.1}$$

where $D_\mu {\psi_\nu}^i \equiv \partial_\mu {\psi_\nu}^i + g\epsilon^{ij} A_\mu \psi^j$. The field equation for ${\psi_\mu}^i$ is[4)]

$$\frac{\delta \mathcal{L}_1}{\delta \overline{\psi}_\mu{}^i} = -(\gamma^{\mu\rho\sigma} D_\rho {\psi_\sigma}^i) \doteq 0 \quad . \tag{1.2}$$

Since the right side is zero, the divergence of the left side is supposed to vanish. However, actual computation shows the opposite:

$$D_\mu \left(\frac{\delta \mathcal{L}_1}{\delta \overline{\psi}_\mu{}^i} \right) = -\tfrac{1}{2} g\epsilon^{ij} (\gamma^{\mu\rho\sigma} {\psi_\sigma}^j) F_{\mu\rho} \neq 0 \quad . \tag{1.3}$$

unless the field strength itself vanishes. This is also known as 'Velo-Zwanziger disease' [7]

In $N = 1$ supergravity theory in 4D [3][4][5][6], this problem does *not* arise. The 1st-order formalism of supergravity has the field content $({e_\mu}^m, \psi_\mu, {\omega_\mu}^{rs})$ with the lagrangian

$$\mathcal{L}_2 = +\tfrac{1}{4} eR - \tfrac{1}{2}(\overline{\psi}_\mu \gamma^{\mu\rho\sigma} D_\rho \psi_\sigma) \quad . \tag{1.4}$$

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4) We use the symbol $\doteq$ for a field equation, distinguished from usual algebraic equality.

Here D_μ is the Lorentz and gauge-covariant derivative: $D_\mu \psi_\nu \equiv \partial_\mu \psi_\nu - (1/4)\,\omega_\mu{}^{rs}(\gamma_{rs}\psi_\nu)$, and R is the scalar curvature constructed out of the Riemann tensor as $R \equiv g^{\mu\nu} R_{\mu\nu} \equiv g^{\mu\nu} R_{\mu\rho\nu}{}^\rho$ with $R_{\mu\nu}{}^{rs} \equiv 2\partial_{[\mu}\omega_{\nu]}{}^{rs} - 2\omega_{[\mu}{}^{rt}\omega_{\nu]t}{}^s$. The ψ_μ-field equation is

$$\frac{\delta \mathcal{L}_2}{\delta \overline{\psi}_\mu{}^i} = -\gamma^{\mu\rho\sigma} D_\rho \psi_\sigma \doteq 0 \quad . \tag{1.5}$$

Most importantly, its divergence is shown to vanish [3][4]:

$$D_\mu \left(\frac{\delta \mathcal{L}_2}{\delta \overline{\psi}_\mu} \right) \doteq + (\gamma^m \psi_\mu) \left(\frac{\delta \mathcal{L}_2}{\delta e_\mu{}^m} \right) \doteq 0 \quad , \tag{1.6}$$

upon the use of the vierbein field equation.

In the present paper, we show that supergravity theory, such as $N = 1$ supergravity theory [3][4][5][6] is *not* singled out as the unique consistent theory for a vector-spinor in 4D. We have an alternative consistent system for a vector-spinor with local nilpotent fermionic symmetry. In other words, a vector-spinor can be the gauge field *not only* for local supersymmetry, *but also* for local nilpotent fermionic symmetry. Moreover, the conventional $N = 1$ supergravity theory [3][4][5][6] is a special sub-system of a larger system only with nilpotent fermionic symmetry.

The usage of a vector-spinor as the gauge field of local nilpotent fermionic symmetry is *not* new. In fact, we have shown in our paper in 2006 [8] that a vector-spinor can be coupled consistently to a non-Abelian gauge field without any problem of consistency. The new result in our present paper is that the conventional $N = 1$ supergravity comes out of such a nilpotent system as a sub-system with supersymmetry as stronger symmetry.

The technique we adopt is based on the compensator mechanism. This is also similar to what we used in [8]. Namely, we need an extra compensator χ in addition to the vector-spinor under question.

Some readers may wonder, if gauging nilpotent fermionic symmetries, especially with compensator fields really makes sense. Such a question is motivated by the following two observations: First, due to unitarity, nilpotent symmetry will have only *zero-norm* states, so that they are associated only with *unphysical* states. Therefore, our system deals only with *unphysical* fields. Second, since compensator fields are gauged away by local symmetry by definition, one can create any theory with compensator that has 'fake' symmetry.

Even though these questions seem legitimate at first glance, they are not actually well supported for the following reasons. For the first point about *unphysical* zero-norm states, we cite the non-trivial series of works on *gauging* BRST symmetry [9] which is also *nilpotent* symmetry. Furthermore, in our aforementioned papers [1][2], we have shown that supersymmetric integrable models in $D = 2 + 1$ and $D = 1 + 1$ are generated by our system in $D = 2 + 2$ with nilpotent fermionic symmetry. Obviously, interacting physical states in $D \leq 3$ are associated with nilpotent fermionic symmetry in $D = 2+2$. Note that even *supersymmetries* in $D \leq 3$ are generated from a *non*-supersymmetric system in $D = 2 + 2$ only with nilpotent fermionic symmetry. For the second point about triviality of compensators, we have to cite the original works by Proca and Stueckelberg [10] which are by no means trivial. It is true that our compensator χ can be gauged away by symmetry, and therefore χ is *unphysical*. However, the vector-spinor gauge field ψ_μ can *not* be gauged away, because components other than the gradient direction remain as *physical* components.

This paper is organized as follows. In the next section, we explain the consistency of compensator mechanism, starting with the case of non-Abelian Proca-Stueckelberg-type compensator field,[5] and show that there is no problem with the divergence of the gauge field equation. In section 3, we give the nilpotent symmetric system in 4D with the field content $(e_\mu{}^m, \psi_\mu, \chi)$, where the usual massive supergravity in 4D [11], comes out as a special case of more general nilpotent symmetric system. Applying a similar technique, we establish nilpotent-symmetric system in 10D with the field content $(e_\mu{}^m, \psi_\mu, \omega_\mu{}^{rs}, A_{\mu\nu\rho}, B_{\mu\nu}, \lambda, \varphi, \chi)$, and show that the massive type-IIA supergravity [12], comes out as a special case of nilpotent-symmetric system.

2. Non-Abelian Gauge Field with a Compensator

As a simple example for a compensator, we consider non-Abelian gauge interactions. Our field content is $(A_\mu{}^I, \varphi^I)$, where I is for the adjoint representation, while φ^I is the Proca-Stueckelberg-type compensator scalar field [10] that is absorbed into the longitudinal component of $A_\mu{}^I$.

[5] The original gauge field by Proca and Stueckelberg [10] was only for Abelian group, but we sometimes call it a 'Proca-Stueckelberg-type compensator' in this paper.

Our total action is $I_3 \equiv \int d^4x \, \mathcal{L}_3$, with the lagrangian[6)]

$$\mathcal{L}_3 = -\frac{1}{4} (F_{\mu\nu}{}^I)^2 - \frac{1}{2} (P_\mu{}^I)^2 \quad , \tag{2.1}$$

where

$$\begin{aligned} F_{\mu\nu}{}^I &\equiv + 2\, \partial_{[\mu} A_{\nu]}{}^I + m f^{IJK} A_\mu{}^J A_\nu{}^K \quad , \\ P_\mu{}^I &\equiv \Big[(D_\mu e^\varphi)\, e^{-\varphi} \Big]^I \equiv + \Big[(\partial_\mu e^\varphi)\, e^{-\varphi} \Big]^I + m A_\mu{}^I \quad . \end{aligned} \tag{2.2}$$

Our action I_3 has the non-Abelian gauge invariance $\delta_T I_3 = 0$, where

$$\delta_T (A_\mu, e^\varphi) = (+D_\mu \alpha \, , \, -m\alpha e^\varphi) \qquad \Longrightarrow \qquad \delta_T (F_{\mu\nu}, P_\mu) = -\big[\alpha \, , (F_{\mu\nu}, P_\mu)\big] \quad . \tag{2.3}$$

These fields and the infinitesimal parameter α carry the implicit anti-hermitian generators T^I, *e.g.*, $\delta_T A_\mu{}^I = D_\mu \alpha^I$.

For getting the φ-field equation, we need a lemma for a general variation of $P_\mu{}^I$:

$$\delta P_\mu{}^I = D_\mu \Big[(\delta e^\varphi) e^{-\varphi} \Big]^I + \big[(\delta e^\varphi) e^{-\varphi}, \, P_\mu \big]^I + m \, \delta A_\mu{}^I \quad . \tag{2.4}$$

Now the field equations of A_μ and φ are

$$\frac{\delta \mathcal{L}_3}{\delta A_\mu} = - D_\nu F^{\mu\nu \, I} - m P^{\mu \, I} \doteq 0 \quad , \tag{2.5a}$$

$$\frac{\delta \mathcal{L}_3}{[\, (\delta e^\varphi) e^{-\varphi} \,]^I} = + D_\mu P^{\mu \, I} \doteq 0 \quad . \tag{2.5b}$$

As expected, the divergence of the A_μ-field equation vanishes upon the use of the φ-field equation:

$$D_\mu \left(\frac{\delta \mathcal{L}_3}{\delta A_\mu{}^I} \right) = -m \, D_\mu P^{\mu \, I} = -m \, \frac{\delta \mathcal{L}_3}{[\, (\delta e^\varphi) e^{-\varphi} \,]^I} \doteq 0 \quad . \tag{2.6}$$

This consistency is also associated with the invariance of the action $\delta_T I = 0$, because

$$\begin{aligned} 0 = & + (D_\mu \alpha^I) \left(\frac{\delta \mathcal{L}_3}{\delta A_\mu{}^I} \right) + (-m\alpha^I) \frac{\delta \mathcal{L}_3}{[\, (\delta e^\varphi) e^{-\varphi} \,]^I} \\ \overset{\nabla}{=} & - \alpha^I \left[+ D_\mu \left(\frac{\delta \mathcal{L}_3}{\delta A_\mu{}^I} \right) + m \, \frac{\delta \mathcal{L}_3}{[\, (\delta e^\varphi) e^{-\varphi} \,]^I} \right] \quad . \end{aligned} \tag{2.7}$$

[6)] Although the formulation in this section is just the repetition of the original paper [10], we repeat it here, due to its importance for understanding our result from the next section, .

Here the symbol $\overset{\nabla}{=}$ is an equality valid up to a total divergence. For any equality in (2.7), *no* field equation has been used. In other words, (2.7) is an *identity* implying (2.6).

As the transformation (2.3) shows, the field φ^I is a Proca-Stueckelberg compensator field [10], which should be absorbed into the longitudinal component of $A_\mu{}^I$ by a field redefinition

$$\widetilde{A}_\mu \equiv e^{-\varphi} A_\mu e^{\varphi} + m^{-1} e^{-\varphi} \partial_\mu e^{\varphi} = m^{-1} e^{-\varphi} P_\mu e^{\varphi} \quad . \tag{2.8}$$

Needless to say, $\delta_T \widetilde{A}_\mu = -[\alpha, \widetilde{A}_\mu]$ and $\delta_T \widetilde{F}_{\mu\nu} = -[\alpha, \widetilde{F}_{\mu\nu}]$ with

$$\widetilde{F}_{\mu\nu}{}^I \equiv 2\, \partial_{[\mu} \widetilde{A}_{\nu]}{}^I + m f^{IJK} \widetilde{A}_\mu{}^J \widetilde{A}_\nu{}^K \quad . \tag{2.9}$$

Under (2.8), the original lagrangian (2.1) is recasted into

$$\mathcal{L}_3 = -\tfrac{1}{4} (\widetilde{F}_{\mu\nu}{}^I)^2 - \tfrac{1}{2} m^2 (\widetilde{A}_\mu{}^I)^2 \quad . \tag{2.10}$$

Now the field equation of $\widetilde{A}_\mu$ is simply

$$\frac{\delta \mathcal{L}_3}{\delta \widetilde{A}_\mu{}^I} = -\, \widetilde{D}_\nu \widetilde{F}^{\mu\nu\, I} - m^2 \widetilde{A}^{\mu I} \doteq 0 \quad . \tag{2.11}$$

Here the covariant derivative $\widetilde{D}_\mu$ is with $\widetilde{A}_\mu$. At the free-field level, (2.11) means nothing but a massive-vector (Klein-Gordon) field equation

$$\partial_\nu^2 \widetilde{A}_\mu \doteq + m^2 \widetilde{A}_\mu + (\text{interactions}) \quad . \tag{2.12}$$

Eq. (2.11) further implies that

$$\widetilde{D}_\mu \left(\frac{\delta \mathcal{L}_3}{\delta \widetilde{A}_\mu{}^I} \right) = -m^2 \widetilde{D}_\mu \widetilde{A}^{\mu\, I} \doteq 0 \quad . \tag{2.13}$$

Even though the middle side of (2.13) is *not* algebraically zero, there arises no problem. However, we can no longer rely on the φ-field equation as in (2.6), because the φ-field is *not* in the new lagrangian (2.10) any longer. Instead, we can interpret (2.13) as the divergence of (2.11) yielding the vanishing of $\widetilde{D}_\mu \widetilde{A}^{\mu\, I}$. This is also equivalent to the e^φ-field equation in the original system in the frame (A_μ, φ). As a matter of fact, the condition $\widetilde{D}_\mu \widetilde{A}^{\mu\, I} \doteq 0$ is needed for a massive vector $\widetilde{A}_\mu{}^I$, because one degree of freedom (DOF) should be eliminated out of the original 4 DOF, so that only $4 - 1 = 3$ DOF survive as a massive vector field.

The reason why (1.3) had a problem, while (2.13) or (2.5b) did *not*, is that the latter has many non-trivial solutions, while the former does *not.* We will see similar situation for a vector-spinor field as the gauge field of nilpotent fermionic symmetry in the next section.

3. Nilpotent Fermionic Symmetry for a Vector a Vector-Spinor in Curved 4D

In a fashion similar to the above massive non-Abelian vector, we can build a system for a vector-spinor ψ_μ in curved 4D with the field content $(e_\mu{}^m,\ \psi_\mu,\ \omega_\mu{}^{rs},\ \chi)$. Our vector-spinor ψ_μ is in the Majorana spinor in 4D, playing a role of the gauge field for nilpotent fermionic symmetry. The χ is a compensator field analogous to φ, and will be absorbed into ψ_μ. The $\omega_\mu{}^{rs}$ is the Lorentz (spinor) connection for the local $SO(3,1)$ Lorentz symmetry.

Our local nilpotent fermionic symmetry generator N_α satisfies the algebra with the translation generator P_m and the Lorentz transformation generator M_{mn}:

$$\{N_\alpha,\ N_\beta\} = 0 \quad , \qquad [M_{mn}, N_\alpha] = -\tfrac{1}{2}\,(\gamma_{mn})_\alpha{}^\beta N_\beta \quad , \tag{3.1a}$$

$$[M_{mn},\ M^{rs}] = +4\delta_{[n}{}^{[r} M_{m]}{}^{s]} \quad , \qquad [M_{mn},\ P^r] = +2\delta_{[n}{}^r P_{m]} \quad , \tag{3.1b}$$

and all other commutators, such as $[P_m,\ N_\alpha]$ are zero. This set of algebra is the curved-space generalization of our algebra in [2]. The only difference from the usual supersymmetry algebra is that the first commutator in (3.1a) vanishes. The generator N_α acts on the fields as

$$\delta_N(e_\mu{}^m,\ \psi_\mu,\ \omega_\mu{}^{rs},\ \chi) = (0,\ D_\mu\beta,\ 0,\ -m\beta) \quad , \tag{3.2}$$

so that the field strengths are all invariant:

$$\delta_N(T_{\mu\nu}{}^m,\ \mathcal{R}_{\mu\nu},\ R_{\mu\nu}{}^{rs},\ L_\mu) = (0,0,0,0) \quad , \tag{3.3}$$

if each field strength is defined by

$$T_{\mu\nu}{}^m \equiv\ +2D_{[\mu}e_{\nu]}{}^m \quad , \tag{3.4a}$$

$$\mathcal{R}_{\mu\nu} \equiv\ +2D_{[\mu}\psi_{\nu]} + \tfrac{1}{4}\, m^{-1}(\gamma_{mn}\chi)R_{\mu\nu}{}^{mn} \quad , \tag{3.4b}$$

$$L_\mu \equiv\ +D_\mu\chi + m\psi_\mu \quad , \tag{3.4c}$$

$$R_{\mu\nu}{}^{rs} \equiv\ +2\,\partial_{[\mu}\omega_{\nu]}{}^{rs} + 2\omega_{[\mu}{}^{rt}\,\omega_{\nu]t}{}^s \quad , \tag{3.4d}$$

where D_μ is the usual Lorentz covariant derivative, *e.g.*, $D_\mu \chi \equiv \partial_\mu \chi + (1/4)\omega_\mu{}^{rs}(\gamma_{rs}\chi)$.

We now set up our total action as $I \equiv \int d^4 x \, \mathcal{L}$ with

$$\mathcal{L} = + \frac{1}{4} eR - \frac{1}{4} m^{-1} e(\overline{L}_\mu \gamma^{\mu\rho\sigma} \mathcal{R}_{\rho\sigma}) + \frac{1}{2} m^{-1} e(\overline{L}_\mu \gamma^{\mu\nu} L_\nu) + a\, m^2 e \quad , \tag{3.5}$$

where a is an arbitrary real constant. Thanks to (3.3), the invariance $\delta_N I = 0$ is manifest.

Similar to (1.6) or (2.6), we have the consistency equation for the divergence of the ψ_μ-field equation

$$D_\mu \left(\frac{\delta \mathcal{L}}{\delta \overline{\psi}_\mu} \right) = - m \left(\frac{\delta \mathcal{L}}{\delta \overline{\chi}} \right) \doteq 0 \quad . \tag{3.6}$$

The first equality is nothing but the δ_N-invariance of our action:

$$\begin{aligned} 0 = \delta_N \mathcal{L} = (\delta_N \overline{\psi}_\mu) \left(\frac{\delta \mathcal{L}}{\delta \overline{\psi}_\mu} \right) + (\delta_N \overline{\chi}) \left(\frac{\delta \mathcal{L}}{\delta \overline{\chi}} \right) &\overset{\nabla}{=} - \overline{\beta} D_\mu \left(\frac{\delta \mathcal{L}}{\delta \overline{\psi}_\mu} \right) - m \overline{\beta} \left(\frac{\delta \mathcal{L}}{\delta \overline{\chi}} \right) \\ = - \overline{\beta} \left[D_\mu \left(\frac{\delta \mathcal{L}}{\delta \overline{\psi}_\mu} \right) + m \left(\frac{\delta \mathcal{L}}{\delta \overline{\chi}} \right) \right] \quad . \end{aligned} \tag{3.7}$$

The first two sides of (3.6) can be confirmed by the direct computations:

$$\begin{aligned} \frac{\delta \mathcal{L}}{\delta \overline{\psi}_\mu} = &- \frac{1}{2} e(\gamma^{\mu\rho\sigma} \check{\mathcal{R}}_{\rho\sigma}) + \frac{1}{2} m^{-1} e(\gamma^{\mu\rho\sigma} L_\rho) \widehat{T}_\sigma \\ &+ \frac{1}{4} e m^{-1} (\gamma^{\mu\rho\sigma} L_\tau) \widehat{T}_{\rho\sigma}{}^\tau - \frac{1}{4} m^{-1} e (\gamma^{\rho\sigma\tau} L_\rho) \widehat{T}_{\sigma\tau}{}^\mu \quad , \end{aligned} \tag{3.8a}$$

$$\frac{\delta \mathcal{L}}{\delta \overline{\chi}} \doteq + \left(a - \frac{3}{2} \right) e \, (\gamma^\mu L_\mu) \quad , \tag{3.8b}$$

where

$$\check{R}_{\mu\nu} \equiv 2 m^{-1} D_{[\mu} L_{\nu]} + \left(\gamma_{[\mu} L_{\nu]} \right) \quad , \tag{3.9b}$$

$$\widehat{T}_{\mu\nu}{}^m \equiv T_{\mu\nu}{}^m - m^{-2} (\overline{L}_\mu \gamma^m L_\nu) \ , \quad T_\mu \equiv T_{\mu\nu}{}^\nu \ , \quad \widehat{T}_\mu \equiv \widehat{T}_{\mu\nu}{}^\nu \ . \tag{3.9b}$$

The symbol $\doteq$ in (3.8b) implies that we have also used other field equations, such as

$$\begin{aligned} e^{-1} \left(\frac{\delta \mathcal{L}}{\delta \omega_\mu{}^{mn}} \right) = &+ \frac{1}{4} \widehat{T}_{mn}{}^\mu + \frac{1}{2} e_{[m}{}^\mu \widehat{T}_{n]} \\ &+ \frac{1}{8} m^{-1} \Big[\overline{\chi} \gamma_{mn} \Big\{ \gamma^{\mu\rho\sigma} \check{R}_{\rho\sigma} - m^{-1} (g^{\mu\rho\sigma} L_\rho) \widehat{T}_\sigma \\ &\qquad - \frac{1}{2} m^{-1} (\gamma^{\mu\rho\sigma} L_\tau) \widehat{T}_{\rho\sigma}{}^\tau + \frac{1}{2} m^{-1} (\gamma^{\rho\sigma\tau} L_\rho) \widehat{T}_{\sigma\tau}{}^\mu \Big\} \Big] \doteq 0 \ , \end{aligned} \tag{3.10a}$$

$$\begin{aligned} e^{-1} \left(\frac{\delta \mathcal{L}}{\delta e_m{}^\mu} \right) = &- \frac{1}{2} \left(R_\mu{}^m - \frac{1}{2} e_\mu{}^m R \right) + \frac{1}{4} m^{-1} e_\mu{}^m (\overline{L}_\nu \gamma^{\nu\rho\sigma} \mathcal{R}_{\rho\sigma}) \\ &- \frac{1}{4} m^{-1} (\overline{L}_\mu \gamma^{m\rho\sigma} \mathcal{R}_{\rho\sigma}) - \frac{1}{2} m^{-1} (\overline{L}_\rho \gamma^{\rho m\sigma} \mathcal{R}_{\mu\sigma}) \\ &- \frac{1}{2} m^{-1} e_\mu{}^m (\overline{L}_\rho \gamma^{\rho\sigma} L_\sigma) + m^{-1} (\overline{L}_\mu \gamma^{m\nu} L_\nu) - a m^2 e_\mu{}^m \doteq 0 \quad . \end{aligned} \tag{3.10b}$$

In particular, the last two lines of (3.10a) vanish upon the ψ_μ-field equation (3.8a). Eventually, (3.10a) yields the torsion condition

$$T_{\mu\nu}{}^m \doteq + m^{-2}(\overline{L}_\mu \gamma^m L_\nu) \quad \Longleftrightarrow \quad \widehat{T}_{\mu\nu}{}^m \doteq 0 \ . \tag{3.11}$$

As in the conventional supergravity [3][4][5][6], (3.11) is equivalent to the usual expression of $\omega_\mu{}^{mn}$ in terms of the anholonomy coefficients $C_{\mu\nu}{}^m$:

$$\omega_{mrs} \doteq - \frac{1}{2}\left(\widehat{C}_{mrs} - \widehat{C}_{msr} - \widehat{C}_{rsm}\right) \ , \quad \widehat{C}_{\mu\nu}{}^m \equiv +2\partial_{[\mu} e_{\nu]}{}^m - m^{-2}(\overline{L}_\mu \gamma^m L_\nu) \ . \tag{3.12}$$

The only difference is that the fermionic-square term in $\widehat{C}_{\mu\nu}{}^m$ are L^2-Terms. However, they are eventually equivalent to the conventional one, because of $L_\mu = m\widetilde{\psi}_\mu$ in (3.6).

As a technical detail, we mention that the Fierz identity

$$(\gamma^{[\rho\sigma|\lambda} L_\rho)(\overline{L}_\sigma \gamma_\lambda{}^{|\tau]} L_\tau) + (\gamma^{[\rho|} L_\rho)(\overline{L}_\sigma \gamma^{|\sigma\tau]} L_\tau) \equiv 0 \ . \tag{3.13}$$

has been used in (3.8). This is derived from the γ-matrix identities

$$(\gamma^{[\rho\sigma|\lambda})_{\alpha(\beta|}(\gamma_\lambda{}^{|\tau]})_{|\gamma\delta)} + (\gamma^{[\rho|})_{\alpha(\beta|}(\gamma^{|\sigma\tau]})_{|\gamma\delta)} \equiv 0 \ , \quad (\gamma^{[\rho|})_{(\alpha\beta|}(\gamma^{|\sigma\tau]})_{|\gamma\delta)} \equiv 0 \ . \tag{3.14}$$

with the indices $\alpha, \ \beta, \ \cdots = 1, \ 2, \ 3, \ 4$ for the spinorial indices of 4-component Majorana spinors.

The last three terms in (3.8a) vanish upon (3.11), yielding the simplified $\widetilde{\psi}_\mu$-field equations

$$\gamma^{\mu\rho\sigma} \check{\mathcal{R}}_{\rho\sigma} \doteq 0 \ , \quad \gamma^\nu \check{\mathcal{R}}_{\mu\nu} \doteq 0 \ , \quad \gamma_{[\mu} \check{\mathcal{R}}_{\nu\rho]} \doteq 0 \ , \quad \check{\mathcal{R}}_{\mu\nu} + i\gamma_5 \widetilde{\check{\mathcal{R}}}_{\mu\nu} \doteq 0 \ . \tag{3.15}$$

where $\widetilde{\check{\mathcal{R}}}_{mn} \equiv (1/2)\, \epsilon_{mn}{}^{rs} \check{\mathcal{R}}_{rs}$. Eq. (3.15) can be used also for other field equations. Note that these ψ-field equations are formally the same as those in $N = 1$ supergravity [3][4][5][6]. Eq. (3.10b) yields the simplified form of the vierbein-field equation

$$R_{\mu\nu} \doteq - m^{-1}(\overline{L}_\rho \gamma_\mu \check{\mathcal{R}}_\nu{}^\rho) + m^{-1}(\overline{L}_\mu \gamma_\nu{}^\rho L_\rho) + 2\, a\, m^2 g_{\mu\nu} \ . \tag{3.16}$$

Using these field equations, we can confirm the consistency (3.6):

$$D_\mu \left(\frac{\delta \mathcal{L}}{\delta \overline{\psi}_\mu}\right) \doteq - \left(a - \tfrac{3}{2}\right) m\, e(\gamma^\mu L_\mu) = -m \left(\frac{\delta \mathcal{L}}{\delta \overline{\chi}}\right) \doteq 0 \ . \tag{3.17}$$

Since the divergence of the ψ_μ-field equation vanishes upon the use of the χ-field equation, there is *no* problem with the consistency of the vector-spinor field equation. Especially, the

χ-field equation starts with $e^{-1}(\delta\mathcal{L}/\delta\overline{\chi}) = (a-3/2)(\gamma^\mu L_\mu) = (a-3/2)(\not{D}\chi + m\gamma^\mu\psi_\mu) \doteq 0$, which is the kinetic term of χ. This implies that there are definitely non-trivial solution for χ, as opposed to the Velo-Zwanziger disease case (1.3). In other words, (3.17) is different from (1.3), while similar to the non-Abelian compensator case (2.6). This fact is also valid *independent* of the value of the real constant a.

As in the Proca-Stueckelberg mechanism [10], the ψ_μ can absorb the compensator χ by

$$\widetilde{\psi}_\mu \equiv \psi_\mu + m^{-1}D_\mu\chi = m^{-1}L_\mu \quad . \tag{3.18}$$

In terms of $(e_\mu{}^m,\, \widetilde{\psi}_\mu,\, \omega_\mu{}^{mn})$, the original lagrangian (3.5) is re-casted into

$$\mathcal{L} = +\,\tfrac{1}{4}\,e\,R - \tfrac{1}{4}\,e\,(\overline{\widetilde{\psi}}_\mu\gamma^{\mu\rho\sigma}\widetilde{\mathcal{R}}_{\rho\sigma}) + \tfrac{1}{2}\,m\,e\,(\overline{\widetilde{\psi}}_\mu\gamma^{\mu\nu}\widetilde{\psi}_\nu) + a\,m^2\,e \quad , \tag{3.19}$$

where $\widetilde{R}_{\mu\nu} \equiv 2D_{[\mu}\widetilde{\psi}_{\nu]}$. We can re-confirm the consistency (3.17) in the frame $(e_\mu{}^m,\, \widetilde{\psi}_\mu,\, \omega_\mu{}^{mn})$. Actually, the field equations become simpler. First, (3.10) stays the same (with L_μ replaced by $m\widetilde{\psi}_\mu$), while the vector-spinor field equation is

$$e^{-1}\left(\frac{\delta\mathcal{L}}{\delta\overline{\widetilde{\psi}}_\mu}\right) = -\,\tfrac{1}{2}\,(\gamma^{\mu\nu\rho}\widetilde{\mathcal{R}}_{\nu\rho}) + m(\gamma^{\mu\nu}\widetilde{\psi}_\nu) = -\,\tfrac{1}{2}\,(\gamma^{\mu\nu\rho}\check{\mathcal{R}}_{\nu\rho}) \doteq 0 \quad , \tag{3.20}$$

The explicit form of the divergence of the ψ_μ-field equation is

$$D_\mu\left(\frac{\delta\mathcal{L}}{\delta\overline{\widetilde{\psi}}_\mu}\right) \doteq -\left(a - \tfrac{3}{2}\right)m^2(\gamma^\mu\widetilde{\psi}_\mu) \quad . \tag{3.21}$$

again by the use of the Fierz identity (3.13), and other field equations.

The most important feature is that even if $a \neq 3/2$, the RHS of (3.21) poses *no* problem for consistency. The last side of (3.21) implies the γ-trance free condition $\gamma^\mu\widetilde{\psi}_\mu \doteq 0$ on the vector-spinor, just as the divergence-less condition $\widetilde{D}_\mu\widetilde{A}^\mu \doteq 0$ (2.13) for the non-Abelian compensator formalism. To put it differently, the RHS of (3.21) is nothing but the reminiscent of the original χ-field equation (3.8b) equivalent to the γ-traceless-ness condition $\gamma^\mu\widetilde{\psi}_\mu = 0$.

In terms of DOF, $\widetilde{\psi}_\mu$ has originally 4×2 DOF, because of 4 for the index μ and 2 for a Majorana spinor. The assignment 4 for the index μ is due to the fact that a massive vector-spinor has *no* gauge invariance with respect to the index μ. However, eventually, a vector-spinor should have 3×2 DOF, so that 1×2 DOF should be eliminated by an extra condition. Eq. (3.21) $\doteq 0$ is exactly such an extra condition. There is also parallel structure between the non-Abelian case (2.13) and (3.21) $\doteq 0$.

In the conventional *massive* $N = 1$ supergravity theory in 4D [11], this point has not been clear. It was concluded in [11], as if the value $a = 3/2$ for anti-de Sitter (AdS) supergravity system *were* singled out for the consistency of the ψ_μ-field equation, and therefore supergravity *would* be the *only* consistent system for a massive vector-spinor. However, the consideration of local nilpotent fermionic symmetry above concludes that *not only* the value $a = 3/2$, *but also any other values* of $a \neq 3/2$ are allowed for the consistency of the vector-spinor field equation. In other words, the conventional $N = 1$ supergravity theory [11][5][6] is just a sub-system of a more larger consistent system of local nilpotent fermionic symmetry in 4D.

4. Application to 10D Case

We can repeat a similar formulation in 10D. The field content in this case is $(e_\mu{}^m,\ \psi_\mu,\ \omega_\mu{}^{rs},\ A_{\mu\nu\rho},\ B_{\mu\nu},\ \lambda,\ \varphi,\ \chi)$, where the first seven fields are the same as the massless [13] or massive type-IIA supergravity [12] in the first-order formalism for the Lorentz connection $\omega_\mu{}^{rs}$, *except for* λ which stands for χ in [12], while we use χ as the compensator for our nilpotent fermionic symmetry just as in 4D. Needless to say, ψ_μ, λ and χ are Majorana spinors in 10D as in type-IIA supergravity [13][12].

Our starting action is $I_{\rm 10D} \equiv \int d^{10}x\, \mathcal{L}_{\rm 10D}$, where[7)]

$$\begin{aligned}
\mathcal{L}_{\rm 10D} = &+ \frac{1}{4}\, e\, R - \frac{1}{4}\, m^{-1} e\, (\overline{L}_\mu \gamma^{\mu\nu\rho} \mathcal{R}_{\nu\rho}) - \frac{1}{2}\, e\, (\overline{\lambda}\gamma^\mu D_\mu \lambda) - \frac{1}{2}\, e\, (D_\mu \varphi)^2 - \frac{1}{48}\, e\, e^{-\varphi} (F_{[4]})^2 \\
&- \frac{1}{12}\, e\, e^{-2\varphi} (G_{[3]})^2 - \frac{1}{4}\, m^2 e\, e^{-3\varphi} B_{\mu\nu}^2 + \frac{1}{\sqrt{2}}\, e\, (\overline{L}_\mu \gamma^\nu \gamma^\mu \lambda) \partial_\nu \varphi \\
&+ \frac{1}{1152}\, \epsilon^{\mu\nu\rho\sigma\tau\lambda\omega\psi\varphi\chi} \Big(F_{\mu\nu\rho\sigma} F_{\tau\lambda\omega\psi} - 8 m F_{\mu\nu\rho\sigma} B_{\tau\lambda} B_{\omega\psi} + \frac{96}{5}\, m^2 B_{\mu\nu} B_{\rho\sigma} B_{\tau\lambda} B_{\omega\psi} \Big) B_{\varphi\chi} \\
&- \frac{1}{96}\, m^{-2} e\, e^{-\varphi/2} \Big[(\overline{L}^\mu \gamma_{[\mu} \gamma^{[4]} \gamma_{\nu]} L^\nu) - \frac{1}{\sqrt{2}}\, m\, (\overline{L}_\mu \gamma^{[4]} \gamma^\mu \lambda) + \frac{3}{4}\, m^2 (\overline{\lambda} \gamma^{[4]} \lambda) \Big] F_{[4]} \\
&- \frac{1}{24}\, m^{-2} e\, e^{\varphi} \Big[(\overline{L}^\mu \gamma_{11} \gamma_{[\mu} \gamma^{[3]} \gamma_{\nu]} L^\nu) - \sqrt{2} m\, (\overline{L}_\mu \gamma_{11} \gamma^{[3]} \gamma^\mu \lambda) \Big] G_{[3]} \\
&+ \frac{1}{8}\, m^{-1}\, e\, e^{-3\varphi/\sqrt{2}} \Big[(\overline{L}^\mu \gamma_{11} \gamma_{[\mu} \gamma^{\rho\sigma} \gamma_{\nu]} L^\nu) - \frac{3}{\sqrt{2}}\, m (\overline{L}_\mu \gamma_{11} \gamma^{\rho\sigma} \gamma^\mu \lambda) + \frac{5}{4}\, m^2 (\overline{\lambda} \gamma_{11} \gamma^{\rho\sigma} \lambda) \Big] B_{\rho\sigma}
\end{aligned}$$

[7)] We use the space-time signature $(-,+,+,\cdots,+)$ used in [12]. The difference in the overall sign in our lagrangian compared with [12] is that the latter has the overall negative sign for the lagrangian. This is obvious from the φ or all other bosonic field kinetic terms in [12]. We sometimes use the symbol $[n]$ for n-totally antisymmetric tensor to avoid messy indices. For example $\gamma^{[3]} G_{[3]} \equiv \gamma^{\mu\nu\rho} G_{\mu\nu\rho}$.

$$- \tfrac{1}{8} m^{-1} e\, e^{-5\varphi/2} m^{-2} (\overline{L}_\mu \gamma^{\mu\nu} L_\nu) + \tfrac{5}{8\sqrt{2}}\, e\, e^{-5\varphi/2} (\overline{L}_\mu \gamma^\mu \lambda) + \tfrac{21}{32}\, m\, e\, e^{-5\varphi/2} (\overline{\lambda}\lambda)$$

$$+ b\, m^2\, e\, e^{-5\varphi} + \mathcal{O}(\mathrm{fm}^4) \ . \tag{4.1}$$

up to quartic-fermion terms $\mathcal{O}(\mathrm{fm}^4)$. The b is a real constant. As in 4D, we define

$$\mathcal{R}_{\mu\nu} \equiv + 2 D_{[\mu} \psi_{\nu]} + \tfrac{1}{4} m^{-1} (\gamma_{mn} \chi) R_{\mu\nu}{}^{mn} \ , \tag{4.2a}$$

$$L_\mu \equiv + D_\mu \chi + m \psi_\mu \ , \tag{4.2b}$$

Note that our action I_{10D} has local nilpotent fermionic symmetry $\delta_N I_{\mathrm{10D}} = 0$, where

$$\delta_N (e_\mu{}^m,\, \psi_\mu,\, \omega_\mu{}^{rs},\, A_{[3]},\, B_{[2]},\, \lambda,\, \varphi,\, \chi) = (0,\, D_\mu \beta,\, 0,\, 0,\, 0,\, 0,\, 0,\, -m\beta) \ , \tag{4.3a}$$

$$\delta_N (T_{\mu\nu}{}^m,\, \mathcal{R}_{\mu\nu},\, R_{\mu\nu}{}^{rs},\, F_{[4]},\, G_{[3]},\, D_\mu \lambda,\, \partial_\mu \varphi, L_\mu) = (0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0) \ . \tag{4.3b}$$

The ψ_μ and χ-field equations are

$$e^{-1} \left(\frac{\delta \mathcal{L}_{\mathrm{10D}}}{\delta \overline{\psi}_\mu} \right) = - \tfrac{1}{2} (\gamma^{\mu\nu\rho} \mathcal{R}_{\nu\rho}) + \tfrac{1}{\sqrt{2}} (\gamma^\nu \gamma^\mu \lambda) D_\nu \varphi$$

$$- \tfrac{1}{48} m^{-1} e^{-\varphi/2} (\gamma_{[\mu} \gamma^{[4]} \gamma_{\nu]} L^\nu) F_{[4]} + \tfrac{1}{96\sqrt{2}}\, e^{-\varphi/2} (\gamma^{[4]} \gamma^\mu \lambda) F_{[4]}$$

$$- \tfrac{1}{12} m^{-1} e^{\varphi} (\gamma_{11} \gamma_{[\mu} \gamma^{[3]} \gamma_{\nu]} L^\nu) G_{[3]} + \tfrac{1}{12\sqrt{2}}\, e^{\varphi} (\gamma_{11} \gamma^{[3]} \gamma^\mu \lambda) G_{[3]}$$

$$+ \tfrac{1}{4} e^{-3\varphi/2} (\gamma_{11} \gamma_{[\mu} \gamma^{[2]} \gamma_{\nu]} L^\nu) B_{[2]} - \tfrac{3}{8\sqrt{2}}\, m\, e^{-3\varphi/2} (\gamma_{11} \gamma^{\rho\sigma} \gamma^\mu \lambda) B_{\rho\sigma}$$

$$- \tfrac{1}{4} m\, e^{-5\varphi/2} (\gamma^{\mu\nu} L_\nu) + \tfrac{5}{8\sqrt{2}}\, m\, e^{-5\varphi/2} (\gamma^\mu \lambda) + \mathcal{O}(\mathrm{fm}^3) \doteq 0 \ , \tag{4.4a}$$

$$e^{-1} \left(\frac{\delta \mathcal{L}_{\mathrm{10D}}}{\delta \overline{\chi}} \right) \doteq - \left(b + \tfrac{1}{8} \right) e^{-5\varphi/2} \left(\gamma^\mu L_\mu - \tfrac{\sqrt{5}}{2} m \lambda \right) + \mathcal{O}(\mathrm{fm}^3) \doteq 0 \ . \tag{4.4b}$$

We can compute the covariant divergence of (4.4a), and find that the consistency corresponding to (3.17) in the 4D case as

$$D_\mu \left(\frac{\delta \mathcal{L}_{\mathrm{10D}}}{\delta \overline{\psi}_\mu} \right) \doteq + \left(b + \tfrac{1}{8} \right) m\, e\, e^{-5\varphi} \left(\gamma^\mu L_\mu - \tfrac{\sqrt{5}}{2} m \lambda \right) = + m\, e\, e^{-5\varphi/2} \left(\frac{\delta \mathcal{L}_{\mathrm{10D}}}{\delta \overline{\chi}} \right) \doteq 0 \ . \tag{4.5}$$

All the equalities here holds for arbitrary values of b.

As in the $N = 1$ case in 4D, we can re-confirm this result in the new frame $(e_\mu{}^m,\, \widetilde{\psi}_\mu,\, \omega_\mu{}^{rs},\, A_{[3]},\, B_{[2]},\, \lambda,\, \varphi)$. The the field redefinition for this purpose is

$$\widetilde{\psi}_\mu \equiv \psi_\mu + m^{-1} D_\mu \chi \qquad (L_\mu = m \widetilde{\psi}_\mu) \ , \tag{4.6}$$

thereby we can simplify our field equations by re-casting the lagrangian (4.1) in the new frame as

$$
\begin{aligned}
\mathcal{L}_{10D} = & + \frac{1}{4}\, e\, R - \frac{1}{2}\, e\, (\overline{\widetilde{\psi}}_\mu \gamma^{\mu\nu\rho} D_\nu \widetilde{\psi}_\rho) - \frac{1}{2}\, e\, (\overline{\lambda}\gamma^\mu D_\mu \lambda) - \frac{1}{2}\, e\, (\partial_\mu \varphi)^2 - \frac{1}{48}\, e\, e^{-\varphi} (F_{[4]})^2 \\
& - \frac{1}{12}\, e\, e^{-2\varphi} (G_{[3]})^2 - \frac{1}{4}\, m^2\, e\, e^{-3\varphi} (B_{\mu\nu})^2 + \frac{1}{\sqrt{2}}\, e\, (\overline{\widetilde{\psi}}_\mu \gamma^\nu \gamma^\mu \lambda) \partial_\nu \varphi \\
& + \frac{1}{1152}\, \epsilon^{\mu\nu\rho\sigma\tau\lambda\omega\psi\varphi\chi} \left(F_{\mu\nu\rho\sigma} F_{\tau\lambda\omega\psi} - 8 m F_{\mu\nu\rho\sigma} B_{\tau\lambda} B_{\omega\psi} + \frac{96}{5}\, m^2 B_{\mu\nu} B_{\rho\sigma} B_{\tau\lambda} B_{\omega\psi} \right) B_{\varphi\chi} \\
& - \frac{1}{96}\, e\, e^{-\varphi/2} \left[(\overline{\widetilde{\psi}}^\mu \gamma_{[\mu} \gamma^{[4]} \gamma_{\nu]} \widetilde{\psi}^\nu) - \frac{1}{\sqrt{2}}\, (\overline{\widetilde{\psi}}_\mu \gamma^{[4]} \gamma^\mu \lambda) + \frac{3}{4}\, (\overline{\lambda} \gamma^{[4]} \lambda) \right] F_{[4]} \\
& - \frac{1}{24}\, e\, e^{\varphi} \left[(\overline{\widetilde{\psi}}^\mu \gamma_{11} \gamma_{[\mu} \gamma^{[3]} \gamma_{\nu]} \widetilde{\psi}^\nu) - \sqrt{2}\, e\, (\overline{\widetilde{\psi}}_\mu \gamma_{11} \gamma^{[3]} \gamma^\mu \lambda) \right] G_{[3]} \\
& + \frac{1}{8}\, m\, e\, e^{-3\varphi/\sqrt{2}} \left[(\overline{\widetilde{\psi}}^\mu \gamma_{11} \gamma_{[\mu} \gamma^{\rho\sigma} \gamma_{\nu]} \widetilde{\psi}^\nu) - \frac{3}{\sqrt{2}}\, (\overline{\widetilde{\psi}}_\mu \gamma_{11} \gamma^{\rho\sigma} \gamma^\mu \lambda) + \frac{5}{4}\, (\overline{\lambda} \gamma_{11} \gamma^{\rho\sigma} \lambda) \right] B_{\rho\sigma} \\
& - \frac{1}{8}\, m\, e\, e^{-5\varphi/2} (\overline{\widetilde{\psi}}_\mu \gamma^{\mu\nu} \widetilde{\psi}_\nu) + \frac{5}{8\sqrt{2}}\, m\, e\, e^{-5\varphi/2} (\overline{\widetilde{\psi}}_\mu \gamma^\mu \lambda) + \frac{21}{32}\, m\, e\, e^{-5\varphi/2} (\overline{\lambda}\lambda) \\
& + b\, m^2\, e\, e^{-5\varphi} + \mathcal{O}(\text{fm}^4) \ . \qquad (4.7)
\end{aligned}
$$

The $\widetilde{\psi}_\mu$-field equation is simply (4.4a) with L_μ replaced by $m\widetilde{\psi}$, so we do not give the explicit form here. It is straightforward to show that the divergence of $\widetilde{\psi}_\mu$-field equation is

$$
D_\mu \left(\frac{\delta \mathcal{L}_{10D}}{\delta \widetilde{\psi}_\mu} \right) \doteq + \left(b + \frac{1}{8} \right) m^2\, e\, e^{-5\varphi} \left[(\gamma^\mu \widetilde{\psi}_\mu) - \frac{5}{\sqrt{2}}\, \lambda \right] \ . \qquad (4.8)
$$

Here this has been confirmed up to cubic terms, corresponding to the skipped $\mathcal{O}(\text{fm}^4)$-terms at the lagrangian level.

Compared with the previous 4D case, the λ-field is also involved linearly. However, this does not pose any problem, because this means that when $b \neq -1/8$, the γ-trace component of $\widetilde{\psi}_\mu$ is mixed up with the λ-field. Eventually, the value $b = -1/8$ for type-IIA massive supergravity in 10D [12][8] is *not* the only consistent theory, but *any other* value of b is also allowed. In any case, the vanishing of the right side of (4.8) implies the modified γ-traceless-ness of $\widetilde{\psi}_\mu$, which is nothing but the condition for a massive vector-spinor in 10D. This is nothing peculiar to 10D case, but parallel to the 4D case (3.21).

We stress again that even for $b \neq -1/8$, there is *no* problem with the non-vanishing divergence (4.9), for the same reason we have explained for the case of non-Abelian Proca-Stueckelberg mechanism (section 2), and for the case of nilpotent fermionic symmetry in 4D

[8] Due to the overall negative sign already mentioned in the lagrangian in [12], our value $b = -1/8$ corresponds to the potential term $+ (1/8) m^2 e^{-5\varphi}$ in [12].

(section 3). In other words, we have not only the conventional type-IIA theory, but also our lagrangian (4.1) with nilpotent fermionic symmetry are the consistent theories for massive vector-spinors in 10D. We conclude that the conventional type-IIA supergravity is a special case $b = -1/8$ of the latter with general value of the constant b.

5. Concluding Remarks

In this paper, we have shown that a 4D system with the field content $(e_\mu{}^m, \psi_\mu, \chi, \omega_\mu{}^{rs})$ only with local nilpotent fermionic symmetry, where ψ_μ is the gauge field for the nilpotent fermionic generator N_α, while χ is the corresponding compensator. Our lagrangian contains a real arbitrary constant a, which coincides with massive $N = 1$ supergravity in 4D, iff $a = 3/2$. This system has not only consistent vector-spinor interactions, but also contains conventional $N = 1$ supergravity [3][4][5] as a special case. The usual vector-spinor consistency condition $D_\mu(\delta\mathcal{L}/\delta\psi_\mu) \doteq 0$ can be satisfied not only in the conventional supergravity [3][4][5][6], but also our system of local nilpotent fermionic symmetry, whose gauge field is the vector-spinor ψ_μ. The consistency condition $D_\mu(\delta\mathcal{L}/\delta\psi_\mu) \doteq 0$ for non-supergravity case $(a \neq 3/2)$ has been confirmed both in the original frame $(e_\mu{}^m, \psi_\mu, \chi, \omega_\mu{}^{rs})$, as well as in the frame $(e_\mu{}^m, \widetilde{\psi}_\mu, \omega_\mu{}^{rs})$ in which the commentator χ is absorbed into $\widetilde{\psi}_\mu$.

In a similar fashion, we have applied this formulation to 10D case for the field content $(e_\mu{}^m, \psi_\mu, \omega_\mu{}^{rs}, A_{\mu\nu\rho}, B_{\mu\nu}, \lambda, \varphi, \chi)$. We have established our lagrangian (4.1) with local nilpotent fermionic symmetry, and with an arbitrary real constant b, where $b = -1/8$ corresponds to type IIA massive supergravity [12]. We confirmed its consistency by inspecting the divergence of the ψ_μ-field equation (4.4a) for the general constant b. Our theory contains the conventional type-IIA massive supergravity [12] only as a special case as $b = -1/8$. There is nothing wrong with non-supergravity theory with nilpotent symmetry with vector-spinor in 10D, similarly to our 4D system.

These are counter-examples against the common notion that any supersymmetric system is generated *only* by a more fundamental *supersymmetric* system, such as dimensional reductions from higher dimensions. Our results also imply that supergravity systems are *not* the only systems that provide consistent interactions for vector-spinors, and this seems true in any space-time dimensions [8]. The only restriction for supergravity is $D \leq 11$, because for a supergravity theory to be realized as a special case of nilpotent symmetry, the restriction

$D \leq 11$ is inevitable. However, it is also true that we can construct a system with local fermionic symmetry in any space-time dimensions [8], so if we forget about supergravity, there is *no limit* for space-time dimensions.

The result in this paper is in a sense very natural, considering the series of results [1][2] about nilpotent symmetry already generating so many supersymmetric systems. The result in [1] already indicates that supersymmetry itself is a sub-system of nilpotent symmetry, while the results in [2] in $D = 2+2$ and/or $D = 8+0$ and $D = 7+0$ suggests the existence of fundamental system with nilpotent symmetry generating conventional $N = 1$ supergravity in $D = 3 + 1$.

One important conclusion of our 4D theory is that *not only* the value $a = 3/2$ corresponding to supergravity is consistent for the lagrangian (3.5) or (3.19), *but also* any other value gives consistent interactions between the vector-spinor ψ_μ (not necessarily 'gravitino') and the vierbein $e_\mu{}^m$. In other words, supergravity theory at $a = 3/2$ is just a special case of a more wider set of consistent theories of vector-spinor in 4D. A similar conclusion can be obtained for the values $b \neq -1/8$ for our 10D theory.

Among those values $a \neq 3/2$ in 4D or $b \neq -1/8$ in 10D, the case of the zero-cosmological constant with $a = 0$ or $b = 0$ is also a special case for consistent vector-spinor interactions with nilpotent fermionic symmetry.

Some readers may wonder about the uniqueness of supergravity with consistent interactions following the Haag-Lopuszański-Sohnius (HLS) theorem [14], which seems to dictate that supergravity is the 'unique' consistent gauge theory of a vector-spinor in 4D. Also, our nilpotent fermionic charge N_α satisfying $\{N_\alpha, N_\beta\} = 0$ may turn out to be just a conventional BRST charge $Q_{\rm B}$ [15] in 'disguise'. Indeed, there are certain similarities, such as the nilpotent feature of our charge N_α, or the resemblance to the gauging of the conventional BRST symmetry [9]. Additionally, the nilpotent charges for topological field theories [16][17] is a kind of BRST charges.

However, there are also essential differences: First, our nilpotent charge N_α carries the spinorial index α, while the conventional BRST charge $Q_{\rm B}$ has a single component. Second, our system is still a *classical* system without Faddeev-Popov [18] or other quantum ghosts. Third, in general topological field theories [16][19], their actions or lagrangians are independent of metrics, while our lagrangians do contain vielbeins, and therefore they do depend on metrics. This is also related to the feature that our system has interactions among physical fields, while topological field theories [16] or their BRST charges are associated with *non*-physical fields such as ghosts. However, this apparent contradiction is known to be compromised by the 'twisting of supersymmetry' [19].

From these considerations, two important points are crystalized: First, due to an essential difference between our fermionic charge N_α and the 'conventional' BRST $Q_{\rm B}$ [15][17], special caution is needed for the interpreting the former as a generalized BRST charge. Second, the superficial contradiction between our system and HLS theorem [14] can be resolved by the twisting of supersymmetry [19]. In other words, our nilpotent fermionic symmetry is a generalized BRST symmetry, also interpreted as twisted supersymmetry [19], evading the conventional HLS theorem [14].

Our results constitute counter-examples against the common notion that any supersymmetric system should come out only from a supersymmetric system, such as higher dimensional supersymmetry *via* compactifications. It is also against the general wisdom that supergravity [3][4][5][6] and supersymmetry are the only systems that involve vector-spinors with consistent interactions [14]. Our results seem to indicate that our local nilpotent symmetry (as a generalized BRST symmetry or twisted supersymmetry) is the *master symmetry* superseding local supersymmetry.

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