Searching for new physics with b[over ¯]→s[over ¯] B_{s}^{0}→V_{1}V_{2} penguin decays

Bhubanjyoti Bhattacharya, Alakabha Datta, Murugeswaran Duraisamy, and David London

Phys. Rev. D **88**, 016007 — Published 10 July 2013

DOI: 10.1103/PhysRevD.88.016007

UMISS-HEP-2013-06
UdeM-GPP-TH-13-224

Searching for New Physics with $\bar{b} \to \bar{s}$ $B_s^0 \to V_1 V_2$ Penguin Decays

Bhubanjyoti Bhattacharya [a,1], Alakabha Datta [b,2], Murugeswaran Duraisamy [b,3] and David London [a,4]

a: *Physique des Particules, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7*

b: *Department of Physics and Astronomy, 108 Lewis Hall, University of Mississippi, Oxford, MS 38677-1848, USA*

Abstract

We present the most general (six-helicity) angular analysis of $B_s^0 \to V_1(\to P_1 P_1')V_2(\to P_2 P_2')$ (V_i is a vector meson, and P_i, P_i' are pseudoscalars). We focus on final states accessible to both B_s^0 and $\bar{B}_s^0$ – these are mainly $\bar{b} \to \bar{s}$ penguin decays. We also derive the most general decay amplitude, and discuss the differences between it and that used by LHCb in its analysis of $B_s^0 \to \phi\phi$. In the standard model, all CP violation is predicted to be small, so that the simple measurement of a sizeable CP-violating observable indicates the presence of new physics. A full fit to the data is not necessary. By determining which of the CP-violating observables are nonzero, one can learn about the structure of the underlying NP. Finally, we apply the angular analysis to $B_s^0 \to K^{*0}\bar{K}^{*0}$, and show that there are numerous CP-violating observables that remain in the untagged data sample.

PACS numbers: 11.30.Er, 12.15.Ji, 13.25.Hw, 14.40.Nd

[1] bhujyo@lps.umontreal.ca
[2] datta@phy.olemiss.edu
[3] duraism@phy.olemiss.edu
[4] london@lps.umontreal.ca

1 Introduction

$B \to V_1 V_2$ decays (V_i is a vector meson) are really three separate decays, one for each polarization of the final state (one longitudinal, two transverse). Here it is useful to use the linear polarization basis, where one decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons are either longitudinal (A_0), or transverse to their directions of motion and parallel ($A_\parallel$) or perpendicular ($A_\perp$) to one another. Many years ago it was shown that one can separate these three helicities by performing an angular analysis of the decay [1].

Recently, it was pointed out that, under certain circumstances, modifications must be made to the angular analysis. In particular, when a neutral vector meson is detected via its decay $V \to PP'$ (P, P' are pseudoscalars), there is usually a background coming from the decay of a scalar resonance $S \to PP'$, or from the scalar non-resonant PP' production [2]. As such, it is necessary to add another (scalar) helicity to the angular analysis. The LHCb Collaboration performed this addition in their studies of the decays $B_s^0 \to J/\psi\phi$ [3] and $B_s^0 \to \phi\phi$ [4]. In both cases the ϕ is detected through its decay to K^+K^-, and there is a resonant (f_0) or non-resonant scalar background. Thus, the angular analyses in Refs. [3] and [4] were performed with four and five helicities, respectively.

However, in the experimental analysis of $B_s^0 \to \phi\phi$ [4], the most general $B_s^0 \to \phi\phi$ amplitude was not used[5]. Rather, simplifications were made based on approximations that hold only within the standard model (SM). This then implied that certain new-physics (NP) signals were absent from the angular analysis. But since the goal is to seek signals of NP, it does not make sense to do only a SM-based angular analysis[6]. Furthermore, not all the SM assumptions were physically well-motivated. We must stress that the main result of Ref. [4] – that there is a potential disagreement with the predictions of the SM – is not in question. Our point is simply that it was not sufficiently precise *what* this disagreement is, and what further NP signals are possible.

In addition, we were informed that LHCb is studying the decay $B_s^0 \to K^{*0}(892)\bar{K}^{*0}(892)$, and that each of these vector mesons has a background coming from the scalar resonance $K^{*0}(1430)$ [8]. It is therefore necessary to perform an angular analysis that takes this background into account. In this case, as one does not have identical particles in the final state (in contrast to $B_s^0 \to \phi\phi$), six helicities must be considered.

In light of all of this, we feel it is useful to present the most general angular analysis of $B_s^0 \to V_1(\to P_1 P_1')V_2(\to P_2 P_2')$. We focus on final states accessible to both B_s^0 and $\bar{B}_s^0$, which are mainly $\bar{b} \to \bar{s}$ penguin decays. Our analysis allows for the presence of NP, and we discuss the possible NP signals. Given the LHCb constraints on NP in B_s^0-$\bar{B}_s^0$ mixing [3], there is little sensitivity to NP of this type. However, $B_s^0 \to V_1 V_2$ decays can probe NP in the decay. In particular, since $\bar{b} \to \bar{s}$ penguin decays are dominated by a single contributing amplitude in the SM, all CP-violating observables are predicted to be small. The observation of sizeable CP violation would then be a smoking-gun signal of NP in the decay. In fact, although experiments aim to search for NP via

[5] In the study of $B_s^0 \to J/\psi\phi$ [3], the most general angular analysis was also not performed. Rather, simplifying assumptions were imposed. The importance of including the most general amplitude was stressed in Ref. [5], and in Ref. [6] it was pointed out that the penguin pollution can be reduced if the assumptions are not made, and the full angular analysis done.

[6] A first attempt at a theoretical analysis of $B_s^0 \to \phi\phi$ with the general amplitude including NP was presented in Ref. [7].

a complete fit to the data, such a fit is not really necessary. A more direct way to detect NP is simply to measure a large CP-violating observable. In addition, one can get an idea about the structure of the underlying NP from the pattern of the measurements (i.e., determining which of the CP-violating observables are nonzero).

We apply these ideas to the decays $B_s^0 \to \phi\phi$ and $B_s^0 \to K^{*0}\bar{K}^{*0}$. For $B_s^0 \to \phi\phi$, we compare the amplitude used in Ref. [4] with the exact amplitude, and examine how the differences can affect the fit. For $B_s^0 \to K^{*0}\bar{K}^{*0}$, we use the full (six-helicity) angular analysis to detail which CP-violating observables remain in the untagged data sample. This decay is particularly interesting since, in addition to triple products, certain direct and indirect CP asymmetries can be observed in untagged decays.

In Sec. 2, we present the full six-helicity angular distrinution. We address the question of new physics in Sec. 3. Here we point out that the best way to search for NP is to measure CP-violating observables, and we discuss the four types of observables. In Secs. 4 and 5, we apply the formalism to the decays $B_s^0 \to \phi\phi$ and $B_s^0 \to K^{*0}\bar{K}^{*0}$. We examine a particular model of NP in Sec. 6. Here we show that different NP operators lead to different patterns of nonzero CP-violating observables. We conclude in Sec. 7.

2 General Angular Distribution

We consider the decay $B_s^0 \to V_1 V_2$. As discussed in the introduction, when either vector meson decays to two pseudoscalar mesons, there is generally a background due to the (resonant or non-resonant) scalar production of the two pseudoscalars. We therefore focus on the decay $B_s^0 \to V_1/S_1(\to P_1 P_1')V_2/S_2(\to P_2 P_2')$, concentrating on final states to which both B_s^0 and $\bar{B}_s^0$ can decay.

In general, there are 6 helicities: $h = VV$ (3), VS, SV, and SS, each with a corresponding amplitude A_h. Thus, when the full amplitude is squared, there are 21 terms. Due to B_s^0-$\bar{B}_s^0$ mixing, the amplitude is time dependent. The angular distribution can be written

$$\frac{d^4\Gamma(t)}{dtd\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{8\pi}\sum_{i=1}^{21} K_i(t) X_i(\theta_1,\theta_2,\phi) \ , \tag{1}$$

where θ_1, θ_2 and ϕ are the helicity angles: θ_1 (θ_2) is the angle between the directions of motion of the P_1 (P_2) in the V_1 (V_2) rest frame and the V_1 (V_2) in the B rest frame, and ϕ is the angle between the normals to the planes defined by $P_1 P_1'$ and $P_2 P_2'$ in the B rest frame.

2.1 $t = 0$

Much can be learned by studying the behaviour at $t = 0$. The individual amplitudes are constructed as follows:

$$\mathcal{A}_{VV} = N \sum_{j=-1}^{1} A_j^{VV} Y_1^{-j}(\theta_1, -\phi) Y_1^j(\pi - \theta_2, 0) \ ,$$

$$\mathcal{A}_{VS} = N\ A_0^{VS} Y_1^0(\theta_1, -\phi) Y_0^0(\pi - \theta_2, 0)\ ,$$

$$\mathcal{A}_{SV} = N\ A_0^{SV} Y_0^0(\theta_1, -\phi) Y_1^0(\pi - \theta_2, 0)\ ,$$

$$\mathcal{A}_{SS} = N\ A_0^{SS} Y_0^0(\theta_1, -\phi) Y_0^0(\pi - \theta_2, 0)\ . \tag{2}$$

N is a normalization constant and the Y_l^m are spherical harmonics. Using the standard expressions for the Y_l^m, as well as $A_\parallel = (A_+ + A_-)/\sqrt{2}$ and $A_\perp = (A_+ - A_-)/\sqrt{2}$, we have

$$\mathcal{A}_{VV} + \mathcal{A}_{VS} + \mathcal{A}_{SV} + \mathcal{A}_{SS} = -\frac{3N}{4\pi}\Bigg(A_0 \cos\theta_1 \cos\theta_2 - \frac{A_S}{3} - \frac{A_{VS}}{\sqrt{3}}\cos\theta_1 + \frac{A_{SV}}{\sqrt{3}}\cos\theta_2$$

$$+ \frac{A_\parallel}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \cos\phi + i\frac{A_\perp}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \sin\phi\Bigg)\ . \tag{3}$$

It is convenient to choose a different notation for the VS modes. We introduce the following amplitude coefficients:

$$A_+^{(VS)} \equiv \frac{A_{VS} + A_{SV}}{\sqrt{2}}\ , \quad A_-^{(VS)} \equiv \frac{A_{VS} - A_{SV}}{\sqrt{2}}\ . \tag{4}$$

Using this notation, Eq. (3) can be rewritten as follows:

$$\mathcal{A}_{VV} + \mathcal{A}_{VS} + \mathcal{A}_{SV} + \mathcal{A}_{SS} = -\frac{3N}{4\pi}\Bigg(A_0 \cos\theta_1 \cos\theta_2 - \frac{A_S}{3} - \frac{A_+^{(VS)}}{\sqrt{6}}(\cos\theta_1 - \cos\theta_2)$$

$$-\frac{A_-^{(VS)}}{\sqrt{6}}(\cos\theta_1 + \cos\theta_2) + \frac{A_\parallel}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \cos\phi + i\frac{A_\perp}{\sqrt{2}} \sin\theta_1 \sin\theta_2 \sin\phi\Bigg)\ . \tag{5}$$

We can now construct the $t = 0$ differential decay rate:

$$\frac{d^4\Gamma}{dt d\cos\theta_1 d\cos\theta_2 d\phi} = \frac{9}{8\pi}|\mathcal{A}_{VV} + \mathcal{A}_{VS} + \mathcal{A}_{SV} + \mathcal{A}_{SS}|^2$$

$$= \frac{9}{8\pi}\sum_{n=1}^{21} K_n X_n(\theta_1, \theta_2, \phi)\ . \tag{6}$$

In Table 1 we list the individual K's and X's. The normalization constant N has been chosen such that the integration of Eq. (6) over the entire phase space gives

$$\frac{d\Gamma}{dt} = \sum_h |A_h|^2 \quad = \quad |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 + |A_+^{(VS)}|^2 + |A_-^{(VS)}|^2 + |A_S|^2\ . \tag{7}$$

The various angular functions can be isolated by performing asymmetric integrals over the three angles [9]. For example, consider $n = 5$: $X_5 = -(1/2\sqrt{2}) \sin 2\theta_1 \sin 2\theta_2 \sin\phi$. If one integrates over $0 \le \phi \le \pi$ with a $+$ sign, and $\pi \le \phi \le 2\pi$ with a $-$ sign, one eliminates all the other X_n except those proportional to $\sin\phi$ ($n = 12, 17, 19$). These can be eliminated by integrating asymmetrically over each of $\theta_{1,2}$: $0 \le \theta_{1,2} \le \pi/2$ with a $+$ sign, and $\pi/2 \le \theta_{1,2} \le \pi$ with a $-$ sign. The other X_n can be isolated similarly. The one exception involves $n = 7, 13, 20$. The difference $X_7 - X_{13}$ is proportional to $\cos\theta_1 \cos\theta_2$, as is X_{20}. These two can therefore not be differentiated. However, apart from this lone exception, all the X_n can be isolated experimentally.

Table 1: Individual K's and X's listed in Eq. (6)

n	K_n	X_n
1	$\lvert A_0\rvert^2$	$\cos^2\theta_1 \cos^2\theta_2$
2	$\lvert A_\parallel\rvert^2$	$\frac{1}{2}\sin^2\theta_1 \sin^2\theta_2 \cos^2\phi$
3	$\lvert A_\perp\rvert^2$	$\frac{1}{2}\sin^2\theta_1 \sin^2\theta_2 \sin^2\phi$
4	$\mathrm{Re}[A_\parallel A_0^*]$	$\frac{1}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \cos\phi$
5	$\mathrm{Im}[A_\perp A_0^*]$	$-\frac{1}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \sin\phi$
6	$\mathrm{Im}[A_\perp A_\parallel^*]$	$-\frac{1}{2}\sin^2\theta_1 \sin^2\theta_2 \sin 2\phi$
7	$\lvert A_+^{(VS)}\rvert^2$	$\frac{1}{6}(\cos\theta_1 - \cos\theta_2)^2$
8	$\mathrm{Re}[A_+^{(VS)} A_S^*]$	$\frac{\sqrt{2}}{3\sqrt{3}}(\cos\theta_1 - \cos\theta_2)$
9	$\mathrm{Re}[A_+^{(VS)} A_-^{(VS)*}]$	$\frac{1}{3}(\cos^2\theta_1 - \cos^2\theta_2)$
10	$\mathrm{Re}[A_+^{(VS)} A_0^*]$	$-\frac{\sqrt{2}}{\sqrt{3}}\cos\theta_1\cos\theta_2(\cos\theta_1 - \cos\theta_2)$
11	$\mathrm{Re}[A_+^{(VS)} A_\parallel^*]$	$-\frac{1}{\sqrt{3}}\sin\theta_1\sin\theta_2\cos\phi(\cos\theta_1 - \cos\theta_2)$
12	$\mathrm{Im}[A_\perp A_+^{(VS)*}]$	$\frac{1}{\sqrt{3}}\sin\theta_1\sin\theta_2\sin\phi(\cos\theta_1 - \cos\theta_2)$
13	$\lvert A_-^{(VS)}\rvert^2$	$\frac{1}{6}(\cos\theta_1 + \cos\theta_2)^2$
14	$\mathrm{Re}[A_-^{(VS)} A_S^*]$	$\frac{\sqrt{2}}{3\sqrt{3}}(\cos\theta_1 + \cos\theta_2)$
15	$\mathrm{Re}[A_-^{(VS)} A_0^*]$	$-\frac{\sqrt{2}}{\sqrt{3}}\cos\theta_1\cos\theta_2(\cos\theta_1 + \cos\theta_2)$
16	$\mathrm{Re}[A_-^{(VS)} A_\parallel^*]$	$-\frac{1}{\sqrt{3}}\sin\theta_1\sin\theta_2\cos\phi(\cos\theta_1 + \cos\theta_2)$
17	$\mathrm{Im}[A_\perp A_-^{(VS)*}]$	$\frac{1}{\sqrt{3}}\sin\theta_1\sin\theta_2\sin\phi(\cos\theta_1 + \cos\theta_2)$
18	$\mathrm{Re}[A_S A_\parallel^*]$	$-\frac{\sqrt{2}}{3}\sin\theta_1\sin\theta_2\cos\phi$
19	$\mathrm{Im}[A_\perp A_S^*]$	$\frac{\sqrt{2}}{3}\sin\theta_1\sin\theta_2\sin\phi$
20	$\mathrm{Re}[A_S A_0^*]$	$-\frac{2}{3}\cos\theta_1\cos\theta_2$
21	$\lvert A_S\rvert^2$	$\frac{1}{9}$

2.2 Time-dependent angular distribution

Due to B_s^0-$\bar{B}_s^0$ mixing, the time evolution of the states $|B_s^0(t)\rangle$ and $|\bar{B}_s^0(t)\rangle$ can be described by the relations

$$\begin{aligned} |B_s^0(t)\rangle &= g_+(t)\ |B_s^0\rangle + \frac{q}{p}\, g_-(t)\ |\bar{B}_s^0\rangle\ , \\ |\bar{B}_s^0(t)\rangle &= \frac{p}{q}\, g_-(t)\ |B_s^0\rangle +\ g_+(t)\ |\bar{B}_s^0\rangle\ , \end{aligned} \tag{8}$$

where $q/p = (V_{tb}^* V_{ts})/(V_{tb} V_{ts}^*) \equiv e^{-i\phi_M}$. (Here we follow the notation of LHCb: ϕ_M is the theoretical phase of B_s^0-$\bar{B}_s^0$ mixing, while ϕ_s is its experimentally-measured value.) In the above, we have

$$\begin{aligned} g_+(t) &= \frac{1}{2}\Big(e^{-(iM_L+\Gamma_L/2)t}\ + e^{-(iM_H+\Gamma_H/2)t}\Big)\ , \\ g_-(t) &= \frac{1}{2}\Big(e^{-(iM_L+\Gamma_L/2)t}\ - e^{-(iM_H+\Gamma_H/2)t}\Big)\ , \end{aligned} \tag{9}$$

where L and H indicate the light and heavy states, respectively. The average mass and width are $m = (M_H + M_L)/2$ and $\Gamma = (\Gamma_L + \Gamma_H)/2$, while the mass and width differences of the B_s^0-meson eigenstates are defined as $\Delta m \equiv M_H - M_L$ and $\Delta\Gamma \equiv \Gamma_L - \Gamma_H$. Δm is positive by definition. For B_s^0 mesons, $\Delta\Gamma_s$ is reasonably large, and is positive in our convention.

Now, the time dependence of the transversity amplitudes A_h is due to B_s^0-$\bar{B}_s^0$ mixing. Their precise form depends on the specific final state. The different helicities of the VV and the SS final states are all CP eigenstates. On the other hand, the VS and SV states are not CP eigenstates. However, their linear combinations, defined as $|\pm\rangle_{VS} \equiv (|VS\rangle \pm |SV\rangle)/\sqrt{2}$, *are* CP eigenstates. Working only with CP eigenstates, the time-dependent amplitudes are given as [10]

$$\begin{aligned} A_h(t) &= \langle f|H_W|B_s^0(t)\rangle_h = \Big[g_+(t)A_h + \eta_h\ q/p\ g_-(t)\ \bar{A}_h\Big]\ , \\ \bar{A}_h(t) &= \langle f|H_W|\bar{B}_s^0(t)\rangle_h = \Big[p/q\ g_-(t)A_h + \eta_h\ g_+(t)\ \bar{A}_h\Big]\ , \end{aligned} \tag{10}$$

where $A_h = \langle f|H_W|B_s^0\rangle_h$, $\bar{A}_h = \langle f|H_W|\bar{B}_s^0\rangle_h$, and $\langle \bar{f}| = \eta_h \langle f|$, with $\eta_h = +1$ for $h = 0, \|, VS_-, SS$ and $\eta_h = -1$ for $h = VS_+, \perp$. These values for the CP eigenvalue η_h can be understood in terms of the total angular momentum of the final state. States with $l = 0$ (SS, VS_-, a combination of $0, \|$) and $l = 2$ (another combination of $0, \|$) are CP even, while those with $l = 1$ (VS_+, $\perp$) are CP odd. It is also important to point out the CP properties of the helicity amplitudes A_h and $\bar{A}_h$:

$$\begin{aligned} CP\ A_h &= CP\ \langle f|H_W|B_s^0\rangle_h = \langle \bar{f}|H_W|\bar{B}_s^0\rangle_h = \eta_h \langle f|\bar{B}_s^0\rangle_h = \eta_h \bar{A}_h\ , \\ CP\ \bar{A}_h &= CP\ \langle f|H_W|\bar{B}_s^0\rangle_h = \langle \bar{f}|H_W|B_s^0\rangle_h = \eta_h \langle f|B_s^0\rangle_h = \eta_h A_h\ . \end{aligned} \tag{11}$$

Thus, in order to go from the B_s^0 decay to the $\bar{B}_s^0$ decay, one simply needs to switch $A_h \leftrightarrow \eta_h \bar{A}_h$.

It is useful to discuss the origin of the η_h factors in Eq. (10) since, naively, such factors are not present. This is understood most easily by considering $B_s^0 \to K^{*0}\bar{K}^{*0}$. As noted above, in the decay $B_s^0 \to V_1 V_2$, the helicity angles are defined with respect to the momenta of V_1 and V_2. In $B_s^0 \to K^{*0}\bar{K}^{*0}$, $V_1 = K^{*0}$ and $V_2 = \bar{K}^{*0}$. On the other hand, in the CP-conjugate decay $\bar{B}_s^0 \to \bar{K}^{*0} K^{*0}$ we have $V_1 = \bar{K}^{*0}$ and $V_2 = K^{*0}$. That is, we have $V_1 \leftrightarrow V_2$ compared to the B_s^0 decay. The effect of this on the helicity angles is to change $\theta_1 \leftrightarrow \theta_2$ and $\phi \to -\phi$. Looking at Table 1, we see that the X_n change sign for $n = 5$, 6, 8-12, 19, i.e., when only one CP-odd state is involved in the K_n. This sign change can be transferred to the $A_h(t)$ by adding the η_h factors in Eq. (10). That is, by using this definition for the $A_h(t)$, the angular functions are the same for B_s^0 and $\bar{B}_s^0$ decays, which makes it easy to compute what is measured in untagged samples.

The expressions for the time-dependent wave functions for the various terms are

$$\begin{aligned}
|A_i(t)|^2 &= \frac{1}{2} e^{-\Gamma t} \Big[\left(|A_i|^2 + |\bar{A}_i|^2 \right) \cosh{(\Delta\Gamma/2)t} \\
&\quad - 2\ \eta_i \ \mathrm{Re}\left(A_i^* \bar{A}_i e^{-i\phi_M} \right) \sinh{(\Delta\Gamma/2)t} \\
&\quad + \ \left(|A_i|^2 - |\bar{A}_i|^2 \right) \cos{\Delta m t} \\
&\quad - 2\ \eta_i \ \mathrm{Im}\left(A_i^* \bar{A}_i e^{-i\phi_M} \right) \sin{\Delta m t} \Big] \ , \\
\mathrm{Im}(A_\perp(t) A_j^*(t)) &= \frac{1}{2} e^{-\Gamma t} \Big[\mathrm{Im}\left(A_\perp A_j^* - \eta_j \bar{A}_\perp \bar{A}_j^* \right) \cosh{(\Delta\Gamma/2)t} \\
&\quad + \mathrm{Im}\left[(\bar{A}_\perp A_j^* + \eta_j A_\perp^* \bar{A}_j) e^{-i\phi_M} \right] \sinh{(\Delta\Gamma/2)t} \\
&\quad + \mathrm{Im}\left(A_\perp A_j^* + \eta_j \bar{A}_\perp \bar{A}_j^* \right) \cos{\Delta m t} \\
&\quad - \mathrm{Re}\left[(\bar{A}_\perp A_j^* + \eta_j A_\perp^* \bar{A}_j) e^{-i\phi_M} \right] \sin{\Delta m t} \Big] \ , \\
\mathrm{Re}(A_k(t) A_l^*(t)) &= \frac{1}{2} e^{-\Gamma t} \Big[\mathrm{Re}\left(A_k A_l^* + \eta_k \eta_l \bar{A}_k \bar{A}_l^* \right) \cosh{(\Delta\Gamma/2)t} \\
&\quad + \mathrm{Re}\left(A_k A_l^* - \eta_k \eta_l \bar{A}_k \bar{A}_l^* \right) \cos{\Delta m t} \\
&\quad - \mathrm{Re}\left[(\eta_k \bar{A}_k A_l^* + \eta_l A_k^* \bar{A}_l) e^{-i\phi_M} \right] \sinh{(\Delta\Gamma/2)t} \\
&\quad - \mathrm{Im}\left[(\eta_k \bar{A}_k A_l^* + \eta_l A_k^* \bar{A}_l) e^{-i\phi_M} \right] \sin{\Delta m t} \Big] \ . \qquad (12)
\end{aligned}$$

Using the above, it is possible to write down the time dependence of the functions K_n listed in Table 1. In general, we have

$$K_n(t) = \frac{1}{2} e^{-\Gamma t} \left[a_n \cosh{(\Delta\Gamma/2)t} + b_n \sinh{(\Delta\Gamma/2)t} + c_n \cos{\Delta m t} + d_n \sin{\Delta m t} \right] \ , \qquad (13)$$

where the individual functions a_n, b_n, c_n, and d_n for $n = 1, \ldots, 21$ are time independent. In Tables 2 and 3 we present the forms of the coefficients a_n-d_n. These are exact and hold even in the presence of NP. Note that not all of the a_n-d_n are independent. There are 23 unknown parameters – 12 magnitudes of A_h and $\bar{A}_h$, and 11 relative phases (ϕ_M can be absorbed into the phases of the A_h and $\bar{A}_h$) – while there are 84 different a_n-d_n. There are therefore many relations among the a_n-d_n.

Table 2: a_n's and c_n's as defined in Eq. (13)

n	$K_n(t)$	a_n	c_n
1	$\lvert A_0(t)\rvert^2$	$\lvert A_0\rvert^2 + \lvert\bar{A}_0\rvert^2$	$\lvert A_0\rvert^2 - \lvert\bar{A}_0\rvert^2$
2	$\lvert A_\parallel(t)\rvert^2$	$\lvert A_\parallel\rvert^2 + \lvert\bar{A}_\parallel\rvert^2$	$\lvert A_\parallel\rvert^2 - \lvert\bar{A}_\parallel\rvert^2$
3	$\lvert A_\perp(t)\rvert^2$	$\lvert A_\perp\rvert^2 + \lvert\bar{A}_\perp\rvert^2$	$\lvert A_\perp\rvert^2 - \lvert\bar{A}_\perp\rvert^2$
4	$\mathrm{Re}[A_\parallel(t)A_0^*(t)]$	$\mathrm{Re}[A_\parallel A_0^* + \bar{A}_\parallel \bar{A}_0^*]$	$\mathrm{Re}[A_\parallel A_0^* - \bar{A}_\parallel \bar{A}_0^*]$
5	$\mathrm{Im}[A_\perp(t)A_0^*(t)]$	$\mathrm{Im}[A_\perp A_0^* - \bar{A}_\perp \bar{A}_0^*]$	$\mathrm{Im}[A_\perp A_0^* + \bar{A}_\perp \bar{A}_0^*]$
6	$\mathrm{Im}[A_\perp(t)A_\parallel^*(t)]$	$\mathrm{Im}[A_\perp A_\parallel^* - \bar{A}_\perp \bar{A}_\parallel^*]$	$\mathrm{Im}[A_\perp A_\parallel^* + \bar{A}_\perp \bar{A}_\parallel^*]$
7	$\lvert A_+^{(VS)}(t)\rvert^2$	$\lvert A_+^{(VS)}\rvert^2 + \lvert\bar{A}_+^{(VS)}\rvert^2$	$\lvert A_+^{(VS)}\rvert^2 - \lvert\bar{A}_+^{(VS)}\rvert^2$
8	$\mathrm{Re}[A_+^{(VS)}(t)A_S^*(t)]$	$\mathrm{Re}[A_+^{(VS)} A_S^* - \bar{A}_+^{(VS)} \bar{A}_S^*]$	$\mathrm{Re}[A_+^{(VS)} A_S^* + \bar{A}_+^{(VS)} \bar{A}_S^*]$
9	$\mathrm{Re}[A_+^{(VS)}(t)A_-^{(VS)*}(t)]$	$\mathrm{Re}[A_+^{(VS)} A_-^{(VS)*} - \bar{A}_+^{(VS)} \bar{A}_-^{(VS)*}]$	$\mathrm{Re}[A_+^{(VS)} A_-^{(VS)*} + \bar{A}_+^{(VS)} \bar{A}_-^{(VS)*}]$
10	$\mathrm{Re}[A_+^{(VS)}(t)A_0^*(t)]$	$\mathrm{Re}[A_+^{(VS)} A_0^* - \bar{A}_+^{(VS)} \bar{A}_0^*]$	$\mathrm{Re}[A_+^{(VS)} A_0^* + \bar{A}_+^{(VS)} \bar{A}_0^*]$
11	$\mathrm{Re}[A_+^{(VS)}(t)A_\parallel^*(t)]$	$\mathrm{Re}[A_+^{(VS)} A_\parallel^* - \bar{A}_+^{(VS)} \bar{A}_\parallel^*]$	$\mathrm{Re}[A_+^{(VS)} A_\parallel^* + \bar{A}_+^{(VS)} \bar{A}_\parallel^*]$
12	$\mathrm{Im}[A_\perp(t)A_+^{(VS)*}(t)]$	$\mathrm{Im}[A_\perp A_+^{(VS)*} + \bar{A}_\perp \bar{A}_+^{(VS)*}]$	$\mathrm{Im}[A_\perp A_+^{(VS)*} - \bar{A}_\perp \bar{A}_+^{(VS)}]$
13	$\lvert A_-^{(VS)}(t)\rvert^2$	$\lvert A_-^{(VS)}\rvert^2 + \lvert\bar{A}_-^{(VS)}\rvert^2$	$\lvert A_-^{(VS)}\rvert^2 - \lvert\bar{A}_-^{(VS)}\rvert^2$
14	$\mathrm{Re}[A_-^{(VS)}(t)A_S^*(t)]$	$\mathrm{Re}[A_-^{(VS)} A_S^* + \bar{A}_-^{(VS)} \bar{A}_S^*]$	$\mathrm{Re}[A_-^{(VS)} A_S^* - \bar{A}_-^{(VS)} \bar{A}_S^*]$
15	$\mathrm{Re}[A_-^{(VS)}(t)A_0^*(t)]$	$\mathrm{Re}[A_-^{(VS)} A_0^* + \bar{A}_-^{(VS)} \bar{A}_0^*]$	$\mathrm{Re}[A_-^{(VS)} A_0^* - \bar{A}_-^{(VS)} \bar{A}_0^*]$
16	$\mathrm{Re}[A_-^{(VS)}(t)A_\parallel^*(t)]$	$\mathrm{Re}[A_-^{(VS)} A_\parallel^* + \bar{A}_-^{(VS)} \bar{A}_\parallel^*]$	$\mathrm{Re}[A_-^{(VS)} A_\parallel^* - \bar{A}_-^{(VS)} \bar{A}_\parallel^*]$
17	$\mathrm{Im}[A_\perp(t)A_-^{(VS)*}(t)]$	$\mathrm{Im}[A_\perp A_-^{(VS)*} - \bar{A}_\perp \bar{A}_-^{(VS)}]$	$\mathrm{Im}[A_\perp A_-^{(VS)*} + \bar{A}_\perp \bar{A}_-^{(VS)*}]$
18	$\mathrm{Re}[A_S(t)A_\parallel^*(t)]$	$\mathrm{Re}[A_S A_\parallel^* + \bar{A}_S \bar{A}_\parallel^*]$	$\mathrm{Re}[A_S A_\parallel^* - \bar{A}_S \bar{A}_\parallel^*]$
19	$\mathrm{Im}[A_\perp(t)A_S^*(t)]$	$\mathrm{Im}[A_\perp A_S^* - \bar{A}_\perp \bar{A}_S^*]$	$\mathrm{Im}[A_\perp A_S^* + \bar{A}_\perp \bar{A}_S^*]$
20	$\mathrm{Re}[A_S(t)A_0^*(t)]$	$\mathrm{Re}[A_S A_0^* + \bar{A}_S \bar{A}_0^*]$	$\mathrm{Re}[A_S A_0^* - \bar{A}_S \bar{A}_0^*]$
21	$\lvert A_S(t)\rvert^2$	$\lvert A_S\rvert^2 + \lvert\bar{A}_S\rvert^2$	$\lvert A_S\rvert^2 - \lvert\bar{A}_S\rvert^2$

3 Searching for New Physics

The result of the previous section is quite theoretical. In order to understand better how to use it to search for NP[7], it is necessary to know what the SM predictions are. Above we focused on final

[7] A different method for searching for NP in $\bar{b} \to \bar{s}$ $B_s^0 \to V_1 V_2$ penguin decays is discussed in Ref. [11]. Here the idea is to use flavor SU(3) symmetry and obtain information from measurements of the SU(3)-related $\bar{b} \to \bar{d}$ B^0 decays.

Table 3: b_n's and d_n's as defined in Eq. (13)

n	$K_n(t)$	b_n	d_n
1	$\|A_0(t)\|^2$	$-2\ \mathrm{Re}[A_0^* \bar{A}_0\ e^{-i\phi_M}]$	$-2\ \mathrm{Im}[A_0^* \bar{A}_0\ e^{-i\phi_M}]$
2	$\|A_\|(t)\|^2$	$-2\ \mathrm{Re}[A_\|^* \bar{A}_\|\ e^{-i\phi_M}]$	$-2\ \mathrm{Im}[A_\|^* \bar{A}_\|\ e^{-i\phi_M}]$
3	$\|A_\perp(t)\|^2$	$2\ \mathrm{Re}[A_\perp^* \bar{A}_\perp\ e^{-i\phi_M}]$	$2\ \mathrm{Im}[A_\perp^* \bar{A}_\perp\ e^{-i\phi_M}]$
4	$\mathrm{Re}[A_\|(t) A_0^*(t)]$	$-\mathrm{Re}[(\bar{A}_\| A_0^* + A_\|^* \bar{A}_0)\ e^{-i\phi_M}]$	$-\mathrm{Im}[(\bar{A}_\| A_0^* + A_\|^* \bar{A}_0)\ e^{-i\phi_M}]$
5	$\mathrm{Im}[A_\perp(t) A_0^*(t)]$	$\mathrm{Im}[(\bar{A}_\perp A_0^* + A_\perp^* \bar{A}_0)\ e^{-i\phi_M}]$	$-\mathrm{Re}[(\bar{A}_\perp A_0^* + A_\perp^* \bar{A}_0)\ e^{-i\phi_M}]$
6	$\mathrm{Im}[A_\perp(t) A_\|^*(t)]$	$\mathrm{Im}[(\bar{A}_\perp A_\|^* + A_\perp^* \bar{A}_\|)\ e^{-i\phi_M}]$	$-\mathrm{Re}[(\bar{A}_\perp A_\|^* + A_\perp^* \bar{A}_\|)\ e^{-i\phi_M}]$
7	$\|A_+^{(VS)}(t)\|^2$	$2\ \mathrm{Re}[A_+^{(VS)*} \bar{A}_+^{(VS)}\ e^{-i\phi_M}]$	$2\ \mathrm{Im}[A_+^{(VS)*} A_+^{(VS)}\ e^{-i\phi_M}]$
8	$\mathrm{Re}[A_+^{(VS)}(t) A_S^*(t)]$	$\mathrm{Re}[(\bar{A}_+^{(VS)} A_S^* - A_+^{(VS)*} \bar{A}_S)\ e^{-i\phi_M}]$	$\mathrm{Im}[(\bar{A}_+^{(VS)} A_S^* - A_+^{(VS)*} \bar{A}_S)\ e^{-i\phi_M}]$
9	$\mathrm{Re}[A_+^{(VS)}(t) A_-^{(VS)*}(t)]$	$\mathrm{Re}[(\bar{A}_+^{(VS)} A_-^{(VS)*}$ $- A_+^{(VS)*} \bar{A}_-^{(VS)})\ e^{-i\phi_M}]$	$\mathrm{Im}[(\bar{A}_+^{(VS)} A_+^{(VS)*}$ $- A_+^{(VS)} \bar{A}_+^{(VS)})\ e^{-i\phi_M}]$
10	$\mathrm{Re}[A_+^{(VS)}(t) A_0^*(t)]$	$\mathrm{Re}[(\bar{A}_+^{(VS)} A_0^* - A_+^{(VS)*} \bar{A}_0)\ e^{-i\phi_M}]$	$\mathrm{Im}[(\bar{A}_+^{(VS)} A_0^* - A_+^{(VS)*} \bar{A}_0)\ e^{-i\phi_M}]$
11	$\mathrm{Re}[A_+^{(VS)}(t) A_\|^*(t)]$	$\mathrm{Re}[(\bar{A}_+^{(VS)} A_\|^* - A_+^{(VS)*} \bar{A}_\|)\ e^{-i\phi_M}]$	$\mathrm{Im}[(\bar{A}_+^{(VS)} A_\|^* - A_+^{(VS)*} \bar{A}_\|)\ e^{-i\phi_M}]$
12	$\mathrm{Im}[A_\perp(t) A_+^{(VS)*}(t)]$	$\mathrm{Im}[(\bar{A}_\perp A_+^{(VS)*} - A_\perp^* \bar{A}_+^{(VS)})\ e^{-i\phi_M}]$	$-\mathrm{Re}[(\bar{A}_\perp A_+^{(VS)*} - A_\perp^* \bar{A}_+^{(VS)})\ e^{-i\phi_M}]$
13	$\|A_-^{(VS)}(t)\|^2$	$-2\ \mathrm{Re}[A_-^{(VS)*} \bar{A}_-^{(VS)}\ e^{-i\phi_M}]$	$-2\ \mathrm{Im}[A_-^{(VS)*} \bar{A}_-^{(VS)}\ e^{-i\phi_M}]$
14	$\mathrm{Re}[A_-^{(VS)}(t) A_S^*(t)]$	$-\mathrm{Re}[(\bar{A}_-^{(VS)} A_S^* + A_-^{(VS)*} \bar{A}_S)\ e^{-i\phi_M}]$	$-\mathrm{Im}[(\bar{A}_-^{(VS)} A_S^* + A_-^{(VS)*} \bar{A}_S)\ e^{-i\phi_M}]$
15	$\mathrm{Re}[A_-^{(VS)}(t) A_0^*(t)]$	$-\mathrm{Re}[(\bar{A}_-^{(VS)} A_0^* + A_-^{(VS)} \bar{A}_0)\ e^{-i\phi_M}]$	$-\mathrm{Im}[(\bar{A}_-^{(VS)} A_0^* + A_-^{(VS)*} \bar{A}_0)\ e^{-i\phi_M}]$
16	$\mathrm{Re}[A_-^{(VS)}(t) A_\|^*(t)]$	$-\mathrm{Re}[(\bar{A}_-^{(VS)} A_\|^* + A_-^{(VS)*} \bar{A}_\|)\ e^{-i\phi_M}]$	$-\mathrm{Im}[(\bar{A}_-^{(VS)} A_\|^* + A_-^{(VS)*} \bar{A}_\|)\ e^{-i\phi_M}]$
17	$\mathrm{Im}[A_\perp(t) A_-^{(VS)*}(t)]$	$\mathrm{Im}[(\bar{A}_\perp A_-^{(VS)*} + A_\perp^* \bar{A}_-^{(VS)})\ e^{-i\phi_M}]$	$-\mathrm{Re}[(\bar{A}_\perp A_-^{(VS)*} + A_\perp^* \bar{A}_-^{(VS)})\ e^{-i\phi_M}]$
18	$\mathrm{Re}[A_S(t) A_\|^*(t)]$	$-\mathrm{Re}[(\bar{A}_S A_\|^* + A_S^* \bar{A}_\|)\ e^{-i\phi_M}]$	$-\mathrm{Im}[(\bar{A}_S A_\|^* + A_S^* \bar{A}_\|)\ e^{-i\phi_M}]$
19	$\mathrm{Im}[A_\perp(t) A_S^*(t)]$	$\mathrm{Im}[(\bar{A}_\perp A_S^* + A_\perp^* \bar{A}_S)\ e^{-i\phi_M}]$	$-\mathrm{Re}[(\bar{A}_\perp A_S^* + A_\perp^* \bar{A}_S)\ e^{-i\phi_M}]$
20	$\mathrm{Re}[A_S(t) A_0^*(t)]$	$-\mathrm{Re}[(\bar{A}_S A_0^* + A_S^* \bar{A}_0)\ e^{-i\phi_M}]$	$-\mathrm{Im}[(\bar{A}_S A_0^* + A_S^* \bar{A}_0)\ e^{-i\phi_M}]$
21	$\|A_S(t)\|^2$	$-2\ \mathrm{Re}[A_S^* \bar{A}_S\ e^{-i\phi_M}]$	$-2\ \mathrm{Im}[A_S^* \bar{A}_S\ e^{-i\phi_M}]$

states to which both B_s^0 and $\bar{B}_s^0$ can decay. This restricts the analysis to $\bar{b} \to \bar{s}$ transitions, so that the quark content of the final state is $s\bar{s}s\bar{s}$ ($\phi\phi$), $s\bar{s}d\bar{d}$ ($K^{*0}\bar{K}^{*0}$), or $s\bar{s}u\bar{u}$ ($K^{*+}K^{*-}$, $\phi\rho$). Decays to $\phi\phi$ and $K^{*0}\bar{K}^{*0}$ are pure gluonic penguin decays, and $K^{*+}K^{*-}$ is dominated by the gluonic penguin (there is a small tree contribution). The decay to $\phi\rho$ has no gluonic penguin component – it arises due to electroweak penguin and tree diagrams. As such, its branching ratio is quite a bit smaller than that of the other decays, so that even if it is eventually measured, it is not clear if an angular analysis can be done.

One quantity that is of interest in such decays is the indirect (mixing-induced) CP-violating asymmetry (CPA). For a given helicity h, the indirect CPA measures

$$\mathrm{Im}\left(e^{-i\phi_M} A_h^* \bar{A}_h\right) \ . \tag{14}$$

Note that the above quantity, which corresponds to the d_n of Table 3, is sensitive to the weak phases of both the mixing and the decay. Now, in the SM the gluonic penguin arises dominantly from the top loop at short distance. Given that B_s^0-$\bar{B}_s^0$ mixing is also dominated by the box diagram with an internal top quark, it is clear that the $e^{-i\phi_M}$ term in Eq. (14) cancels the weak phase in $A_h^* \bar{A}_h$, so that the indirect CPA vanishes. This is a common argument.

However, things are a bit more complicated. In particular, one also has to consider the new up and charm penguin amplitudes that are generated at the b mass scale. These contributions can arise from the tree-level operators $\bar{b} \to \bar{c}c\bar{s}$ and $\bar{b} \to \bar{u}u\bar{s}$ that produce the final-state particles through rescattering: $\bar{c}c \to \bar{q}q$ and $\bar{u}u \to \bar{q}q$ where $q = d, s$. For a given helicity h, the $\bar{b} \to \bar{s}$ gluonic penguin amplitude can be written

$$\begin{aligned} A_h &= V_{tb}^* V_{ts} P'_{t,h} + V_{cb}^* V_{cs} P'_{c,h} + V_{ub}^* V_{us} P'_{u,h} \\ &= |V_{tb}^* V_{ts}| e^{-i\phi_M/2} P'_{tc,h} + |V_{ub}^* V_{us}| e^{i\gamma} P'_{uc,h} \ . \end{aligned} \tag{15}$$

(As this is a $\bar{b} \to \bar{s}$ transition, the diagrams are written with primes.) In the second line, we have used the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix ($V_{tb}^* V_{ts} + V_{cb}^* V_{cs} + V_{ub}^* V_{us} = 0$) to eliminate the c-quark contribution: $P'_{tc,h} \equiv P'_{t,h} - P'_{c,h}$, $P'_{uc} \equiv P'_{u,h} - P'_{c,h}$. We have also explicitly written the weak-phase dependence, while $P'_{tc,h}$ and $P'_{uc,h}$ contain strong phases.

Now, we know that $|V_{tb}^* V_{ts}|$ and $|V_{ub}^* V_{us}|$ are $O(\lambda^2)$ and $O(\lambda^4)$, respectively, where $\lambda = 0.22$ is the sine of the Cabibbo angle. This implies that the $|V_{ub}^* V_{us}| P'_{uc,h}$ term is much smaller in magnitude than $|V_{tb}^* V_{ts}| P'_{tc,h}$. If $|V_{ub}^* V_{us}| P'_{uc,h}$ is neglected, then the $e^{-i\phi_M}$ term in Eq. (14) cancels the weak phase in $A_h^* \bar{A}_h$, so that the indirect CPA vanishes. However, while the result (a vanishing indirect CPA) is correct, the argument leading to it is not. The easiest way to see this is to use CKM unitarity to eliminate the t-quark contribution in the first line of Eq. (15). The amplitude now reads

$$A_h = |V_{cb}^* V_{cs}| P'_{ct,h} + |V_{ub}^* V_{us}| e^{i\gamma} P'_{ut,h} \ , \tag{16}$$

where $P'_{ct,h} \equiv P'_{c,h} - P'_{t,h}$, $P'_{ut} \equiv P'_{u,h} - P'_{t,h}$. Now if $|V_{ub}^* V_{us}| P'_{ut,h}$ is neglected, there is no cancellation of the $e^{-i\phi_M}$ term in Eq. (14), and the indirect CPA is (apparently) nonzero. So there appears to be a contradiction.

What is really going on is the following. $|V_{ub}^* V_{us}|$ is $O(\lambda^4)$. If it is neglected, for consistency one must neglect *all* $O(\lambda^4)$ terms. One of these is $\mathrm{Im}(V_{tb}^* V_{ts})$, so that $V_{tb}^* V_{ts}$ is real. And since $\phi_M \propto \arg(V_{tb}^* V_{ts})$, it too vanishes in the limit that $O(\lambda^4)$ terms are neglected. So, at the end of the day, we recover the result of a vanishing indirect CPA. The difference is that here the up and charm penguin contributions have been properly taken into account.

The weak phase of A_h is therefore generated by keeping the $|V_{ub}^* V_{us}|$ term. The amplitude can then be written

$$\begin{aligned} A_h &= |V_{tb}^* V_{ts}| e^{-i\phi_M/2} P'_{tc,h} + |V_{ub}^* V_{us}| e^{i\gamma} P'_{uc,h} \\ &= e^{-i\phi_M/2} \left[|V_{tb}^* V_{ts}| P'_{tc,h} + |V_{ub}^* V_{us}| e^{i(\gamma + \phi_M/2)} P'_{uc,h} \right] \ . \end{aligned} \tag{17}$$

Writing the strong phases explicitly, we have

$$A_h = e^{-i\phi_M/2} \left[P'_{tc,h} e^{i\delta_{tc,h}} + P'_{uc,h} e^{i(\gamma + \phi_M/2)} e^{i\delta_{uc,h}} \right] \ . \tag{18}$$

In the above, $P'_{tc,h}$ and $P'_{uc,h}$ have been redefined to absorb $|V_{tb}^* V_{ts}|$ and $|V_{ub}^* V_{us}|$, respectively, so that $R_h \equiv P'_{uc,h}/P'_{tc,h} = O(\lambda^2)$.

The SM a_n-d_n can be calculated using Tables 2 and 3 with the above expression for A_h. These take the general form $P'_{tc,h} P'_{tc,h'}$ multiplied by either quantities of $O(1)$ or $R_h \equiv P'_{uc,h}/P'_{tc,h}$. In the SM, those of the second type are expected to be smaller than those of the first type since $R_h = O(\lambda^2)$. In fact, the coefficients proportional to R_h are all CP-violating observables: direct CP asymmetries, indirect CP asymmetries, triple products, and mixing-induced triple products. Physically, this makes sense – CP violation is due to the interference of two amplitudes. But in the SM one of the amplitudes ($P'_{uc,h}$) is quite small, so that all CP-violating observables are also small. As shown below, this is a key point in the search for new physics.

New physics can enter in two different places – in B_s^0-$\bar{B}_s^0$ mixing or in the decay. We discuss these in turn below.

3.1 NP in the mixing

If there is NP in B_s^0-$\bar{B}_s^0$ mixing, this has two consequences. First, ϕ_M, which is predicted to be $\simeq 0$ in the SM, could be large. Second, the weak phase associated with $P'_{tc,h}$ will not, in general, be equal to $e^{-i\phi_M/2}$, so a nonzero indirect CPA could appear. However, LHCb has already measured the phase of B_s^0-$\bar{B}_s^0$ mixing in $B_s^0 \to J/\psi\phi$ [12]. They find

$$\phi_s = 0.07 \pm 0.09 \text{ (stat)} \pm 0.01 \text{ (syst) rad} , \tag{19}$$

in agreement with the SM. While the errors are large enough that NP cannot be excluded, a very large deviation from 0 is ruled out.

Given this, it appears that, at present, $\bar{b} \to \bar{s}$ penguin decays are not sensitive to NP in B_s^0-$\bar{B}_s^0$ mixing. Put another way, it is probably best to search for such NP using the decay mode $B_s^0 \to J/\psi\phi$.

3.2 NP in the decay

The second, more interesting possibility is that there is NP in the decay. In this case the amplitude takes the form (we neglect $P'_{uc,h}$ and ϕ_M)

$$A_h = P'_{tc,h} e^{i\delta_{tc,h}} + P'_{NP,h} e^{i\phi_{NP}} e^{i\delta_{NP,h}} . \tag{20}$$

This has the same form as Eq. (18), with $P'_{uc,h} \to P'_{NP,h}$, $(\gamma + \phi_M/2) \to \phi_{NP}$ and $\delta_{uc,h} \to \delta_{NP,h}$. As a result, the expressions for the a_n-d_n are the same as in the SM, with these substitutions.

The signal for NP in the decay is then evident. In the presence of NP, R_h is equal to $P'_{NP,h}/P'_{tc,h}$, which can be significantly larger than its SM value, $O(\lambda^2)$. As noted above, the a_n-d_n proportional to R_h all correspond to CP-violating observables. These can only be large in the presence of a sizeable second amplitude, i.e., NP in the decay.

3.3 Measuring CP-violating observables

The bottom line is that the angular analysis of $B_s^0 \to V_1/S_1(\to P_1 P_1')V_2/S_2(\to P_2 P_2')$ is sensitive to NP in the decay. In order to search for this NP, measurements must be made of the CP-violating observables. Here we discuss these in more detail, referring to Tables 2 and 3.

As noted above, there are four such observables. In general, the direct CP asymmetries take the form $\mathrm{Re}[A_h A_{h'}^* - \bar{A}_h \bar{A}_{h'}^*]$ (for $h = h'$, this becomes the familiar $|A_h|^2 - |\bar{A}_h|^2$). The indirect (mixing-induced) CP asymmetries are $\mathrm{Im}[(A_h^* \bar{A}_{h'} + \bar{A}_h A_{h'}^*)\, e^{-i\phi_M}]$.

The triple products (TPs) [13] are a little more complicated. For a B_s^0 decay, the TP takes the form $\mathrm{Im}[A_\perp A_h^*]$. Now, the amplitudes possess both weak and strong phases. However, the TP can be nonzero even in the absence of any weak phases, as long as the strong-phase difference is nonzero. Thus, a nonzero TP is not necessarily a signal of CP violation. In order to obtain a true CP-violating signal, one has to compare the TPs in B_s^0 and $\bar{B}_s^0$ decays. This latter TP is given by $\mathrm{Im}[\bar{A}_\perp \bar{A}_h^*]$. One combination of B_s^0 and $\bar{B}_s^0$ TPs is nonzero only if the weak phases are nonzero, and so is called a true (CP-violating) TP. The second combination can be nonzero even if the weak phases are zero, and so it is not a signal of CP violation – it is called a fake TP. The true TP takes the form $\mathrm{Im}[A_\perp A_h^* - \bar{A}_\perp \bar{A}_h^*]$. One can also have TPs induced by B_s^0-$\bar{B}_s^0$ mixing. The true mixing-induced TP is $\mathrm{Im}[(\bar{A}_\perp A_h^* + A_\perp^* \bar{A}_h)\, e^{-i\phi_M}]$.

The coefficients corresponding to the four CP-violating observables are

1. direct CP asymmetries: c_n ($n = $ 1-4, 7, 13-16, 18, 20, 21), a_n ($n = $ 8-11);

2. indirect CP asymmetries: d_n ($n = $ 1-4, 7, 13-16, 18, 20, 21), b_n ($n = $ 8-11);

3. triple products: a_n ($n = 5, 6, 17, 19$), c_{12};

4. mixing-induced triple products: b_n ($n = 5, 6, 17, 19$), d_{12}.

If any coefficient is measured to be significantly larger than the SM prediction ($O(\lambda^2)$), this would be a sign of NP in the decay. Note that, in general, the direct CP asymmetries, indirect CP asymmetries, TPs, and mixing-induced TPs are represented by the c_n, d_n, a_n and b_n, respectively. However, this pattern is broken for $n = $ 8-12. The reason is that these observables involve $A_+^{(VS)}$, which is CP odd. That is, in going from the B_s^0 to $\bar{B}_s^0$ decay, $A_+^{(VS)} \to -\bar{A}_+^{(VS)}$. This additional minus sign leads to the pattern breaking above.

As noted in Sec. 2.1, it is *not* necessary to do the full angular analysis to measure these observables. Rather, by performing asymmetric integrals over the three angles, one can isolate (almost) any angular function, i.e., value of n. Then one uses the time-dependence of Eq. (13) to distinguish among the a_n-d_n. Indeed, this has already been done in Refs. [14, 15] for the TPs a_5 and a_6 in the simpler case of time-integrated untagged $B_s^0 \to \phi\phi$ decays.

In fact, it should be stressed that at this stage there is no point in trying to perform a full angular analysis. The aim of such an analysis would be to determine the NP parameters. However, unless a NP signal is found, this is irrelevant. We therefore suggest that experiments concentrate on measuring the a_n-d_n that are expected to be small in the SM.

In the following two sections we discuss the above formalism in the context of the specific decays $B_s^0 \to \phi\phi$ and $B_s^0 \to K^{*0}\bar{K}^{*0}$.

4 $B_s^0 \to \phi\phi$

The LHCb Collaboration studied $B_s^0 \to \phi\phi$ in Ref. [4]. Their analysis uses the following logic. They argue that, since $B_s^0 \to \phi\phi$ proceeds via a gluonic $\bar{b} \to \bar{s}s\bar{s}$ diagram with a t quark in the loop, the mixing and decay weak phases cancel identically. QCD factorization calculations, which take into account the up and charm penguin contributions, find an upper limit of 0.02 rad for $|\theta_s|$ [16], where θ_s is the phase in the $B_s^0 \to \phi\phi$ decay. Putting this all together, LHCb writes each helicity amplitude as

$$A_h = |A_h| e^{i\theta_s/2} e^{i\delta_h} \ , \tag{21}$$

where δ_h is the strong phase. Here θ_s is taken to be a weak phase and is assumed to be helicity-independent.

The angular analysis of $B_s^0 \to \phi\phi$ is as given in Sec. 2, except that, for this final state, we have $|VS\rangle = -|SV\rangle$ so that the VS_+ state vanishes. This implies that the $A_+^{(VS)}(t)$ amplitude is not present. The expressions for the coefficients a_n-d_n are found using the formulae in Tables 2 and 3, in which A_h is given in Eq. (21). These are listed in Table 4 for $n =$ 1-6 (in total, n goes to 15, but LHCb finds that the VS and SS contributions are very small). We define $\delta_1 \equiv \delta_\perp - \delta_\parallel$, $\delta_2 \equiv \delta_\perp - \delta_0$, and $\delta_{2,1} \equiv \delta_2 - \delta_1$. Table 4 agrees with Ref. [4].

Table 4: a_n-d_n's ($n =$ 1-6) for $B_s^0 \to \phi\phi$. A_h takes the form in Eq. (21).

n	N	a_n/N	b_n/N	c_n/N	d_n/N
1	$2\|A_0\|^2$	1	$-\cos\theta_s$	0	$\sin\theta_s$
2	$2\|A_\parallel\|^2$	1	$-\cos\theta_s$	0	$\sin\theta_s$
3	$2\|A_\perp\|^2$	1	$\cos\theta_s$	0	$-\sin\theta_s$
4	$2\|A_0\|\|A_\parallel\|$	$\cos\delta_{2,1}$	$-\cos\delta_{2,1}\cos\theta_s$	0	$\cos\delta_{2,1}\sin\theta_s$
5	$2\|A_0\|\|A_\perp\|$	0	$-\cos\delta_2\sin\theta_s$	$\sin\delta_2$	$-\cos\delta_2\cos\theta_s$
6	$2\|A_\parallel\|\|A_\perp\|$	0	$-\cos\delta_1\sin\theta_s$	$\sin\delta_1$	$-\cos\delta_1\cos\theta_s$

Of course, the form assumed for the A_h [Eq. (21)] has specific implications for the expressions for the a_n-d_n. For example, the fact that $c_n = 0$ ($n =$ 1-4) and $a_n = 0$ ($n =$ 5,6) is a direct consequence. In addition, the quantity $\sin\theta_s$ appears explicitly in a number of entries in Table 4, so that $\sin\theta_s = \pm d_n/a_n$ ($n =$ 1-4). Also, $\tan\theta_s = b_n/d_n$ ($n = 5, 6$). This allows LHCb to restrict the value of θ_s to the interval of $[-2.46, -0.76]$ rad at 68% C.L. As explained above, θ_s is expected to be quite small in the SM, so this result is an intriguing hint of NP.

The problem is that the logic leading to Eq. (21) is somewhat faulty, and the form assumed for A_h is not the most general. Indeed, a rather strong assumption has been made, one that is not well-motivated physically. The exact amplitude is given in Eq. (18). One can rephase it by $e^{i\phi_M/2}$, giving

$$\begin{aligned} A_h &= P'_{tc,h} e^{i\delta_{tc,h}} + P'_{uc,h} e^{i(\gamma+\phi_M/2)} e^{i\delta_{uc,h}} \\ &= P'_{tc,h} e^{i\delta_{tc,h}} \left[1 + R_h e^{i(\gamma+\phi_M/2) e^{i\Delta_h}} \right] \ , \end{aligned} \tag{22}$$

where $\Delta_h = \delta_{uc,h} - \delta_{tc,h}$. If we assume that $\Delta_h = 0$ and that R_h is helicity-independent, we can write the piece in square brackets as $Xe^{i\theta_s/2}$, which yields Eq. (21). Here we see that θ_s is indeed small, $O(\lambda^2)$, and this is due to the smallness of R_h.

Unfortunately, the assumption that $\Delta_h = 0$ is generally false and is not supported by any computation. Given that the starting point, Eq. (21), is questionable, this raises serious warning signs about the result. Is the large measured value of θ_s really a hint of NP, or is it just a consequence of an incorrect assumption?

Still, one question remains: although Eq. (21) is not an exact parametrization of the amplitude, could it be taken as an approximation? And here the answer is yes. The key point is that, even if $\Delta_h = 0$ is not assumed, the piece in square brackets in Eq. (22) can be written as $Xe^{i\theta_{s,h}/2}$. However, as $\theta_{s,h}$ is small in the SM, the assumptions that it is helicity independent and purely a weak phase are acceptable provided one is not measuring quantities of the expected size of θ_s, $O(\lambda^2)$. In other words, if a value of θ_s is measured that is much larger than the SM expectation, then this would be a sign of NP. However, if the measured θ_s is small, then even if it deviates from the SM expectation, one cannot reliably claim the presence of NP.

But this then raises another question: if such large NP effects are present, are they best detected by performing a fit to the data? Here the answer is clearly no. The only way one can find a large value of θ_s is if there are CP-violating observables whose values are much larger than in the SM. But in this case, the simple measurement of these observables will reveal the presence of NP – it is not necessary to perform a full fit to the data.

The bottom line is that if one wishes to perform a fit, it is best to do the analysis using the exact amplitude of Eq. (18). This will be difficult, as there are considerably more unknown parameters than in Eq. (21). However, as stated previously, at this stage a full angular analysis is not even warranted. Experiments measuring $B_s^0 \to \phi\phi$ should simply focus on measuring the CP-violating observables, since these are expected to be small in the SM.

5 $B_s^0 \to K^{*0}\bar{K}^{*0}$

As noted in the previous section, LHCb finds that the VS and SS contributions to $B_s^0 \to \phi\phi$ are very small. This is not surprising. The resonant scalar background comes from the decay $f_0 \to K^+K^-$. However, the dominant f_0 decay is to $\pi\pi$ – the Particle Data Group notes only that $f_0 \to K\bar{K}$ has been "seen" [17]. The $f_0 \to \pi\pi$ decay is an important background for decays in which a final-state ρ^0 is produced. Indeed, measurements of the decay $B^0 \to \rho^0\rho^0$ [18] had to take this background into account. However, B_s^0 decays to final states involving a ρ^0 are rare.

One decay for which the scalar-background contributions are clearly significant is $B_s^0 \to K^{*0}\bar{K}^{*0}$ [8]. Here the final-state vector meson is the $K^{*0}(892)$, identified through its decay to $K^+\pi^-$. However, the scalar meson $K^{*0}(1430)$ decays almost exclusively to the same final state. And since its width is (270 ± 80) MeV [17], it constitutes an important background. Finally, since the final state does not involve identical particles, both additional amplitudes A_{VS} and A_{SV} (or equivalently $A_+^{(VS)}$ and $A_-^{(VS)}$ [Eq. (4)]) must be considered.

When this angular analysis is done, the first step will be to examine the untagged decays. In order to find which observables are present in these decays, one proceeds as follows. The time-dependent transversity amplitudes for the $\bar{B}_s^0$ decay ($\bar{K}_n$'s) can be obtained by interchanging $A_h \leftrightarrow \eta_h \bar{A}_h$ and changing the sign of the weak phase ϕ_M. One can write an equation similar to Eq. (13) in the $\bar{B}_s^0$ case as follows:

$$\bar{K}_n(t) = \frac{1}{2} e^{-\Gamma t} \left[\bar{a}_n \cosh{(\Delta\Gamma/2)t} + \bar{b}_n \sinh{(\Delta\Gamma/2)t} + \bar{c}_n \cos{\Delta m t} + \bar{d}_n \sin{\Delta m t} \right] \ , \tag{23}$$

where once again $\bar{a}_n, \bar{b}_n, \bar{c}_n$, and $\bar{d}_n$ for $n = 1, \ldots, 21$ are time-independent functions of A_h and $\bar{A}_h$.

It is straightforward to show that

$$\bar{a}_n = a_n \quad , \quad \bar{c}_n = -c_n \quad , \quad \bar{b}_n = b_n \quad , \quad \bar{d}_n = -d_n \ . \tag{24}$$

With these results the transversity amplitudes for the *untagged* decay are

$$K_n^{\text{untagged}}(t) = K_n(t) + \bar{K}_n(t) = e^{-\Gamma t} \left[a_n \cosh{(\Delta\Gamma/2)t} + b_n \sinh{(\Delta\Gamma/2)t} \right] \ . \tag{25}$$

As stressed above, experiments should focus on measuring the CP-violating observables. In the untagged case, these are (see Sec. 3.3)

1. triple products: a_n ($n = 5, 6, 17, 19$);
2. mixing-induced triple products: b_n ($n = 5, 6, 17, 19$);
3. direct CP asymmetries: a_n ($n =$ 8-11);
4. indirect CP asymmetries: b_n ($n =$ 8-11).

As can be seen, all four types of CP-violating observables are accessible in untagged $B_s^0 \to K^{*0} \bar{K}^{*0}$ decays, given that the scalar-background contributions are important.

It is perhaps surprising to find direct CP asymmetries in the untagged sample. After all, we usually think that such observables require tagging. However, their presence in the above list can be understood as follows. As mentioned earlier, the general form for a direct CP asymmetry is $\text{Re}[A_h A_{h'}^* - \bar{A}_h \bar{A}_{h'}^*]$. Now, the B-decay contribution is $\text{Re}[A_h A_{h'}^*]$, while that for the $\bar{B}$ is obtained by taking $A_{h,h'} \to \eta_{h,h'} \bar{A}_{h,h'}$, where $\eta_{h,h'} = +1$ (-1) if $A_{h,h'}$ is CP even (CP odd). The observable in untagged decays is then the sum of the B and $\bar{B}$ contributions:

$$\text{Re}[A_h A_{h'}^* + \eta_h \eta_{h'} \bar{A}_h \bar{A}_{h'}^*] \ . \tag{26}$$

Suppose first that both A_h and $A_{h'}$ are CP even. In this case $\eta_h \eta_{h'} = +1$ and the observable in Eq. (26) is not a direct CP asymmetry. This is the case for $n = 4$, for example. On the other hand, suppose that one of A_h and $A_{h'}$ is CP odd. Now we have $\eta_h \eta_{h'} = -1$ and the observable in Eq. (26) *is* a direct CP asymmetry. This is what is occurring for $n =$ 8-11. Here the CP-odd amplitude $A_+^{(VS)}$ is involved, and this leads to the direct CP asymmetries in the untagged sample. A similar logic applies to the indirect CP asymmetries.

6 New Physics

Above we have stressed that measurements should be made of the CP-violating observables. Such measurements are sensitive to NP in the decay. In this section we examine a model of NP that can yield such effects. Although we focus on a particular NP scenario, our analysis is easily applicable to other NP models.

The recent discovery of a Higgs-like resonance at the LHC [19], along with supporting evidence for its existence from Fermilab [20], have renewed interest in models with an extended Higgs sector. An accurate determination of the couplings of the new state to quarks, including flavor-changing neutral-current (FCNC) couplings, is clearly very important. Constraints on possible FCNC couplings of this state have recently been examined [21].

We consider a model with an extended Higgs sector in which the neutral scalars have FCNC couplings. We identify the lowest-mass state as the X particle, and assume that X mediates the decay $\bar{b} \to \bar{s} q \bar{q}$, where $q = d, s$. (There may be contributions to the decay from heavier states, but in order to retain predictive power, we assume that that the dominant contribution comes from the lowest-lying state X.) X may be identified with the newly-discovered particle of mass ~ 125 GeV, but this is not important for our discussion. However, it should be pointed out that the penguin $B_s^0 \to V_1 V_2$ decays have the potential to explore the coupling of the new scalar state to light quarks, which is not possible at collider experiments.

After integrating out the X state, we generate the effective Hamiltonian [22, 23]

$$H_{NP} = \frac{4G_F}{\sqrt{2}} \sum_{A,B=L,R} f_q^{AB}\, \bar{b}\gamma_A s\, \bar{q}\gamma_B q \ , \tag{27}$$

with a total of four contributing operators ($A, B = L, R$, $q = d, s$). In order to determine the contribution of each operator to the various observables, it is necessary to calculate the hadronic matrix elements. However, instead of computing these using a particular model, we prefer to simply make some general observations. To do this, we introduce two small parameters:

1. We assume that the NP contribution to any observable is smaller than that of the dominant SM amplitude, but larger than the subdominant SM amplitude of $O(\lambda^2)$. This is reasonable since larger NP contributions would likely have already been seen in experiments. We therefore define $\epsilon \equiv |NP|/|SM|$, and take its value to be $\sim 20\%$.

2. We introduce the heavy-quark expansion parameter $\epsilon_b \sim \Lambda_{QCD}/m_b$. Generically, we expect that $\epsilon_b \sim$ 10-20 %.

We also make the assumption that the NP matrix elements can be estimated by naive factorization. This is very reasonable since any correction to naive factorization would typically be $O(\epsilon\epsilon_b)$. Note that, with this assumption, the scalar operators cannot directly produce the VV or VS final states – they can only do so after a Fierz transformation. Moreover, as the NP operators do not contain charm quarks, possible large nonperturbative rescattering effects are absent [24]. A consequence of this is that the NP amplitudes have strong phases that are 0 or π. (This can be justified on more general grounds [22, 25].) Since we are considering NP effects with large new weak phases, we can neglect the small weak phases in the SM amplitudes. Hence the SM and NP amplitudes, respectively s_h and n_h, take the following forms:

$$s_h = \bar{s}_h = |s_h| e^{i\delta_h} \quad , \qquad n_h = \bar{n}_h^* = |n_h| e^{i\phi_h} \ , \tag{28}$$

where ϕ_h are the NP weak phases and δ_h are the SM strong phases.

To simplify things we concentrate on one NP operator at a time, and consider its effect on the process $B_s^0 \to K^{*0} \bar{K}^{*0}$ (for this decay, a Fierz transformation is also needed to produce the SS final state). The procedure is to obtain the form of each helicity amplitude A_h in the presence of the NP operator, and then to compute the CP-violating observables that appear in the untagged distribution. We focus on the triple products a_n ($n = 5, 6, 17, 19$) and the direct CP asymmetries a_n (n = 8-11). Assuming the mixing phase to be small, the mixing-induced triple products b_n ($n = 5, 6, 17, 19$) do not provide additional information over that already contained in the triple products, and so we do not calculate them. Similarly, the indirect CP asymmetries b_n (n = 8-11) contain the same information as the a_n (n = 8-11) when the mixing phase is neglected. The triple products arise from the interference of $A_\perp$ with the other amplitudes, while the direct CP asymmetries arise from the interference of $A_+^{(VS)}$ with the other amplitudes. Note that, while there are SM contributions to all the A_h, $s_\| = -s_\perp$ in the heavy-quark limit [26].

We consider the following three cases.

- Case a: We begin with the NP operator of $\bar{b} \to \bar{s} d \bar{d}$ whose coefficient is f_d^{RR} [Eq. (27)]:

$$\frac{4G_F}{\sqrt{2}} f_d^{RR} \, \bar{b}\gamma_R s \, \bar{d}\gamma_R d \ . \tag{29}$$

  For this operator to contribute to the decay, we perform a Fierz transformation (both fermions and colors):

$$-\frac{4}{N_c}\frac{G_F}{\sqrt{2}} f_d^{RR} \left[\frac{1}{2}\, \bar{b}\gamma_R d \, \bar{d}\gamma_R s + \frac{1}{8}\, \bar{b}\sigma^{\mu\nu}\gamma_R d \, \bar{d}\sigma_{\mu\nu}\gamma_R s \right] \ . \tag{30}$$

  Under the factorization assumption the currents produce the final-state mesons, so that the scalar currents cannot produce vector mesons and the tensor currents cannot produce scalar mesons. Thus the first term contributes only to the SS state within factorization and in the heavy-quark limit, while the second term contributes only to the VV states. We will make use of the following factorization results.

  – To leading order in $1/m_b$ we have

$$\langle VV | \, \bar{b}\gamma_R d \, \bar{d}\gamma_R s \, | B \rangle = 0 \quad , \qquad \langle (VS)_\pm | \, \bar{b}\gamma_R d \, \bar{d}\gamma_R s \, | B \rangle = 0 \ . \tag{31}$$

  The results above are due to $\langle V | \, \bar{b}\gamma_R d \, | B \rangle = 0$ [27] and $\langle V | \, \bar{d}\gamma_R s \, | 0 \rangle = 0$.

  – The matrix element $\langle VV | \, \bar{b}\sigma^{\mu\nu}\gamma_R d \, \bar{d}\sigma_{\mu\nu}\gamma_R s \, | B \rangle$ was worked out in Refs. [23, 26], with the result that the contribution to the longitudinal amplitude is $\sim 1/m_b$ while the transverse amplitudes are unsuppressed. We also note that for the VS states, the amplitude in which the scalar is produced from the vacuum vanishes as tensor operators cannot produce a scalar meson from the vacuum. The amplitude in which the vector state is produced from the vacuum can be shown to be suppressed by $\sim 1/m_b$. It is also true for reasons stated above that the tensor operators cannot produce the SS state.

Using the results discussed above, and keeping terms up to linear in ϵ and ϵ_b, we can write the amplitudes as

$$\begin{aligned} A_0 = s_0 \quad &, \quad A_\perp = s_\perp + n_\perp^{RR} \ , \\ A_\| = -s_\perp + n_\perp^{RR} + O(\epsilon_b) \quad &, \quad A_+^{(VS)} = s_+^{(VS)} \ , \\ A_-^{(VS)} = s_-^{(VS)} \quad &, \quad A_{SS} = s_{SS} + n_{SS}^{RR} \ . \end{aligned} \tag{32}$$

The prediction for this operator is that one should observe nonzero values for all the triple products while the direct CP-violation terms a_9 and a_{10} should be small.

- Case b: Similar to the example above, for the f_d^{LL} operator the amplitudes are

$$\begin{aligned} A_0 = s_0 \quad &, \quad A_\perp = s_\perp + n_\perp^{LL} \ , \\ A_\| = -s_\perp - n_\perp^{LL} + O(\epsilon_b) \quad &, \quad A_+^{(VS)} = s_+^{(VS)} \ , \\ A_-^{(VS)} = s_-^{(VS)} \quad &, \quad A_{SS} = s_{SS} + n_{SS}^{LL} \ . \end{aligned} \tag{33}$$

  Hence the prediction is that all true triple products have similar sizes, except for a_6 which should be small. As in the previous case, the direct CP-violation terms a_9 and a_{10} should be small.

- Case c: Finally, we consider the case when we have the operators, f_d^{LR} and f_d^{RL} [Eq. (27)]. The Fierz transformation produces $(V-A)\times(V+A)$ and $(V+A)\times(V-A)$ operators. In this case the NP transverse amplitudes are suppressed by ϵ_b. We can write the amplitudes as,

$$\begin{aligned} A_0 = s_0 + n_0^{LR} - n_0^{RL} \quad &, \quad A_\perp = s_\perp \ , \\ A_\| = -s_\perp + O(\epsilon_b) \quad &, \quad A_+^{(VS)} = s_+^{(VS)} + n_+^{LR,VS} - n_+^{RL,VS} \ , \\ A_-^{(VS)} = s_-^{(VS)} + n_-^{LR,VS} - n_-^{RL,VS} \quad &, \quad A_{SS} = s_{SS} + n_{SS}^{LR} - n_{SS}^{RL} \ . \end{aligned} \tag{34}$$

  In this case the triple product a_6 is small, but the other TPs will generally be nonzero. This case is different from Case b above as the direct CP-violation terms a_9 and a_{10} are not small.

We therefore see that the three cases make different predictions for the CP-violating terms in the untagged distribution. As a result, one can learn about the nature of the underlying NP from the pattern of the measurements. If the tagged measurements are also available, then the additional CP-violating observables can be used to further pinpoint the structure of the NP.

7 Conclusions

It is well known that the amplitude for $B \to V_1 V_2$ (V_i is a vector meson) can be decomposed in terms of three helicities – A_0, $A_\|$, $A_\perp$ – and that these can be separated experimentally by performing an angular analysis of the decay. Recently it was pointed out that if a neutral vector meson is detected via its decay $V \to PP'$ (P, P' are pseudoscalars), there is usually a background coming from scalar resonant or non-resonant PP' production. This can be taken into account by adding another (scalar) helicity to the angular analysis.

Since the ϕ is detected through its decay to K^+K^-, LHCb performed this addition in their studies of $B_s^0 \to J/\psi\phi$ [3] and $B_s^0 \to \phi\phi$ [4]. For the first decay there were four helicities in the angular analysis, while in the second there were five. LHCb is also examining $B_s^0 \to K^{*0}(892)\bar{K}^{*0}(892)$. In this case, the angular analysis requires six helicities since there are no identical particles in the final state.

Also, in its analysis of $B_s^0 \to \phi\phi$, LHCb did not use the most general decay amplitude. This raises the question of whether the result of the analysis (an intriguing hint of NP) is due to the chosen form of the amplitude.

In this paper, we address the above issues. We present the most general (six-helicity) angular analysis of $B_s^0 \to V_1(\to P_1P_1')V_2(\to P_2P_2')$. We focus on final states to which both B_s^0 and $\bar{B}_s^0$ can decay. These are mainly $\bar{b} \to \bar{s}$ penguin transitions. We also derive the most general decay amplitude. We show that the amplitude used by LHCb in Ref. [4] makes an assumption regarding the strong phases that is not reproduced by direct calculation.

One of the reasons that LHCb used its form of the decay amplitude is that it contains a small number of unknown parameters. This permits a search for NP via a full fit to the data. However, the most general amplitude contains more unknowns, so that a full fit is considerably more difficult. Fortunately, a fit is not necessary to detect NP. Since $\bar{b} \to \bar{s}$ penguin decays are dominated by a single contributing amplitude in the SM, all CP-violating observables are predicted to be small. The presence of NP would then be clearly indicated by the simple measurement of a sizeable CP-violating observable. There are four such observables – direct CP asymmetries, indirect CP asymmetries, triple products, and mixing-induced triple products – and we discuss all of these in the context of the six-helicity angular analysis.

We apply this analysis to the decay $B_s^0 \to K^{*0}\bar{K}^{*0}$. In particular, we examine which CP-violating observables remain in the untagged data sample. Triple products and mixing-induced triple products are of course present. In addition, because this decay has a CP-odd background, certain direct and indirect CP asymmetries can be observed in untagged decays. This is a particuliarly interesting aspect of $B_s^0 \to K^{*0}\bar{K}^{*0}$.

Finally, one can learn about the nature of the underlying NP by determining which of the CP-violating observables are nonzero. To demonstrate this, we consider a particular model of NP and show that different NP operators make different predictions for the pattern of sizeable CP-violating observables.

Acknowledgments: A special thanks goes to Bernardo Adeva for his invaluable input to this project. We also thank Paula Álvarez, Brais Sanmartin, Antonio Romero, Sean Benson, Franz Muheim, Stéphane Monteil and Olivier Leroy for helpful communications. This work was financially supported by NSERC of Canada (BB, DL), and by the National Science Foundation under Grant No. NSF PHY-1068052 (AD, MD).

References

[1] A. S. Dighe, I. Dunietz, H. J. Lipkin and J. L. Rosner, Phys. Lett. B **369**, 144 (1996) [hep-ph/9511363]; I. Dunietz and R. Fleischer, Phys. Rev. D **55**, 259 (1997) [hep-ph/9605220]; A. S. Dighe, I. Dunietz and R. Fleischer, Eur. Phys. J. C **6**, 647 (1999) [hep-ph/9804253]; C. -W. Chiang and L. Wolfenstein, Phys. Rev. D **61**, 074031 (2000) [hep-ph/9911338].

[2] S. Stone and L. Zhang, Phys. Rev. D **79**, 074024 (2009) [arXiv:0812.2832 [hep-ph]]; Y. Xie, P. Clarke, G. Cowan and F. Muheim, JHEP **0909**, 074 (2009) [arXiv:0908.3627 [hep-ph]].

[3] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **108**, 101803 (2012) [arXiv:1112.3183 [hep-ex]].

[4] R. Aaij *et al.* [LHCb Collaboration], arXiv:1303.7125 [hep-ex].

[5] A. Datta and S. Khalil, Phys. Rev. D **80**, 075006 (2009) [arXiv:0905.2105 [hep-ph]], C. -W. Chiang, A. Datta, M. Duraisamy, D. London, M. Nagashima and A. Szynkman, JHEP **1004**, 031 (2010) [arXiv:0910.2929 [hep-ph]].

[6] B. Bhattacharya, A. Datta and D. London, arXiv:1209.1413 [hep-ph].

[7] A. Datta, M. Duraisamy and D. London, Phys. Rev. D **86**, 076011 (2012) [arXiv:1207.4495 [hep-ph]].

[8] B. Adeva, private communication.

[9] For example, see Ref. [7] and references therein.

[10] I. Dunietz and R. Fleischer, Ref. [1].

[11] R. Fleischer and M. Gronau, Phys. Lett. B **660**, 212 (2008) [arXiv:0709.4013 [hep-ph]].

[12] R. Aaij *et al.* [LHCb Collaboration], arXiv:1304.2600 [hep-ex].

[13] A. Datta and D. London, Int. J. Mod. Phys. A **19**, 2505 (2004) [hep-ph/0303159].

[14] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **107**, 261802 (2011) [arXiv:1107.4999 [hep-ex]].

[15] R. Aaij *et al.* [LHCb Collaboration], arXiv:1204.2813 [hep-ex].

[16] M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B **774**, 64 (2007) [hep-ph/0612290]; M. Bartsch, G. Buchalla and C. Kraus, arXiv:0810.0249 [hep-ph]; H. -Y. Cheng and C. -K. Chua, Phys. Rev. D **80**, 114026 (2009) [arXiv:0910.5237 [hep-ph]].

[17] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).

[18] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **78**, 071104 (2008) [arXiv:0807.4977 [hep-ex]]; I. Adachi *et al.* [Belle Collaboration], arXiv:1212.4015 [hep-ex].

[19] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[20] T. Aaltonen *et al.* [CDF and D0 Collaborations], Phys. Rev. Lett. **109** (2012) 071804 [arXiv:1207.6436 [hep-ex]].

[21] G. Blankenburg, J. Ellis and G. Isidori, Phys. Lett. B **712**, 386 (2012) [arXiv:1202.5704 [hep-ph]]; R. Harnik, J. Kopp and J. Zupan, arXiv:1209.1397 [hep-ph]; D. Boubaa, A. Datta, M. Duraisamy and S. Khalil, arXiv:1211.5168 [hep-ph].

[22] A. Datta and D. London, Phys. Lett. B **595**, 453 (2004) [hep-ph/0404130].

[23] S. Baek, A. Datta, P. Hamel, O. F. Hernandez and D. London, Phys. Rev. D **72**, 094008 (2005) [hep-ph/0508149].

[24] M. Duraisamy and A. L. Kagan, Eur. Phys. J. C **70**, 921 (2010) [arXiv:0812.3162 [hep-ph]].

[25] A. Datta, M. Imbeault, D. London, V. Page, N. Sinha and R. Sinha, Phys. Rev. D **71**, 096002 (2005) [hep-ph/0406192].

[26] A. Datta, M. Duraisamy and D. London, Phys. Lett. B **701**, 357 (2011) [arXiv:1103.2442 [hep-ph]].

[27] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **60**, 014001 (1999) [hep-ph/9812358].