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Lattice calculation of composite dark matter form factors

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Composite dark matter candidates, which can arise from new strongly-coupled sectors, are well-motivated and phenomenologically interesting, particularly in the context of asymmetric generation of the relic density. In this work, we employ lattice calculations to study the electromagnetic form factors of electroweak-neutral dark-matter baryons for a three-color, QCD-like theory with $N_f = 2$ and 6 degenerate fermions in the fundamental representation. We calculate the (connected) charge radius and anomalous magnetic moment, both of which can play a significant role for direct detection of composite dark matter. We find minimal $N_f$ dependence in these quantities. We generate mass-dependent cross-sections for dark-matter-nucleon interactions and use them in conjunction with experimental results from XENON100, excluding dark matter candidates of this type with masses below 10 TeV.

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Introduction

Experimental bounds on the interaction of the dark matter with Standard-Model (SM) particles have strengthened by many orders of magnitude in recent years. In particular, dark-matter particles cannot have SM-strength couplings to electroweak gauge bosons, based on direct-detection constraints [1, 2]. At the same time, there is a strong motivation for the dark matter to couple to the SM in some way for the purpose of relic density generation, either as a thermal relic via the so-called “WIMP miracle” (see [3] for a recent review) or through an asymmetric scenario which may be related to the creation of baryon asymmetry [4–11]. Construction of dark matter models thus requires a careful balance between the presence and absence of dark-sector interactions with the SM.

Composite dark matter models provide a simple mechanism for attaining this balance, one which can lead to interesting and unique phenomenology. By hypothesizing a new, confining gauge force in the dark sector, an electroweak-neutral composite dark matter candidate can be constructed as a bound state of electroweak-charged constituents. In this way, electroweak interactions can be active in the early Universe for the generation of relic density, but only neutral bound states survive to the present day. Electroweak coupling to the constituents is still possible, leading to form-factor suppressed interactions with the neutral composites. They can be roughly estimated from QCD analogs, but in general can be determined quantitatively only by lattice calculations.

In this paper, we consider an underlying $SU(3)$ gauge theory with fermions in the fundamental representation, but focus on fermions not associated with electroweak breaking. We use $SU(3)$ because much is known about it from lattice QCD and because we have already generated lattice vacuum states of $SU(3)$ with 2 and 6 fundamental flavors on large lattices [12, 13]. We take the fermions to be mass-degenerate $SU(2)_L$ singlets such that $Q = Y$. We consider a two-fermion theory ($N_f = 2$) with $Q_u = 2/3$ and $Q_d = -1/3$, as well as a six-fermion theory ($N_f = 6$) with three such pairs of fermions. In either case, the lightest baryon is expected to be electrically neutral, and will therefore also have vanishing weak charge. The dominant contribution to its interaction with ordinary nuclei will be due to single photon exchange, which can be parameterized primarily in terms of its magnetic moment and charge radius. In these initial lattice calculations we consider only quark-line connected contributions to the charge radius and magnetic moment. We compute the electromagnetic form factors of this particle to extract these quantities, describe their dependence on $N_f$, and discuss consequences for direct detection.

One could also modify or enlarge the fermion content of the $SU(3)$ gauge theory to include $SU(2)_L$-doublet fermions. This would be a necessary modification in order to consider composite dark matter arising in a theory of dynamical electroweak symmetry breaking [4–9, 14–20]. Careful model building is then required to ensure that the lightest baryon is net electroweak neutral. We do not discuss this possibility here.

Model setup

For the theory with $SU(2)_L$-singlet fermions carrying charges $Q_u = 2/3$ and $Q_d = -1/3$, the fundamental SU(3) representation is decomposed as

$$L = (1, 0, 0) + (3, 0, 0) + (1, 0, 0) + (0, 1, 0) + (0, 0, 1),$$

where the first and third terms of the fundamental representation are the weak isoscalar and isovector mesons, respectively. The mass dependence of these quantities is negligible compared to their theoretical form factors.

$$M^2 = m^2 + rac{1}{2} m^2 + rac{1}{2} m^2 + m^2 + m^2,\quad \text{for} \quad L = (1, 0, 0) + (3, 0, 0) + (1, 0, 0) + (0, 1, 0) + (0, 0, 1).$$

The form factors are calculated using the lattice QCD and because we have already generated lattice QCD and because we have already generated lattice QCD and because we have already generated lattice QCD and because we have already generated lattice QCD and because we have already generated lattice QCD.
with $N_f = 2$ or 6, the analogue of the neutron ($N \sim udd$) will be the dark matter candidate, with mass $M_B$ and carrying no net electroweak charge. It is stabilized by conservation of dark baryon number. The other charged baryons are expected to be heavier due to electromagnetic mass corrections of order $\Delta M \sim \alpha M_B/4\pi$. We include a fermion mass $m_f$, essential for lattice calculation purposes, and examine dependence on $m_f$ for a range $m_f \ll M_B$.

Our dark sector also contains $N_f^2 - 1$ pseudo-Nambu-Goldstone-boson (PNGB) states. We assume that these states are unstable, decaying to Standard-Model particles with a sufficient rate that their presence does not influence the cosmological history of the Universe.

As our focus is on direct-detection signatures, we do not consider the dark matter generation in detail here. The confinement scale $\Lambda$, or equivalently the dark matter mass $M_B$, is a free parameter in our construction.

### Electromagnetic Form Factors

Since the neutral baryon in the $SU(2)$-singlet theory is the dark matter candidate of interest [39], the baryon mass $M_B$ (degenerate in the absence of other interactions) is the dark matter mass. This mass and all other dimensionful quantities are expressed in lattice units here.

The quantities of central interest here are the Dirac and the Pauli electromagnetic form factors of a neutral dark-matter baryon $|N(p)\rangle$. For the $N_f = 2$ case, they can be expressed in terms of matrix elements of the vector currents of individual quarks as follows:

$$\langle N(p')|\overline{\psi}\gamma^\mu\gamma^5\psi|N(p)\rangle$$

$$= U(p')\left[F_1^\psi(Q^2)\gamma^\mu + F_2^\psi(Q^2)\frac{i\sigma^\mu\nu q_\nu}{2M_B}\right] U(p),$$

where $\psi = u, d$ are quark fields, $U, \overline{U}$ are on-shell baryon spinors, $q = p' - p$, and $Q^2 = -q^2 > 0$ is the momentum transfer. In the forward limit $Q^2 = 0$, the Dirac form factors are equal to the numbers of the valence quarks: $F_1^u(0) = 1$ and $F_1^d(0) = 2$.

From these one constructs the isovector and isoscalar form factors [40]:

$$F_{1,2}^u(Q^2) = F_{1,2}^d(Q^2) - F_{1,2}^s(Q^2),$$

$$F_{s,2}^s(Q^2) = F_{1,2}^s(Q^2) + F_{1,2}^s(Q^2).$$

Both of these quantities can be extracted from lattice calculations, but the isoscalar contribution contains expensive disconnected lattice quark contractions, which cancel in the isovector case, and as a result, isovector form factors are far more tractable. While we ultimately will calculate the disconnected pieces of the isoscalar form factor as well, this work focuses on only the connected contractions.

For the $N_f = 6$ case, with three pairs of $u(Q = 2/3)$ and $d(Q = -1/3)$ fermions, we take the $|N(p)\rangle$ state to be composed of fermions from only one pair. Since we omit disconnected lattice quark contractions in our calculation, it is only the currents $\overline{\psi}\gamma^\mu\gamma^5\psi$ composed of the fermion fields from the same pair that contribute to the computed electromagnetic form factors. Therefore, in our calculation the other two pairs play a role in only the strong dynamics of the $SU(3)$ gauge theory.

The full electromagnetic form factors of the neutral dark baryon [41] are given by

$$F_{1,2;\text{neut}}(Q^2) = Q_u F_{1,2}^u(Q^2) + Q_d F_{1,2}^d(Q^2)$$

$$= \frac{1}{6}F_1^s(Q^2) - \frac{1}{2} F_2^s(Q^2);$$

since $F_1^s(0) = 3$ and $F_2^s(0) = 1$, the total charge $F_{1,\text{neut}}(0) = 0$. For soft single-photon exchange scattering, only the forward ($Q^2 \to 0$) behavior of the electromagnetic form factors is relevant. Since the electric charge $F_{1,\text{neut}}(0)$ is zero, only the magnetic moment $\mu_{\text{neut}} = \kappa_{\text{neut}}$ and the Dirac radius $\langle r_{1,\text{neut}}^2 \rangle$ contribute to the scattering amplitude to the lowest order in $Q^2$:

$$F_{1,\text{neut}}(Q^2) = \frac{1}{6} Q^2 \langle r_{1,\text{neut}}^2 \rangle + \mathcal{O}(Q^4),$$

$$F_{2,\text{neut}}(Q^2) = \kappa_{\text{neut}} + \mathcal{O}(Q^2),$$

The Dirac charge radius $\langle r_{1,\text{neut}}^2 \rangle$ determines the slope of the form factor in the $Q^2 \to 0$ limit:

$$\langle r_{1,\text{neut}}^2 \rangle \equiv \frac{-6dF_{1,\text{neut}}(Q^2)}{dQ^2} \bigg|_{Q^2=0}.\tag{5}$$

The definition of the radius (5) is motivated by the algebraic identity

$$\int d^3r \ r^2 \rho(r) \equiv -\frac{6dF_{1}(Q^2)}{dQ^2} \bigg|_{Q^2=0},\tag{6}$$

where $\rho(r)$ is the “charge density”:

$$\int d^3r \ e^{i\vec{q}\cdot\vec{r}} \rho(r) = F_1(Q^2), \quad Q^2 \sim \rho^2,$$\tag{7}

which has physical meaning if and only if the spatial extent of this distribution is much larger than the Compton wave length of the composite particle, $\langle r^2 \rangle \gg M_B^{-2}$. Since the total charge, $\int d^3r \rho(r) = F_1(0)$, is zero, the charge density must have alternating sign (or be exactly zero), and the integral in Eq. (6) can be either positive or negative.

For the following, we also need to define the mean squared charge radius $\langle r_{E}^2 \rangle$, or the “radius” of the charge form factor $G_E(Q^2)$:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_B^2} F_2(Q^2).\tag{8}$$

Similar to Eq. (5), the charge radius of the neutral baryon is equal to

$$\langle r_{E;\text{neut}}^2 \rangle \equiv \frac{-6dG_{E;\text{neut}}(Q^2)}{dQ^2} \bigg|_{Q^2=0} = \langle r_{1;\text{neut}}^2 \rangle + \frac{3\kappa_{\text{neut}}}{2M_B^2},\tag{9}$$

differing from the Dirac radius by only the relativistic correction $\sim M_B^{-2}$ (the Foldy term). This correction is important if the size of the particle is comparable to its Compton
wave length, which is the case for the neutron and the proton in QCD.

The (anomalous) magnetic moment of the neutral baryon is related to the isovector and isoscalar moments as

\[ \kappa_{\text{neut}} = \frac{1}{6} \kappa_s - \frac{2}{3} \kappa_v . \]  

(10)

The isovector and isoscalar Dirac form factors are not zero in the forward limit. Their radii are defined to be independent of their overall normalization,

\[ F_{1,n}^v(Q^2) = F_{1,n}^v(0) \left[ 1 - \frac{1}{6} Q^2 \langle r_1^2 \rangle_v + O(Q^4) \right] . \]  

(11)

The radii of the neutral baryon are related to the isovector and isoscalar radii as follows:

- \[ r_{1,v}^2 = \frac{1}{2} \langle r_{1,v}^2 \rangle_v \]  

and

- \[ r_{1,s}^2 = \frac{1}{2} \langle r_{1,s}^2 \rangle_v \]  

(12)

**Simulation Details**  
Lattice calculations are performed using 32\(^3\) \times 64 domain-wall lattices with the Ising improved gauge action and a fifth-dimensional length \( L_s = 16 \) and a domain-wall height of \( m_0 = 1.8 \). By using domain-wall fermions, the calculation preserves exact flavor symmetry, and chiral-breaking lattice spacing artifacts are suppressed. The calculation is performed for \( N_f = 2 \) at \( \beta = 2.70 \) and \( N_f = 6 \) at \( \beta = 2.10 \). The beta values are tuned to match the confinement scale of both theories relative to the lattice spacing, including \( M_B \) as we shall see below. For both \( N_f = 2 \) and \( N_f = 6 \), five separate mass points are analyzed with \( m_f = 0.010, 0.015, 0.020, 0.025, 0.030 \). The pion masses (in units of the nucleon mass) are \( 0.41 \leq m_\pi/M_B \leq 0.52 \) and \( 0.44 \leq m_\pi/M_B \leq 0.52 \) for \( N_f = 2 \) and \( N_f = 6 \), respectively. Further details and other results from these ensembles are given in [12, 13, 21].

**Calculation and Fitting**  
The parameters of interest are extracted from two sets of correlation functions: two-point correlation functions given by

\[ C_{NN}(\tau, p) = \sum_x e^{-i p \cdot x} \langle N(x, \tau) \bar{N}(0) \rangle , \]  

(13)

and three-point correlation functions

\[ C_{NON}(\tau, T, p, p') = \sum_{x,y} e^{-i p \cdot x + (p' - p) \cdot y} \times \langle N(x, T) \bar{O}(y, \tau) \bar{N}(0) \rangle , \]  

(14)

where \( \bar{O}(y, \tau) \) is the quark vector current density operator.

The long-distance limit of the Euclidean time behavior of these correlation functions is given by

\[ C_{NN}(\tau, p) \xrightarrow{\tau \gg \frac{1}{T}} \frac{Z(p)e^{-E_\tau}}{2E} \text{Tr} \left[ \Gamma_{\text{pol}}(i \bar{p} + M_B) \right] , \]  

(15)

\[ C_{NON}(\tau, T, p, p') \xrightarrow{T, \tau \gg \frac{1}{T}} \frac{\sqrt{Z(p)Z(p')}}{4E_\tau} \frac{e^{-E_\tau(T-\tau) - E_\tau}}{4E_e} \times \text{Tr} \left[ \Gamma_{\text{pol}}(i \bar{p}' + M_B) \Gamma^\mu(i \bar{p} + M_B) \right] \]  

(16)

where \( \Gamma_{\text{pol}} \) is the polarization matrix of the initial and final baryon spin states corresponding to Eq. (13,14), \( \Gamma^\mu \) is the fermion vertex function (cf. Eq.(1)),

\[ \Gamma^\mu = F_1(Q^2)\gamma^\mu + F_2(Q^2) \frac{q^\mu q_e}{2M_B} , \]  

(17)

and \( \Delta \) is the difference in energy between the ground and the first excited state of the baryon. More details on the form factor calculation on the lattice can be found in Ref. [22].

We use the standard widely adopted “ratio” method in order to extract hadron matrix elements from corresponding two- and three-point functions,

\[ R_{O}(\tau, T, p, p') = \frac{C_{NON}(\tau, T, p, p')}{\sqrt{C_{NN}(T, p)C_{NN}(T, p')}} \times \frac{\sqrt{C_{NN}(T - \tau, p)C_{NN}(\tau, p')}}{C_{NN}(T - \tau, p')C_{NN}(\tau, p')} , \]  

(18)

where the long Euclidean time behavior yields

\[ R_{O}(\tau, T, p, p') \xrightarrow{\tau \gg \frac{1}{T}} \langle N(p')|\bar{O}|N(p)\rangle \]  

(19)

We analyze these ratios for multiple initial and final momentum combinations and vector current components in order to extract form factors \( F_1 \) and \( F_2 \). Their values form a reasonable “plateau” as functions of \( \tau \), the timeslice of the current operator insertion, indicating absence of significant excited-state contaminations (see Fig. 1).

In general, excited states can cause significant systematic errors in three-point functions and hadron matrix elements [23]. We compute our form factor values as averages of three central points in the plateaus. The form factors \( F_{1,2}(Q^2) \) are calculated at discrete values of the momentum transfer \( Q^2 \approx (p' - p)^2 \) determined by the lattice volume. We interpolate the Dirac and isovector Pauli form factors using a dipole formula fit

\[ F_{1,2}(Q^2) \sim \frac{A_{1,2}}{(1 + B_{1,2}Q^2)^2} \]  

(20)

motivated by nucleon form factor phenomenology. The isoscalar Pauli form factor turns out to be very close to zero, and the dipole form that has definite sign does not necessarily yield a stable fit to the data; therefore, we use the linear fit \( F_i(Q^2) \sim F_i(0) + F_i'(0)Q^2 \). Examples of fits are shown on Fig. 2. We use these fits to interpolate (extrapolate in the case of Pauli form factors) near the forward limit \( Q^2 = 0 \) in order to determine \( \kappa \) and \( \langle r_1^2 \rangle \).
Baryon Mass The dark-matter baryon mass is plotted as a function of the fermion mass $m_f$ in Fig. 3. A linear dependence of the baryon mass on $m_f$ can be seen for both theories, as expected in the calculation regime where the fermion masses are small. In the absence of additional interactions, a finite value of $m_f$ is required to give mass to the PNGB’s of the theory, but we nevertheless perform a linear fit in order to extract the chiral-limit baryon mass $M_{B_0}$. This scale, which can be taken as a proxy for the confinement scale of the theory, serves as a common reference scale for the calculation results with $m_f \geq 0$.

Anomalous magnetic moment The anomalous magnetic moment is the most important for direct detection experiments. It enters at the dimension-5 level in the baryon effective field theory and arises as the zero-momentum value of the Pauli form factor, $F_2(0)$. The isovector Pauli form factor, giving $k_{\pi}$, is under most control since all expensive disconnected contributions cancel due to isospin symmetry. The isoscalar channel, which is also necessary to determine $k_{\text{neut}}$, has both connected and disconnected contributions to the three-point correlation function. In this initial work, we omit the disconnected contributions and assume the connected pieces dominate the isoscalar contribution.

We plot the anomalous magnetic moment $k_{\text{neut}}$, computed as described above, versus $M_B/M_{B_0}$ in Fig. 4. It shows little dependence on the mass and little dependence on the number of fermions. The $N_f = 2$ results $k_{\text{neut}} \approx -(1.71 \ldots 2.09)$ are consistent with the measured neutron value $k = -1.91$ [24]. Calculations of nucleon structure with $N_f = 2$ Wilson fermions were previously reported in Ref. [25], which found values $k_{\text{neut}} \approx -(1.30 \ldots 1.45)$, with the difference coming predominantly from the isovector Pauli form factor; our results for this form factor more closely match the more recent results of [26, 27].

Charge radius While the charge radius is expected to lead to a smaller effect on the spin-independent cross section as compared to the magnetic moment, it could have a significant effect if its value depends significantly on $N_f$. It is therefore informative to explore the relative size of the charge radius contribution to the spin-independent cross section. As with the magnetic moment, only the isovector charge radius is absent of disconnected lattice quark contractions, but we omit them for the isoscalar channel as well.

The results for the mean square charge radius $\langle r_{E, \text{neut}}^2 \rangle$
of an electroweak-neutral dark-matter baryon are presented in Fig. 5. Note that the results are negative (see discussion after Eq. (6)). As in the case of the anomalous moment, our results show little dependence on \( N_f \) and little dependence on the dark-baryon mass as it varies due to changes in the underlying fermion mass. If the fermion mass is reduced further, bringing \( M_B/M_{B_0} \) closer to unity, the magnitude \( \langle r_{E,neut}^2 \rangle \) is expected to grow. This is because the PNGB cloud mass drops, and the charge radius is quite sensitive to the size of the PNGB cloud.

For \( N_f = 2 \), this point can be made more precisely by comparison to QCD. There, the mean squared charge radius of the neutron is also negative, \( \langle r_{E,n}^2 \rangle = -0.1161(22) \text{ fm}^2 \) [24]. Our \( N_f = 2 \) calculation corresponds to QCD with \( M_B \approx 1 \text{ GeV} \), but with relatively heavy underlying quarks, and thus relatively heavy pions: the pion mass in units of \( M_B \) ranges between the lightest \( m_\pi/m_B = 0.41 \) to the heaviest \( m_\pi/m_B = 0.52 \). In QCD units, our lattice spacing is given by \( a \approx 0.055 \text{ fm} \), so our result is \( \langle r_{E,neut}^2 \rangle \approx -(0.009 \ldots 0.025) \text{ fm}^2 \), substantially less than the observed result. Previous calculations of nucleon structure with \( N_f = 2 \) Wilson fermions [25] yielded similar values \( \langle r_{E,neut}^2 \rangle = -(0.011 \ldots 0.023) \text{ fm}^2 \). These results, too, employed relatively heavy underlying quarks. In our case, further studies with smaller fermion mass can shed light on the range of direct detection allowed values for the mean square charge radius.

Direct detection exclusion plots We next compare our calculations of dark-matter parameters with the current experimental bounds on the dark-matter-nucleus cross-sections in direct detection experiments. Currently, the most stringent bound is provided by the XENON100 experiment [28], in which hypothetical dark-matter particles are detected through their collisions with xenon nuclei with \( Z = 54 \) and \( A = 124 \ldots 136 \), and which has accumulated
2323.7 kg-days of effective exposure. Two of the isotopes, $^{129}$Xe and $^{131}$Xe, have non-zero spin and are sensitive to the spin-dependent $M1$ interaction. Their combined abundance constitutes approximately 1/2 in natural xenon [29].

In this section, we adopt a more conventional notation $M_{\chi}$ for the mass of the dark-matter particle, and also denote its radius and magnetic moment with a subscript \( \chi \). Figs. 4 and 5 show that the anomalous moment and mean square charge radius vary little with the amount of the dark-matter mass coming from the underlying fermion mass (and also vary little as $N_f$ is increased from 2 to 6).

The differential cross-section of a dark-matter fermion and a nucleus, to leading order in the non-relativistic dark-matter velocity $v \ll 1$ is

$$\frac{d\sigma}{dE_R} = \frac{1}{16\pi(M_X + M_T)^2} \left| \mathcal{M}_{\text{SI}} \right|^2, \quad (21)$$

where $M_T$ is the mass of the target nucleus, and $E_R^{\text{max}} = 2M_{\chi}^2M_v^2v^2/(M_X + M_T)$ is the maximal recoil energy for given collision velocity $v$. The quantities and $\left| \mathcal{M}_{\text{SLSD}} \right|^2$ are spin-independent amplitudes squared, averaged over initial and summed over final states:

$$\left| \mathcal{M}_{\text{SI}} \right|^2 = e^4 \left[ Z_F e(Q) \right]^2 \left( \frac{M_T}{M_X} \right)^2 \frac{4}{9} M_{\chi}^4 \left( \frac{2}{9} \right)^2 \frac{\kappa_{\text{CM}}^2 \cot^2 \theta_{\text{CM}}}{2},$$

$$\left| \mathcal{M}_{\text{SD}} \right|^2 = e^4 \frac{2}{3} \left( \frac{J + 1}{J} \right) \left( \left( A_{\text{MF}} \mu_n \right) F_s(Q) \right)^2 \kappa_{\chi}^2.$$

Here, $Z$ and $A$ are the charge and atomic numbers of a specific xenon isotope, $(\mu_T/\mu_n)$ is the nuclear magnetic moment expressed in Bohr magnetons $\mu_n = e/2m_n$. $F_{c,s}(Q)$ are its nuclear charge and spin form factors, respectively, at the momentum transfer $Q \approx \sqrt{Q^2} = \sqrt{2M_T E_R}$, and $\theta_{\text{CM}}$ is the scattering angle in the center-of-mass frame [42]. For non-relativistic velocities, $\cot^2 \theta_{\text{CM}} = (E_R^{\text{max}}/E_R - 1)$.

For the nuclear response form factors $F_{c,s}(Q^2)$, we use the following commonly accepted phenomenological expressions [18, 30]:

$$|F_c(Q)|^2 = \frac{1}{\sin(Q R_c) - (Q R_c) \cos(Q R_c)} [\sin(Q R_c)]^2 e^{-(Q S)^2},$$

$$|F_s(Q)|^2 = \begin{cases} 0.047, & 2.55 \leq Q R_s \leq 4.5, \\ \sin(Q R_s)^2, & \text{otherwise}, \end{cases}$$

where $R_c = 1.14A^{1/3}$ fm, $R_s = 1.00A^{1/3}$ fm, and $S = 0.9$ fm. The nuclear response functions $F_{c,s}(Q^2)$ can also be evaluated using nuclear models, as was done in Ref. [31, 32].

Following Refs. [18, 28], we compute the scattering rate for a range of dark-matter particle masses with the recoil energies $E_R = 6.6 \ldots 43.3$ keV:

$$R = \frac{M_{\text{detector}} \rho_{DM}}{M_X} \int_{E_{\text{min}}}^{E_{\text{max}}} dE_R \mathcal{A}(E_R) \left\langle \langle v' \frac{d\sigma}{dE_R} \rangle \right\rangle_f,$$ \quad (26)

where \( \langle \rangle_f \) denotes averaging over the DM velocity distribution (27), $v' = |\vec{v} - \vec{v}_{\text{Earth}}|$ is the dark-matter velocity with respect to the detector (the Earth), and $\mathcal{A}(E_R)$ is the recoil energy-dependent acceptance rate of the detector [28]. We assume the thermal distribution of velocities in the galactic dark-matter halo [30],

$$\frac{d^3n}{d^3v} = f(v) = \frac{1}{\pi^{3/2}v_0^2} e^{-v^2/v_0^2}, \quad \int_{|\vec{v}| < v_{\text{esc}}} d^3 v f(v) \equiv 1,$$ \quad (27)

with $v_0 = v_{\text{Earth}} = 220$ km/s, $v_{\text{esc}} = 544$ km/s, and the dark-matter mass density $\rho_{DM} = 0.3$ GeV/cm$^3$. Finally, we average the expected scattering rate over the natural xenon isotopic abundances.

We show computed scattering rates in Fig. 6 with solid lines. The accumulated XENON100 statistics [28] exclude composite dark matter particles with $M_{\chi} \lesssim 10$ TeV. With fixed values of the dimensionless quantities $M_{\chi}^2/r_s^2$ and $K$ computed on a lattice, the differential cross-section scales as

$$\frac{d\sigma}{dE_R} \sim A \left( \frac{M_{\chi}^2}{r_{s\chi}^2} \right)^2 + B \frac{K}{r_{s\chi}^2}.$$ \quad (28)

The charge radius contribution is suppressed as $M_{\chi}^2/r_{s\chi}^2$ relative to the magnetic moment contribution and becomes negligible with growing $M_{\chi}$. In the scattering rate shown in Fig. 6, both contributions are additionally suppressed by the DM particle number density $\rho_{DM}/M_{\chi}$ as $M_{\chi} \to \infty$ (see Eq.(26)). The large-$M_{\chi}$ scaling of the charge radius term is shown in Fig. 6 with the dashed lines; it is evident that the total scattering rate (solid lines) is dominated by the magnetic moment term for dark matter masses $M_{\chi} \gtrsim 25$ GeV. Even if one were to make the charge radius as much as an order of magnitude larger by reducing the PNGB mass (see the discussion following Fig. 5), its contribution would still be negligible at $M_B = 10$ TeV, the lower limit of the allowed region.

**Discussion** We have studied the electromagnetic form factors of electroweak-neutral dark-matter baryons in an SU(3) gauge theory with $N_f = 2$ and $6$ SU(2)$_L$-singlet fermions, with charge assignments +2/3 and $-1/3$ (one pair or three pairs). These baryons have the desired properties of dark matter since they are stable, electroweak neutral, and can explain the relic density through the same early universe sphaleron process that describes baryogenesis.

Of particular interest to direct detection experiments are the anomalous magnetic moment and mean square charge radius of the dark-matter baryon. These parameters determine the observed cross-section with nuclei (in this work,
we primarily focus on Xenon) due to (dominant) single-photon anomalous moment. The contribution from the dark-matter anomalous moment dominates the charge-radius contribution for $M_\chi \gtrsim 25\text{GeV}$. However, in our calculation the charge radius $\langle r_E^2 \rangle$ turns out to be particularly small, much smaller than it is in QCD. Exploring smaller quark mass regions may change this balance and make the charge radius more relevant for the direct detection of the dark matter.

Examining the dark matter exclusion plots in light of the latest Xenon100 results [28], we conclude that in these theories, dark-matter masses less than 10 TeV are excluded. We have so far seen little dependence on $N_f$. It will be interesting to see whether this begins to change, even continuing to neglect disconnected quark contractions, as $N_f$ is increased toward the edge of the conformal window ($N_f \approx 10 - 12$ for an $SU(3)$ gauge theory with fermions in the fundamental representation). When the disconnected contractions are included, additional $N_f$ dependence will arise simply from the counting of these loop contributions.

As we have shown in this work, with non-zero magnetic moment the experimental constraints on the dark matter mass are quite stringent. This naturally motivates the consideration of even $N_f$ theories, in which the baryons are bosonic and thus have no magnetic moment. Interactions can be further suppressed if the charge assignments are symmetric in such a way that the charge radius vanishes (see e.g. [19]), making the electromagnetic polarizabilities the dominant interactions. Some initial lattice work on the zero-temperature dynamics of such theories has been carried out in [33, 34], and we are currently planning similar calculations with an eye towards dark matter form factors.

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Sign conventions of the isovector and isoscalar form factors are chosen to agree with nuclear physics notations; since we use the neutron (udd) as our in- and out-states, we have to swap $u \leftrightarrow d$ in the r.h.s of Eq. 2.

We denote the observables associated with the neutral dark baryon in our calculations with the subscript “neut” to avoid confusion with the QCD neutron, for which we reserve the subscript “n”.

In comparison to the differential cross-section in Ref. [18], Eq. (22) is the full spin-independent amplitude taking into account the non-trivial electric form factor of a DM particle, $G_E(Q^2) = -\frac{1}{6}Q^2r^2_E + O(Q^4)$. With the Dirac radius set to zero, $r^2_{1\chi} = r^2_E - \frac{3\kappa^2}{2m^2_\chi} = 0$, Eq. (22) reproduces identically the results in Ref. [18] with $g_M = \frac{1}{2}\kappa_\chi$ and zero electric dipole moment ($g_E = 0$).

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[42] In comparison to the differential cross-section in Ref. [18], Eq. (22) is the full spin-independent amplitude taking into account the non-trivial electric form factor of a DM particle, $G_E(Q^2) = -\frac{1}{6}Q^2r^2_E + O(Q^4)$. With the Dirac radius set to zero, $r^2_{1\chi} = r^2_E - \frac{3\kappa^2}{2m^2_\chi} = 0$, Eq. (22) reproduces identically the results in Ref. [18] with $g_M = \frac{1}{2}\kappa_\chi$ and zero electric dipole moment ($g_E = 0$).