# Matrix element analyses of dark matter scattering and annihilation

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# Matrix element analyses of dark matter scattering and annihilation

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We provide a compendium of results at the level of matrix elements for a systematic study of dark matter scattering and annihilation. We identify interactions that yield spin-dependent and spin-independent scattering and specify whether the interactions are velocity- and/or momentum-suppressed. We identify the interactions that lead to $s$-wave or $p$-wave annihilation, and those that are chirality-suppressed. We also list the interaction structures that can interfere in scattering and annihilation processes. Using these results, we point out situations in which deviations from the standard lore are obtained.

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## I. INTRODUCTION

Recently, several experiments have reported signals that may be interpreted as hints of dark matter interactions [1–6]. However, since none of these signals have been recognized as smoking guns for weakly interacting massive particles (WIMPs) of the Minimal Supersymmetric Standard Model (MSSM), there has been renewed interest in more general studies of dark matter models. One particular area of recent interest is in an effective operator analysis [7–10], where the detailed microscopic physics underlying interactions between the dark sector and Standard Model (SM) sector are abstracted away, leaving a description in terms of effective 4-point contact interaction operators.

Thus far, this type of analysis has been carried forward in a somewhat piecemeal manner. For example, many analyses assume that dark matter interactions involve a single contact interaction operator, without accounting for possible effects arising from interference between multiple operators. Only initial steps have been taken toward complementary studies of effective operators using direct, indirect and collider search strategies. Although it is well recognized that the effective operator approximation can break down if the mediating particles are not heavy enough, there has been little study of the features of the effective operator analysis which are robust.

Our goal is to provide tools needed for a systematic matrix element study of dark matter interactions with the Standard Model sector, and results relevant for direct, indirect and collider searches. We address the following questions:

1. Which interaction structures yield spin-dependent (SD) or spin-independent (SI) scattering? Are these matrix elements unsuppressed, or suppressed by factors of the relative velocity or momentum transfer?

2. Which interaction structures permit $s$-wave annihilation or $p$-wave annihilation, and which are chirality-suppressed?

3. Which interaction structures can interfere with each other in a scattering process? Which can interfere in an annihilation process?

4. What unique signals arise from interaction structures that are $CP$-violating?

5. How may we distinguish between spin-0, spin-1/2 (Majorana or Dirac) and spin-1 dark matter by utilizing signals in direct, indirect and collider searches?

Terms in the scattering matrix element can be suppressed by factors proportional to the relative velocity, or to the ratio of the momentum transfer to dark matter or nucleus mass. For cold dark matter, these factors are all very small. It is common to focus on scattering matrix elements with no velocity or momentum suppressions, since these terms will typically dominate the scattering cross section. However, velocity- or momentum-suppressed terms can dominate if the unsuppressed terms have very small coefficients. We therefore provide a complete treatment of the velocity- and momentum-suppressed terms as well.

The basic structure of a dark matter-SM interaction can be written in terms of a dark matter bilinear $\Gamma_X$, and a SM bilinear $\Gamma_f$:

$$\mathcal{O} = \Gamma_X \Gamma_f F(s,t,u)\,. \tag{1}$$

$F$ is a form factor which describes deviations from the structure of a pure contact interaction (if the form of $F$ is determined for a scattering interaction, it is determined for an annihilation process by crossing symmetry); for a contact interaction, $F =$ constant. $F$ depends on the details of the particle physics model, including the mass of the mediating particles, as well as nuclear form factors. On the other hand, at lowest dimension, $\Gamma_X$ and $\Gamma_f$ are somewhat more restricted and can be characterized by their Lorentz structure. If the interaction structure mediates a process such as $t$-channel scattering, then the Lorentz structure will be determined by the spin and parity of the mediating particle which is exchanged. But if the mediating particle is exchanged in the $u$- or $s$-channel for a scattering process, then the interaction structure will be more complicated, and can be determined through use of Fierz transformations. Our focus will be on the features that can be determined from knowledge of these bilinears. In the following, we denote a quark field by $q$, a spin-0 dark matter field by $\phi$, a spin-1/2 dark matter field by $X$, and a spin-1 dark matter field by $B_\mu$. A general fermion field (either dark matter or Standard Model) will be represented by $\psi$.

For simplicity, we focus on interactions only between dark matter and SM fermions, or with the Higgs. In Sections II and III, we describe our computation of the scattering matrix elements and dark matter annihilation matrix elements, respectively. In section IV we compile our results. In section V, we conclude with a discussion of interesting features and deviations from standard lore that arise from the application of our analysis.

## II. SCATTERING

The kinematics of a scattering process in the center of mass frame are determined by the relative velocity $v$ and the momentum transfer $\overrightarrow{q}$. In addition to these kinematic variables, each bilinear can contain terms that are either independent of spin, or depend on the spin matrix element. If the spin matrix element is a vector, then it can be projected on any of three orthogonal axes. It is useful to define these three axes as $\hat{q} = \overrightarrow{q}/|\overrightarrow{q}|$, $\hat{v}^\perp = (\overrightarrow{v} - \overrightarrow{v} \cdot \hat{q})/|\overrightarrow{v} - \overrightarrow{v} \cdot \hat{q}|$ and $\hat{\eta} = \hat{q} \times \hat{v}^\perp$. In other words, each bilinear will be a sum of terms of the form

$$(...)\langle \zeta_{out}|\Gamma|\zeta_{in}\rangle \,, \tag{2}$$

where $(...)$ is a function of $\overrightarrow{q}$ and $\overrightarrow{v}$, $\zeta_{in,out}$ is the spin state of the incoming and outgoing particle, respectively, and $\Gamma = 1, S_{\hat{q}}\hat{q}, S_{\hat{v}^\perp}\hat{v}^\perp, S_{\hat{\eta}}\hat{\eta}$ (if dark matter is spin-1, then there can also be spin matrix elements which transform as a symmetric traceless tensor). Terms with $\Gamma = 1$ are spin-independent, while the others are spin-dependent.

The spin and kinematic dependence of these bilinears can most easily be understood from their transformation properties under rotation and parity. Each bilinear can depend on only the incoming and outgoing momenta (which are odd under parity) and the spin matrix element (which is even under parity). A bilinear with a single spatial index transforms as a vector under rotations, while a bilinear with only timelike indices transforms as a scalar. A bilinear must then consist of a sum of terms in which the momenta and spin are contracted in such a way as to have the correct rotation and parity transformation properties.

For each bilinear interaction structure, these matrix elements are computed in Appendix A, and listed for convenience in Table X. Also computed there are squared matrix elements, summed over all initial and final state spins. By contracting a dark matter bilinear with a SM bilinear, one gets a possible interaction structure. From Table X, one can determine the full momentum- and velocity-dependence of the spin-dependent and spin-independent matrix elements for all such interaction structures.

Also from Table X, we see that there are only a few Lorentz structures for the Standard Model coupling such that the nucleon matrix element is momentum- and velocity-independent. These are $\bar{q}q$ (SI), $\bar{q}\gamma^0 q$ (SI), $\bar{q}\gamma^i\gamma^5 q$ (SD), and $\bar{q}\sigma^{ij} q$ (SD).

## III. ANNIHILATION

For the annihilation process, we are guided by the $C$ and $P$ quantum numbers of the initial and final state. We assume that both the initial state and final state consist of a particle and its anti-particle, which may be identical to the particle. For a fermion/anti-fermion state, the transformations under charge conjugation and parity are given by

$$C : (-1)^{L+S} \qquad P : (-1)^{L+1} \,, \tag{3}$$

while for a boson/anti-boson initial states, the transformations are given by

$$C : (-1)^{L+S} \qquad P : (-1)^{L} \,. \tag{4}$$

The only allowed $s$-wave states are $L = 0$, $S = 0$ $(J = 0)$; $L = 0$, $S = 1$ $(J = 1)$; and $L = 0$, $S = 2$ $(J = 2)$. Consequently, we are primarily interested in initial and final states with $J = 0, 1, 2$. In Table I, we list the $C$ and $P$

| S | L | J | C | P |
|---|---|---|---|---|
| 0 | 0 | 0 | + | - |
| 0 | 1 | 1 | - | + |
| 1 | 0 | 1 | - | - |
| 1 | 1 | 0,1,2 | + | + |
| 1 | 2 | 1,2,3 | - | - |
| 1 | 3 | 2,3,4 | + | + |

| S | L | J | C | P |
|---|---|---|---|---|
| 0 | 0 | 0 | + | + |
| 0 | 1 | 1 | - | - |
| 1 | 0 | 1 | - | + |
| 1 | 1 | 0,1,2 | + | - |
| 1 | 2 | 1,2,3 | - | + |
| 2 | 0 | 2 | + | + |
| 2 | 1 | 1,2,3 | - | - |
| 2 | 2 | 0,1,2,3,4 | + | + |
| 2 | 3 | 1,2,3,4,5 | - | - |
| 2 | 4 | 2,3,4,5,6 | + | + |

TABLE I. The $C$ and $P$ transformation properties of a fermion/anti-fermion (left) or boson/anti-boson (right) state for given $S$, $L$ and $J$ quantum numbers.

| bilinear | C | P | J | state |
|---|---|---|---|---|
| $\bar{\psi}\psi$ | + | + | 0 | $S=1,\ L=1$ |
| $\imath\bar{\psi}\gamma^5\psi$ | + | - | 0 | $S=0,\ L=0$ |
| $\bar{\psi}\gamma^0\psi$ | - | + | 0 | none |
| $\bar{\psi}\gamma^i\psi$ | - | - | 1 | $S=1,\ L=0,2$ |
| $\bar{\psi}\gamma^0\gamma^5\psi$ | + | - | 0 | $S=0,\ L=0$ |
| $\bar{\psi}\gamma^i\gamma^5\psi$ | + | + | 1 | $S=1,\ L=1$ |
| $\bar{\psi}\sigma^{0i}\psi$ | - | - | 1 | $S=1,\ L=0,2$ |
| $\bar{\psi}\sigma^{ij}\psi$ | - | + | 1 | $S=0,\ L=1$ |
| $\phi^\dagger\phi$ | + | + | 0 | $S=0,\ L=0$ |
| $\imath Im(\phi^\dagger\partial^0\phi)$ | - | + | 0 | none |
| $\imath Im(\phi^\dagger\partial^i\phi)$ | - | - | 1 | $S=0,\ L=1$ |
| $B^\dagger_\mu B^\mu$ | + | + | 0 | $S=0,\ L=0;\ S=2,\ L=2$ |
| $\imath Im(B^\dagger_\nu\partial^0 B^\nu)$ | - | + | 0 | none |
| $\imath Im(B^\dagger_\nu\partial^i B^\nu)$ | - | - | 1 | $S=0,\ L=1;\ S=2,\ L=1,3$ |
| $\imath(B^\dagger_i B_j - B^\dagger_j B_i)$ | - | + | 1 | $S=1,\ L=0,2$ |
| $\imath(B^\dagger_i B_0 - B^\dagger_0 B_i)$ | - | - | 1 | $S=0,\ L=1;\ S=2,\ L=1,3$ |
| $\epsilon^{0ijk}B_i\partial_j B_k$ | + | - | 0 | $S=1,\ L=1$ |
| $-\epsilon^{0ijk}B_0\partial_j B_k$ | + | + | 1 | $S=2,\ L=2$ |
| $B^\nu\partial_\nu B_0$ | + | + | 0 | $S=0,\ L=0;\ S=2,\ L=2$ |
| $B^\nu\partial_\nu B_i$ | + | - | 1 | $S=1,\ L=1$ |

TABLE II. The $C$, $P$ and $J$ quantum numbers of any state that can be either created or annihilated by the bilinear. For each possible state, the $S$ and $L$ quantum numbers are also given.

eigenvalues for a fermion/anti-fermion state (left) or boson/anti-boson state (right) in terms of the angular momentum quantum numbers.

For any bosonic or fermionic bilinear, the transformation of the bilinear under rotations determines the total angular momentum of the state that this bilinear either creates or annihilates. This information, along with the $C$ and $P$ quantum numbers of the bilinear, are thus sufficient to determine (from Table I) the spin and orbital angular momentum of the initial and final state. The $S$ and $L$ quantum numbers of the states created (annihilated) by every lowest-dimension bilinear are listed in Table II.

We see that the only dark matter bilinears that can couple to an $s$-wave initial state are $\imath\bar{X}\gamma^5 X$, $\bar{X}\gamma^i X$, $\bar{X}\gamma^0\gamma^5 X$, $\bar{X}\sigma^{0i}X$, $\phi^\dagger\phi$, $B^\dagger_\mu B^\mu$, $\imath(B^\dagger_i B_j - B^\dagger_j B_i)$ and $B^\nu\partial_\nu B^0$. Note that the structures $\bar{\psi}\gamma^0\psi$, $\imath Im(\phi^\dagger\partial^0\phi)$ and $\imath Im(B^\dagger_\nu\partial^0 B^\nu)$ cannot couple to any state and cannot contribute to any non-zero annihilation matrix element.

The Standard Model fermion bilinear must be able to produce a final state with the same $J$ quantum number as the initial state (though the $C$ and $P$ transformations need not be the same, since a general interaction structure can

| $S$ | $L$ | $J$ | $J_z = S_z$ | fermion helicities |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $f_L$, $\bar{f}_R$; $f_R$, $\bar{f}_L$ |
| 1 | 0 | 1 | 1 | $f_R$, $\bar{f}_R$ |
| 1 | 0 | 1 | 0 | $f_L$, $\bar{f}_R$; $f_R$, $\bar{f}_L$ |
| 1 | 0 | 1 | -1 | $f_L$, $\bar{f}_L$ |
| 0 | 1 | 1 | 0 | $f_L$, $\bar{f}_R$; $f_R$, $\bar{f}_L$ |
| 1 | 1 | 0 | 0 | $f_L$, $\bar{f}_R$; $f_R$, $\bar{f}_L$ |
| 1 | 1 | 1 | 1 | $f_R$, $\bar{f}_R$ |
| 1 | 1 | 1 | 0 | - |
| 1 | 1 | 1 | -1 | $f_L$, $\bar{f}_L$ |
| 1 | 2 | 1 | 1 | $f_R$, $\bar{f}_R$ |
| 1 | 2 | 1 | 0 | $f_L$, $\bar{f}_R$; $f_R$, $\bar{f}_L$ |
| 1 | 2 | 1 | -1 | $f_L$, $\bar{f}_L$ |

TABLE III. The possible fermion and anti-fermion helicities of a fermion/anti-fermion state with given $S$, $L$, $J$ and $J_z$ quantum numbers. It is assumed that the fermion is travelling on the $+z$-axis, and the anti-fermion is travelling on the $-z$-axis. $\bar{f}_{L,R}$ denotes the $CP$-conjugate of $f_{L,R}$ (so, for example, $\bar{f}_L$ is a right-handed anti-fermion ).

violate either symmetry). Thus, the spin and orbital angular momentum of the final and initial state may be different.

Finally, we address the question of whether or not there is a chirality-suppression ($\propto m_f^2/m_X^2$) of the annihilation matrix element. This suppression arises if a SM mass insertion is required to produce a final state with the appropriate spin. An outgoing state of SM fermions $\bar{f}f$ can only be in a $S_z = 0$ state if the fermion and anti-fermion are from different Weyl spinors ($f_L$ and $f_R$). They are in an $S_z = \pm 1$ state if the fermion and anti-fermion are from the same Weyl spinor. We take the $z$-axis to lie along the direction of motion of the outgoing fermion and anti-fermion, so $L_z = 0$, and $J_z = S_z$. (Note that for particles moving along the $z$-axis it is clear that $L_z = 0$, because $Y_{lm}(\theta = 0, \phi) \neq 0$ only if $m = 0$.)

In Table III, we list the possible fermion and anti-fermion helicities for final states with fixed choices of $S$, $L$, $J$ and $J_z$. We assume that the fermion moves along the $+z$-axis and the anti-fermion along the $-z$-axis, and that the initial state is written in a basis with angular momentum projected along the $z$-axis. In our notation, $\bar{f}_L$ is a right-handed anti-fermion, the $CP$-conjugate of $f_L$. For a SM bilinear to produce one of the listed final states, it must be able to produce a state with appropriate $S$, $L$ and $J$ quantum numbers. The helicities of the produced fermion and anti-fermion are then determined by the number of Dirac matrices in the bilinear; a bilinear with an even number of Dirac matrices will produce a fermion/anti-fermion pair from the same Weyl spinor, while a bilinear with an odd number of Dirac matrices will produce a pair from different Weyl spinors. If a bilinear does not produce a fermion and anti-fermion of the needed helicities, then there will be a chirality flip arising from a mass-insertion.

We can now bring together all of the pieces which contribute to an understanding of the annihilation matrix element. The procedure is as follows:

- For each interaction structure, we find the $C$ and $P$ transformations and $J$ quantum number of the dark matter bilinear, and from this identify the initial state that can couple to this bilinear; $s$-wave annihilation is only permitted if this state has $L = 0$.

- We then determine if the Standard Model bilinear can create a final state with the same $J$ as the initial state. If so, the matrix element for annihilation from the initial state to the appropriate final state is non-zero.

- We then check if the matrix element has an additional $m_f/m_X$ chirality suppression. For each $J_z$ projection of the final state, we find the helicities of the final state fermion and anti-fermion. If there is no choice of $J_z$ for which the SM bilinear can produce fermions with the appropriate helicities, then the annihilation cross section is suppressed by $m_f^2/m_X^2$.

In Appendix B, we list the matrix elements arising from fermion/anti-fermion creation or annihilation, for all choices of interaction structure. In the interest of generality, the anti-fermion is not assumed to be the anti-particle of the fermion, and the two particles are allowed to have different masses. These matrix elements can thus be used for the case of dark matter co-annihilation, or if dark matter annihilates through a flavor-violating process. The standard case can be obtained by setting the masses of the two particles to be equal.

## IV. RESULTS

We summarize our results in the following four tables. In Table IV, we list the dependence of the spin-independent and spin-dependent scattering matrix elements on $\overrightarrow{S}$, $\overrightarrow{q}$ and $v^\perp$. For each structure, we indicate whether the momentum or velocity dependence arises from the dark matter or Standard Model bilinear. For interactions structures that yield several matrix element terms with different kinematic dependence, the kinematic dependence of each term is listed on a separate line. We also list if each interaction permits $s$-wave annihilation, and (if so) whether or not $s$-wave annihilation is chirality-suppressed. Note that, using Lorentz gauge, one can rewrite $B_\nu \partial^\nu B_\mu$ as $\partial^\nu(B_\nu B_\mu) = \partial^\nu[(1/4)g_{\mu\nu}B^\rho B_\rho + B_\nu B_\mu(sym)]$, where "$(sym)$" means symmetric and traceless in the $\mu\nu$ indices.

| Name | Interaction Structure | $\sigma_{\rm SI}$ suppression | $\sigma_{\rm SD}$ suppression | $s$-wave? |
|---|---|---|---|---|
| F1 | $\bar{X}X\bar{q}q$ | 1 | $q^2v^{\perp 2}$ (SM) | No |
| F2 | $\bar{X}\gamma^5 X\bar{q}q$ | $q^2$ (DM) | $q^2v^{\perp 2}$ (SM); $q^2$ (DM) | Yes |
| F3 | $\bar{X}X\bar{q}\gamma^5 q$ | 0 | $q^2$ (SM) | No |
| F4 | $\bar{X}\gamma^5 X\bar{q}\gamma^5 q$ | 0 | $q^2$ (SM); $q^2$ (DM) | Yes |
| F5 | $\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$ (vanishes for Majorana $X$) | 1 | $q^2v^{\perp 2}$ (SM)<br>$q^2$ (SM); $q^2$ or $v^{\perp 2}$ (DM) | Yes |
| F6 | $\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q$ | $v^{\perp 2}$ (SM or DM) | $q^2$ (SM) | No |
| F7 | $\bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5 q$ (vanishes for Majorana $X$) | $q^2v^{\perp 2}$ (SM); $q^2$ (DM) | $v^{\perp 2}$ (SM)<br>$v^{\perp 2}$ or $q^2$ (DM) | Yes |
| F8 | $\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu\gamma^5 q$ | $q^2v^{\perp 2}$ (SM) | 1 | $\propto m_f^2/m_X^2$ |
| F9 | $\bar{X}\sigma^{\mu\nu}X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana $X$) | $q^2$ (SM); $q^2$ or $v^{\perp 2}$ (DM)<br>$q^2v^{\perp 2}$ (SM) | 1 | Yes |
| F10 | $\bar{X}\sigma^{\mu\nu}\gamma^5 X\bar{q}\sigma_{\mu\nu}q$ (vanishes for Majorana $X$) | $q^2$ (SM) | $v^{\perp 2}$ (SM)<br>$q^2$ or $v^{\perp 2}$ (DM) | Yes |
| S1 | $\phi^\dagger\phi\bar{q}q$ or $\phi^2\bar{q}q$ | 1 | $q^2v^{\perp 2}$ (SM) | Yes |
| S2 | $\phi^\dagger\phi\bar{q}\gamma^5 q$ or $\phi^2\bar{q}\gamma^5 q$ | 0 | $q^2$ (SM) | Yes |
| S3 | $\phi^\dagger\partial_\mu\phi\bar{q}\gamma^\mu q$ | 1 | $q^2v^{\perp 2}$ (SM)<br>$q^2$ (SM); $v^{\perp 2}$ (DM) | No |
| S4 | $\phi^\dagger\partial_\mu\phi\bar{q}\gamma^\mu\gamma^5 q$ | 0 | $v^{\perp 2}$ (SM or DM) | No |
| V1 | $B^\dagger_\mu B^\mu\bar{q}q$ or $B_\mu B^\mu\bar{q}q$ | 1 | $q^2v^{\perp 2}$ (SM) | Yes |
| V2 | $B^\dagger_\mu B^\mu\bar{q}\gamma^5 q$ or $B_\mu B^\mu\bar{q}\gamma^5 q$ | 0 | $q^2$ (SM) | Yes |
| V3 | $B^\dagger_\nu\partial_\mu B^\nu\bar{q}\gamma^\mu q$ | 1 | $q^2v^{\perp 2}$ (SM)<br>$q^2$ (SM); $v^{\perp 2}$ (DM) | No |
| V4 | $B^\dagger_\nu\partial_\mu B^\nu\bar{q}\gamma^\mu\gamma^5 q$ | 0 | $v^{\perp 2}$ (SM or DM) | No |
| V5 | $(B^\dagger_\mu B_\nu - B^\dagger_\nu B_\mu)\bar{q}\sigma^{\mu\nu}q$ | $q^2v^{\perp 2}$ (SM) | 1 | Yes |
| V6 | $(B^\dagger_\mu B_\nu - B^\dagger_\nu B_\mu)\bar{q}\sigma^{\mu\nu}\gamma^5 q$ | $q^2$ (SM) | $v^{\perp 2}$ (SM) | Yes |
| V7 | $B^\dagger_\nu\partial^\nu B_\mu\bar{q}\gamma^\mu q$ or $B_\nu\partial^\nu B_\mu\bar{q}\gamma^\mu q$ | $v^{\perp 2}$ (SM); $q^2$ (DM) | $q^2$ (SM); $q^2$ (DM) | No |
| V8 | $B^\dagger_\nu\partial^\nu B_\mu\bar{q}\gamma^\mu\gamma^5 q$ or $B_\nu\partial^\nu B_\mu\bar{q}\gamma^\mu\gamma^5 q$ | $q^2v^{\perp 2}$ (SM); $q^2$ (DM) | $q^2$ (DM) | $\propto m_f^2/m_X^2$ |
| V9 | $\epsilon^{\mu\nu\rho\sigma}B^\dagger_\nu\partial_\rho B_\sigma\bar{q}\gamma_\mu q$ or $\epsilon^{\mu\nu\rho\sigma}B_\nu\partial_\rho B_\sigma\bar{q}\gamma_\mu q$ | $v^{\perp 2}$ (DM or SM) | $q^2$ (SM) | No |
| V10 | $\epsilon^{\mu\nu\rho\sigma}B^\dagger_\nu\partial_\rho B_\sigma\bar{q}\gamma_\mu\gamma^5 q$ or $\epsilon^{\mu\nu\rho\sigma}B_\nu\partial_\rho B_\sigma\bar{q}\gamma_\mu\gamma^5 q$ | $q^2v^{\perp 2}$ (SM) | 1 | No |

TABLE IV. The kinematic suppression of the spin-independent and spin-dependent scattering cross sections for all possible interaction structures. F1-F10 correspond to fermionic dark matter (with F5, F7, F9 and F10 absent for Majorana fermions), S1-S4 correspond to real or complex scalar dark matter, V1-V10 to real or complex vector dark matter. Each suppression is labelled to indicate if it arises from the SM or dark matter (DM) bilinear. If a cross section contains several terms with different kinematic suppressions, each is listed on a separate line. We also list if $s$-wave annihilation is permitted and unsuppressed, if it is chirality-suppressed by a factor $\propto m_f^2/m_X^2$, or if it is not permitted at all; although the interactions are expressed in terms of quark fields $q$, by a slight abuse of notation we allow for annihilation to any pair of SM fermions $\bar{f}f$, each of mass $m_f$.

| $J$ | $S_{init}$ | $L_{init}$ | $S_{final}$ | $L_{final}$ | Interaction structure |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | $\bar{X}\gamma^5 X\bar{q}\gamma^5 q$, $\bar{X}\gamma^0\gamma^5 X\bar{q}\gamma^0\gamma^5 q$ |
| 0 | 0 | 0 | 1 | 1 | $\bar{X}\gamma^5 X\bar{q}q$ |
| 0 | 1 | 1 | 0 | 0 | $\bar{X}X\bar{q}\gamma^5 q$ |
| 0 | 1 | 1 | 1 | 1 | $\bar{X}X\bar{q}q$ |
| 1 | 0 | 1 | 0 | 1 | $\bar{X}\sigma^{ij}X\bar{q}\sigma^{ij}q$ |
| 1 | 0 | 1 | 1 | 0 | $\bar{X}\sigma^{ij}X\bar{q}\sigma^{ij}\gamma^5 q$ |
| 1 | 1 | 0 | 0 | 1 | $\bar{X}\sigma^{ij}\gamma^5 X\bar{q}\sigma^{ij}q$ |
| 1 | 1 | 0 | 1 | 0 | $\bar{X}\gamma^i X\bar{q}\gamma^i q$, $\bar{X}\sigma^{ij}\gamma^5 X\bar{q}\sigma^{ij}\gamma^5 q$ |
| 1 | 1 | 1 | 1 | 1 | $\bar{X}\gamma^i\gamma^5 X\bar{q}\gamma^i\gamma^5 q$ |
| 1 | 1 | 0 | 1 | 1 | $\bar{X}\gamma^i X\bar{q}\gamma^i\gamma^5 q$ |
| 1 | 1 | 1 | 1 | 0 | $\bar{X}\gamma^i\gamma^5 X\bar{q}\gamma^i q$ |
| 0 | 0 | 0 | 0 | 0 | $B^\dagger_\mu B^\mu\bar{q}\gamma^5 q$, $B^\nu\partial_\nu B_0\bar{q}\gamma^0\gamma^5 q$ |
| 0 | 0 | 0 | 1 | 1 | $B^\dagger_\mu B^\mu\bar{q}q$ |
| 0 | 1 | 1 | 0 | 0 | $\epsilon^{0ijk}B_i\partial_j B_k\bar{q}\gamma^0\gamma^5 q$ |
| 1 | 0 | 1 | 0 | 1 | $\imath(B^\dagger_i B_0 - B^\dagger_i B_0)\bar{q}\sigma^{0i}\gamma^5 q$ |
| 1 | 0 | 1 | 1 | 0 | $\imath(B^\dagger_i B_0 - B^\dagger_i B_0)\bar{q}\sigma^{0i}q$, $\imath Im(B^\dagger_\nu\partial_i B^\nu)\bar{q}\gamma^i q$ |
| 1 | 0 | 1 | 1 | 1 | $\imath Im(B^\dagger_\nu\partial_i B^\nu)\bar{q}\gamma^i\gamma^5 q$ |
| 1 | 1 | 0 | 0 | 1 | $\imath(B^\dagger_i B_j - B^\dagger_i B_j)\bar{q}\sigma^{ij}q$ |
| 1 | 1 | 0 | 1 | 0 | $\imath(B^\dagger_i B_j - B^\dagger_i B_j)\bar{q}\sigma^{ij}\gamma^5 q$ |
| 1 | 1 | 1 | 1 | 0 | $B^\nu\partial_\nu B_i\bar{q}\gamma^i q$ |
| 1 | 1 | 1 | 1 | 1 | $B^\nu\partial_\nu B_i\bar{q}\gamma^i\gamma^5 q$ |
| 1 | 2 | 2 | 1 | 0 | $\epsilon^{0ijk}B_j\partial_0 B_k\bar{q}\gamma_i q$ |
| 1 | 2 | 2 | 1 | 1 | $\epsilon^{0ijk}B_j\partial_0 B_k\bar{q}\gamma_i\gamma^5 q$ |

TABLE V. The interaction structures that can annihilate an initial state with quantum numbers $S_{init}$, $L_{init}$ and $J$ and create a final state with quantum numbers $S_{final}$, $L_{final}$ and $J$. If two interaction structures are listed on the same line, then they can interfere in an annihilation process.

### A. Interference

Of course, it is certainly possible for dark matter to couple to Standard Model matter through a sum of several effective interaction structures. In that case, it is important to understand if these operators can interfere. For the annihilation process, interference can only occur between structures that annihilate states of the same quantum numbers ($S$, $L$ and $J$) and create states of the same quantum numbers. Table V indicates the interaction structures that can connect initial and final states for all possible combinations of quantum numbers; interaction operators that appear on the same line can interfere with one another in annihilation processes. In particular, interference between two interactions structures can only occur for $s$-wave annihilation. It can be seen from Table II that if dark matter is spin-0, then there are no interference terms.

We now consider interference between different interaction structures in scattering processes. As we have seen, each of the SM or dark matter bilinears depends on a spin matrix element which is either spin-independent (1) or depends on a spin projection ($S_{\hat{q}}$, $S_{\hat{v}^\perp}$ or $S_{\hat{\eta}}$, if the dark matter spin matrix element is a vector). For the full interaction structure, there are sixteen possible choices of the full spin matrix element. The four choices that are independent of the quark spin (but may or may not depend on the dark matter spin) yield spin-independent scattering, while the remaining twelve choices yield spin-dependent scattering. Two interaction structures can interfere in a scattering process only if they have the same full spin matrix element. Two operators that couple to different spin projections will not interfere as the interference terms vanish on summing over spins.

We denote the four choices of the spin-independent matrix element by the numbers 1-4, and the twelve choices of the spin-dependent matrix element by the letters A-L. We list in Table VI, for each interaction structure for spin-1/2 dark matter, the leading spin matrix elements. If an interaction structure contains terms with multiple spin matrix elements, then they are listed on separate lines. Note, it is possible for two operators to each interfere with a third, even if they cannot interfere with each other. In Table VII we list the leading spin matrix elements if dark matter is spin-0, and in Table VIII we list the spin matrix elements for spin-1 dark matter. Note that, for spin-0 dark matter, it is not necessary to list the dark matter spin matrix element, which is always trivial. Thus, all interaction structures can

| | Interaction Structure | SI ($S_X$-dep.) | SD ($S_X$-dep.) | SD ($S_{SM}$-dep.) | SI Class | SD Class |
|---|---|---|---|---|---|---|
| F1 | $\bar{X}X\bar{q}q$ | 1 | 1 | $S_{\hat{\eta}}$ | 1 | C |
| F2 | $\bar{X}\gamma^5 X\bar{q}q$ | $S_{\hat{q}}$ | $S_{\hat{q}}$ | $S_{\hat{\eta}}$ | 2 | F |
| F3 | $\bar{X}X\bar{q}\gamma^5 q$ | - | 1 | $S_{\hat{q}}$ | - | A |
| F4 | $\bar{X}\gamma^5 X\bar{q}\gamma^5 q$ | - | $S_{\hat{q}}$ | $S_{\hat{q}}$ | - | D |
| F5 | $\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$ | 1 | 1 | $S_{\hat{\eta}}$ | 1 | C |
| | (vanishes for Majorana $X$) | | $S_{\hat{v}^\perp}$ | $S_{\hat{v}^\perp}$ | | H |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{\eta}}$ | | L |
| F6 | $\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q$ | $S_{\hat{v}^\perp}$ | $S_{\hat{\eta}}$ | $S_{\hat{v}^\perp}$ | 3 | K |
| | | | $S_{\hat{v}^\perp}$ | $S_{\hat{\eta}}$ | | I |
| F7 | $\bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5 q$ | $S_{\hat{v}^\perp}$ | 1 | $S_{\hat{v}^\perp}$ | 3 | B |
| | (vanishes for Majorana $X$) | | $S_{\hat{v}^\perp}$ | $S_{\hat{\eta}}$ | | I |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{v}^\perp}$ | | K |
| F8 | $\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu\gamma^5 q$ | $S_{\hat{\eta}}$ | $S_{\hat{q}}$ | $S_{\hat{q}}$ | 4 | D |
| | | | $S_{\hat{v}^\perp}$ | $S_{\hat{v}^\perp}$ | | H |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{\eta}}$ | | L |
| F9 | $\bar{X}\sigma^{\mu\nu} X\bar{q}\sigma_{\mu\nu} q$ | 1 , $S_{\hat{\eta}}$ | $S_{\hat{q}}$ | $S_{\hat{q}}$ | 1, 4 | D |
| | (vanishes for Majorana $X$) | | $S_{\hat{v}^\perp}$ | $S_{\hat{v}^\perp}$ | | H |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{\eta}}$ | | L |
| F10 | $\bar{X}\sigma^{\mu\nu}\gamma^5 X\bar{q}\sigma_{\mu\nu} q$ | $S_{\hat{q}}$ | 1 | $S_{\hat{q}}$ | 2 | A |
| | (vanishes for Majorana $X$) | | $S_{\hat{q}}$ | $S_{\hat{\eta}}$ | | F |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{q}}$ | | J |

TABLE VI. For each interaction structure, we indicate if the scattering matrix element is independent of the dark matter or SM spin (1), or if it depends on the projection of the spin on any of three orthogonal axes: the direction of momentum transfer ($\hat{q}$), the direction of the relative velocity transverse to the momentum transfer ($\hat{v}^\perp$), or the direction perpendicular to $\hat{q}$ and $\hat{v}^\perp$ ($\hat{\eta} = \hat{q} \times \hat{v}^\perp$). If an interaction structure yields several terms with different spin component dependence, they are listed on separate lines. The sixteen possible couplings to dark matter and nucleon spin are divided into four classes (1-4) that are independent of the nucleon spin, and twelve classes (A-L) that are nucleon spin-dependent. For each structure, all of its coupling classes are listed; if two interaction structures are listed in the same class, then they can interfere.

| | Interaction Structure | SD ($S_{SM}$-dep.) |
|---|---|---|
| S1 | $\phi^\dagger\phi\bar{q}q$ or $\phi^2\bar{q}q$ | $S_{\hat{\eta}}$ |
| S2 | $\phi^\dagger\phi\bar{q}\gamma^5 q$ or $\phi^2\bar{q}\gamma^5 q$ | $S_{\hat{q}}$ |
| S3 | $\phi^\dagger\partial_\mu\phi\bar{q}\gamma^\mu q$ | $S_{\hat{\eta}}$ |
| S4 | $\phi^\dagger\partial_\mu\phi\bar{q}\gamma^\mu\gamma^5 q$ | $S_{\hat{v}^\perp}$ |

TABLE VII. Similar to Table VI, but for spin-0 dark matter. Thus, there is no dependence on the dark matter spin. All of these structures can interfere for spin-independent scattering. For spin-dependent scattering, two interaction structures can interfere if they couple to the same projection of the nucleon spin.

interfere for spin-independent scattering of spin-0 dark matter. There is interference in the spin-dependent scattering matrix element if two structures couple to the same nucleon spin matrix element. For spin-1 DM, the interaction structures V7 and V8 couple to a dark matter spin matrix element that transforms as a traceless symmetric tensor, denoted by $\Pi$. We represent it by its components in the orthogonal basis defined by $\hat{q}$, $\hat{v}^\perp$ and $\hat{\eta}$.

If two interaction structures can interfere, but their matrix elements scale with different powers of $q$ and $v^\perp$, then the interference terms will be small unless one of the structures has a very small coefficient. But if two interfering interaction structures are suppressed by the same number of powers of $q$ and $v^\perp$, then the interference terms will be significant as long as the coefficients are comparable. In Table IX, we list each interaction structure according to the number of powers of $q$ or $v^\perp$ that appear in the SI (top) or SD (bottom) matrix element. Interaction structures that appear within parentheses can interfere.

| | Interaction Structure | SI ($S_X$-dep.) | SD ($S_X$-dep.) | SD ($S_{SM}$-dep.) | SI Class | SD Class |
|---|---|---|---|---|---|---|
| V1 | $B^\dagger_\mu B^\mu \bar{q}q$ or $B_\mu B^\mu \bar{q}q$ | 1 | 1 | $S_{\hat{\eta}}$ | 1 | C |
| V2 | $B^\dagger_\mu B^\mu \bar{q}\gamma^5 q$ or $B_\mu B^\mu \bar{q}\gamma^5 q$ | - | 1 | $S_{\hat{q}}$ | - | A |
| V3 | $B^\dagger_\nu \partial_\mu B^\nu \bar{q}\gamma^\mu q$ | 1 | 1 | $S_{\hat{\eta}}$ | 1 | C |
| V4 | $B^\dagger_\nu \partial_\mu B^\nu \bar{q}\gamma^\mu\gamma^5 q$ | - | 1 | $S_{\hat{v}^\perp}$ | 1 | B |
| V5 | $(B^\dagger_\mu B_\nu - B^\dagger_\nu B_\mu)\bar{q}\sigma^{\mu\nu}q$ | $S_{\hat{\eta}}$ | $S_{\hat{q}}$ | $S_{\hat{q}}$ | 4 | D |
| | | | $S_{\hat{v}^\perp}$ | $S_{\hat{v}^\perp}$ | | H |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{\eta}}$ | | L |
| V6 | $(B^\dagger_\mu B_\nu - B^\dagger_\nu B_\mu)\bar{q}\sigma^{\mu\nu}\gamma^5 q$ | $S_{\hat{q}}$ | $S_{\hat{q}}$ | $S_{\hat{\eta}}$ | 2 | F |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{q}}$ | | J |
| V7 | $B^\dagger_\nu \partial^\nu B_\mu \bar{q}\gamma^\mu q$ or $B_\nu \partial^\nu B_\mu \bar{q}\gamma^\mu q$ | $\Pi_{\hat{q}\hat{v}^\perp}$ | $\Pi_{\hat{q}\hat{v}^\perp}$ | $S_{\hat{\eta}}$ | | |
| | | | $\Pi_{\hat{q}\hat{\eta}}$ | $S_{\hat{v}^\perp}$ | | |
| V8 | $B^\dagger_\nu \partial^\nu B_\mu \bar{q}\gamma^\mu\gamma^5 q$ or $B_\nu \partial^\nu B_\mu \bar{q}\gamma^\mu\gamma^5 q$ | $S_{\hat{q}}$ | $\Pi_{\hat{q}\hat{v}}$ | $S_{\hat{v}}$ | 2 | |
| | | $\Pi_{\hat{q}\hat{\eta}}$ | $\Pi_{\hat{q}\hat{q}}$ | $S_{\hat{q}}$ | | |
| | | | $\Pi_{\hat{q}\hat{\eta}}$ | $S_{\hat{\eta}}$ | | |
| V9 | $\epsilon^{\mu\nu\rho\sigma} B^\dagger_\nu \partial_\rho B_\sigma \bar{q}\gamma_\mu q$ or $\epsilon^{\mu\nu\rho\sigma} B_\nu \partial_\rho B_\sigma \bar{q}\gamma_\mu q$ | $S_{\hat{v}^\perp}$ | $S_{\hat{v}^\perp}$ | $S_{\hat{\eta}}$ | 3 | I |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{v}^\perp}$ | | K |
| V10 | $\epsilon^{\mu\nu\rho\sigma} B^\dagger_\nu \partial_\rho B_\sigma \bar{q}\gamma_\mu\gamma^5 q$ or $\epsilon^{\mu\nu\rho\sigma} B_\nu \partial_\rho B_\sigma \bar{q}\gamma_\mu\gamma^5 q$ | $S_{\hat{\eta}}$ | $S_{\hat{q}}$ | $S_{\hat{q}}$ | 4 | D |
| | | | $S_{\hat{v}^\perp}$ | $S_{\hat{v}^\perp}$ | | H |
| | | | $S_{\hat{\eta}}$ | $S_{\hat{\eta}}$ | | L |

TABLE VIII. Similar to Table VI, but for spin-1 dark matter. For structures V7 and V8, the dark matter spin bilinear is a traceless symmetric tensor represented by $\Pi$ with components in the orthogonal basis defined by $\hat{q}$, $\hat{v}^\perp$ and $\hat{\eta}$. Note that V7 and V8 cannot interfere with any other structure.

| | Powers of $q$ and $v^\perp$ | Interaction structures |
|---|---|---|
| SI | 0 | $(\bar{X}X\bar{q}q,\ \bar{X}\gamma^\mu X\bar{q}\gamma_\mu q)$ |
| | 2 | $(\bar{X}\gamma^5 X\bar{q}q,\ \bar{X}\sigma^{\mu\nu}\gamma^5 X\bar{q}\sigma_{\mu\nu}q),\ \bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q$ |
| | 4 | $(\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu\gamma^5 q,\ \bar{X}\sigma^{\mu\nu}X\bar{q}\sigma_{\mu\nu}q)$ |
| | 6 | $\bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5 q$ |
| SD | 0 | $(\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu\gamma^5 q\ ,\ \bar{X}\sigma^{\mu\nu}X\bar{q}\sigma_{\mu\nu}q)$ |
| | 2 | $(\bar{X}X\bar{q}\gamma^5 q,\ \bar{X}\sigma^{\mu\nu}\gamma^5 X\bar{q}\sigma_{\mu\nu}q),\ (\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q,\ \bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5 q)$ |
| | 4 | $(\bar{X}X\bar{q}q,\ \bar{X}\gamma^\mu X\bar{q}\gamma_\mu q),\ \bar{X}\gamma^5 X\bar{q}\gamma^5 q$ |
| | 6 | $\bar{X}\gamma^5 X\bar{q}q$ |

TABLE IX. The number of powers of $q$ and $v^\perp$ that appear in the spin-independent and spin-dependent scattering cross section for each interaction structure (if dark matter is spin-1/2). Interaction structures that are listed together in parentheses can interfere and have the same kinematic suppression.

## V. INTERESTING FEATURES AND DEVIATIONS FROM THE STANDARD LORE

These results lead to some interesting observations, including deviations from the standard lore which arise from consideration of more general models than WIMPs of a constrained version of the MSSM. We find:

1. The standard lore is that neutralino annihilation to the light Higgs ($XX \rightarrow hh$) is necessarily $p$-wave suppressed [11]. In fact, we see from our analysis that the annihilation of Majorana fermion dark matter to identical scalars is either $p$-wave suppressed or suppressed by $CP$-violating phases [12]. Since the final state consists of identical scalars with $S = 0$, symmetry of the wavefunction requires that $L$ must be even. If the initial state is $S = 0$, $L = 0$, $J = 0$, $CP$-odd, then the final state of identical bosons must be $S = 0$, $L = 0$, $J = 0$, $CP$-even, and there must be $CP$-violation in the annihilation matrix element. The relevant interaction structure is then $\bar{X}\gamma^5 Xhh$. If the initial state is $S = 1$, $L = 1$, $CP$-even, then the matrix element is $p$-wave suppressed.

More generally, there are interesting interaction structures which are $CP$-violating and usually ignored – so some "suppressed" annihilation channels can be open if new physics is $CP$-violating.

2. The lore is that, if dark matter is a Majorana fermion, then $s$-wave annihilation to SM fermions is chirality suppressed. In fact, we find that this is only true if the term in the dark matter bilinear which annihilates the $s$-wave initial state couples to the time component of a pseudovector Standard Model bilinear. For other interaction structures, there need not be any chirality suppression. From the point of view of the microscopic theory, these interaction structures can arise from any new physics which interacts with both Weyl spinors, including sfermion-mixing, heavy fermions, etc. Although sfermion-mixing contributions to the matrix element are often assumed to scale as $m_f/m_X$, this is only true if one makes certain assumptions (such as minimal flavor violation) about the flavor structure of the theory.

   As a concrete example, consider models of isospin-violating dark matter [13, 14] that have been entertained in the context of recent signals of low-mass dark matter [14]. The contribution to the spin-independent matrix element from $s$- and $u$-channel squark exchange can be sizable if left-handed and right-handed squarks mix; squark-mixing for first generation squarks can therefore contribute to isospin violation. A consequence of this squark mixing is the presence of interaction structures other than pseudovector exchange [15], that can contribute to $s$-wave annihilation to fermions which is not chirality-suppressed.

3. If the SM fermion bilinear is pseudoscalar ($\bar{q}\gamma^5 q$), then the spin-independent scattering matrix element vanishes, including velocity- or momentum-suppressed terms [9]. This can be understood simply from the Lorentz structure of the interaction; one cannot construct a nucleon matrix element which is invariant under rotations and odd under parity unless it depends on the nucleon spin.

   Interestingly, if the SM fermion bilinear is pseudovector ($\bar{q}\gamma^\mu\gamma^5 q$) and the dark matter is spin-0, then the spin-independent scattering matrix element is zero. Again, this can be understood from the Lorentz structure of the interaction. A spin-independent fermion bilinear matrix element with the rotation and parity transformation properties needed for a pseudovector coupling must be a vector proportional to $\vec{v}^{\perp} \times \vec{q}$. A non-vanishing scalar can only be produced if this vector is contracted with a dark matter spin polarization, which is not present for spin-0 dark matter. In fact, if dark matter is spin-1, then the V4 interaction structure will also have exactly vanishing SI matrix element, because this interaction structure does not depend on the dark matter spin. Note that these interaction structures involving $\bar{q}\gamma^\mu\gamma^5 q$ with exactly vanishing SI-matrix element are all $CP$-violating.

   An interesting corollary of this result is that, if it can be shown that SM quarks couple to dark matter through exchange of a pseudovector, and if a spin-independent scattering cross section can be measured (even if velocity- or momentum-suppressed), then dark matter cannot be spin-0. As dark matter direct detection experiments increase in sensitivity, this result may have useful applications.

4. For spin-1/2 dark matter, only two sets of interaction structures can interfere in an annihilation process, and both sets annihilate an $L = 0$ state. Only one is relevant for Majorana fermion dark matter. If interference occurs for $p$-wave annihilation, then dark matter must be spin-1. Spin-0 dark matter does not exhibit interference in annihilation processes.

5. For both SI and SD scattering processes, the interaction structures whose matrix elements have no velocity- or momentum-suppression can interfere with each other. But for interaction structures with momentum or velocity-suppressed scattering matrix elements, interference effects may be small. For example, the structure $\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q$ yields a spin-independent scattering cross section which is suppressed by $v^{\perp 2}$, but can only interfere with interaction structures whose SI matrix element is suppressed by even more powers of $q$ or $v^\perp$. For this interaction structure, interference effects will be small unless it has a very small coefficient.

6. Note that these results do not depend on whether or not the dark matter-SM interaction is short-ranged. A non-contact interaction can induce additional form factors in the scattering matrix element, but the kinematic suppressions found here will always be present. Similarly, although dark matter annihilation can receive additional suppression if the interaction is non-contact, the results regarding which interaction structures yield $s$-wave or $p$-wave annihilation are independent of whether or not the interaction is short-ranged.

7. Most of these interaction structures are dimension 6. In a collider production process ($pp \to \bar{X}X$), these operators will receive an $\sim (E/m_X)^2$ enhancement, where $E$ is the energy scale of the dark matter production process. The dimension 5 interaction structures are S1, S2, V1, V2, V5 and V6; these dimension 5 structures will only receive an $E/m_X$ enhancement. But if dark matter is a spin-1 particle, then the matrix element can receive additional $E/m_X$ enhancement from the longitudinal polarization tensor. In particular, all interaction structures for spin-1 dark matter except V7 and V9 can couple to an $S = 0$ or $S = 2$ state, which will yield terms with

an additional $(E/m_X)^2$ enhancement with both dark matter particles longitudinally polarized. Since structures V7 and V9 couple to an $S=1$ state, they will yield terms with an additional $E/m_X$ enhancement. Scalar dark matter which interacts through interaction structures S1 and S2 will have suppressed monojet/monophoton production rates at the LHC [10]. These features can be used to distinguish between different interaction structures.

8. One can potentially distinguish the dark matter-SM interaction structure by combining information from indirect, direct and collider search strategies. As an example, we see that if dark matter is a Majorana fermion, then the only interaction structure which permits spin-independent scattering without velocity- or momentum-suppression is $\bar{X}X\bar{q}q$ (F1). But if dark matter is a Dirac fermion, then there is another interaction structure which permits unsuppressed SI-scattering; $\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$ (F5). It is difficult to distinguish these possibilities with direct detection experiments, but they can be distinguished by the event rates at indirect detection experiments [10], since the first operator permits only $p$-wave annihilation (which is highly suppressed) while the second operator allows annihilation from an $s$-wave state. However, if dark matter is a real scalar, then the operator $\phi^2\bar{q}q$ (S1) also permits unsuppressed SI-scattering and $s$-wave annihilation. This structure can be distinguished from the previous two by monojet and monophoton searches at the LHC; since this operator is dimension 5, it does not receive as large an energy enhancement as the other operators [10]. But if dark matter is spin-1 or is a complex spin-0 particle, then there are other interaction structures which can yield unsuppressed SI-scattering (S3, V1, V3). Interaction structures V1 and V3 will yield an LHC production rate with a large enhancement due to the longitudinal polarization tensors. This may permit them to be distinguished from the other interactions structures (and may be distinguished from each other because V1 allows $s$-wave annihilation, while V3 does not). However, it is difficult to distinguish structures F1 and S3 without a more detailed analysis. Unfortunately, the spin-dependent scattering cross section is not very useful in distinguishing these two possibilities, since both interaction structures yield spin-dependent cross sections suppressed by the factor $q^2 v^{\perp 2}$.

9. If the mass of the mediating particle is small compared to the dark matter mass or the collider production energy scale, then the form factor $F$ will scale as $E^{-2}$ for a production or annihilation process. Then, the rate of dark matter annihilation or production at colliders will be suppressed. On the other hand, striking signals of the mediating particle at a collider experiment may then be possible [16]. Combined studies of indirect detection and collider production rates can thus provide independent probes of the mass of the particles which mediate the dark matter interaction.

10. Many directional detection experiments are either operating or under construction [17]. For such experiments, the magnitude and direction of $\vec{q}$ can be measured on an event by event basis. With a sufficient number of events at such a detector, one can potentially distinguish a dependence on $q$ from a dependence on $v^\perp$. One interesting question has been the possibility of *dark matter astronomy*: the possibility of probing the dark matter velocity distribution using the event rate of direct detection experiments. A difficulty is that the event rate really probes the integral of the velocity distribution. But since the $v$-dependence of the spin-dependent and spin-independent matrix elements are generally different, measurements from directional detectors sensitive to SI and SD scattering can potentially probe two independent moments of the velocity distribution. This may permit a more detailed study of the velocity distribution of dark matter.

These are immediate and general results which arise from a study of generic dark matter interaction structures in the formalism which we have described here. An interesting long-term program for future study is the use of this formalism to determine the prospects for distinguishing the nature of dark matter interactions from the many data sets that are becoming available though direct, indirect and collider experiments. And if a clear indication of dark matter interactions is discovered, the next step would be to utilize this formalism to piece together the full dark matter-SM interaction structure.

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| bilinear | spin-independent | spin-dependent |
|---|---|---|
| $\bar{\psi}\psi$ | $2m_X\left(\xi'^{\dagger}\xi\right)$ | $\imath\frac{\mu}{m_X}\epsilon^{ijk}q^i v^{\perp j}\left(\xi'^{\dagger}\hat{S}^k\xi\right)$ |
| $\bar{\psi}\gamma^5\psi$ | 0 | $-2q^i\left(\xi'^{\dagger}\hat{S}^i\xi\right)$ |
| $\phi^{\dagger}\phi$ | 1 | 0 |
| $B^{\dagger}_{\mu}B^{\mu}$ | $\epsilon'^{\dagger}\cdot\epsilon$ | 0 |
| $\bar{\psi}\gamma^0\psi$ | $2m_X\left(\xi'^{\dagger}\xi\right)$ | $-\imath\frac{\mu}{m_X}\epsilon^{ijk}q^i v^{\perp j}\left(\xi'^{\dagger}\hat{S}^k\xi\right)$ |
| $\bar{\psi}\gamma^0\gamma^5\psi$ | 0 | $-4\mu v^{\perp i}\left(\xi'^{\dagger}\hat{S}^i\xi\right)$ |
| $-Im(\phi^{\dagger}\partial^0\phi)$ | $2m_X$ | 0 |
| $-Im(B^{\dagger}_{\nu}\partial^0 B^{\nu})$ | $2m_X\epsilon'^{\dagger}\cdot\epsilon$ | 0 |
| $\epsilon^{0ijk}B_i\partial_j B_k$ | 0 | $-2\imath\mu\epsilon^{ijk}v^{\perp}_i\epsilon_j\epsilon'_k$ |
| $\bar{\psi}\gamma^i\psi$ | $2\mu v^{\perp i}\left(\xi'^{\dagger}\xi\right)$ | $2\imath\epsilon^{ijk}q^j\left(\xi'^{\dagger}\hat{S}^k\xi\right)$ |
| $\bar{\psi}\gamma^i\gamma^5\psi$ | $-\imath\frac{\mu}{2m_X}\epsilon^{ijk}v^{\perp j}q^k\left(\xi'^{\dagger}\xi\right)$ | $-4m_X\left(\xi'^{\dagger}\hat{S}^i\xi\right)$ |
| $-Im(\phi^{\dagger}\partial^i\phi)$ | $2\mu v^{\perp i}$ | 0 |
| $-Im(B^{\dagger}_{\nu}\partial^i B^{\nu})$ | $2\mu v^{\perp i}\epsilon'^{\dagger}\cdot\epsilon$ | 0 |
| $\epsilon^{0ijk}B_j\partial_0 B_k$ | 0 | $2\imath m_X\epsilon^{ijk}\epsilon_j\epsilon'_k$ |
| $\bar{\psi}\sigma^{0i}\psi$ | $q^i\left(\xi'^{\dagger}\xi\right)$ | $4\imath\mu\epsilon^{ijk}v^{\perp j}\left(\xi'^{\dagger}\hat{S}^k\xi\right)$ |
| $\bar{\psi}\sigma^{ij}\psi$ | $-\frac{\mu}{2m_X}(q^i v^{\perp j}-q^j v^{\perp i})\left(\xi'^{\dagger}\xi\right)$ | $-4\imath m_X\epsilon^{ijk}\left(\xi'^{\dagger}\hat{S}^k\xi\right)$ |
| $\imath(B^{\dagger i}B^j - B^{\dagger j}B^i)$ | 0 | $\imath(\epsilon'^{\dagger i}\epsilon^j-\epsilon'^{\dagger j}\epsilon^i)$ |
| $B^iB^j(sym)$ | 0 | $\epsilon'^i\epsilon^j(sym)$ |

TABLE X. For each bilinear for spin-0 ($\phi$), spin-1/2 ($\psi$) and spin-1 ($B_\mu$) particles, we list the spin-independent and spin-dependent scattering matrix element at leading order. In the last row, "$(sym)$" means symmetric and traceless in the $ij$ indices. The spinor bilinears of the Standard Model can be obtained by the substitutions, $q^i \to -q^i$, $v^{\perp i} \to -v^{\perp i}$, $m_X \to m_f$, $\xi \to \zeta$.

### Appendix A: Scattering matrix elements

In Table X, we summarize the bilinear spin matrix elements for an incoming dark matter particle of mass $m_X$ with momentum $\vec{k} = \mu\vec{v}$ and an outgoing particle with momentum $\vec{k}' = \vec{k} - \vec{q}$, where $\mu = m_X m_A/(m_X + m_A)$ is the reduced mass of the dark matter-nucleus system, $\vec{v}$ is the relative velocity of the dark matter and the target nucleus, and the $\xi$'s are two-component spinors; $v^\perp$ is defined below. If dark matter is spin-1, then $\epsilon^\mu$ is its polarization vector. The spinor bilinears of the Standard Model are related to those for the dark matter bilinears by $\vec{q} \to -\vec{q}$, $\vec{v} \to -\vec{v}$, $m_X \to m_f$, $\xi \to \zeta$. We have grouped together terms that have the same Lorentz structure, but including the possibility of parity violation. The entries of the table are derived below.

In the center of mass frame, the incoming and outgoing four-momenta of the parton, $p$ and $p'$, and the incoming and outgoing four-momenta of the dark matter, $k$ and $k'$, to first order in the three-momenta, are

$$\begin{aligned}
p &= \left(\sqrt{m_f^2 + \vec{p}^{\,2}}, -\vec{p}\right) \sim \left(m_f, -\vec{p}\right), \\
p' &= \left(\sqrt{m_f^2 + (\vec{q} - \vec{p})^2}, \vec{q} - \vec{p}\right) \sim \left(m_f, \vec{q} - \vec{p}\right), \\
k &= \left(\sqrt{m_X^2 + \vec{p}^{\,2}}, \vec{p}\right) \sim \left(m_X, \vec{p}\right), \\
k' &= \left(\sqrt{m_X^2 + (\vec{p} - \vec{q})^2}, \vec{p} - \vec{q}\right) \sim \left(m_X, \vec{p} - \vec{q}\right).
\end{aligned} \tag{A1}$$

It is useful to define $v^\perp$ via

$$2\vec{k} - \vec{q} = -(2\vec{p} + \vec{q}) = 2\mu v^\perp \equiv 2\mu\frac{(\vec{q}\times\vec{v})\times\vec{q}}{|\vec{q}|^2}. \tag{A2}$$

We compute spinor bilinear matrix elements of the form

$$\bar{X}\Gamma X, \tag{A3}$$

where $\Gamma$ is a Dirac structure. Where helpful, we also compute the square of the matrix element, summed over initial and final spins.

We write our spinors as

$$u(p) = \left( \frac{p \cdot \sigma + m_f}{\sqrt{2(p^0 + m_f)}} \zeta \qquad \frac{p \cdot \bar{\sigma} + m_f}{\sqrt{2(p^0 + m_f)}} \zeta \right)^T ,$$
$$u(k) = \left( \frac{k \cdot \sigma + m_X}{\sqrt{2(k^0 + m_X)}} \xi \qquad \frac{k \cdot \bar{\sigma} + m_X}{\sqrt{2(k^0 + m_X)}} \xi \right)^T . \tag{A4}$$

### 1. Scalar

For a scalar Lorentz structure ($\Gamma = 1$), the matrix element is invariant under rotations and even under parity. It can therefore only contain terms that are either constant, or proportional to $\overrightarrow{p'} \cdot \overrightarrow{p}$ or $\overrightarrow{p'} \times \overrightarrow{p} \cdot \overrightarrow{S}$.

We get

$$\begin{aligned} \mathcal{M}_s &= \bar{u}(p')u(p) \\ &= \frac{1}{\sqrt{(p'^0 + m)(p^0 + m)}} \zeta'^\dagger \left[ m^2 + p' \cdot p - \imath \epsilon^{ijk} p'^i p^j \sigma^k + m(p'^0 + p^0) \right] \zeta \\ &\sim 2m \left( \zeta'^\dagger \zeta \right) - \frac{\imath}{m} \epsilon^{ijk} p'^i p^j \left( \zeta'^\dagger \hat{S}^k \zeta \right) . \end{aligned} \tag{A5}$$

The dominant term is spin-independent, and the spin-dependent term is suppressed.

Then, the Standard Model matrix element is

$$\begin{aligned} \mathcal{M}_{s(SM)} &\sim 2m_f \left( \zeta'^\dagger \zeta \right) - \frac{\imath}{m_f} \epsilon^{ijk} q^i p^j \left( \zeta'^\dagger \hat{S}^k \zeta \right) \\ &\sim 2m_f \left( \zeta'^\dagger \zeta \right) + \imath \frac{\mu}{m_f} \epsilon^{ijk} q^i v^{\perp j} \left( \zeta'^\dagger \hat{S}^k \zeta \right) , \end{aligned} \tag{A6}$$

and the dark matter matrix element is

$$\begin{aligned} \mathcal{M}_{s(X)} &\sim 2m_X \left( \xi'^\dagger \xi \right) + \frac{\imath}{m_X} \epsilon^{ijk} q^i k^j \left( \xi'^\dagger \hat{S}^k \xi \right) \\ &\sim 2m_X \left( \xi'^\dagger \xi \right) + \imath \frac{\mu}{m_X} \epsilon^{ijk} q^i v^{\perp j} \left( \xi'^\dagger \hat{S}^k \xi \right) . \end{aligned} \tag{A7}$$

If dark matter is spin-0 and the bilinear has a scalar Lorentz structure ($\phi^\dagger \phi$), then the matrix element is

$$\mathcal{M}^{spin-0}_{s(X)} = 1 . \tag{A8}$$

Similarly, if dark matter is a real or complex spin-1 and the bilinear has a scalar Lorentz structure ($B^\dagger_\mu B^\mu$), then the matrix element is

$$\mathcal{M}^{spin-1}_{s(X)} = \epsilon'^\dagger \cdot \epsilon , \tag{A9}$$

where $\epsilon$ and $\epsilon'$ are polarization vectors.

### 2. Pseudoscalar

For a bilinear with pseudoscalar Lorentz structure, the matrix element is invariant under rotations, but odd under parity. It must then be proportional to either $\overrightarrow{p} \cdot \overrightarrow{S}$ or $\overrightarrow{p'} \cdot S$. The matrix element is given by

$$\begin{aligned} \mathcal{M}_{ps} &= \bar{u}(p')\gamma^5 u(p) \\ &= \frac{1}{\sqrt{(p'^0 + m)(p^0 + m)}} \zeta'^\dagger \left[ (p^0 + m)(\overrightarrow{p'} \cdot \overrightarrow{\sigma}) - (p'^0 + m)(\overrightarrow{p} \cdot \overrightarrow{\sigma}) \right] \zeta \\ &\sim (\overrightarrow{p'} - \overrightarrow{p}) \cdot \zeta'^\dagger \overrightarrow{\sigma} \zeta \sim 2(p' - p)^i \left( \zeta'^\dagger \hat{S}^i \zeta \right) , \end{aligned} \tag{A10}$$

which gives

$$\mathcal{M}_{ps(SM)} = \bar{u}(p')\gamma^5 u(p) \sim 2q^i \left(\zeta'^\dagger \hat{S}^i \zeta\right) ,$$
$$\mathcal{M}_{ps(X)} = \bar{u}(k')\gamma^5 u(k) \sim -2q^i \left(\xi'^\dagger \hat{S}^i \xi\right) . \tag{A11}$$

This structure is spin-dependent and velocity-dependent; interestingly, there is no spin-independent term at all.

### 3. Vector

For a bilinear with vector Lorentz structure, the time-like component is invariant under rotations and parity. So it can either be a constant, or be proportional to $\overrightarrow{p'} \times \overrightarrow{p} \times \overrightarrow{S}$. The space-like components must rotate as a vector, but be odd under parity. They may then contain terms that are proportional to either $\overrightarrow{p'}$, $\overrightarrow{p}$, $\overrightarrow{p'} \times \overrightarrow{S}$ or $\overrightarrow{p} \times \overrightarrow{S}$.

We get

$$\begin{aligned}\mathcal{M}_v^0 &= \bar{u}(p')\gamma^0 u(p) \\ &= \frac{1}{\sqrt{(p'^0+m)(p^0+m)}}\zeta'^\dagger \left[(p'^0+m)(p^0+m) + \overrightarrow{p'}\cdot\overrightarrow{p} + \imath\epsilon^{ijk}p'^i p^j \sigma^k\right]\zeta \\ &\sim 2m\left(\zeta'^\dagger\zeta\right) + \frac{\imath}{m}\epsilon^{ijk}p'^i p^j \left(\zeta'^\dagger \hat{S}^k \zeta\right) ,\end{aligned} \tag{A12}$$

$$\begin{aligned}\mathcal{M}_v^i &= \bar{u}(p')\gamma^i u(p) \\ &= \frac{1}{\sqrt{(p'^0+m)(p^0+m)}}\zeta'^\dagger \left[(m+p^0)p'^i + (m+p'_0)p^i + \imath(m+p'^0)\epsilon^{ijk}p^j\sigma^k - \imath(m+p^0)\epsilon^{ijk}p'^j\sigma^k\right]\zeta \\ &\sim (p'+p)^i\left(\zeta'^\dagger\zeta\right) + 2\imath\epsilon^{ijk}(p-p')^j\left(\zeta'^\dagger \hat{S}^k\zeta\right) .\end{aligned} \tag{A13}$$

The leading term of this matrix element is spin-independent, but there are also momentum-suppressed spin-dependent terms.

We thus find, for the SM matrix elements,

$$\begin{aligned}\mathcal{M}^0_{v(SM)} &\sim 2m_f\left(\zeta'^\dagger\zeta\right) + \frac{\imath}{m_f}\epsilon^{ijk}q^i p^j\left(\zeta'^\dagger\hat{S}^k\zeta\right) \\ &\sim 2m_f\left(\zeta'^\dagger\zeta\right) - \imath\frac{\mu}{m_f}\epsilon^{ijk}q^i v^{\perp j}\left(\zeta'^\dagger\hat{S}^k\zeta\right) , \\ \mathcal{M}^i_{v(SM)} &\sim (2p+q)^i\left(\zeta'^\dagger\zeta\right) - 2\imath\epsilon^{ijk}q^j\left(\zeta'^\dagger\hat{S}^k\zeta\right) \\ &\sim -2\mu v^{\perp i}\left(\zeta'^\dagger\zeta\right) - 2\imath\epsilon^{ijk}q^j\left(\zeta'^\dagger\hat{S}^k\zeta\right) ,\end{aligned} \tag{A14}$$

and for the dark matter matrix elements,

$$\begin{aligned}\mathcal{M}^0_{v(X)} &\sim 2m_X\left(\xi'^\dagger\xi\right) - \frac{\imath}{m_X}\epsilon^{ijk}q^i k^j\left(\xi'^\dagger\hat{S}^k\xi\right) \\ &\sim 2m_X\left(\xi'^\dagger\xi\right) - \imath\frac{\mu}{m_X}\epsilon^{ijk}q^i v^{\perp j}\left(\xi'^\dagger\hat{S}^k\xi\right) , \\ \mathcal{M}^i_{v(X)} &\sim (2k-q)^i\left(\xi'^\dagger\xi\right) + 2\imath\epsilon^{ijk}q^j\left(\xi'^\dagger\hat{S}^k\xi\right) \\ &\sim 2\mu v^{\perp i}\left(\xi'^\dagger\xi\right) + 2\imath\epsilon^{ijk}q^j\left(\xi'^\dagger\hat{S}^k\xi\right) .\end{aligned} \tag{A15}$$

The squared matrix elements can be written as the tensors,

$$\begin{aligned}T_v^{00} &= 2m(2m+E_R)\sum_{spins}\zeta^\dagger\zeta = 8m^2 + 2\overrightarrow{q}^2 , \\ T_v^{0i} &= -m\sum_{spins}\zeta^\dagger[(\sigma^\alpha\sigma^i - \bar{\sigma}^\alpha\sigma^i)p'_\alpha]\zeta = -2mq_k\sum_{spins}\zeta^\dagger\sigma^k\sigma^i\zeta = -2mq^i\sum_{spins}\zeta^\dagger\zeta = -4mq^i , \\ T_v^{ij} &= m\sum_{spins}\zeta^\dagger[(\sigma^i\sigma^\alpha\sigma^j + \sigma^i\bar{\sigma}^\alpha\sigma^j)p'_\alpha - 2m\sigma^i\sigma^j]\zeta = 2m\sum_{spins}\zeta^\dagger[\sigma^i\sigma^j(m+E_R) - m\sigma^i\sigma^j]\zeta = 2\overrightarrow{q}^2\delta^{ij} .\end{aligned} \tag{A16}$$

So $T_v^{00} \sim 8m^2$, with all other components momentum-suppressed.

If dark matter is a spin-0 particle and the bilinear has a vector Lorentz structure ($-Im(\phi^\dagger \partial^\mu \phi)$), then

$$\mathcal{M}_v^{(spin-0)\mu} = (p+p')^\mu \,. \tag{A17}$$

Similarly, if dark matter is a complex spin-1 particle and the bilinear has a vector Lorentz structure ($-Im(B_\nu^\dagger \partial^\mu B^\nu)$), then

$$\mathcal{M}_v^{(spin-1)\mu} = (p+p')^\mu \epsilon'^\dagger \cdot \epsilon \,. \tag{A18}$$

Thus,

$$\begin{aligned}
\mathcal{M}_{v(X)}^{(spin-0)0} &\sim 2m_X \,, \\
\mathcal{M}_{v(X)}^{(spin-0)i} &\sim 2\mu v^{\perp i} \,, \\
\mathcal{M}_{v(X)}^{(spin-1)0} &\sim 2m_X \epsilon'^\dagger \cdot \epsilon \,, \\
\mathcal{M}_{v(X)}^{(spin-1)i} &\sim 2\mu v^{\perp i} \epsilon'^\dagger \cdot \epsilon \,.
\end{aligned} \tag{A19}$$

### 4. Pseudovector

For a bilinear with a pseudovector Lorentz structure, the time-component is rotation-invariant and odd under parity. It must therefore be a sum of terms that are proportional to either $\vec{p} \cdot \vec{S}$ or $\vec{p'} \cdot \vec{S}$. The spacelike components must rotate like a vector, but be even under parity. They must then be a sum of terms proportional to either $\vec{S}$ or $\vec{p'} \times \vec{p}$. We find

$$\begin{aligned}
\mathcal{M}_{pv}^0 &= \bar{u}(p')\gamma^0\gamma^5 u(p) \\
&= -\frac{1}{\sqrt{(p'^0+m)(p^0+m)}} \zeta'^\dagger \left[ (p'^0+m)(\vec{p} \cdot \vec{\sigma}) + (p^0+m)(\vec{p'} \cdot \vec{\sigma}) \right] \zeta \\
&\sim -2(p'+p)^i \left( \zeta'^\dagger \hat{S}^i \zeta \right) \,,
\end{aligned} \tag{A20}$$

$$\begin{aligned}
\mathcal{M}_{pv}^i &= \bar{u}(p')\gamma^i\gamma^5 u(p) \\
&= -\frac{1}{\sqrt{(p'^0+m)(p^0+m)}} \zeta'^\dagger \left[ (p' \cdot p + m(p^0+p'^0) + m^2)\sigma^i + p^i p'^j \sigma^j + p'^i p^j \sigma^j + \imath \epsilon^{ijk} p^j p'^k \right] \zeta \\
&\sim -4m \left( \zeta'^\dagger \hat{S}^i \zeta \right) + \frac{\imath}{2m} \epsilon^{ijk} p^j p'^k \left( \zeta'^\dagger \zeta \right) \,.
\end{aligned} \tag{A21}$$

Thus, the leading term of the pseudovector structure is spin-dependent. Interestingly, while the timelike component has no spin-independent contribution, the spacelike components have suppressed spin-independent terms.

Then, the SM matrix elements are

$$\begin{aligned}
\mathcal{M}_{pv(SM)}^0 &\sim -2(2p+q)^i \left( \zeta'^\dagger \hat{S}^i \zeta \right) \\
&\sim 4\mu v^{\perp i} \left( \zeta'^\dagger \hat{S}^i \zeta \right) \,, \\
\mathcal{M}_{pv(SM)}^i &\sim -4m_f \left( \zeta'^\dagger \hat{S}^i \zeta \right) + \frac{\imath}{2m_X} \epsilon^{ijk} p^j q^k \left( \zeta'^\dagger \zeta \right) \\
&\sim -4m_f \left( \zeta'^\dagger \hat{S}^i \zeta \right) - \imath \frac{\mu}{2m_X} \epsilon^{ijk} v^{\perp j} q^k \left( \zeta'^\dagger \zeta \right) \,,
\end{aligned} \tag{A22}$$

and the dark matter matrix elements are

$$\begin{aligned}
\mathcal{M}_{pv(X)}^0 &\sim -2(2k-q)^i \left( \xi'^\dagger \hat{S}^i \xi \right) \\
&\sim -4\mu v^{\perp i} \left( \xi'^\dagger \hat{S}^i \xi \right) \,, \\
\mathcal{M}_{pv(X)}^i &\sim -4m_X \left( \xi'^\dagger \hat{S}^i \xi \right) - \frac{\imath}{2m_X} \epsilon^{ijk} k^j q'^k \left( \xi'^\dagger \xi \right) \\
&\sim -4m_X \left( \xi'^\dagger \hat{S}^i \xi \right) - \imath \frac{\mu}{2m_X} \epsilon^{ijk} v^{\perp j} q'^k \left( \xi'^\dagger \xi \right) \,.
\end{aligned} \tag{A23}$$

We can write the squared matrix elements as tensors:

$$\begin{aligned}
T^{00}_{pv} &= m\sum_{spins}\zeta^\dagger[(\sigma^\alpha+\bar\sigma^\alpha)p'_\alpha - 2m]\zeta = 2mE_R\sum_{spins}\zeta^\dagger\zeta = 2\overrightarrow{q}^2\,,\\
T^{0i}_{pv} &= -m\sum_{spins}\zeta^\dagger[(\sigma^\alpha\sigma^i-\bar\sigma^\alpha\sigma^i)p'_\alpha]\zeta = -2mq_k\sum_{spins}\zeta^\dagger\sigma^k\sigma^i\zeta = -2mq^i\sum_{spins}\zeta^\dagger\zeta = -4mq^i\,,\\
T^{ij}_{pv} &= m\sum_{spins}\zeta^\dagger[(\sigma^i\sigma^\alpha\sigma^j+\sigma^i\bar\sigma^\alpha\sigma^j)p'_\alpha + 2m\sigma^i\sigma^j]\zeta\\
&= 2m\sum_{spins}\zeta^\dagger[\sigma^i\sigma^j(m+E_R)+m\sigma^i\sigma^j]\zeta = (4m^2+\overrightarrow{q}^2)\delta^{ij}\sum_{spins}\zeta^\dagger\zeta = \frac{16}{3}J(J+1)(m^2+\frac{\overrightarrow{q}^2}{4})g^{ij}\,. \qquad \text{(A24)}
\end{aligned}$$

Thus, $T^{ij}_{pv} = \frac{16}{3}J(J+1)m^2g^{ij}$ is the dominant component, with all the others momentum-suppressed.

If dark matter is spin-1, then there is one other possible structure, $\epsilon^{\mu\nu\rho\sigma}B_\nu\partial_\rho B_\sigma$, which gives

$$\mathcal{M}^{spin-1}_{pv} = \imath\epsilon^{\mu\nu\rho\sigma}(p+p')_\rho\epsilon_\nu\epsilon'_\sigma\,, \qquad \text{(A25)}$$

so that

$$\begin{aligned}
\mathcal{M}^{(spin-1)0}_{pv(X)} &\sim -2\imath\mu\epsilon^{ijk}v^\perp_i\epsilon_j\epsilon'_k\,,\\
\mathcal{M}^{(spin-1)i}_{pv(X)} &\sim 2\imath m_X\epsilon^{ijk}\epsilon_j\epsilon'_k\,. \qquad \text{(A26)}
\end{aligned}$$

#### 5. Tensor

Under parity, $\bar\psi\sigma^{\mu\nu}\psi$ transforms with the sign $(-1)^\mu(-1)^\nu$, where $(-1)^\mu \equiv 1$ for $\mu = 0$ and $(-1)^\mu \equiv -1$ for $\mu = 1,2,3$. The structure $\bar\psi\sigma^{0i}\psi$ thus rotates as a vector, but is odd under parity. It thus therefore be a sum of terms proportional to $\overrightarrow{p'}$, $\overrightarrow{p}$, $\overrightarrow{p'}\times\overrightarrow{S}$ or $\overrightarrow{p}\times\overrightarrow{S}$. Similarly, the structure $\bar\psi\sigma^{ij}\psi$ should transform under rotations as a tensor, and be invariant under parity. So it should contain terms which are proportional to $p'^ip^j$ or $S^iS^j$. We find

$$\begin{aligned}
\mathcal{M}^{0i}_t &= \bar u(p')\gamma^0\gamma^i u(p)\\
&= \frac{1}{\sqrt{(p'^0+m)(p^0+m)}}\zeta'^\dagger\left[-(p^0+m)(p'^i-\imath\epsilon^{ijk}p'^j\sigma^k)+(p'^0+m)(p^i+\imath\epsilon^{ijk}p^j\sigma^k)\right]\zeta\\
&\sim -(p'-p)^i\left(\zeta'^\dagger\zeta\right)+2\imath\epsilon^{ijk}(p'+p)^j\left(\zeta'^\dagger\hat S^k\zeta\right)\,, \qquad \text{(A27)}
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}^{ij}_t &= \bar u(p')\gamma^i\gamma^j u(p)\\
&= -\frac{\imath\epsilon^{ijk}}{\sqrt{(p'^0+m)(p^0+m)}}\zeta'^\dagger\left[[(p'^0+m)(p^0+m)-\overrightarrow{p'}\cdot\overrightarrow{p}]\sigma^k - p^kp'^l\sigma^l+\imath\epsilon^{kml}p'^lp^m+p'^kp^m\sigma^m\right]\zeta\\
&\sim -4\imath m\epsilon^{ijk}\left(\zeta'^\dagger\hat S^k\zeta\right)+\frac{1}{2m}(p'^ip^j-p'^jp^i)\left(\zeta'^\dagger\zeta\right)\,. \qquad \text{(A28)}
\end{aligned}$$

This structure is spin-dependent, and also has a momentum-suppressed spin-independent term.

For the SM matrix elements, we get

$$\begin{aligned}
\mathcal{M}^{0i}_{t(SM)} &= -q^i\left(\zeta'^\dagger\zeta\right)+2\imath\epsilon^{ijk}(2p+q)^j\left(\zeta'^\dagger\hat S^k\zeta\right)\\
&= -q^i\left(\zeta'^\dagger\zeta\right)-4\imath\mu\epsilon^{ijk}v^{\perp j}\left(\zeta'^\dagger\hat S^k\zeta\right)\,,\\
\mathcal{M}^{ij}_{t(SM)} &= -4\imath m_f\epsilon^{ijk}\left(\zeta'^\dagger\hat S^k\zeta\right)+\frac{1}{2m_f}(q^ip^j-q^jp^i)\left(\zeta'^\dagger\zeta\right)\\
&= -4\imath m_f\epsilon^{ijk}\left(\zeta'^\dagger\hat S^k\zeta\right)-\frac{\mu}{2m_f}(q^iv^{\perp j}-q^jv^{\perp i})\left(\zeta'^\dagger\zeta\right)\,, \qquad \text{(A29)}
\end{aligned}$$

and for the dark matter matrix elements, we get

$$\begin{aligned}
\mathcal{M}^{0i}_{t(X)} &= q^i\left(\xi'^\dagger\xi\right) + 2\imath\epsilon^{ijk}(2k-q)^j\left(\xi'^\dagger\hat{S}^k\xi\right) \\
&= q^i\left(\xi'^\dagger\xi\right) + 4\imath\mu\epsilon^{ijk}v^{\perp j}\left(\xi'^\dagger\hat{S}^k\xi\right), \\
\mathcal{M}^{ij}_{t(X)} &= -4\imath m_X\epsilon^{ijk}\left(\xi'^\dagger\hat{S}^k\xi\right) - \frac{1}{2m_X}(q^ik^j - q^jk^i)\left(\xi'^\dagger\xi\right) \\
&= -4\imath m_X\epsilon^{ijk}\left(\xi'^\dagger\hat{S}^k\xi\right) - \frac{\mu}{2m_X}(q^iv^{\perp j} - q^jv^{\perp i})\left(\xi'^\dagger\xi\right) \\
&= -4\imath m_X\epsilon^{ijk}\left(\xi'^\dagger\hat{S}^k\xi\right).
\end{aligned} \tag{A30}$$

The squared matrix elements are

$$\begin{aligned}
T_t^{\mu\nu\rho\sigma} &= \sum_{spins}\bar{u}(p)\sigma^{\mu\nu}(\not{p}+m)\sigma^{\rho\sigma}u(p) = m\sum_{spins}\bar{u}(p)\sigma^{\mu\nu}(1+\gamma^0)\sigma^{\rho\sigma}u(p), \\
T_t^{\mu\nu kl} &= m\sum_{spins}\bar{u}(p)\sigma^{\mu\nu}\sigma^{kl}(1+\gamma^0)u(p), \\
T_t^{ijkl} &= \frac{16J(J+1)}{3}m^2(g^{ik}g^{jl} - g^{il}g^{jk}),
\end{aligned} \tag{A31}$$

with all other components vanishing.

If dark matter is a complex spin-1 particle with tensor Lorentz structure ($\imath(B^\dagger_\mu B_\nu - B^\dagger_\nu B_\mu)$) then the matrix element is

$$\mathcal{M}^{(cpx.\ spin-1)\mu\nu}_{t(X)} = \imath(\epsilon'^{*\mu}\epsilon^\nu - \epsilon'^{*\nu}\epsilon^\mu). \tag{A32}$$

For a real spin-1 particle with tensor Lorentz structure ($B^\mu B^\nu(sym)$, where "$(sym)$" means symmetric and traceless in the $\mu\nu$ indices), the matrix element is

$$\mathcal{M}^{(real\ spin-1)\mu\nu}_{t(X)} = \epsilon'^\mu\epsilon^\nu(sym). \tag{A33}$$

## Appendix B: Annihilation matrix elements

We begin by listing the (exact) spinor bilinears for a dark matter creation or annihilation process. For the sake of generality, we allow the particles to have different masses. In terms of two-component spinors $\xi_i$, the Dirac spinors may be written as

$$\begin{aligned}
u(k_1) &= \begin{pmatrix} \frac{k_1\cdot\sigma + m_1}{\sqrt{2(k_1^0+m_1)}}\xi_1 & \frac{k_1\cdot\bar{\sigma}+m_1}{\sqrt{2(k_1^0+m_1)}}\xi_1 \end{pmatrix}^T, \\
v(k_2) &= \begin{pmatrix} \frac{k_2\cdot\sigma + m_2}{\sqrt{2(k_2^0+m_2)}}\xi_2 & -\frac{k_2\cdot\bar{\sigma}+m_2}{\sqrt{2(k_2^0+m_2)}}\xi_2 \end{pmatrix}^T,
\end{aligned} \tag{B1}$$

where the particles have four-momenta,

$$\begin{aligned}
k_1^\mu &= (E_1, \vec{k}_1) = \left(\sqrt{m_1^2 + \vec{k}^2}, \vec{k}\right) \\
k_2^\mu &= (E_2, \vec{k}_2) = \left(\sqrt{m_2^2 + \vec{k}^2}, -\vec{k}\right).
\end{aligned} \tag{B2}$$

The bilinears for an outgoing fermion/anti-fermion pair are then,

$$\begin{aligned}
\bar{u}(k_1)v(k_2) &= \left[\sqrt{\frac{E_1+m_1}{E_2+m_2}} + \sqrt{\frac{E_2+m_2}{E_1+m_1}}\right]\vec{k}\cdot\left(\xi_1^\dagger\vec{\sigma}\xi_2\right), \\
\bar{u}(k_1)\gamma^5v(k_2) &= -\frac{1}{\sqrt{(E_1+m_1)(E_2+m_2)}}\left[(E_1+m_1)(E_2+m_2) + \vec{k}^2\right]\left(\xi_1^\dagger\xi_2\right),
\end{aligned}$$

| bilinear | annihilation matrix element |
|---|---|
| $\phi^\dagger \phi$ | $1$ |
| $\imath Im(\phi^\dagger \partial^0 \phi)$ | $0$ |
| $\imath Im(\phi^\dagger \partial^i \phi)$ | $-\imath k^i$ |
| $B_\mu B^\mu$ | $\epsilon_1 \cdot \epsilon_2$ |
| $\imath Im(B_\nu^\dagger \partial^0 B^\nu)$ | $0$ |
| $\imath Im(B_\nu^\dagger \partial^i B^\nu)$ | $-\imath k^i \epsilon_1^\dagger \cdot \epsilon_2$ |
| $\imath(B_i^\dagger B_j - B_j^\dagger B_i)$ | $\imath(\epsilon_{1i}^\dagger \epsilon_{2j} - \epsilon_{2i}^\dagger \epsilon_{1j})$ |
| $\imath(B^{i\dagger} B^0 - B^{0\dagger} B^i)$ | $\imath(\epsilon_1^{i\dagger} \epsilon_2^0 - \epsilon_2^{i\dagger} \epsilon_1^0)$ |
| $\epsilon^{0ijk} B_i \partial_j B_k$ | $\imath \epsilon_{ijk} k^i (\epsilon_2^j \epsilon_1^k - \epsilon_2^k \epsilon_1^j)$ |
| $\epsilon^{0ijk} B_j \partial_0 B_k$ | $0$ |
| $-\epsilon_{0ijk} B^0 \partial^j B^k$ | $\imath \epsilon_{ijk} k^j (\epsilon_2^0 \epsilon_1^k - \epsilon_2^k \epsilon_1^0)$ |
| $B^\nu \partial_\nu B_0$ | $-2\imath E \epsilon_1^0 \epsilon_2^0 - \imath k_i (\epsilon_2^i \epsilon_1^0 - \epsilon_2^0 \epsilon_1^i)$ |
| $B^\nu \partial_\nu B^i$ | $-\imath E(\epsilon_2^0 \epsilon_1^i + \epsilon_1^0 \epsilon_2^i) - \imath k_j (\epsilon_2^j \epsilon_1^i - \epsilon_1^j \epsilon_2^i)$ |

TABLE XI. The annihilation matrix elements for spin-0 and spin-1 dark matter bilinears.

$$
\begin{aligned}
\bar{u}(k_1)\gamma^0\gamma^5 v(k_2) &= -\frac{1}{\sqrt{(E_1+m_1)(E_2+m_2)}}\left[(E_1+m_1)(E_2+m_2) - \overrightarrow{k}^2\right]\left(\xi_1^\dagger \xi_2\right) , \\
\bar{u}(k_1)\gamma^i\gamma^5 v(k_2) &= \imath \epsilon^{ijk} k^j \left(\sqrt{\frac{E_2+m_2}{E_1+m_1}} + \sqrt{\frac{E_1+m_1}{E_2+m_2}}\right)\left(\xi_1^\dagger \sigma^k \xi_2\right) + k^i \left(\sqrt{\frac{E_1+m_1}{E_2+m_2}} - \sqrt{\frac{E_2+m_2}{E_1+m_1}}\right)\left(\xi_1^\dagger \xi_2\right) , \\
\bar{u}(k_1)\gamma^0 v(k_2) &= \frac{E_1+m_1-E_2-m_2}{\sqrt{(E_1+m_1)(E_2+m_2)}} k^i \left(\xi_1^\dagger \sigma^i \xi_2\right) , \\
\bar{u}(k_1)\gamma^i v(k_2) &= -\frac{(E_1+m_1)(E_2+m_2) + \overrightarrow{k}^2}{\sqrt{(E_1+m_1)(E_2+m_2)}}\left(\xi_1^\dagger \sigma^i \xi_2\right) + \frac{2k^i k^j}{\sqrt{(E_1+m_1)(E_2+m_2)}}\left(\xi_1^\dagger \sigma^j \xi_2\right) , \\
\bar{u}(k_1)\sigma^{0i} v(k_2) &= -\imath \frac{(E_1+m_1)(E_2+m_2) - \overrightarrow{k}^2}{\sqrt{(E_1+m_1)(E_2+m_2)}}\left(\xi_1^\dagger \sigma^i \xi_2\right) - 2\imath \frac{k^i k^j}{\sqrt{(E_1+m_1)(E_2+m_2)}}\left(\xi_1^\dagger \sigma^j \xi_2\right) , \\
\bar{u}(k_1)\sigma^{0i}\gamma^5 v(k_2) &= \imath k^i \left(\sqrt{\frac{E_2+m_2}{E_1+m_1}} + \sqrt{\frac{E_1+m_1}{E_2+m_2}}\right)\left(\xi_1^\dagger \xi_2\right) + \epsilon^{ijk} k^j \left(\sqrt{\frac{E_2+m_2}{E_1+m_1}} - \sqrt{\frac{E_1+m_1}{E_2+m_2}}\right)\left(\xi_1^\dagger \sigma^k \xi_2\right) . \quad \text{(B3)}
\end{aligned}
$$

The bilinears of the initial state fermion/anti-fermion pair can be obtained by conjugating the above expressions.

In Table XI we provide the annihilation matrix elements for various dark matter bilinears in the case of spin-0 and spin-1 dark matter. For spin-1 dark matter, the two particles have polarization vectors $\epsilon_1$ and $\epsilon_2$.

One can verify that the structure $(\epsilon_1^0 \epsilon_2^k - \epsilon_1^k \epsilon_2^0)$ is only non-zero for an initial state with total spin and $z$-axis spin projection $|S, S_z\rangle$ given by $|2,1\rangle$, $|2,0\rangle$, $|2,-1\rangle$ and $|0,0\rangle$. Similarly, the structure $(\epsilon_1^j \epsilon_2^k - \epsilon_1^k \epsilon_2^j)$ is only non-zero for initial spin states $|1,1\rangle$, $|1,0\rangle$ and $|1,-1\rangle$, and the structure $(\epsilon_1^i \epsilon_2^0 + \epsilon_1^0 \epsilon_2^i)$ is only non-zero for initial spin states $|1,1\rangle$ and $|1,-1\rangle$. The structure $\epsilon_1 \cdot \epsilon_2$ is only non-zero for initial spin states $|0,0\rangle$ and $|2,0\rangle$. Finally, $\epsilon_1^0 \epsilon_2^0$ is only non-zero for initial spin states $|0,0\rangle$ and $|2,0\rangle$.


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