The pesky power asymmetry

Liang Dai, Donghui Jeong, Marc Kamionkowski, and Jens Chluba

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The Pesky Power Asymmetry

Liang Dai, Donghui Jeong, Marc Kamionkowski and Jens Chluba
Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles St., Baltimore, MD 21218

Physical models for the hemispherical power asymmetry in the cosmic microwave background (CMB) reported by the Planck Collaboration must satisfy CMB constraints to the homogeneity of the Universe and quasar constraints to power asymmetries. We survey a variety of models for the power asymmetry and show that consistent models include a modulated scale-dependent isocurvature contribution to the matter power spectrum or a modulation of the reionization optical depth, gravitational-wave amplitude, or scalar spectral index. We propose further tests to distinguish between the different scenarios.

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The Planck Collaboration has reported a hemispherical asymmetry in the cosmic microwave background (CMB) fluctuations [1], thus confirming a similar power asymmetry seen in the Wilkinson Microwave Anisotropy Probe (WMAP) data [2]. The new data, with far better multi-frequency component separation, make it more difficult to attribute the asymmetry to foregrounds. The asymmetry is also seen in the Planck data to extend to smaller scales, and it is thus of greater statistical significance than in the WMAP data.

If the asymmetry is modeled as a dipolar modulation of an otherwise statistically isotropic CMB sky [3–5], the best-fit dipole has direction $(l, b) = (227, -27)$ and amplitude (in terms of r.m.s. temperature fluctuations on large angular scales, multipoles $\ell < 64$) of $A = 0.072 \pm 0.022$, although the asymmetry extends to higher ℓ [1, 6].

This power asymmetry is, as we explain below, extremely difficult to reconcile with inflation. Given the plenitude of impressive successes of inflation (the nearly, but not precisely, Peebles-Harrison-Zeldovich spectrum, adiabatic rather than isocurvature perturbations, the remarkable degree of Gaussianity), the result requires the deepest scrutiny. While there are some who may wave away the asymmetry as a statistical fluctuation [7], evidence is accruing that it is statistically significant. There is moreover the tantalizing prospect that it may be an artifact of some superhorizon pre-inflationary physics. Here we investigate physical explanations for the origin of the asymmetry and put forward new tests of those models.

We begin by reviewing the tension between the asymmetry and inflation. We then provide a brief survey of prior hypotheses and discuss the very stringent constraints imposed by the CMB temperature quadrupole and by upper limits to hemispherical asymmetries in quasar abundances. We review some existing models for the asymmetry and then posit that the asymmetry may be due to spatial variation of standard cosmological parameters (e.g., the reionization optical depth, the scalar spectral index, and gravitational-wave amplitude) that affect CMB fluctuations without affecting the total density nor the matter power spectrum. We show how the different scenarios can be distinguished by the ℓ dependence of the asymmetry, and we discuss other possible tests of the models.

The CMB power asymmetry is modeled as a dipole modulation of the power; i.e., the temperature fluctuation in direction $\hat{n}$ is $(\Delta T/T)(\hat{n}) = s(\hat{n})[1 + A\,\hat{n}\cdot\hat{p}]$, where $s(\hat{n})$ is a statistically isotropic map, A is the power-dipole amplitude, and $\hat{p}$ is its direction. However, the asymmetry cannot arise due to a preferred direction in the three-dimensional spectrum $P(k)$ [8], as reality of the fractional matter-density perturbation $\delta(\mathbf{r})$ relates the Fourier component $\tilde{\delta}(\mathbf{k})$ for wavevector $\mathbf{k}$ to that, $\tilde{\delta}(-\mathbf{k}) = \tilde{\delta}^*(\mathbf{k})$, of $-\mathbf{k}$. The asymmetry must therefore be attributed to a *spatial modulation* of three-dimensional power across the observable Universe. The modulation required to explain the asymmetry can then be written in terms of a spatially-varying power spectrum, $P(k, \mathbf{r}) = P(k)[1 + 2A\,\hat{p}\cdot\mathbf{r}/r_{ls}]$, where r_{ls} is the distance to the last-scattering surface, for modes inside the comoving horizon at present ($k \gtrsim H_0^{-1}$).

Any model that modulates the power must do so without introducing a modulation in the density of the Universe. A long-wavelength isocurvature density fluctuation with an amplitude $O(10\%)$ must generates a temperature dipole two orders of magnitude greater than is observed. If the density fluctuation is adiabatic, then the intrinsic temperature dipole is cancelled by a Doppler dipole due to our infall toward the denser side. Even so, in this case, the small temperature quadrupole and octupoles constrain the density in the observed Universe to vary by no more than $O(10^{-3})$ [9].

These considerations make it unlikely that the power asymmetry could arise in single-clock models for inflation. In these models, the inflaton controls both the power-spectrum amplitude and the total density, thus making it difficult to introduce an $O(10\%)$ modulation in the power with a $\lesssim O(10^{-3})$ modulation of the total density. These arguments were made precise for slow-roll inflation with a standard kinetic term in Ref. [10]. We surmise that it may be difficult to get to work also with nontrivial kinetic terms [11], especially given the increasingly tight constraints to such models from Planck.

There are additional and very stringent constraints to power modulation—even in models that can do so without modulating the density—from the Sloan Digital Sky Survey (SDSS) quasar sample [12]. These quasars are found at distances nearly half of the comoving distance to the CMB surface of last scatter, and their abundances depend very sensitively on the amplitude of the primordial power spectrum. A detailed analysis [12] finds an upper limit $A < 0.0153$ (99% C.L.) to the amplitude of an asymmetry oriented in the direction of the CMB dipole. While this is consistent with the 3σ lower limit $A \gtrsim 0.006$ inferred from Ref. [13], it is roughly five times smaller than the central value $A = 0.072$. Of course, quasars probe the power spectrum primarily at wavenumbers $k \sim 1$ Mpc^{-1}, while the $\ell \lesssim 60$ CMB power probes $k \lesssim 0.035$ Mpc^{-1}. A model that produces a power asymmetry with a sufficiently strong scale dependence may (neglecting the possible extension of the CMB asymmetry to higher ℓ) allow the central CMB value to be consistent with the quasar constraint.

To summarize: Any mechansim that accounts for the CMB power asymmetry at $\ell \lesssim 60$ must (1) do so while leaving the amplitude of the long-wavelength adiabatic or isocurvature density fluctuation across the observable Universe to be $\lesssim 10^{-3}$, and (2) leave the power asymmetry at $k \sim 1$ Mpc^{-1} small. Given that the asymmetries are seen at $\ell < 60$, it is also likely that any causal mechanism must involve inflation. We now run through a number of scenarios for the power asymmetry.

Ref. [10] arranged for a scale-independent power modulation, while keeping the total density fixed, by introducing a curvaton during inflation. The curvaton can then contribute appreciably to a modulation of the density-perturbation amplitude without modulating the mean density. This model is inconsistent with the quasar bound for the best-fit value $A = 0.072$. This model also predicts a non-Gaussianity parameter $f_{\rm NL}^{\rm (local)} \gtrsim 25(A/0.07)^2$, and would thus be ruled out for $A = 0.072$ by Planck constraints [14] to $f_{\rm NL}^{\rm (local)}$, even if there were no quasar constraint.

Ref. [15] also presented a modified inflationary theory wherein the curvaton decays after dark matter freezes out thus giving rise to an isocurvature perturbation that is subdominant relative to the usual adiabatic perturbations from inflaton decay. A postulated superhorizon perturbation to the curvaton field then modulates the amplitude of the isocurvature contribution across the observable Universe. The model parameters can be chosen so that this spatially-varying isocurvature contribution is scale-dependent, with a CMB power spectrum that peaks around $\ell \sim 10$, falls off rapidly from $\ell \sim 10$ to $\ell \sim 100$, and is then negligibible at higher ℓ. The model predicts an isocurvature contribution to primordial perturbations that may be in tension with new Planck upper limits [16], although more analysis may be required, given the asymmetric nature of the contribution, to determine the consistency of the model with current Planck data.

Ref. [17] recently postulated that the power asymmetry may arise in single-field inflation through some sort of non-Gaussianity that increases the bispectrum in the squeezed limit. In this way, the homogeneity constraint imposed by the CMB can be evaded. This model, however, gives rise to a roughly scale-independent power asymmetry and thus conflicts with the quasar constraint. It may still be possible though, to adjust the parameters to reduce the asymmetry on small scales.

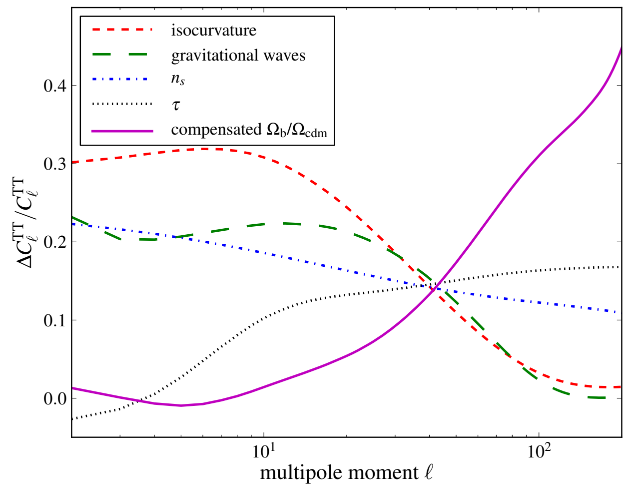

FIG. 1: The fractional change $\Delta C_\ell^{\rm TT}/C_\ell^{\rm TT}$ in the CMB power spectrum due to the inflationary model of Ref. [15] and modulation of the gravitational-wave amplitude, scalar spectral index, reionization optical depth, and baryon density (compenstated by the dark-matter density so that the total density is fixed). Each curve is normalized so that $A = 0.072$.

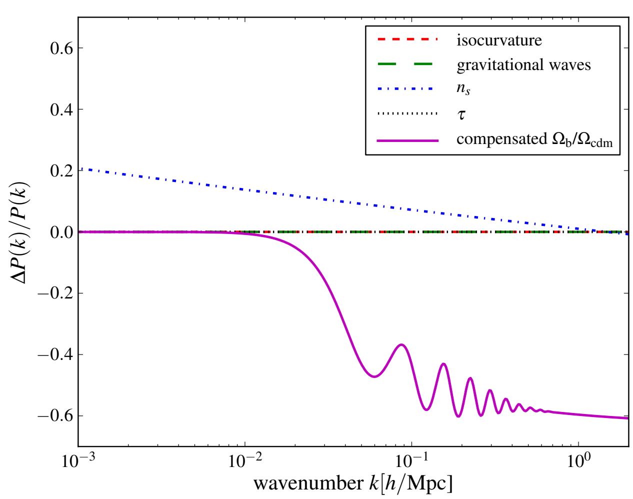

FIG. 2: The fractional change $\Delta P(k)/P(k)$ in the matter power spectrum for each of the models shown in Fig. 1.

We now suppose that the power asymmetry may arise from a modulation of one of the cosmological parameters that affects the CMB power spectrum. There are a number of cosmological parameters—abundances of cosmic constituents, inflationary observables, fundamental-physics parameters [18]—that affect the CMB power spectrum [19]. If there is a difference between the value of one of these cosmological parameters on one side of the sky and the value on the other side, then the CMB power spectrum one side may differ from that on the other side. For each of these parameters p, we calculate $\Delta C_\ell^{\rm TT} = (\partial C_\ell^{\rm TT}/\partial p)\Delta p$, where the amplitude Δp is chosen so that it produces an asymmetry $A = 0.072$. We assume here that this asymmetry is determined from the data by weighting the asymmetry in all spherical-harmonic modes equally up to $\ell_{\rm max} = 64$; i.e.,

$$A = \frac{(1/2)\sum_{\ell=2}^{\ell_{\rm max}}(2\ell+1)(\Delta C_\ell^{\rm TT}/C_\ell^{\rm TT})}{\sum_{\ell=2}^{\ell_{\rm max}}(2\ell+1)}. \qquad (1)$$

The fractional power-spectrum differences, normalized to give $A = 0.072$ are given in Fig. 1. We then plot in Fig. 2 the fractional change $\Delta P(k)$ in the matter power spectrum induced by the modulations of each of the parameters considered in Fig. 2. We do not consider parameters that only affect the recombination history (e.g., the helium abundance, fine-structure constant, etc.), as these modify CMB power only on small scales (high ℓ), not at the lower ℓ where the asymmetry is best seen.

Ref. [6] considers, among other possibilities, a modulation in $\Omega_{\rm b}$, the baryon density. However, if $\Omega_{\rm b}$ is modulated by $O(10\%)$, while holding all other parameters fixed, it will introduce a large-scale inhomogeneity in conflict with the CMB dipole/quadrupole/octupole. These constraints can be evaded through a compensated isocurvature perturbations (CIP), wherein $\Omega_{\rm b}$ and $\Omega_{\rm cdm}$, the dark-matter density, are both modulated in such a way that the total matter density $\Omega_{\rm b} + \Omega_{\rm cdm}$ remains constant across the observable Universe [20]. Such a hypothesis results, as Fig. 2 shows, in a power asymmetry at small scales larger than allowed by the quasar constraint. It can therefore be ruled out.

Ref. [4] considered a dark-energy density that varies linearly with position along the direction picked out by the power dipole. The dark-energy density is negligible at the surface of last scatter, but it affects at later times CMB fluctuations in two different ways. First of all, changes to $\Omega_{\rm de}$ change the ISW contribution to low-ℓ power, but this is a relatively small effect. The other consequence is a change to the angle subtended by a given comoving scale. The effect of a dipolar modulation, across the sky, of this mapping is equivalent to, and indistinguishable from, that induced by a peculiar velocity. A variation of $\Omega_{\rm de}$ large enough to account for the $A \simeq 0.072$ asymmetry would thus yield a CMB temperature dipole considerably larger than that observed. This explanation can thus be learned out.

More generally, a modulation of any of the parameters that affects the total density of the Universe that is large enough to account for the power asymmetry will give rise to a large-angle CMB fluctuation in gross conflict with observations. We thus now consider modulations to several parameters that affect the CMB power spectrum without changing the matter densities.

We begin with a variation to the scalar spectral index $n_{\rm s}$. In considering a modulation of n_s, we must specify a pivot wavenumber k_0, at which the power on both sides of the sky is equal. Here we choose this pivot point to be $k_0 = 1$ Mpc^{-1} so that the quasar constraint is satisfied. Doing so allows us accommodate a large-scale power-asymmetry amplitude $A = 0.072$ with a value of $n_{\rm s} \simeq 0.93$ on one side of the sky and $n_{\rm s} \simeq 0.99$ on the other. This model then predicts that the CMB power asymmetry should decrease, but relatively slowly, with higher ℓ, as shown in Fig. 1.

Along similar lines, one can vary the gravitational-wave amplitude from one side of the sky to the other. The gravitational-wave energy-density flucutation required to account for the low-ℓ power asymmetry is small enough to satisfy the homogeneity constraints, and gravitational waves contribute nothing to $P(k)$. This hypothesis can thus explain the CMB power asymmetry without violating other constraints. The only difficulty with the model is that an asymmetry amplitude $A = 0.072$ requires a gravitational-wave amplitude on one side of the Universe ten times larger than the homogeneous upper limit, a magnitude that may be not only unpalatable, but also inconsistent with current data. Still, an asymmetry of smaller amplitude, say $A \sim 0.015$ may be consistent.

We finally consider a dipolar modulation of τ, the reionization optical depth. The optical depth primarily suppresses power at $\ell \gtrsim 20$, but there is also a small increase in power at lower ℓ from re-scattering of CMB photons. We find from Figs. 1 and 2 that a modulation of τ can account for the asymmetry in the CMB without affecting (by assumption, really) $P(k)$ at quasar scales. An asymmetry $A = 0.072$ can be obtained by taking $\tau = 0.017$ on one side of the Universe and $\tau = 0.21$ on the other side. A value of $\tau \simeq 0.017$ implies a reionization redshift $z \simeq 3$, which is lower than quasar absorption spectra allow. Still, an asymmetry $A \simeq 0.05$ could be accommodated while preserving a reionization redshift $z \gtrsim 6$ everywhere. We surmise, without developing a detailed microphysical model that a spatial modulation in the primordial $P(k)$ at $k \gg$Mpc^{-1} could give rise to such a τ modulation, or perhaps one of slightly lower amplitude, given the highly uncertain physics reionization and the possibly strong dependence to small changes in initial conditions.

To summarize, we have four models that can account for the CMB asymmetry while leaving the Universe homogeneous and without affecting $P(k)$ on quasar scales. There is the inflation model of Ref. [15] whose possible phenomenological weakness may be in an isocurvature contribution in tension with current upper limits, and there is modulation of n_s, which is phenomenologically quite attractive. The other two models—variable gravitational-wave amplitude and variable τ—require variations in the parameters that are probably larger than are allowed. Still, we continue to consider them for illustrative purposes and in case the asymmetry amplitude is found in the future to be smaller but still nonzero. We now discuss measurements that can distinguish between the different scenarios.

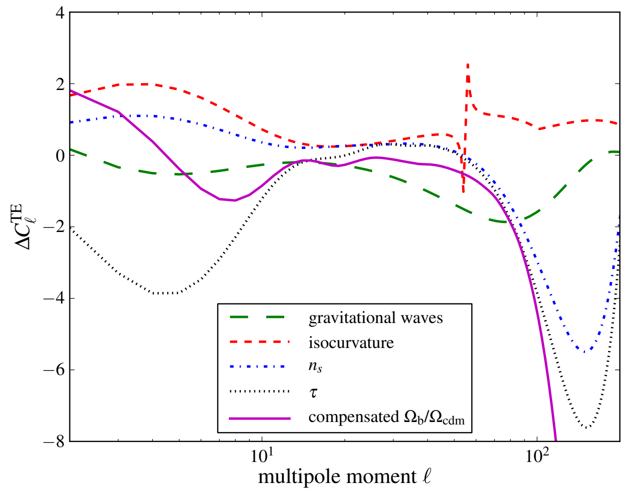

FIG. 3: The changes $\Delta C_\ell^{\rm TE}$ of the CMB temperature-polarization power spectrum for each of the models shown in Fig. 1. To facilitate the comparison better, we present $\Delta C_\ell^{\rm TE}$ instead of the fractional changes.

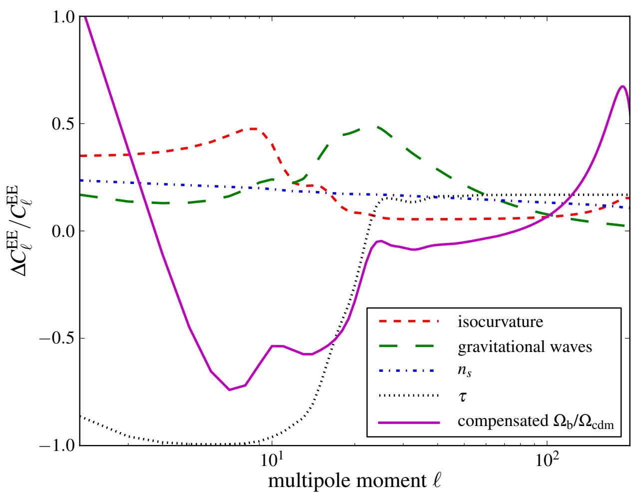

FIG. 4: The fractional changes $\Delta C_\ell^{\rm EE}/C_\ell^{\rm EE}$ in the CMB polarization power spectrum for each of the models shown in Fig. 1.

First of all, with Plank data we should be able to measure the difference $\Delta C_\ell^{\rm TT}$ in the power spectra between the two hemispheres, as a function of ℓ. The model of Ref. [15] and a modulation of the gravitational-wave amplitude both predict little or no asymmetry at $\ell \gtrsim 100$, while the power asymmetry should extend to much higher ℓ if it is due to a modulation of $n_{\rm s}$ or τ. There are also the temperature-polarization correlations ($C_\ell^{\rm TE}$) and polarization autocorrelations ($C_\ell^{\rm EE}$) shown in Figs. 3 and 4. The TE difference power spectrum, in particular, should help distinguish, through the sign at low ℓ of $\Delta C_\ell^{\rm TE}$, modulation of gravitational waves from modulation of τ. An asymmetry in $P(k)$ can be probed at lower k than quasars probe through all-sky lensing, Compton-y, and/or cosmic-infrared-background (CIB) maps, such as those recently made by Planck [21]. Asymmetries at $\sim 0.1-1$ Mpc^{-1} scales may also be probed via number counts in populations of objects other than quasars [22]. Probing asymmetries in $P(k)$ at $k \gg$Mpc^{-1}, as required if τ is modulated, may be more difficult in the near term, although future 21-cm maps of the neutral-hydrogen distribution during the dark ages [23] and/or epoch of reionization [24] may do the trick, as may maps of the μ distortion to the CMB frequency spectrum [25]. A modulation in the gravitational-wave background may show up as an asymmetry in CMB B modes [26]. In fact, if the asymmetry is attributed to a gravitational-wave asymmetry, suborbital B-mode searches may do better to search on one side of the sky than on the other! If the asymmetry has an origin in the coupling of an inflaton to some other field, a "fossil" field, during inflation, there may be higher-order correlation functions at smaller scales that can be sought [28]. It may also be instructive to perform a bipolar spherical harmonic (BiPoSH) [27] analysis and consider the odd-parity dipolar (i.e., $L=1$, with $\ell+\ell'+L$=odd) BiPoSH [29], which may shed light on the nature of a fossil field [30] that would give rise to the asymmetry.

While a modulation in $n_{\rm s}$ can account for the CMB power asymmetry, and a modulation to the gravitational-wave amplitude may do so for a smaller-amplitude asymmetry, these are both no more than phenomenological hypotheses, and there may be difficult work ahead to accommodate them within an inflationary model. Similar comments apply to variable optical depth.

There may also well be a completely different explanation, like bubble collisions [31] or non-trivial topology of the Universe [32], that is not well described by the modulation models we have considered here. Any such model must still, however, satisfy the CMB constraints to homogeneity and the quasar limit to a small-scale power asymmetry. Finally, if the asymmetry does indeed signal something beyond the simplest inflationary models, then it may be possible to draw connections between it and the tension between CMB and local models of the Hubble constant, between different suborbital CMB experiments, and perhaps other anomalies in current data.

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