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We provide a formula for the scaling behaviour of the inflationary bispectrum in the 'squeezed' limit where one momentum becomes much smaller than the other two. This determines the scaling of the halo bias at low wavenumber and will be an important observable for the next generation of galaxy surveys. Our formula allows it to be predicted for the first time for a generic inflationary model with multiple light, canonically-normalized scalar fields.

For the last twenty years, our information about the very early universe has primarily come from the large-scale anisotropies of the cosmic microwave background (CMB). But over the next decade fresh data will come from a new generation of large-scale structure surveys such as Euclid and LSST, and complementary small-scale CMB observations such as PIXIE [1].

These surveys will probe a new suite of observables, requiring the development of techniques by which they can be predicted from models of the early universe. In this *Letter* we focus on an observable which will be a target for future surveys—the *scale-dependent halo bias*, which is sensitive to the scaling of the primordial bispectrum in the so-called squeezed limit. It is a discriminant of inflationary models with more than one active field [2, 3].

Scale-dependent bias.—We cannot observe the primordial density field itself, but only correlations between populations of objects which trace its properties. One such population are halos of mass M, which form from collapse of over-dense regions. The clustering of these halos is described by the halo–halo or matter–halo power spectra P^{hh} and $P^{\delta h}$, where δ is the primordial density contrast. At lowest order the relationship between δ and the halo number density is linear $P^{\delta h}(k) \approx b_L P_R^{\delta\delta}(k)$ and therefore $P^{hh}(k) \approx b_L^2 P_R^{\delta\delta}(k)$. The superscript '$R$' denotes smoothing over a lengthscale R sufficiently large to enclose the halo mass M.

Corrections to b_L come from higher-order correlations of δ. They can be expressed using the n-point functions of the curvature perturbation ζ, which is typically used to characterize predictions of inflationary models. The leading effect comes from the three-point function and induces a k-dependent shift $b_L \to b_L + \delta b(k)$, where [4, 5]

$$\delta b(k) = \frac{M_R^{-1}(k,z)}{4\pi^2 \sigma_M^2} \left(\frac{\delta_c^2}{\sigma_M^2} - 1 \right) \int_0^\infty q^2 \mathrm{d}q \, M_R(q,z) \\ \times \int_{-1}^1 \mathrm{d}\mu \, M_R(Q,z) \frac{B_\zeta(k,q,Q)}{P_\zeta(k)}. \tag{1}$$

Here, z is the redshift and $Q^2 \equiv k^2 + q^2 + 2\mu kq$; the critical density for spherical collapse is $\delta_c \simeq 1.69$; and

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σ_M^2 is the variance of the tracing population of mass-M halos [6]. The kernel $M_R(q,z)$ is related to the linearized Poisson equation connecting the density contrast $\delta(\mathbf{k},z)$ and $\zeta(\mathbf{k})$. It can be written as

$$M(k,z) = \frac{2}{5} \frac{k^2}{H_0^2} \frac{T(k)}{\Omega_{m,0}} \frac{g(z)}{1+z}. \tag{2}$$

The transfer function $T(k)$ is normalized to unity as $k \to 0$, and $g(z)$ accounts for suppression of growth during Λ-domination. The present-day matter density is $\Omega_{m,0}$. To smooth $\delta(\mathbf{k},z)$ on the scale R we introduce a window function $W_R(k)$ and define $M_R(k,z) \equiv W_R(k)M(k,z)$. Finally, the spectrum and bispectrum of the curvature perturbation satisfy

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta \tag{3}$$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta. \tag{4}$$

Scaling of δb.—For k smaller than the wavenumber k_{eq} corresponding to the horizon scale at matter–radiation equality, we have $T(k) \sim 1$ and $M(k,z)$ proportional to k^2. For $k < k_{eq}$ this sets the dominant scaling of δb. It is the complete result whenever the integral over B_ζ does not depend on k, as for the case of the local model with constant amplitude f_{NL} [4, 5].

In this *Letter* we focus on the possibility that the integral in (1) also scales nontrivially with k. The precise scaling depends on details of the underlying inflationary model, and may constitute a useful observable. Because $M(q)$ grows steeply with q for $q < k_{eq}$, and the window function restricts the integral to wavenumbers $q \lesssim R^{-1}$, the dominant contribution will come from the region $k_{eq} \lesssim q \lesssim R^{-1}$. If $k \ll k_{eq}$ then this is the 'squeezed' region, where one of the momenta in the bispectrum is much less than the other two. Therefore, for $k \ll k_{eq}$, the halo bias is a probe of the squeezed limit of the bispectrum.

To study this limit we denote the 'squeezed' momentum k_L. The other two momenta are taken to be of order k_S, with $k_L \ll k_S$. We will see below that it has not yet been possible to obtain reliable predictions for the bispectrum in this limit, because the presence of multiple large hierarchies causes perturbation theory to break down. But if k_L is not too much smaller than k_S, the bispectrum can be approximated by a power law with

spectral index $n_{\rm sq}$,

$$B_\zeta \approx \frac{\mathcal{B}_\zeta(k_{\rm S})}{4k_{\rm L}^3 k_{\rm S}^3} \left(\frac{k_{\rm L}}{k_t}\right)^{n_{\rm sq}(k_{\rm S})} \left\{1 + {\rm O}\left(\frac{k_{\rm L}^2}{k_{\rm S}^2}\right)\right\}, \qquad (5)$$

where $\mathcal{B}_\zeta(k_{\rm S})$ is an overall amplitude depending weakly on $k_{\rm S}$ and $k_t = \sum_i k_i$ is the perimeter of the momentum triangle $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$. The spectral index $n_{\rm sq}$ measures the response of B_ζ to a change of $k_{\rm L}$ and may also depend weakly on $k_{\rm S}$. In what follows, our aim is to provide a precise formula for the spectral index $n_{\rm sq}$ and relate it to the scaling of δb.

The integral over q in (1) has been obtained for a variety of simple templates [3, 5, 7]. Here we focus on the scale dependence inherited from the squeezed limit of the bispectrum. The amplitude and scale-dependence of the spectrum and bispectrum can be expressed in terms of an arbitrary reference scale k_*. We write

$$P_\zeta(k) = \frac{\mathcal{P}_\zeta(k)}{2k^3} = \frac{\mathcal{P}_\zeta(k_*)}{2k^3} \left(\frac{k}{k_*}\right)^{n_s(k_*)-1}, \qquad (6)$$

where $n_s - 1 \equiv {\rm d}\ln\mathcal{P}_\zeta/{\rm d}\ln k$ is the spectral index of P_ζ, and $\mathcal{B}_\zeta(k_{\rm S}) = \mathcal{B}_\zeta(k_*)(k_{\rm S}/k_*)^\alpha$. We assume n_s and $n_{\rm sq}$ to be approximately constant over the range $k_{\rm eq} \lesssim q \lesssim R^{-1}$. Because this is comparatively small, with $Rk_{\rm eq} \sim 0.1$ on cluster scales, we expect this to be reasonable. We conclude that δb has approximate scaling behaviour

$$\delta b \approx A_{\rm NL} \times M_R(k,z)^{-1} \left(\frac{k}{k_*}\right)^{n_{\rm sq}-(n_s-1)}, \qquad (7)$$

where the amplitude $A_{\rm NL}$ depends on details of the model, the scale R and the background cosmology. In general, it may be difficult to calculate. For the purposes of this paper we do not require a precise estimate, because the observable is the scaling behaviour of δb rather than its amplitude. This is a strength of the approach.

We define the combination $n_{\delta b} \equiv n_{\rm sq} - n_s + 1$ as the *spectral index of the bias*. The main result of this *Letter* is a prescription to compute it for any inflationary model. Constraints on $n_{\delta b}$ have been obtained from present-day data [2], although the results show sensitivity to priors. In particular, the first Planck results impose strong constraints on the value of $f_{\rm NL}$ local [8] and therefore on the range of models that can show an observable $n_{\delta b}$. Improvements are expected from a future Euclid- or LSST-like survey [9].

The k_t-dependence of the bispectrum B_ζ was first studied by Chen [10]. Formulae for a multiple-field model were given by Byrnes et al. [11, 12]. Our analysis differs because of its focus on scaling in the squeezed limit, rather than scaling of the amplitude for nearly equilateral momenta. Later, Tzavara & van Tent studied the k_t-dependence and the scaling behavior of (5) in a two-field model using a Green's function formalism [13]. In this *Letter*, we give expressions using the separate universe formalism, in both its 'variational' and 'transport'

versions, which are valid for any number of scalar fields. These expressions are only valid when super-horizon evolution of curvature perturbations has ceased, in other words, when the adiabatic limit has been reached. Otherwise, if isocurvature perturbations persist at the end of inflation, it is necessary to augment the model by providing a prescription for their subsequent evolution. Our formulae apply when it is sufficient to evaluate the curvature perturbation at or before the end of the inflationary phase.

The scalar spectral index.—Sasaki & Stewart used the separate universe method to give an expression for n_s [14]. More recently an alternative (but equivalent) prescription was given in Ref. [15], which we follow here [16]. According to the separate universe picture, the power spectrum evaluated at time t can be written as

$$\mathcal{P}_\zeta(k)|_t = N_\alpha N_\beta \Sigma_{\alpha\beta}(k)|_{t_0}, \qquad (8)$$

where $N_\alpha \equiv \partial N(t, t_0)/\partial\phi_\alpha(t_0)$, and N measures the number of efolds elapsed from a spatially flat slice at time t_0 to a uniform density hypersurface at later time t; indices α, β, ..., label the different species of scalar field; and $\Sigma_{\alpha\beta}$ defines the two-point function of scalar field fluctuations,

$$\langle\delta\phi_\alpha(\mathbf{k}_1)\delta\phi_\beta(\mathbf{k}_2)\rangle_{t_0} = (2\pi)^3\delta(\mathbf{k}_1 + \mathbf{k}_2)\frac{\Sigma_{\alpha\beta}|_{t_0}}{2k^3}, \qquad (9)$$

with $k = |\mathbf{k}_1| = |\mathbf{k}_2|$. The time t_0 can be chosen to coincide with the horizon crossing time of k, which we will denote with a subscript 'k'.

The spectral index can be obtained from (8) provided we know $\Sigma_{\alpha\beta}$ to next-order in slow-roll, which was first obtained by Nakamura & Stewart [17]. At this order and for light, canonically-normalized fields, we find

$$\Sigma_{\alpha\beta} = H_*^2 \left(\delta_{\alpha\beta} + 2r_{\alpha\beta}^* - 2u_{\alpha\beta}^*\ln(-k_*\tau) - 2M_{\alpha\beta}^*\ln\frac{2k}{k_*}\right), \qquad (10)$$

where '$*$' denotes evaluation at the horizon-crossing time for an arbitrary scale k_*; the conformal time τ satisfies ${\rm d}t = a(t)\,{\rm d}\tau$; $r_{\alpha\beta}$ is a constant; $u_{\alpha\beta} = -m_{\alpha\beta}/3H^2$ is a rescaled mass matrix for the fluctuations, equivalent to the expansion tensor of the inflationary flow field [18]; $M_{\alpha\beta} \equiv \varepsilon\delta_{\alpha\beta} + u_{\alpha\beta}$, and $\varepsilon \equiv -\dot{H}/H^2$.

The k-dependence can be extracted from the coefficient of $\ln k/k_*$, provided we know how to choose k_*. To do so, note that the $\ln(-k_*\tau)$ term is the lowest power in a series expansion which describes the time dependence of the fluctuations. Since we wish to estimate the scale dependence at a fixed time t_0 we can choose the arbitrary scale so that $k_*\tau = -1$, making all powers of $\ln(-k_*\tau)$ negligible. With this choice the evaluation point of H, $r_{\alpha\beta}$, $u_{\alpha\beta}$ and $M_{\alpha\beta}$ becomes coincident with t_0 and the spectral index simplifies to

$$n_s - 1 = -2\frac{N_\alpha N_\beta M_{\alpha\beta}|_k}{N_\lambda N_\lambda}. \qquad (11)$$

Squeezed bispectrum.—A similar approach can be used to obtain the spectral index $n_{\rm sq}$ in (5). We require a next-order expression for the bispectrum, in analogy with (10). The requisite corrections are known for general single-field models [19], and were recently obtained for multiple canonically-normalized fields in Ref. [20]. In the squeezed limit they can be written

$$\langle \delta\phi_\alpha(\mathbf{k}_{\rm L})\delta\phi_\beta(\mathbf{k}_{\rm S})\delta\phi_\gamma(\mathbf{k}'_{\rm S})\rangle = (2\pi)^3\delta(\mathbf{k}_{\rm L}+\mathbf{k}_{\rm S}+\mathbf{k}'_{\rm S})B_{\alpha\beta\gamma} \xrightarrow{k_{\rm L}\to 0} (2\pi)^3\delta(\mathbf{k}_{\rm S}+\mathbf{k}'_{\rm S})\frac{1}{4k_{\rm L}^3 k_{\rm S}^3}\Big(b_{\alpha|\beta\gamma}+{\rm O}\Big(\frac{k_{\rm L}^2}{k_{\rm S}^2}\Big)\Big). \tag{12}$$

Corrections of order $\sim k_{\rm L}^2/k_{\rm S}^2$ may be relevant in the 'not-so squeezed' limit where the hierarchy $k_{\rm L}/k_{\rm S}$ is modest [21], but in the present case we expect their effect to be negligible. The coefficient $b_{\alpha|\beta\gamma}$ is symmetric under interchange of the indices β and γ, but not necessarily under other permutations. In what follows it is helpful to break it into terms of lowest-order and next-order in slow-roll, which we label 'LO' and 'NLO'. Using the results of Ref. [20], we find

$$b_{\alpha|\beta\gamma}^{\rm LO} = -H_*^4\frac{\dot\phi_\alpha}{H_*}\delta_{\beta\gamma}+\cdots, \tag{13a}$$

$$b_{\alpha|\beta\gamma}^{\rm NLO} = \Big(-2H_*^4 u_{\alpha\beta\gamma}-u_{\alpha\lambda}^* b_{\lambda|\beta\gamma}^{\rm LO}-u_{\beta\lambda}^* b_{\alpha|\lambda\gamma}^{\rm LO}-u_{\gamma\lambda}^* b_{\alpha|\beta\lambda}^{\rm LO}\Big)\ln(-k_*\tau)-2M_{\alpha\lambda}^* b_{\lambda|\beta\gamma}^{\rm LO}\ln\frac{2k_{\rm L}}{k_*}+\cdots \tag{13b}$$

The omitted terms are not logarithmically enhanced, or are proportional to $\ln(k_{\rm S}/k_*)$ or $\ln(k_t/k_*)$. In (13a) and (13b) we have chosen $k_* \sim k_t$, making these contributions negligible Therefore, if we intend to estimate this expression at some fixed time, only $\ln(k_{\rm L}/k_*)$ can generate dangerously large contributions. In analogy with the $\ln(-k_*\tau)$ term in (10), this is the lowest term in a series expansion, and, if uncontrolled, the series will diverge when $k_{\rm L}/k_* \approx k_{\rm L}/k_{\rm S} \to 0$. It is for this reason that there is no analytic formula for the squeezed limit of the bispectrum in a multiple-field model.

Ref. [20] developed a method based on the dynamical renormalization group to control the series of $\ln(-k_*\tau)$ terms in the limit $\tau \to 0$. It is not yet clear whether a similar scheme could be used to deal with the series expansion of $\ln(k_{\rm L}/k_{\rm S})$. For this reason our results are untrustworthy for $k_{\rm L} \ll k_{\rm S}$. Our approach may be reliable up to a hierarchy $|\ln(k_{\rm L}/k_{\rm S})|$ of order a few.

Using the separate universe formula, the bispectrum B_ζ can be written (for an arbitrary momentum triangle)

$$B_\zeta(k_1,k_2,k_3)|_t = N_\alpha N_\beta N_\gamma B_{\alpha\beta\gamma}(k_1,k_2,k_3)|_{t_0}$$
$$+ N_{\alpha\beta}N_\gamma N_\delta\Big(\frac{\Sigma_{\alpha\gamma}(k_1)}{k_1^3}\frac{\Sigma_{\beta\delta}(k_2)}{k_2^3}+\frac{\Sigma_{\alpha\gamma}(k_1)}{k_1^3}\frac{\Sigma_{\beta\delta}(k_3)}{k_3^3}+\frac{\Sigma_{\alpha\gamma}(k_2)}{k_2^3}\frac{\Sigma_{\beta\delta}(k_3)}{k_3^3}\Big)|_{t_0}, \tag{14}$$

where $N_{\alpha\beta} \equiv \partial^2 N(t,t_0)/\partial\phi_\alpha(t_0)\partial\phi_\beta(t_0)$. The squeezed spectral index, $n_{\rm sq}$, can be computed using this expression. We take t_0 to be the horizon crossing time for k_t and denote evaluation at this time with a subscript 'k_t'. Choosing $k_* \sim k_t$, the term proportional to $\ln(-k_*\tau)$ in (13b) vanishes and the scale dependence of $B_{\alpha\beta\gamma}$ can be read off. In combination with (11), we obtain

$$n_{\rm sq} = -2\frac{N_\alpha N_\beta N_\gamma(M_{\alpha\lambda}b_{\lambda|\beta\gamma}^{\rm LO})|_{k_t}+N_{\alpha\beta}N_\gamma N_\delta\big(M_{\alpha\lambda}\Sigma_{\lambda\gamma}+M_{\gamma\lambda}\Sigma_{\alpha\lambda}\big)|_{k_t}\Sigma_{\beta\delta}|_{k_t}}{N_\lambda N_\mu N_\nu b_{\lambda|\mu\nu}^{\rm LO}|_{k_t}+2N_{\lambda\mu}N_\nu N_\pi(\Sigma_{\lambda\nu}\Sigma_{\mu\pi})|_{k_t}}. \tag{15}$$

Whenever the bispectrum in a multiple-field inflationary model is large enough to be observable, the 'nonlinear' terms in the second line of (14) dominate the 'intrinsic' term in the first line [22, 23]. In this limit, since $\Sigma_{\alpha\beta}|_{k_t} \approx H_{k_t}^2\delta_{\alpha\beta}$, we can write $n_{\rm sq}$ in the simpler form

$$n_{\rm sq} \approx -2\frac{N_{\alpha\beta}N_\gamma N_\beta M_{\alpha\gamma}|_{k_t}}{N_{\lambda\mu}N_\lambda N_\mu}, \tag{16}$$

and under the same assumptions we conclude that

$$n_{\delta b} \approx -2\frac{N_{\alpha\beta}N_\gamma N_\beta M_{\alpha\gamma}|_{k_t}}{N_{\lambda\mu}N_\lambda N_\mu}+2\frac{N_\alpha N_\beta M_{\alpha\beta}|_{k_t}}{N_\lambda N_\lambda}. \tag{17}$$

This constitutes one of the principal results of this *Letter*.

Transport equations.—Eqs. (11) and (15) are framed in terms of the 'variational' formulation of the separate universe method, which involves 'variational' derivatives

such as N_α. A successful numerical implementation requires sufficient resolution to extract the variation of N against a background of numerical noise accumulated during the integration from t_0 to t. An alternative approach to solving (8) and (14) consists in integrating the correlators of field fluctuations using Jacobi evolution equations [18]. Mathematically, both methods are equivalent, but the Jacobi approach is simpler because it is more tolerant to numerical noise. For details on this formalism we refer to Refs. [18, 24–28].

To write a Jacobi-type equation which defines the time evolution of the coefficient matrix $b_{\alpha|\beta\gamma}$ in (12), we define $u_{\alpha\beta\gamma} \equiv \partial u_{\alpha\beta}/\partial\phi_\gamma$. Using the prescription of Ref. [20], it follows that

$$-\frac{\mathrm{d}b_{\alpha|\beta\gamma}}{\mathrm{d}\ln\tau} = u_{\alpha\lambda}b_{\lambda|\beta\gamma}+u_{\beta\lambda}b_{\alpha|\lambda\gamma}+u_{\gamma\lambda}b_{\alpha|\beta\lambda}+2u_{\alpha\lambda\mu}\Sigma_{\lambda\beta}\Sigma_{\mu\gamma}.$$
(18)

Note that we should regard one of the two-point functions $\Sigma_{\alpha\beta}$ to be evaluated at k_L and the other at k_s. A suitable initial condition at horizon-crossing is provided by the lowest-order value (13a), after setting $k_* \sim k_t$.

To compute the scale dependence in the squeezed limit, we adjust k_L while keeping k_s fixed. Defining $n_{\alpha|\beta\gamma} \equiv \mathrm{d}b_{\alpha|\beta\gamma}/\mathrm{d}\ln k_\mathrm{L}$, it follows that

$$-\frac{\mathrm{d}n_{\alpha|\beta\gamma}}{\mathrm{d}\ln\tau} = u_{\alpha\lambda}n_{\lambda|\beta\gamma} + u_{\beta\lambda}n_{\alpha|\lambda\gamma} + u_{\gamma\lambda}n_{\alpha|\beta\lambda}$$
$$+ u_{\alpha\lambda\mu}n_{\lambda\beta}\Sigma_{\mu\gamma} + u_{\alpha\lambda\mu}\Sigma_{\lambda\beta}n_{\mu\gamma},$$
(19)

where $n_{\alpha\beta} \equiv \mathrm{d}\Sigma_{\alpha\beta}/\mathrm{d}\ln k$. The initial condition is given by $n_{\alpha|\beta\gamma} = -2M_{\alpha\lambda}b_{\lambda|\beta\gamma}^\mathrm{LO}$ near horizon-exit for k_t.

Likewise $\Sigma_{\alpha\beta}$ and $n_{\alpha\beta}$ satisfy [15]

$$\frac{\mathrm{d}\Sigma_{\alpha\beta}}{\mathrm{d}N} = u_{\alpha\lambda}\Sigma_{\lambda\beta} + u_{\beta\lambda}\Sigma_{\alpha\lambda},$$
(20a)

$$\frac{\mathrm{d}n_{\alpha\beta}}{\mathrm{d}N} = u_{\alpha\lambda}n_{\lambda\beta} + u_{\beta\lambda}n_{\alpha\lambda},$$
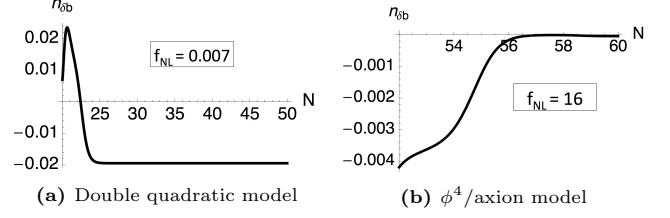(20b)

with respective initial conditions $\Sigma_{\alpha\beta} = H_*^2\delta_{\alpha\beta}$ and $n_{\alpha\beta} = -M_{\alpha\lambda}\Sigma_{\lambda\beta} - M_{\beta\lambda}\Sigma_{\alpha\lambda}$ near the horizon-crossing time for k_t, supplied by (10).

To relate $n_{\alpha\beta}$ and $n_{\alpha|\beta\gamma}$ to the spectral indices n_s and n_sq requires the gauge transformation to ζ. We label these coefficients N_α^t because they are obtained by computing a perturbation in the e-folding number N, but the label t emphasizes that they involve only local quantities at t. Explicit expressions for these gauge transformations are tabulated in Anderson et al. [26]. The spectral index of the bias can be written

$$n_{\delta b} = \frac{N_\alpha^t N_\beta^t N_\gamma^t n_{\alpha|\beta\gamma} + 2N_\alpha^t N_\beta^t N_{\gamma\delta}^t n_{\alpha\gamma}\Sigma_{\beta\delta}}{N_\alpha^t N_\beta^t N_\gamma^t b_{\alpha|\beta\gamma} + 2N_\alpha^t N_\beta^t N_{\gamma\delta}^t \Sigma_{\alpha\gamma}\Sigma_{\beta\delta}} - \frac{N_\alpha^t N_\beta^t n_{\alpha\beta}}{N_\lambda^t N_\mu^t \Sigma_{\lambda\mu}}.$$
(21)

Focusing on multiple-field models with a bispectrum large enough to be detectable, in analogy with Eq. (16), this expression reduces to

$$n_{\delta b} = \frac{N_\alpha^t N_\beta^t N_{\gamma\delta}^t n_{\alpha\gamma}}{N_\alpha^t N_\beta^t N_{\gamma\delta}^t \Sigma_{\alpha\gamma}} - \frac{N_\alpha^t N_\beta^t n_{\alpha\beta}}{N_\lambda^t N_\mu^t \Sigma_{\lambda\mu}}.$$
(22)

FIG. 1: Evolution of $n_{\delta b}$ some efolds before the end of inflation in two different models. The values of f_NL shown are estimated at the end of inflation.

As an example, Fig. 1 depicts the evolution of $n_{\delta b}$ in double quadratic inflation [29–32] and in the ϕ^4-plus-axion model studied by Elliston et al. [33]. To generate a significant $n_{\delta b}$, the bispectrum must scale differently to the power spectrum. If $|f_\mathrm{NL}|$ is small this is typical, as in Fig. 1a. Alternatively, if $|f_\mathrm{NL}|$ is $\mathcal{O}(1)$ or larger the scaling is more similar, as in Fig. 1b. Where the spectral index of the bias is large we find that it is often negative, giving the squeezed limit a red tilt. This is the opposite of quasi-single-field inflation (QSFI), where $n_{\delta b} > 0$. This can be understood heuristically. QSFI contains massive modes, which mediate finite-range forces and tend to soften long-range correlations. In comparison, models with multiple active fields tend to add tachyons to the spectrum which mediate long-range forces and therefore enhance correlations.

Conclusions.—The halo bias is known to inherit a scale dependence from the underlying inflationary model. It is therefore an important observable which is likely to be measured by future surveys of the galaxy distribution. In this *Letter* we focus on its scaling, for which we do not require detailed knowledge of the amplitude A_NL. We provide a formula to compute $n_{\delta b}$ for any model of multiple-field inflation. To obtain a significant effect, the squeezed limit (characterized by the spectral index n_sq) should scale differently to the power spectrum (characterized by $n_s - 1$). This can be measured most easily when $n_{\delta b} < 0$, making $k^{-2+n_{\delta b}}$ diverge more strongly in the limit $k \to 0$. Therefore, a slightly red-tilted bias will be easier to constrain than a blue-tilted one.

The principal drawback of our method is the use of a spectral index to parametrize the behavior of the squeezed bispectrum. For a sufficiently large variation of k_L the bispectrum may have a shape which cannot be approximated by a power law. Nevertheless, our formula should give a good approximation provided the hierarchy $\ln(k_\mathrm{L}/k_\mathrm{s})$ does not exceed $\sim$ a few. Near-future surveys such as DES may probe a hierarchy of order $\ln(k_\mathrm{L}/k_\mathrm{s}) \sim -2$, but surveys arriving in the medium- to long-term, such as Euclid, may probe $\ln(k_\mathrm{L}/k_\mathrm{s}) \sim -8$. Small-scale observables such as μ-distortions may even probe $\ln(k_\mathrm{L}/k_\mathrm{s}) \sim -19$. For such cases a more precise description of the bispectrum will be required, perhaps using numerical methods.

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