CMB and matter power spectra from cross correlations of primordial curvature and magnetic fields

Kerstin E. Kunze

Phys. Rev. D **87**, 103005 — Published 10 May 2013

DOI: 10.1103/PhysRevD.87.103005

CMB and matter power spectra from cross correlations of primordial curvature and magnetic fields

Kerstin E. Kunze [1]

Departamento de Física Fundamental and *IUFFyM*,
Universidad de Salamanca,
Plaza de la Merced s/n, 37008 Salamanca, Spain

Abstract

A complete numerical calculation of the temperature anisotropies and the polarization of the cosmic microwave background (CMB) is presented for a non zero cross correlation of a stochastic magnetic field with the primordial curvature perturbation. Such a cross correlation results, for example, if the magnetic field is generated during inflation by coupling electrodynamics to a scalar field which is identified with the curvaton. For a nearly scale invariant magnetic field of 1 nG it is found that the contribution due to the cross correlation dominates over that of the pure magnetic mode. A similar behaviour on large scales is found for the linear matter power spectrum.

1 Introduction

The presence of magnetic fields on very different scales has been confirmed by observations [1]. On galactic scales there is an abundance of magnetic field detections with typical field strengths in the μG range at present time. Over recent years there have been claims of truly cosmological magnetic fields, that are not associated with any virialized structures such as galaxies or clusters of galaxies [2]. However, this has been challenged in [3].

There are many models to explain the origin of large scale magnetic fields, for reviews see, e.g., [4]. In particular, in the early universe conditions could have been such as to naturally generate magnetic fields strong enough to seed a galactic dynamo. The subsequent amplification since galaxy formation then results in μG magnetic fields today. Inflationary models seem to be especially attractive to generate the initial seed magnetic field since the correlation lengths can be very large. The problem usually is with the magnetic field strength which in spatially flat models is too small to account for the initial magnetic seed field due to the global conformal invariance of standard electrodynamics in these backgrounds. However, in spatially curved models this is not the case and thus not a problem [5] (see however, [6]).

Therefore, when considering the standard flat ΛCDM model conformal invariance of electrodynamics has to be broken which in the simplest case is realized by coupling to a scalar field [7]. However, there are possible problems with back reaction effects onto inflation [8, 9]. Other possibil-

[1] E-mail: kkunze@usal.es

ities include coupling electrodynamics to curvature [10] of which some models also have problems due to ghost instabilities [11]. There are models in which magnetic fields of cosmologically relevant strengths can be generated during inflation [10, 12]. However, since in this case inflation is of power-law type there is yet another constraint coming from the fact that the curvature perturbation for the relevant parameters cannot be created during inflation. It is possible to use the simplest curvaton scenario to generate the curvature perturbation after inflation [13]. This, however, results in a further restriction of the parameter space but still allows to generate magnetic fields strong enough to act as seed fields for the galactic dynamo [14].

In [15] a particular model has been studied in which magnetic fields are generated during inflation by coupling electrodynamics to a scalar field which is not the inflaton. In this case it has been shown that there is a non trivial correlation between the scalar field perturbation and the magnetic field [15]. Moreover, it was pointed out that identifying the (spectator) scalar field with the curvaton determines the cross correlation of the magnetic field with the curvature perturbation. This is the case to be considered here. The aim is to calculate the anisotropies of the temperature and polarization of the cosmic microwave background (CMB) due to the cross correlation between the magnetic field and the primordial curvature perturbation. Upto now, in general, the effect of magnetic fields present before decoupling on the CMB have been calculated assuming either complete correlation [16]-[19] or no correlation at all [20, 19] between the primordial curvature perturbation and the magnetic field. In most works the magnetic field is assumed to be non helical. However, the helical case has also been studied which is particularly interesting since it induces odd parity correlations of the CMB modes [21]. In [22] the effect of a non vanishing cross correlation between the curvature and the magnetic field due the evolution of the magnetic field has been studied. In that case the effect on the CMB temperature anisotropies and polarization has been found to be much below that of the compensated mode for a magnetic field which only redshifts with the expansion of the universe.

The magnetic field does not enter linearly in the perturbation equations but rather in form of its energy density and anisotropic stress which are quadratic in the magnetic field implying that non gaussianity is induced even at linear order. The resulting bispectra have been calculated [23]. Thereby the cross correlations between the curvature mode and the magnetic field contributions are determined by the bispectra of the magnetic field and the curvature mode. In [24] the bispectra induced by coupling of the inflaton to electromagnetism have been calculated. This model has been also considered in [25] where the corresponding CMB anisotropies have been calculated.

2 The model

The background metric is assumed to be of the form

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \delta_{ij}dx^i dx^j\right), \tag{2.1}$$

where $a(\eta)$ is the corresponding scale factor.

The model of magnetic field generation during de Sitter inflation used in [15] which is based on [8, 26] is described by the action

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{4}W(\phi)F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) \tag{2.2}$$

where the scalar field ϕ takes the role of a spectator field during inflation. Its potential is assumed to be of the form $V(\phi) = -3nMH_I^2\phi$ and the coupling to the Maxwell tensor field, $F_{\mu\nu}$, is chosen to be

$W(\phi) = e^{2\phi/M}$. Moreover, $H_I = const.$ is the Hubble parameter and the scale factor is determined by $a(\eta) = -1/(\eta H_I)$ for $-\infty < \eta \leq \eta_I < 0$ upto the end of inflation at η_I. The resulting magnetic field is non helical and its power spectrum at the end of inflation on superhorizon scales is given by [15]

$$P_B(k,\eta_I) \simeq \frac{\Gamma\left(\frac{5-n_B}{2}\right)^2}{\pi}\left(\frac{-k\eta_I}{2}\right)^{n_B-4} k\,(H_I\eta_I)^4\,, \tag{2.3}$$

where

$$n_B = \begin{cases} 4+2(n+1) & n \;<\; -1, \\ 4-2n & n \;\geq\; 0, \end{cases} \tag{2.4}$$

where n is one of the parameters specifying the potential $V(\phi)$ as given just after equation (2.2). In the numerical solutions the magnetic field values are taken at present time. Thus assuming that the magnetic field only redshifts with expansion, so that $B_i \sim 1/a^2$. Then the two point function of the magnetic field,

$$\langle B_i^*(\vec{k},\eta)B_j(\vec{k}',\eta')\rangle = P_B(k,\eta)\delta_{\vec{k},\vec{k}'}\delta_{\eta,\eta'}\left(\delta_{ij} - \frac{k_ik_j}{k^2}\right) \tag{2.5}$$

determines that the power spectrum scales as a^{-4}, so that at present time $a_0^4 P_B(k,\eta_0) = (H_I\eta_I)^{-4}P_B(k,\eta_I)$. Then together with the parameters chosen as in [15], $-\eta_I = 10^{-24}$ Mpc for the end of inflation, $a_0 = 1$, and introducing a pivot scale $k_L = 1\ \mathrm{Mpc}^{-1}$ for the magnetic field spectrum and a gaussian window function $W(k,k_m)$ to model damping of the magnetic field on small scales thereby imposing an upper cut-off of the wave numbers, k_m, the spectral function at present time is given by

$$P_B(k,\eta_0) = 10^{97.2-24.3n_B}\frac{\Gamma\left(\frac{5-n_B}{2}\right)^2}{\pi}\left(\frac{k_L}{1\mathrm{Mpc}^{-1}}\right)^{n_B-3}\left(\frac{k}{k_L}\right)^{n_B-3} W(k,k_m)\mathrm{Mpc}^{-1} \tag{2.6}$$

where $W(k,k_m) = \pi^{-3/2}e^{-\left(\frac{k}{k_m}\right)^2}$, so that $\int d^3k W(k,k_m) = k_m^3$. The maximal wave number is given by [27]

$$k_\mathrm{m} \simeq 198.454\left(\frac{B}{\mathrm{nG}}\right)^{-1}\mathrm{Mpc}^{-1}, \tag{2.7}$$

for the values of the bestfit ΛCDM model of WMAP9, $\Omega_b = 0.02264h^{-2}$ and $h = 0.7$ [28]. Comparing the spectrum (2.6) with the form of the magnetic field spectra used in, e.g., in [19] the spectral index there becomes $n_B \to n_B + 3$, so that the scale invariant cases correspond here to $n_B = 0$ and in [19] to $n_{B*} = -3$. Thus the magnetic field strength today smoothed over the diffusion scale is determined by

$$\langle \vec{B}^2(\vec{x},\eta_0)\rangle = 10^{-19.8-24.3n_B}\Gamma^2\left(\frac{5-n_B}{2}\right)\Gamma\left(\frac{n_B}{2}\right)\left(\frac{k_m}{1\mathrm{Mpc}^{-1}}\right)^{n_B}\mathrm{G}^2. \tag{2.8}$$

Defining the magnetic field strength $B \equiv \sqrt{\langle \vec{B}^2(\vec{x},\eta_0)\rangle}$ and using equation (2.7) yields to

$$\left(\frac{B}{\mathrm{nG}}\right)^{\frac{n_B}{2}+1} = 10^{-0.9-11n_B}\Gamma\left(\frac{5-n_B}{2}\right)\Gamma^{\frac{1}{2}}\left(\frac{n_B}{2}\right), \tag{2.9}$$

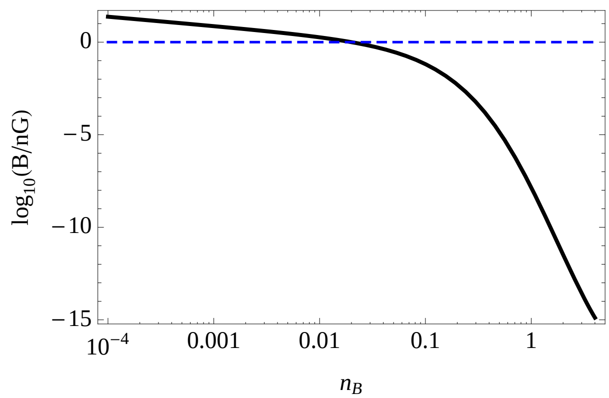

Figure 1: The root mean square magnetic field strength smoothed over the diffusion scale as a function of the spectral index n_B.

which is shown in figure 1. Therefore for $n_B = 0.02$ corresponding to $n_{B*} = -2.98$ the smoothed magnetic field strength is of the order of 1 nG. The scale invariant case $n_B = 0$ corresponds to $n = 2$ or $n = -3$. As shown in [15] only the case $n = 2$ is allowed since in the other case strong back reactions would develop since the electromagnetic energy density dominates over the inflaton energy density.

3 Cross correlation functions of primordial curvature and magnetic fields

In [15] the 3-point cross correlation between the magnetic field energy density proportional to $B^2$and the fluctuations of the spectator field $\delta\phi$ has been calculated. However, since the anisotropies of the CMB are determined by the magnetic energy density contrast Δ_B and the magnetic anisotropic stress π_B it is necessary to obtain the 3-point cross correlation functions allowing for arbitrary components of the magnetic field. Starting with the expression found for the cross correlation of the gauge potential A_i of the electromagnetic field and $\delta\phi$ [15] the 3-point cross correlation involving arbitrary components of the magnetic field is determined to be, in Fourier space,

$$\langle\frac{\delta\phi(\vec{k}_1,\eta_I)}{M}B_i(\vec{k}_2,\eta_0)B_j(\vec{k}_3,\eta_0)\rangle = -(2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)\epsilon_{ilm}k_{2l}\epsilon_{jnq}k_{3n}U_{mq}, \tag{3.1}$$

where [15]

$$U_{mq} = -\frac{\pi^2}{8}\left(\frac{H_I}{M}\right)^2\frac{1}{k_1^4}\left[\delta_{mq}I_1 + \left(\hat{k}_2\cdot\hat{k}_3\delta_{mq} - \hat{k}_{2q}\hat{k}_{3m}\right)I_2\right], \tag{3.2}$$

where I_1 and I_2 are integrals depending, in particular, on the parameter n which can be found in [15]. The special case of $n = 2$ will be presented in the notation used here below. This corresponds to a scale invariant magnetic field spectrum which is the cosmologically relevant case [15].

In order to relate the perturbations in the scalar field to the curvature perturbation the spectator field is identified with a curvaton field as suggested in [15]. In particular, we assume the simplest curvaton model [13] so that the total curvature perturbation ζ is generated by the curvaton after inflation and that any other contributions to the curvature perturbation are negligible. Moreover, it is assumed that the curvaton dominates the energy density before its decay. Then following [13]

$$\frac{\delta\phi(\vec{k})}{M}=\frac{3}{2}\left(\frac{\phi_*}{M}\right)\zeta(\vec{k}), \tag{3.3}$$

where the asterisk denotes the epoch when the mode k leaves the horizon during inflation assuming no nonlinear evolution on large scales [29]. Therefore calculating $\langle\zeta^*_{\vec{k}}\zeta_{\vec{k}'}\rangle$ at the pivot scale of WMAP9 $k_p=0.002$ Mpc^{-1} yields to

$$\left(\frac{H_I}{\phi_*}\right)=3\pi\mathcal{P}_\zeta^{\frac{1}{2}}(k_p) \tag{3.4}$$

where the dimensionless power spectrum of two random variables X and Y, $\mathcal{P}_{\langle XY\rangle}$, defined by

$$\langle X^*_{\vec{k}}Y_{\vec{k}'}\rangle=\frac{2\pi^2}{k^3}\mathcal{P}_{\langle XY\rangle}(k)\delta_{\vec{k},\vec{k}'}. \tag{3.5}$$

has been used.

The magnetic energy density contrast and anisotropic stress are defined in terms of the photon energy density ρ_γ and pressure $p_\gamma=\frac{1}{3}\rho_\gamma$. This is due to the weakness of the magnetic field which does not contribute to the background quantities (for a different approach see [30]). The energy density contrast Δ_B in k-space is defined by (for details, e.g., cf. [19])

$$\Delta_B(\vec{k})=\frac{1}{2\rho_{\gamma 0}}\sum_{\vec{q}}B_i(\vec{q})B^i(\vec{k}-\vec{q}). \tag{3.6}$$

So that one of the contributions to the cross correlation between the primordial curvature mode and the magnetic mode is determined by the two-point correlation function $\langle\zeta(\vec{k}')\Delta_B(\vec{k})\rangle$. Taking the continuum limit $\sum_{\vec{k}}\to\int\frac{d^3k}{(2\pi)^3}$ and using the expressions for the integrals I_1 and I_2 for $n=2$ on superhorizon scales as given in [15] yields to

$$\begin{aligned}\mathcal{P}_{\langle\zeta\Delta_B\rangle}(k) &= \frac{9}{16\pi^{\frac{9}{2}}}\frac{\mathcal{P}_\zeta^{\frac{1}{2}}(k_p)}{\rho_{\gamma 0}\eta_I^4}\left(\frac{H_I}{M}\right)e^{-\frac{1}{2}\left(\frac{k}{k_m}\right)^2}\\ &\times\int_0^\infty dz z^3 e^{-\left(\frac{k}{k_m}\right)^2 z^2}\int_{-1}^1 dx e^{\left(\frac{k}{k_m}\right)^2 zx}\vartheta\left[2\frac{x-z}{\vartheta}J_1+\left[1+\left(\frac{x-z}{\vartheta}\right)^2\right]J_2\right],\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}J_1 &\equiv \frac{(1+\vartheta)(1+\vartheta+\vartheta^2)+2(1+\vartheta+\vartheta^2)z+2(1+\vartheta)z^2+z^3}{z^3\vartheta^3(1+z+\vartheta)^2},\\ J_2 &\equiv \big[-3(1+z+\vartheta)^2(\gamma+\ln(1+z+\vartheta))+(1+\vartheta)^2(3+3\vartheta+\vartheta^3)\\ &+(1+\vartheta)(9+6\vartheta+2\vartheta^3)z+(9+6\vartheta+2\vartheta^2+2\vartheta^3)z^2\\ &+2(2+\vartheta+\vartheta^2)z^3+2(1+\vartheta)z^4+z^5\big]\,z^{-4}\vartheta^{-4}(1+z+\vartheta)^{-2}.\end{aligned} \tag{3.8}$$

Moreover,

$$x\equiv\frac{\vec{k}\cdot\vec{q}}{kq},\qquad z\equiv\frac{q}{k},\qquad \vartheta\equiv\vartheta(x,z)\equiv(1-2zx+z^2)^{\frac{1}{2}} \tag{3.9}$$

The two-point correlation function $\langle\zeta(\vec{k})\pi_B(\vec{k}')\rangle$ is calculated using the expression for the anisotropic stress of the scalar mode (e.g., [19])

$$\pi_B(\vec{k}) = \frac{3}{2\rho_{\gamma 0}}\left[\sum_{\vec{q}}\frac{3}{k^2}B_i(\vec{q})(k^i - q^i)B_j(\vec{k}-\vec{q})q^j - \sum_{\vec{q}}B_m(\vec{q})B^m(\vec{k}-\vec{q})\right]. \tag{3.10}$$

For the cosmologically interesting case $n = 2$ this yields to

$$\begin{aligned}\mathcal{P}_{\langle\zeta\pi_B\rangle}(k) &= -\frac{27}{16\pi^{\frac{9}{2}}}\frac{\mathcal{P}_\zeta^{\frac{1}{2}}}{\rho_{\gamma 0}\eta_I^4}\left(\frac{H_I}{M}\right)e^{-\frac{1}{2}\left(\frac{k}{k_m}\right)^2}\\ &\times\int_0^\infty dz z^3 e^{-\left(\frac{k}{k_m}\right)^2 z^2}\int_{-1}^1 dx e^{\left(\frac{k}{k_m}\right)^2 zx}\Big[\left(2x + (1-3x^2)z\right)J_1\\ &+\left(\vartheta + \frac{x-z}{\vartheta}\left[x + (2-3x^2)z\right]\right)J_2\Big]. \end{aligned}\tag{3.11}$$

The dimensionless power spectra determining the cross correlation between the curvature mode and the magnetic modes are shown in figure 2 (*Upper Panel (left)*) for $\frac{H_I}{M} = 5\times 10^{-4}$ corresponding to the upper bound on this parameter inferred in [15]. As can be appreciated from figure 2 (*Upper Panel (left)*) whereas the curvature perturbation is correlated with the magnetic energy density contrast, it is anticorrelated with the magnetic anisotropic stress. For comparison, in figure 2 (*Upper Panel (right)*) the auto- and cross correlations of the magnetic modes are shown for magnetic field strength $B = 1$ nG and spectral index $n_{B*} = -2.98$. The expressions for the correlation functions of the magnetic modes, $\mathcal{P}_{\langle\Delta_B\Delta_B\rangle}(k)$, $\mathcal{P}_{\langle\pi_B\pi_B\rangle}(k)$ and $\mathcal{P}_{\langle\Delta_B\pi_B\rangle}(k)$ can be found, e.g., in [19]. Comparing the figures of the (*Upper Panel*) of figure 2 shows that the amplitudes of the spectral functions of the cross correlations between curvature and the magnetic modes are larger than the auto- and cross correlation functions of the magnetic modes. In figure 2 (*Lower Panel*) the ratio

$$X_\Xi = \frac{|\mathcal{P}_{\langle\zeta\Xi\rangle}|}{\sqrt{|\mathcal{P}_{\zeta\zeta}||\mathcal{P}_{\langle\Xi\Xi\rangle}|}}, \qquad \Xi = \Delta_B, \pi_B \tag{3.12}$$

is reported which determines the generalized Schwarz inequality and as can be appreciated is well below the unity bound.

4 Results

The CMB angular power spectra are determined by the brightness function which in the line-of-sight approach [31] is written for each component, for the scalar mode, [32]

$$\frac{\Theta_\ell^X(\eta_0, k)}{2\ell+1} = \int_0^{\eta_0} d\eta S_\Theta^X(k,\eta)j_\ell\left[k(\eta_0 - \eta)\right] \tag{4.1}$$

where S_Θ^X is the source function and X denotes ζ, Δ_B and π_B. This determines the temperature auto correlation function

$$(2\ell+1)^2 C_\ell^{TT,\langle XY\rangle} = \frac{1}{2\pi^2}\int_0^\infty \frac{dk}{k}\mathcal{P}_{\langle XY\rangle}(k)\Theta_\ell^X(\eta_0,k)\Theta_\ell^Y(\eta_0,k) \tag{4.2}$$

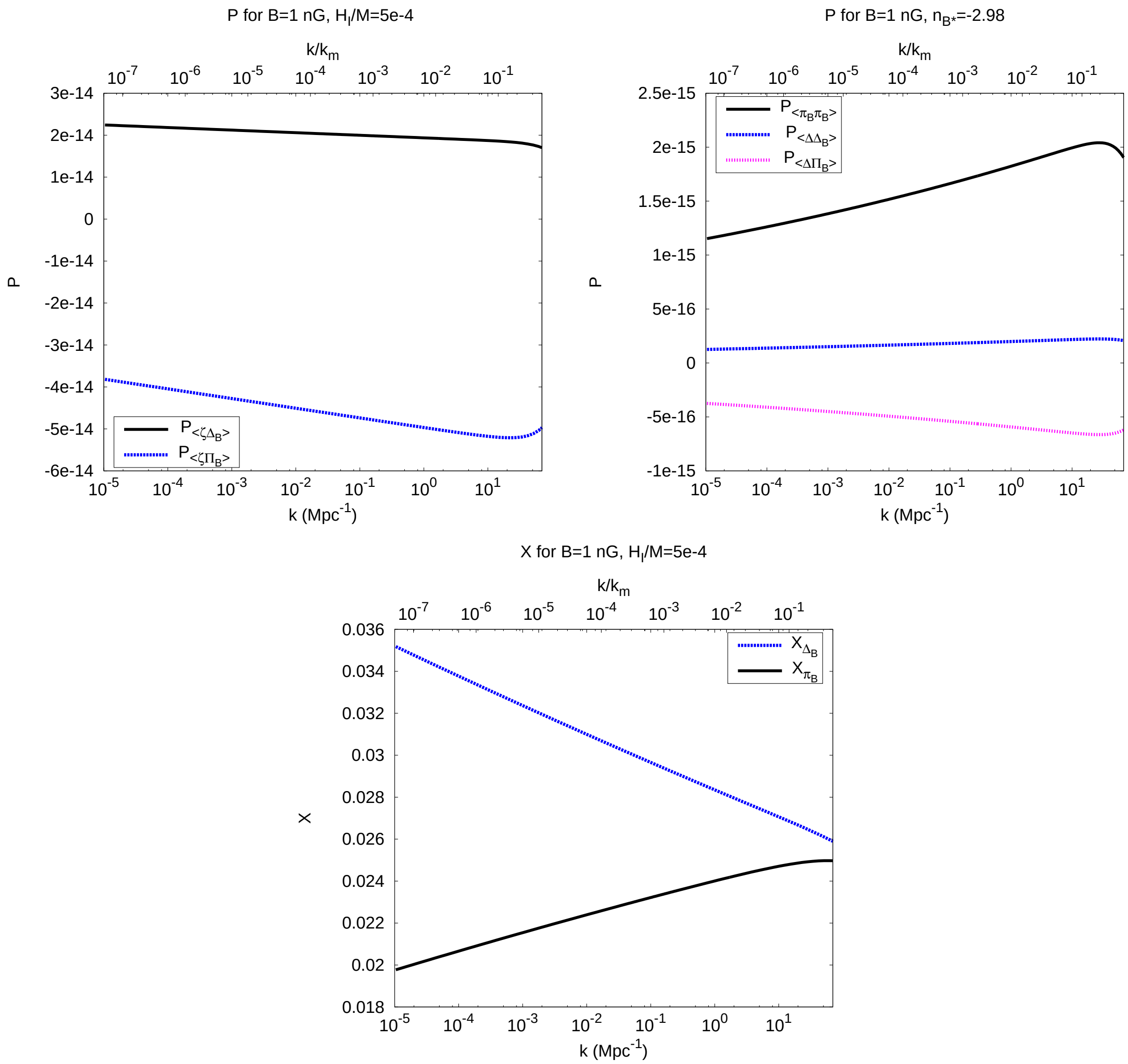

Figure 2: *Upper Panel:* (Left): The dimensionless spectra determining the cross correlation function of the primordial curvature perturbation and the magnetic energy density contrast as well as the magnetic anisotropic stress. (Right): The dimensionless spectra determining the auto- and cross correlations of the pure magnetic mode for $B = 1$ nG for a nearly scale invariant magnetic field, which corresponds to $n_B = 0.02$ or $n_{B*} = -2.98$ in the conventions used, e.g., in [19]. *Lower Panel:* The ratios determining the generalized Schwarz inequality for $\langle\zeta\Delta_B\rangle$ and $\langle\zeta\pi_B\rangle$. The numerical solutions are calculated for the parameters $\frac{H_I}{M} = 5\times 10^{-4}$, $\eta_I = -10^{-24}$ Mpc [15].

and similarly for the auto correlation function of the E-mode and the temperature polarization cross correlation. The resulting angular power spectra are calculated using a modified version of CMBEASY [33]. It is based on the modified version of [19] where the initial conditions and evolution equations including the magnetic field contributions can be found. In all solutions the best fit values of the 6-parameter ΛCDM model of WMAP9 [28] have been used, in particular, $\Omega_b = 0.0463$, $\Omega_\Lambda = 0.721$, $\mathcal{P}_\zeta(k_p) = 2.41\times 10^{-9}$, $n_s = 0.972$ and the reionization optical depth

$\tau = 0.089$. Moreover, the nearly scale invariant magnetic field has field strength $B = 1$ nG and spectral index $n_B* = -2.98$. The factor $\frac{H_I}{M}$ determining the cross correlation functions is $\frac{H_I}{M} = 5 \times 10^{-4}$. There are two types of magnetic modes. On the one hand there is the compensated magnetic mode for which the initial conditions are such that the contributions from the neutrino anisotropic stress and the magnetic anisotropic stress cancel each other. On the other hand there is the so called passive mode [20]. This is due to the presence of a magnetic field before neutrino decoupling at some conformal time η_ν which causes an additional contribution to the curvature perturbation amplitude ζ proportional to the magnetic anisotropic stress given by [20]

$$\zeta \simeq -\frac{1}{3} R_\gamma \pi_B \beta, \qquad \beta = \ln \frac{\eta_\nu}{\eta_B} \tag{4.3}$$

where $R_\gamma \equiv \frac{\Omega_\nu}{\Omega_\gamma + \Omega_\nu}$ and η_B is the conformal time corresponding to the instant of generation of the magnetic field which would correspond to the time of phase transition or to reheating if generated during inflation. As way of example, in the numerical solutions it is assumed that the magnetic field is generated during inflation and that the reheat temperature is $T_{RH} = 10^{10}$ GeV. So that $\beta = \ln \frac{T_{RH}}{T_\nu}$ with $T_\nu = 1$ MeV is determined to be $\beta = 30$. The compensated mode corresponds to $\beta = 0$ in figures 3 to 6. In figure 3 the angular power spectra determining the temperature auto correlation of the CMB are shown. For $\beta = 30$ the contribution due to the cross correlation between

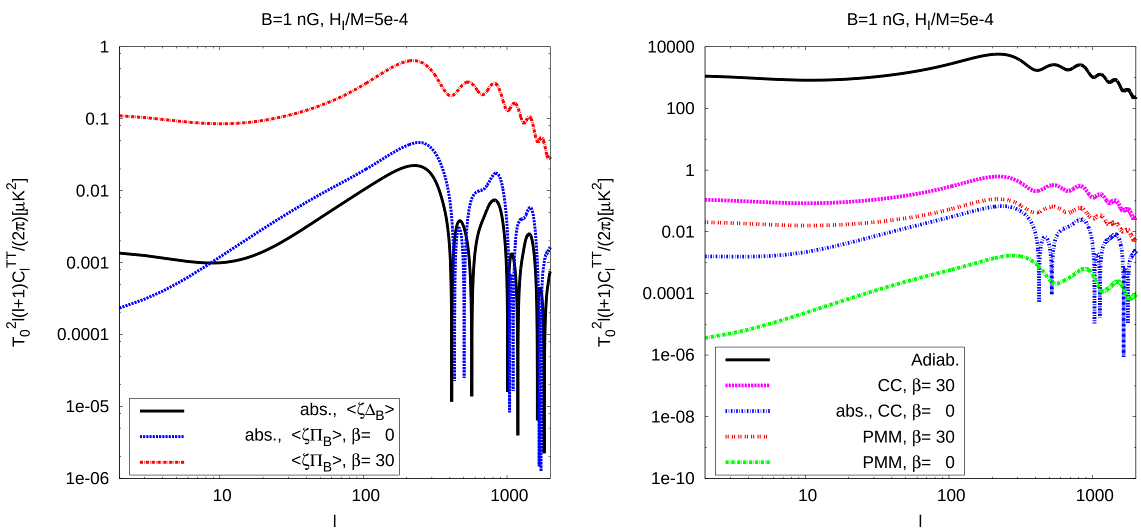

Figure 3: Angular power spectra determining the temperature autocorrelation of the CMB due to the cross correlation between the curvature mode ζ and the Δ_B mode and due to the cross correlation between the curvature mode and the π_B mode and *abs.* indicates that the absolute value is shown (*left*). The contributions to the total angular power spectrum include the adiabatic mode (*Adiab.*), the pure magnetic mode (*PMM*) and the total cross-correlated magnetic-curvature mode (*CC*) (*right*). Numerical solutions have been calculated for WMAP9 ΛCDM best fit parameters.

the curvature mode and the magnetic mode dominates on all scales over the one due to the pure magnetic mode. Moreover, the shapes of the curves are very similar. This is to be expected since for $\beta = 30$ the magnetic mode is adiabatic-like. On the contrary, due to the very different shapes of the compensated magnetic mode and the curvature mode the final cross-correlated curvature-magnetic mode is quite distinct in the case $\beta = 0$. Though, again its amplitude is dominant over that of the pure magnetic mode for almost all scales. A similar behaviour is also found for the contribution

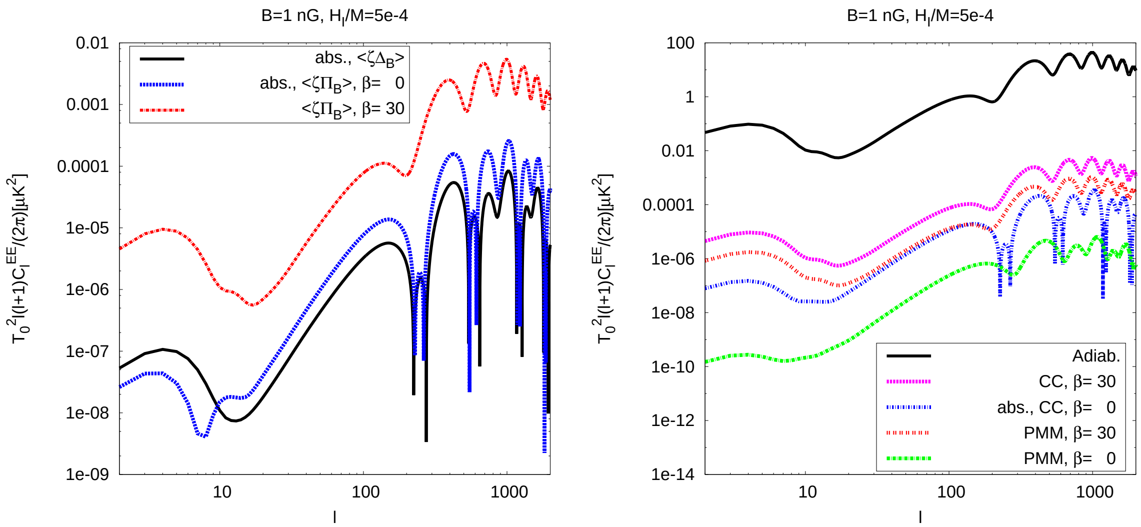

Figure 4: Angular power spectra determining the autocorrelation of the polarization E-mode of the CMB due to the cross correlation between the curvature mode ζ and the Δ_B mode and due to the cross correlation between the curvature mode and the π_B mode and *abs.* indicates that the absolute value is shown (*left*). The contributions to the total angular power spectrum include the adiabatic mode (*Adiab.*), the pure magnetic mode (*PMM*) and the total cross-correlated magnetic-curvature mode (*CC*) (*right*). These have been calculated for WMAP9 ΛCDM best fit parameters.

due to the curvature-magnetic mode cross correlation to the polarization autocorrelation (cf. figure 4) as well as the temperature polarization cross correlation (cf. figure 5). In figure 6 the linear matter power spectrum is reported. On large scales the contribution due to the curvature-magnetic cross correlation dominates over that of the pure magnetic mode, in both cases, $\beta = 0$ as well as $\beta = 30$. This behaviour is reversed on small scales, where the pure magnetic modes show the characteristic rise of the linear matter power spectrum in the presence of a magnetic field. This is due to the presence of the Lorentz term in the baryon velocity equation which on small scales dominates [20, 19]. Thereby leading to a local maximum on small scales in the total linear matter power spectrum taking into account all contributions, that is the adiabatic and magnetic modes and their cross correlations. For wave numbers much larger than the maximal wave number k_m determined by the damping of the magnetic field the linear matter power spectrum rapidly decays. In the case under consideration here $k_m = 198$ Mpc^{-1} (cf. equation (2.7)).

5 Conclusions

Observations of the CMB of unprecedented precision allow to test physics upto very early times. Observational evidence of large scale magnetic fields is abundant. There is however the open question of their origin. Assuming magnetic fields have been created in the very early universe, long before decoupling, they influence the spectra of the anisotropies and polarization of the CMB as well the linear matter power spectrum. There is clear observational evidence in the CMB pointing

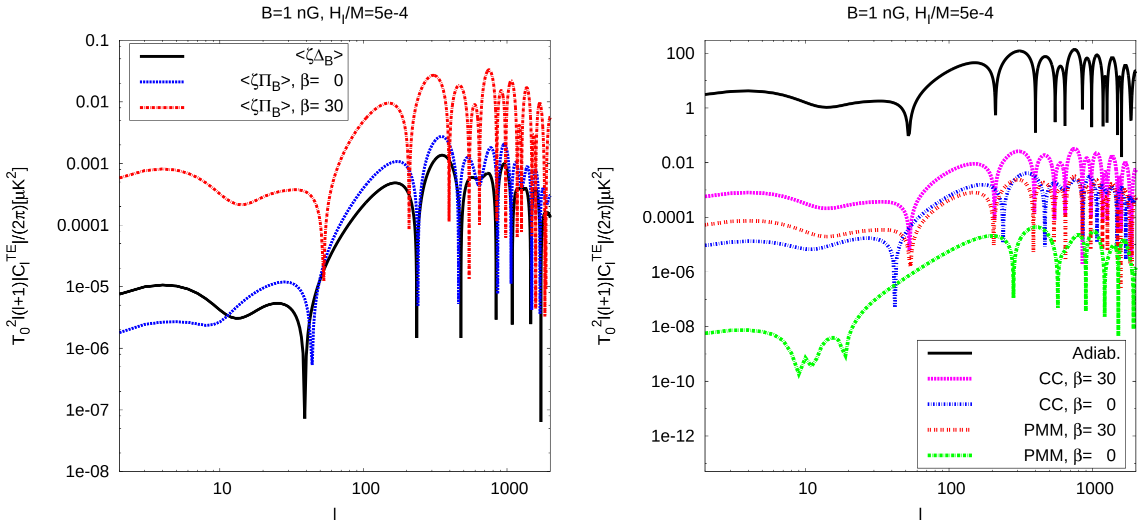

Figure 5: Absolute values of the angular power spectra determining the cross correlation of temperature and polarization E-mode of the CMB due to the cross correlation between the curvature mode ζ and the Δ_B mode and due to the cross correlation between the curvature mode and the π_B mode (*left*). The contributions to the total angular power spectrum include the adiabatic mode (*Adiab.*), the pure magnetic mode (*PMM*) and the total cross-correlated magnetic-curvature mode (*CC*) (*right*). These have been calculated for WMAP9 ΛCDM best fit parameters.

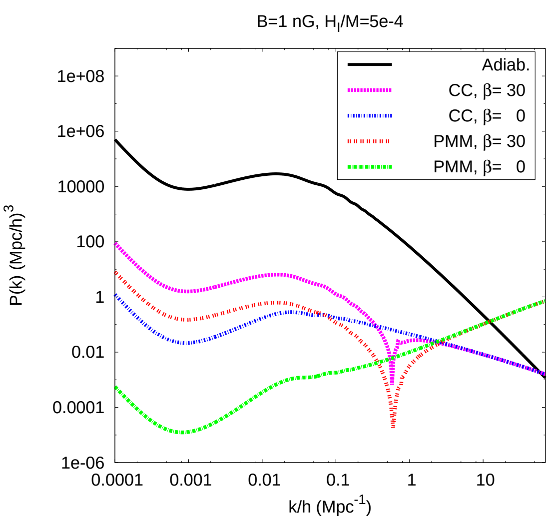

Figure 6: The total linear matter power spectrum due to the pure magnetic modes (*PMM*), the cross correlation between the curvature mode and the magnetic modes (*CC*) and the pure adiabatic mode ($B = 0$) (*Adiab.*) for the best-fit ΛCDM model of WMAP9.

to the presence of an adiabatic mode which could be a primordial curvature mode generated during inflation (cf. e.g. [28]). In general it has been assumed that the curvature mode and the magnetic field modes are uncorrelated. However, even at lowest order, that is not taking into account nonlinear effects due to the evolution of the magnetic field [22], it seems natural to assume that non vanishing correlations exist depending on the particular model of magnetic field generation during inflation, such as the coupling of electrodynamics to the inflaton [24, 25] or, as the case considered here, to a spectator field which is identified with the curvaton [15]. In [15] the bispectrum determining the cross correlation between the magnetic field and a scalar field was calculated. Identifying the scalar field with the curvaton here the cross correlations between the curvature and the magnetic modes is determined. Taking into account that the magnetic modes in the CMB are proportional to the magnetic field energy density and the magnetic anisotropic stress, respectively, both of which are quadratic in the magnetic field, the bispectrum or three-point function calculated in [15] induces a spectrum or two-point function relevant for the calculation of the CMB and matter power spectrum. The angular power spectra of the temperature anisotropies and polarization of the CMB have been calculated for a nearly scale invariant field with field strength of 1 nG. It has been found that the contribution due to the cross correlation between the curvature mode and the magnetic modes dominates over the contribution due to the pure magnetic mode. Similarly, the curvature magnetic mode cross correlated contribution dominates on large scales over the pure magnetic mode in the linear matter power spectrum. While on the contrary, on small scales the pure magnetic mode dominates, not only over the cross correlated contribution but also the adiabatic contribution which is due to the presence of the Lorentz term in the baryon velocity equation.

6 Acknowledgments

I would like to thank the CERN Theory Division as well as the Max-Planck-Institute for Astrophysics for hospitality where part of this work was done. Financial support by Spanish Science Ministry grants FPA2009-10612, FIS2012-30926 and CSD2007-00042 is gratefully acknowledged. Furthermore I acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by the NASA Office of Space Science.

References

[1] P. P. Kronberg, Rept. Prog. Phys. **57** (1994) 325; R. Wilebinski and R. Beck, Lect. Notes Phys. **664** (2005); C. L. Carilli and G. B. Taylor, Ann. Rev. Astron. Astrophys. **40** (2002) 319.

[2] I. Vovk, A. M. Taylor, D. Semikoz and A. Neronov, Astrophys. J. **747** (2012) L14; A. Neronov and I. Vovk, Science **328** (2010) 73; S. 'i. Ando and A. Kusenko, Astrophys. J. **722** (2010) L39.

[3] T. C. Arlen, V. V. Vassiliev, T. Weisgarber, S. P. Wakely and S. Y. Shafi, "Intergalactic Magnetic Fields and Gamma Ray Observations of Extreme TeV Blazars," arXiv:1210.2802 [astro-ph.HE].

[4] D. Grasso, H. R. Rubinstein, Phys. Rept. **348** (2001) 163; L. M. Widrow, Rev. Mod. Phys. **74** (2002) 775; M. Giovannini, Int. J. Mod. Phys. D **13** (2004) 391; A. Kandus, K. E. Kunze, C. G. Tsagas, Phys. Rept. **505** (2011) 1; L. M. Widrow, D. Ryu, D. Schleicher, K. Subramanian, C. G. Tsagas and R. A. Treumann, Space Sci. Rev. **166** (2012) 37; D. Ryu, D. R. G. Schleicher, R. A. Treumann, C. G. Tsagas and L. M. Widrow, Space Sci. Rev. **166** (2012) 1.

[5] C. G. Tsagas and A. Kandus, Phys. Rev. D **71** (2005) 123506; C. G. Tsagas, Phys. Rev. D **72** (2005) 123509; J. D. Barrow, C. G. Tsagas and K. Yamamoto, Phys. Rev. D **86** (2012) 023533.

[6] J. Adamek, C. de Rham and R. Durrer, Mon. Not. Roy. Astron. Soc. **423** (2012) 2705.

[7] B. Ratra, Astrophys. J. **391** (1992) L1.

[8] V. Demozzi, V. Mukhanov and H. Rubinstein, JCAP **0908** (2009) 025.

[9] S. Kanno, J. Soda and M. -a. Watanabe, JCAP **0912** (2009) 009; N. Bartolo, S. Matarrese, M. Peloso and A. Ricciardone, Phys. Rev. D **87** (2013) 023504.

[10] M. S. Turner and L. M. Widrow, Phys. Rev. D **37** (1988) 2743.

[11] B. Himmetoglu, C. R. Contaldi and M. Peloso, Phys. Rev. D **80** (2009) 123530.

[12] K. E. Kunze, Phys. Rev. D **81** (2010) 043526.

[13] D. H. Lyth and D. Wands, Phys. Lett. B **524** (2002) 5.

[14] K. E. Kunze, Phys. Rev. D **87** (2013) 063505.

[15] R. R. Caldwell, L. Motta and M. Kamionkowski, Phys. Rev. D **84** (2011) 123525.

[16] D. Yamazaki, K. Ichiki, T. Kajino and G. J. Mathews, Astrophys. J. **646** (2006) 719; D. Yamazaki, K. Ichiki, T. Kajino and G. J. Mathews, Phys. Rev. D **77** (2008) 043005; K. Kojima, K. Ichiki, D. G. Yamazaki, T. Kajino and G. J. Mathews, Phys. Rev. D **78** (2008) 045010.

[17] M. Giovannini and K. E. Kunze, Phys. Rev. D **77** (2008) 061301; M. Giovannini and K. E. Kunze, Phys. Rev. D **77** (2008) 063003.

[18] F. Finelli, F. Paci and D. Paoletti, Phys. Rev. D **78** (2008) 023510. D. Paoletti, F. Finelli and F. Paci, Mon. Not. Roy. Astron. Soc. **396** (2009) 523; D. Paoletti and F. Finelli, Phys. Rev. D **83** (2011) 123533; D. Paoletti and F. Finelli, "Constraints on a Stochastic Background of Primordial Magnetic Fields with WMAP and South Pole Telescope data," arXiv:1208.2625 [astro-ph.CO].

[19] K. E. Kunze, Phys. Rev. **D83** (2011) 023006.

[20] J. R. Shaw and A. Lewis, Phys. Rev. D **81** (2010) 043517; J. R. Shaw and A. Lewis, Phys. Rev. D **86** (2012) 043510.

[21] L. Pogosian, T. Vachaspati and S. Winitzki, Phys. Rev. D **65** (2002) 083502; C. Caprini, R. Durrer and T. Kahniashvili, Phys. Rev. D **69** (2004) 063006; T. Kahniashvili and B. Ratra, Phys. Rev. D **71** (2005) 103006; K. E. Kunze, Phys. Rev. D **85** (2012) 083004.

[22] K. E. Kunze, JCAP **1302** (2013) 009.

[23] T. R. Seshadri and K. Subramanian, Phys. Rev. Lett. **103** (2009) 081303; C. Caprini, F. Finelli, D. Paoletti and A. Riotto, JCAP **0906** (2009) 021; P. Trivedi, K. Subramanian and T. R. Seshadri, Phys. Rev. D **82** (2010) 123006; I. A. Brown, Astrophys. J. **733** (2011) 83; M. Shiraishi, D. Nitta, S. Yokoyama, K. Ichiki and K. Takahashi, Phys. Rev. D **83** (2011) 123523; M. Shiraishi, D. Nitta, S. Yokoyama, K. Ichiki and K. Takahashi, Phys. Rev. D **83** (2011) 123003; M. Shiraishi, D. Nitta, S. Yokoyama and K. Ichiki, JCAP **1203** (2012) 041; M. Shiraishi, JCAP **1206** (2012) 015; R. K. Jain and M. S. Sloth, Phys. Rev. D **86** (2012) 123528; R. K. Jain and M. S. Sloth, JCAP **1302** (2013) 003.

[24] L. Motta and R. R. Caldwell, Phys. Rev. D **85** (2012) 103532.

[25] M. Shiraishi, S. Saga and S. Yokoyama, JCAP **1211** (2012) 046.

[26] K. Bamba and J. Yokoyama, Phys. Rev. D **69** (2004) 043507.

[27] K. Subramanian and J. D. Barrow, Phys. Rev. D **58** (1998) 083502.

[28] G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett, J. Dunkley, M. R. Nolta and M. Halpern *et al.*, "Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results," arXiv:1212.5226 [astro-ph.CO].

[29] M. Sasaki, J. Valiviita and D. Wands, Phys. Rev. D **74** (2006) 103003.

[30] C. G. Tsagas and J. D. Barrow, Class. Quant. Grav. **14** (1997) 2539 C. G. Tsagas and J. D. Barrow, Class. Quant. Grav. **15** (1998) 3523; J. D. Barrow, R. Maartens and C. G. Tsagas, Phys. Rept. **449** (2007) 131.

[31] U. Seljak and M. Zaldarriaga, Astrophys. J. **469** (1996) 437.

[32] W. Hu and M. J. White, Phys. Rev. D **56** (1997) 596

[33] M. Doran, JCAP **0510** (2005) 011;
http://www.thphys.uni-heidelberg.de/~robbers/cmbeasy/