Sigma terms from an SU(3) chiral extrapolation

P. E. Shanahan, A. W. Thomas, and R. D. Young

Phys. Rev. D **87**, 074503 — Published 11 April 2013

DOI: 10.1103/PhysRevD.87.074503

Sigma terms from an SU(3) chiral extrapolation

P.E. Shanahan, A.W. Thomas, and R.D. Young
ARC Centre of Excellence in Particle Physics at the Terascale and CSSM,
School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005, Australia

We report a new analysis of lattice simulation results for octet baryon masses in 2+1-flavor QCD, with an emphasis on a precise determination of the pion-nucleon and strangeness nucleon sigma terms. A controlled chiral extrapolation of a recent PACS-CS Collaboration data set yields baryon masses which exhibit remarkable agreement both with experimental values at the physical point and with the results of independent lattice QCD simulations at unphysical meson masses. Using the Feynman-Hellmann relation, we evaluate sigma commutators for all octet baryons. The small statistical uncertainty and considerably smaller model-dependence allows a significantly more precise determination of the pion-nucleon sigma commutator and the strangeness sigma term than hitherto possible, subject to an unresolved issue concerning the lattice scale setting.

PACS numbers: 12.38.Gc, 12.39.Fe, 14.20.Pt
Keywords: Lattice QCD, Chiral Symmetry, Extrapolation

The light-quark sigma terms provide critical information concerning the nature of explicit chiral symmetry breaking in QCD, as well as the decomposition of the mass of the nucleon [1]. While these physical observables are difficult to measure with conventional probes, an accurate knowledge of the sigma terms is of essential importance in the interpretation of experimental searches for dark matter [2–6]. Dark matter candidates, such as the favoured neutralino, a weakly interacting fermion with mass of order 100 GeV or more, have interactions with hadronic matter which are essentially determined by couplings to the light and strange quark sigma commutators.

Experimentally, $\sigma_{\pi N}$ is determined from πN scattering through a dispersion relation analysis [7]. Traditionally, the strange scalar form factor has then been evaluated indirectly using $\sigma_{\pi N}$ and a best-estimate for the non-singlet contribution $\sigma_0 = m_l\langle N|\overline{u}u + \overline{d}d - 2\overline{s}s|N\rangle$. These traditional evaluations have yielded a value for σ_s as large as 300 MeV, compared to 50 MeV for the light quark commutator, indicating that as much as one third of the nucleon mass might be attributed to non-valence quarks. This suggestion appears to be incompatible with widely-used constituent quark models, and has generated considerable theoretical interest.

The traditional method of determination of σ_s is severely limited because it involves the small difference between $\sigma_{\pi N}$ (with its uncertainty) and σ_0 which is usually deduced in terms of SU(3) symmetry breaking. Even given a perfect determination of $\sigma_{\pi N}$, σ_s will have an uncertainty of order 90 MeV [8]. For that reason σ_s has been considered notoriously difficult to pin down. In recent years, the best value for σ_s has seen an enormous revision. Advances in lattice QCD have revealed a strange sigma term of 20-50 MeV [9–21], an order of magnitude smaller than was previously believed.

Here we use the finite-range regularization (FRR) technique to effectively resum the chiral perturbation theory expansion of the quark mass dependence of octet baryons. Fitting the resulting functions to recent lattice data, we extract the scalar form factors by simple differentiation using the Feynman-Hellmann theorem. Our technique allows comparison with recent direct lattice QCD calculations of the flavor-singlet matrix elements at unphysical meson masses [11–15].

B	$C_{Bl}^{(1)}$	$C_{Bs}^{(1)}$
N	$2\alpha + 2\beta + 4\sigma$	2σ
Λ	$\alpha + 2\beta + 4\sigma$	$\alpha + 2\sigma$
Σ	$\frac{5}{3}\alpha + \frac{2}{3}\beta + 4\sigma$	$\frac{1}{3}\alpha + \frac{4}{3}\beta + 2\sigma$
Ξ	$\frac{1}{3}\alpha + \frac{4}{3}\beta + 4\sigma$	$\frac{5}{3}\alpha + \frac{2}{3}\beta + 2\sigma$

TABLE I. Coefficients for the terms in Eq. 6 linear in the light and strange quark masses, $bm_l \to m_\pi^2/2$ and $bm_s \to (m_K^2 - m_\pi^2/2)$.

Because of the indirect evaluation of the sigma terms by differentiation, the analysis of the strangeness sigma term in particular suffers from an uncertainty due to the choice of scale setting scheme. With two prescriptions for setting the lattice scale, namely using 'mass dependent', or 'mass independent' schemes, described further in the text, we find $\sigma_s = 59 \pm 6$ MeV and $\sigma_s = 21 \pm 6$ MeV respectively. Comparison with recent direct lattice calculations suggests a slight preference for the latter value [11–14]. Our results for the pion-nucleon sigma term are consistent irrespective of scale setting prescription. We report a value of $\sigma_{\pi N} = 45 \pm 6$ MeV at the physical point.

The sigma terms of a baryon B are defined as scalar form factors, evaluated in the limit of vanishing momentum transfer. For each quark flavor q,

$$\sigma_{Bq} = m_q\langle B|\overline{q}q|B\rangle; \quad \overline{\sigma}_{Bq} = \sigma_{Bq}/M_B. \tag{1}$$

For the nucleon, the so-called πN sigma commutator and the strange sigma commutator are defined by

$$\sigma_{\pi N} = m_l\langle N|\overline{u}u + \overline{d}d|N\rangle, \tag{2}$$
$$\sigma_s = m_s\langle N|\overline{s}s|N\rangle, \tag{3}$$

where $m_l = (m_u + m_d)/2$.

Following the technique described in Refs. [9, 23], we fit octet baryon mass data recently published by the PACS-CS Collaboration [24] using a chiral expansion:

$$M_B = M^{(0)} + \delta M_B^{(1)} + \delta M_B^{(3/2)} + \dots \quad (4)$$

Here, $M^{(0)}$ denotes the degenerate mass of the baryon octet in the SU(3) chiral limit, and $\delta M_B^{(1)}$ gives the correction linear in the quark masses. This may be derived by considering the relevant terms of the usual lowest-order effective Lagrangian:

$$2\alpha b \mathrm{Tr}(\overline{B}B\mathcal{M}) + 2\beta b \mathrm{Tr}(\overline{B}\mathcal{M}B) + 2\sigma b \mathrm{Tr}(\overline{B}B)\mathrm{Tr}(\mathcal{M}), \quad (5)$$

where B represents the usual tensor of octet baryon fields, and $\mathcal{M}$ indicates the quark mass matrix (see, for example, Ref. [25]). One finds

$$\delta M_B^{(1)} = -C_{Bl}^{(1)} b m_l - C_{Bs}^{(1)} b m_s, \quad (6)$$

with the coefficients given in Table I.

According to the Gell-Mann–Oakes-Renner (GMOR) relation,

$$m_\pi^2 = 2bm_l + \mathcal{O}(m_q^2), \quad (7)$$

$$m_K^2 = b(m_l + m_s) + \mathcal{O}(m_q^2), \quad (8)$$

and hence we substitute the quark masses in Eq. 6 by $bm_l \to m_\pi^2/2$ and $bm_s \to (m_K^2 - m_\pi^2/2)$. We note that the corrections to the leading-order GMOR result will only amount to a modification of the chiral series at $\mathcal{O}(m_q^2)$, which is beyond the order of the expansion considered here.

The term at next order, following the linear mass insertions, represents quantum corrections corresponding to one-loop contributions from the pseudo-Goldstone bosons $\phi = \pi, K, \eta$. These loops take the form:

$$\delta M_B^{(3/2)} = -\frac{1}{16\pi f^2} \sum_\phi \left[\chi_{B\phi} I_R(m_\phi, 0, \Lambda) + \chi_{T\phi} I_R(m_\phi, \delta, \Lambda)\right], \quad (9)$$

where the coefficients $\chi_{B\phi}$, $\chi_{T\phi}$, corresponding to octet-baryon–meson and decuplet-baryon–meson loops, are given in Table II.

The meson loops involve the integrals:

$$I_R = \frac{2}{\pi} \int dk \frac{k^4}{\sqrt{k^2 + m_\phi^2}(\delta + \sqrt{k^2 + m_\phi^2})} u^2(k) - b_0 - b_2 m_\phi^2 \quad (10)$$

where the subtraction constants, $b_{0,2}$, are defined so that the parameters $M^{(0)}, C_{Bl}^{(1)}$ and $C_{Bs}^{(1)}$ are renormalized (explicit expressions may be found in Ref. [26], or can be readily evaluated numerically by Taylor expanding the integrand in m_ϕ^2).

Following Ref. [9], we retain the octet-decuplet mass difference δ in numerical evaluations to properly account for the branch structure near $m_\phi \sim \delta$. The loop contribution parameters are set to appropriate experimental and phenomenological values; $D + F = g_A = 1.27$, $F = \frac{2}{3}D$, $C = -2D$, $f = 0.0871$ GeV, and $\delta = 0.292$ GeV. Within the framework of FRR, we introduce a mass scale Λ, through a regulator $u(k)$. Λ is related to the scale beyond which a formal expansion in powers of the Goldstone boson masses breaks down. This allows for the suppression of short-distance physics from the loop integrals of the effective theory. Here, Λ is chosen by fitting to the lattice data itself. We note that the demonstrated benefit of FRR is to incorporate the non-analytic behaviour associated with chiral symmetry breaking in QCD, while maintaining a robust fit to lattice data over a wide range of quark masses. In particular, higher-order terms are implicit in the structure of FRR, and essentially sum to zero in the region of large quark masses. For this reason, the chiral series is stable under the truncation of such terms. We refer to Refs. [26–30] for further discussions of the FRR regularization scheme.

The model-dependent uncertainty in the result is estimated by the consideration of a variety of forms of the regulator $u(k)$, namely monopole, dipole, and Gaussian, as well as a sharp cutoff. The uncertainty due to the choice of regulator is small and below the resolution of the figures. In addition, we allow f, the meson decay constant in the chiral limit, the baryon-baryon-meson coupling constants F and C, and δ to vary by $\pm 10\%$ from the central values given above; see Ref. [31] for details. The effects of these variations, as well as the effect of a 2% uncertainty in the physical value of r_0, are included in the final quoted errors. Statistical uncertainties are accounted for by a covariance matrix analysis which includes the effect of correlations between all of the fit parameters M^0, α, β, σ, as well as the regulator mass Λ.

The PACS-CS results have been corrected for small, model-independent, finite volume effects before fitting. These finite volume corrections were evaluated by considering the leading one-loop results of chiral EFT [9, 32–34]. We note that the largest shift was -0.022 ± 0.002 GeV for the nucleon at the lightest pion mass.

The fit to the PACS-CS baryon octet data is shown in Figure 1. We find an optimal dipole regularization scale of $\Lambda = 1.0 \pm 0.1$ GeV, in close agreement with the value deduced from an analysis of nucleon magnetic moment data [35] and, from the phenomenological point of view, remarkably close to the value preferred from comparison of the nucleon's axial and induced pseudoscalar form factors [36]. The minimum χ^2_{dof} is 0.45 (6.8 divided by (20-5 $\equiv$ 15)) for the dipole, and varies between 0.44 and 0.426 for the other regulators. This value is somewhat lower than unity, as correlations between the lattice data cannot be accounted for without access to the original data.

Clearly, the fit is very satisfactory over the entire range of quark masses explored in the simulations. Further-

		$\chi_{B\phi}$			$\chi_{T\phi}$	
	π	K	η	π	K	η
N	$\frac{3}{2}(D+F)^2$	$\frac{1}{3}(5D^2-6DF+9F^2)$	$\frac{1}{6}(D-3F)^2$	$\frac{4}{3}\mathcal{C}^2$	$\frac{1}{3}\mathcal{C}^2$	0
Λ	$2D^2$	$\frac{2}{3}(D^2+9F^2)$	$\frac{2}{3}D^2$	$\mathcal{C}^2$	$\frac{2}{3}\mathcal{C}^2$	0
Σ	$\frac{2}{3}(D^2+6F^2)$	$2(D^2+F^2)$	$\frac{2}{3}D^2$	$\frac{2}{9}\mathcal{C}^2$	$\frac{10}{9}\mathcal{C}^2$	$\frac{1}{3}\mathcal{C}^2$
Ξ	$\frac{3}{2}(D-F)^2$	$\frac{1}{3}(5D^2+6DF+9F^2)$	$\frac{1}{6}(D+3F)^2$	$\frac{1}{3}\mathcal{C}^2$	$\mathcal{C}^2$	$\frac{1}{3}\mathcal{C}^2$

TABLE II. Chiral SU(3) coefficients for the octet baryons to octet (B) and decuplet (T) baryons through the pseudoscalar octet meson ϕ.

more, the masses of the octet baryons agree remarkably well with experiment at the physical point (with a χ^2/point close to one). A comparison of the extrapolated baryon masses with the best experimental values is given in Table III. The first error quoted is statistical and includes the correlated uncertainty of all of the fit parameters including the regulator mass Λ, while the second is an estimate of model-dependence. This includes the full variation over dipole, monopole, sharp cutoff and Gaussian regulator forms, as well as accounting for the variation of the phenomenologically-set parameters f, F, $\mathcal{C}$ and δ described earlier.

As we fit baryon mass functions to lattice data over a range of pseudoscalar masses significantly larger than the physical values, it is prudent to check the consistency of our results as the analysis moves outside the power-counting regime (PCR), where higher order terms may become significant. By performing our fit to progressively fewer data points, that is, by dropping the heaviest mass points, we test the scheme dependence of our evaluation. The results are consistent, and largely independent of the truncation of the data. This can be seen clearly in Figure 2, which shows the variation of the dimensionless baryon sigma terms as progressively fewer data points are used for the fit to the octet masses. The points shown correspond to an evaluation with a dipole regulator, and error bars are purely statistical.

B	Mass (GeV)	Experimental	$\overline{\sigma}_{Bl}$	$\overline{\sigma}_{Bs}$
N	0.959(24)(9)	0.939	0.047(6)(5)	0.022(6)(0)
Λ	1.129(15)(6)	1.116	0.026(3)(2)	0.141(8)(1)
Σ	1.188(11)(6)	1.193	0.020(2)(2)	0.171(8)(1)
Ξ	1.325(6)(2)	1.318	0.0089(7)(4)	0.239(8)(1)

TABLE III. Extracted masses and sigma terms for the physical baryons, with the lattice scale set using the 'mass-dependent' prescription. The first uncertainty quoted is statistical, while the second results from the variation of various chiral parameters and the form of the UV regulator as described in the text. The experimental masses are shown for comparison.

To further test our claim that the fitted mass functions accurately describe the variation of the baryon masses

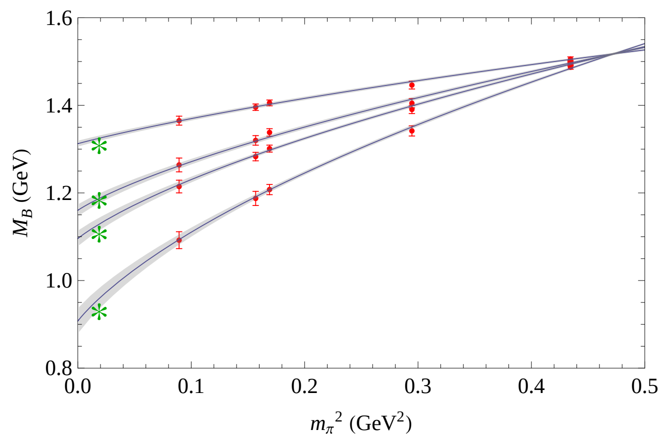

FIG. 1. Fit to the PACS-CS baryon octet data. Error bands shown are purely statistical, and incorporate correlated uncertainties between all fit parameters. Note that the data shown has been corrected for finite volume and the simulation strange quark mass, which was somewhat larger than the physical value. The green stars show experimental values.

with quark mass, we compare our extrapolation with independent lattice data along a very different trajectory in the $m_l - m_s$ plane, as compared to the fit domain. Most lattice simulations, including that of the PACS-CS Collaboration, hold the simulation strange quark mass fixed near the physical value, and progressively lower the light quark mass to approach the physical point. These simulations necessarily sample a range of singlet masses $(2m_K^2 + m_\pi^2)$. As an alternative, the QCDSF-UKQCD Collaboration has recently presented a different method of tuning the quark masses, in which the singlet mass is held fixed along the simulation trajectory [37]. This procedure constrains the simulation kaon mass to always be smaller than the physical value. In comparison, the traditional trajectory in the $m_\pi - m_K$ plane necessarily keeps the kaon mass larger than the physical value.

The close match between our fit to the PACS-CS points and the QCDSF-UKQCD lattice data, shown in Figure 3, is extremely encouraging. We emphasize that the lines in Figure 3 are *not* a fit to the data shown, but rather a prediction, resulting from the described fit to the PACS-CS octet data being evaluated along the QCDSF-UKQCD simulation trajectory.

All lattice points shown in Figure 3 have been shifted, by the procedure described for the PACS-CS data, to ac-

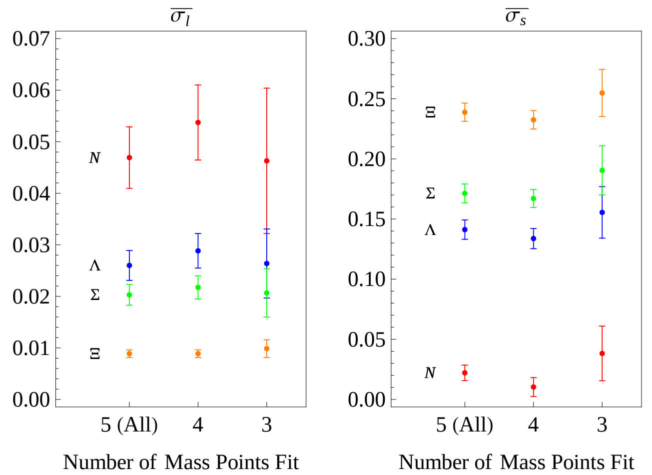

FIG. 2. Dimensionless baryon sigma terms, evaluated using a dipole regulator, based on fits to the PACS-CS results at the lightest 5 (all), 4, and 3 pseudoscalar mass points.

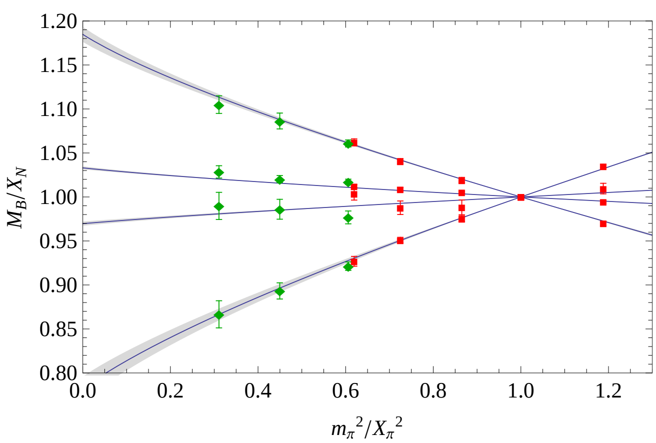

FIG. 3. Prediction of QCDSF-UKQCD lattice data, based on our fit to the PACS-CS octet baryon mass simulation. Red (square) and green (diamond) points correspond to 24^3 and 32^3 lattice volumes respectively. Error bands shown are purely statistical, and incorporate correlated uncertainties between all fit parameters.

count for finite-volume effects. We chose to use the lattice spacing $a = 0.078$ fm deduced by the QCDSF-UKQCD Collaboration. For further details of the QCDSF-UKQCD data set, and the normalizations X_N, X_π, we refer to Ref. [37].

It should be noted that the leading-order term in a chiral expansion for the strangeness sigma commutator is determined by the parameter σ, as seen in Table I. This parameter is common to all baryons in the octet, and by Eq. 5 is sensitive only to the singlet combination of the quark masses. The contours in Fig. 4 show that, across the PACS-CS ensemble, the variation of the singlet quark mass is relatively large, with only a relatively small extrapolation necessary to reach the physical point.

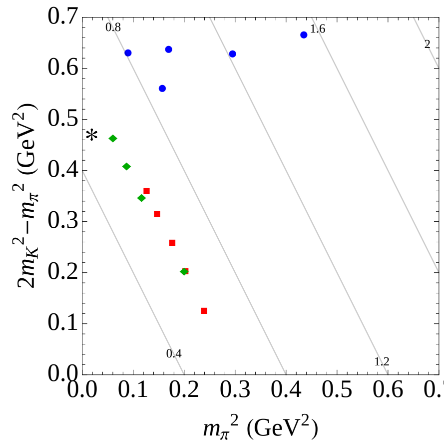

FIG. 4. Locations of lattice QCD simulations by the PACS-CS Collaboration (blue circles), and QCDSF-UKQCD Collaboration (red and green squares and diamonds) in the $m_l - m_s$ plane. The star denotes the physical point. Contours indicate lines of constant singlet quark mass $(2m_K^2 + m_\pi^2)$, in units of $(\mathrm{GeV})^2$. Figure 3 shows the fit to the PACS-CS data only, evaluated at the QCDSF-UKQCD simulation quark masses.

With respect to the variation of the quark masses orthogonal to the singlet direction, Fig. 4 acts to emphasize the extrapolation distance required in the prediction of the QCDSF-UKQCD results seen in Fig. 3. While a powerful check of the robustness of the chiral expansions, it should be noted that an analysis of the QCDSF-UKQCD results on their own cannot give a meaningful determination of the parameter σ, since (by design) these simulations have only been performed at a single value of the singlet quark mass.

To extract the sigma commutators from our baryon mass functions, we use the Feynman-Hellman relation [38],

$$\sigma_{Bq} = m_q \frac{\partial M_B}{\partial m_q}, \tag{11}$$

and, as above, replace quark masses by meson masses squared: $bm_l \to m_\pi^2/2$ and $bm_s \to (m_K^2 - m_\pi^2/2)$. For the case of the nucleon, we recall the alternative conventional notation to quantify the strangeness content, namely the kaon sigma term

$$\sigma_{KN} = \frac{1}{2}(m_l + m_s)\langle N|\overline{u}u + \overline{d}d + \overline{s}s|N\rangle. \tag{12}$$

A direct measure of the magnitude of the strange quark content of the nucleon relative to its light quark content:

$$y = \frac{2\langle N|\overline{s}s|N\rangle}{\langle N|\overline{u}u + \overline{d}d|N\rangle} = \frac{m_l}{m_s}\frac{2\sigma_s}{\sigma_{\pi N}}, \tag{13}$$

can be trivially evaluated given the strange and light quark sigma terms. At the physical point, we find $\sigma_{\pi N} = 45 \pm 6$ MeV, $\sigma_{KN} = 300 \pm 40$ MeV and $\sigma_s = 21 \pm 6$ MeV, corresponding to a y-value of 0.04 ± 0.01

			σ_s (MeV)		
		(m_π, m_K) MeV	direct	mass-dep scale	mass-indep scale
QCDSF Collaboration [11]	3-point	(281,547)	12^{+23}_{-16}	16(5)(1)	56(6)(1)
ETM Collaboration [12]	3-point	(390,580)	13(5)(1)	12(5)(1)	58(6)(1)
Engelhardt [13]	3-point	physical (chiral extrap)	43(10)	21(6)(0)	59(6)(1)
JLQCD Collaboration [14]	3-point	physical (chiral extrap)	8(14)(15)	21(6)(0)	59(6)(1)
MILC Collaboration [15]	hybrid	physical (chiral extrap)	59(6)(8)	21(6)(0)	59(6)(1)

TABLE IV. Recent direct lattice calculations of σ_s compared with the results of our analysis. Columns labelled 'mass-dep scale' and 'mass-indep scale' correspond to our analysis of the PACS-CS Collaboration lattice results, evaluated at the indicated (m_π, m_K) values, with the scale set using the relevant scale setting prescriptions.

for $m_l/m_s = 0.039(6)$ [39]. This analysis also constrains $\sigma_{\pi N} - \sigma_0$ to be 1.64 ± 0.53 MeV. The quoted errors include all systematic and model-dependent uncertainties combined in quadrature. Results for the other octet baryons are made explicit in Table III.

While the method used here leads to relatively small uncertainties for all sigma terms calculated, we point out that there is a systematic effect which arises because of the need to set the scale for the lattice data. As the Feynman-Hellman theorem relates the sigma commutators to the octet baryon masses via a derivative with respect to quark mass, a spectral determination of these terms will necessarily make reference to the scale away from the physical point, and hence depend on the scale setting scheme.

Precisely, the application of the Feynman-Hellman relation requires taking a partial derivative of a baryon mass with respect to quark mass. That is, all other parameters must be held fixed, including the strong coupling α (or, equivalently, $\Lambda_{\rm QCD}$). In lattice QCD, there is an apparent ambiguity as to how to define a fixed renormalized coupling α [40, 41].

For example, for the analysis in this work the scale for the PACS-CS lattice data was set assuming that the dimensionful Sommer scale r_0 is independent of quark mass. This choice is based on the assumption that r_0, which is related to the force between static quarks at relatively short distance, is essentially disconnected from chiral physics and should therefore vary slowly with changes in quark mass.

An alternative scale setting method is to assume that the lattice scale, at constant bare coupling (e.g., β), is independent of the bare quark mass (e.g., κ). In this 'mass independent' approach, one identifies a single lattice scale with an entire lattice ensemble at constant bare coupling by extrapolation of some dimensional observable to the physical quark masses. For instance, at fixed β the lattice Sommer scale could be extrapolated to the physical point. The extrapolated quantity, denoted r_0^*/a^*, is then matched to experiment to determine the lattice scale.

Using the mass-dependent scale setting scheme, applying the Feynman-Hellman relation amounts to evaluating

$$\frac{\partial(\frac{r_0}{a} a M_B)}{\partial m_q}, \tag{14}$$

which requires one to assume that $\partial r_0/\partial m_q = 0$. The mass-independent scheme amounts to calculating

$$\frac{\partial(\frac{r_0^*}{a^*} a M_B)}{\partial m_q}, \tag{15}$$

which, in contrast, requires the assumption that $a/a^* = 1$ (or equivalently, $\partial a/\partial m_q = 0$). We extend our described analysis to investigate the latter method of scale determination, and describe the consequences for the determination of the sigma terms.

Repeating the analysis described above with a 'mass independent' scale setting scheme gives $\sigma_{\pi N} = 51(3)(6)$ MeV and $\sigma_s = 59(6)(1)$ MeV (compared to $\sigma_{\pi N} = 45(5)(4)$ and $\sigma_s = 21(6)(0)$ with the 'mass dependent' prescription). We emphasize that our results for the pion-nucleon sigma term using each scale setting method are precise and compatible within uncertainties, and that we are for this reason extremely confident in that result.

With a view to finding a physically significant result for σ_s, we point out that *direct* lattice calculations of this quantity should not have a large dependence on the scale setting scheme. An advantage of the method used here is that we can easily evaluate sigma terms from our fit at any pion or kaon mass. In particular, we may compare with the results of recent direct lattice calculations, including preliminary calculations performed at only one set of pseudoscalar masses. Such a comparison is given for the strangeness sigma term σ_s in Table IV. The available direct calculations include $2-$ and $2+1+1-$flavor simulations [11, 12] at a single set of pion and kaon masses, and $2+1-$flavor calculations which have been chirally extrapolated to the physical point [13, 14]. The MILC Collaboration calculation is not a direct three-point calculation, but rather uses a 'hybrid' method to find the sigma term [15]. In comparison with spectral results, the Collaboration indicates that the mass-independent scale setting scheme is relevant to their results [42]. We emphasize that this summary does not include the results of calculations which use the Feynman-Hellman theorem, as these may suffer from the same source of scale setting ambiguity as in our own work. We also note that several of these calculations are preliminary, with results at only one lattice spacing and volume.

The results of our calculation using the 'mass dependent' scale setting approach agree extremely well with the direct QCDSF and TMC calculations at the simulation values of m_π and m_K. A similar level of agreement is found with the JLQCD result. Finally, the Engelhardt result sits between the values of σ_s given by the two scale setting schemes, while the MILC result favors the 'mass independent' scheme.

The conclusion of our analysis is clear. By developing closed-form functions for baryon mass as a function of quark mass based on a fit to PACS-CS Collaboration lattice data, we were able to determine precise baryonic sigma terms by simple differentiation. This method allows us to achieve small statistical and model-dependent uncertainties. Considerable effort was made to check the sensitivity of the final results to variations in low energy constants as well as the lattice spacing, with correlations being consistently included in the evaluation of the quoted errors. The only significant systematic uncertainty we find, which is discussed in detail, is that arising from the choice of scale setting method. As an important additional check, we tested our predictions for the sigma terms against recent direct lattice calculations of these values at unphysical pseudoscalar masses, finding excellent agreement when the scale is set using the 'mass dependent' prescription in particular. With this scale setting scheme, we find the pion-nucleon sigma term to be $\sigma_{\pi N} = 45 \pm 6$ MeV at the physical point, in close agreement with other recent lattice determinations of this value [17, 18, 43]. This result is within uncertainties of the value $\sigma_{\pi N} = 51 \pm 7$ MeV found within the 'mass independent' scheme. We also determine the strangeness nucleon sigma term very precisely within each scale setting scheme. Using a 'mass independent' scheme, we find $\sigma_s = 59 \pm 6$ MeV. Comparison with direct calculations of σ_s suggests a slight preference for using the 'mass dependent' prescription to set the lattice spacing. That yields $\sigma_s = 21 \pm 6$ MeV at the physical point.

ACKNOWLEDGMENTS

We acknowledge helpful discussions with J. Zanotti and P. Guichon. This work was supported by the University of Adelaide and the Australian Research Council through the ARC Centre of Excellence for Particle Physics at the Terascale and grants FL0992247 (AWT) and DP110101265 (RDY).

[1] X. -D. Ji, Phys. Rev. Lett. **74**, 1071 (1995)
[2] A. Bottino *et al.*, Astropart. Phys. **13**, 215 (2000)
[3] J. R. Ellis, K. A. Olive and C. Savage, Phys. Rev. **D77**, 065026 (2008)
[4] J. Giedt, A. W. Thomas and R. D. Young, Phys. Rev. Lett. **103**, 201802 (2009)
[5] S. J. Underwood *et al.*, Phys. Rev. **D86**, 035009 (2012)
[6] R. J. Hill and M. P. Solon, Phys. Lett. **B707**, 539 (2012)
[7] J. M. Alarcon, J. Martin Camalich and J. A. Oller, Phys. Rev. **D85**, 051503 (2012)
[8] R. D. Young and A. W. Thomas, Nucl. Phys. **A844**, 266C (2010)
[9] R. D. Young, A. W. Thomas, Phys. Rev. **D81**, 014503 (2010)
[10] H. Ohki *et al.*, PoS **LAT2009**, 124 (2009)
[11] G. S. Bali *et al.* [QCDSF Collaboration], Phys. Rev. **D85**, 054502 (2012)
[12] S. Dinter *et al.*, JHEP **1208**, 037 (2012)
[13] M. Engelhardt, Phys. Rev. **D86**, 114510 (2012)
[14] H. Ohki *et al.* [JLQCD Collaboration], [arXiv:1208.4185 [hep-lat]]
[15] D. Toussaint, W. Freeman [MILC Collaboration], Phys. Rev. Lett. **103**, 122002 (2009)
[16] S. Dinter, V. Drach and K. Jansen, Int. J. Mod. Phys. Proc. Suppl. **E20**, 110 (2011)
[17] R. Horsley *et al.*, Phys. Rev. **D85**, 034506 (2012)
[18] S. Durr *et al.*, Phys. Rev. **D85**, 014509 (2012)
[19] R. Babich *et al.*, Phys. Rev. **D85**, 054510 (2012)
[20] K. Takeda *et al.* [JLQCD Collaboration], Phys. Rev. **D83**, 114506 (2011)
[21] J. Martin Camalich, L. S. Geng and M. J. Vicente Vacas, Phys. Rev. **D82**, 074504 (2010)
[22] J. Ellis, K. A. Olive, P. Sandick New J. Phys. **11**, 105015 (2009)
[23] P. E. Shanahan, A. W. Thomas, R. D. Young, Phys. Rev. Lett. **107**, 092004 (2011)
[24] S. Aoki *et al.* [PACS-CS Collaboration], Phys. Rev. **D79**, 034503 (2009)
[25] A. Walker-Loud, Nucl. Phys. A **747**, 476 (2005)
[26] D. B. Leinweber, A. W. Thomas, R. D. Young, Phys. Rev. Lett. **92**, 242002 (2004)
[27] J. M. M. Hall *et al.* Phys. Rev. **D84**, 114011 (2011)
[28] R. E. Stuckey, M. C. Birse, J. Phys. **G23**, 29-40 (1997)
[29] J. F. Donoghue, B. R. Holstein, B. Borasoy, Phys. Rev. **D59**, 036002 (1999)
[30] D. B. Leinweber, D. -H. Lu, A. W. Thomas, Phys. Rev. **D60**, 034014 (1999)
[31] A. W. Thomas, P. E. Shanahan, R. D. Young, Few Body Systems (2012), [DOI:10.1007/s00601-012-0372-8]
[32] S. R. Beane *et al.*, Phys. Rev. **D84**, 014507 (2011)
[33] Li-sheng Geng *et al.*, Phys. Rev. **D84**, 074024 (2011)
[34] A. Ali Khan *et al.* [QCDSF-UKQCD Collaboration], Nucl. Phys. **B689**, 175 (2004)
[35] J. M. M. Hall, D. B. Leinweber and R. D. Young, Phys. Rev. **D85**, 094502 (2012)
[36] P. A. M. Guichon, G. A. Miller, A. W. Thomas, Phys. Lett. **B124**, 109 (1983)
[37] W. Bietenholz *et al.* [QCDSF-UKQCD Collaboration], Phys. Rev. **D84**, 054509 (2011)
[38] R. P. Feynman, Phys. Rev. **56**, 340 (1939)
[39] K. Nakamura *et al.* [Partice Data Group], J. Phys. **G37**, 075021 (2010)
[40] P. E. Shanahan *et al.*, PoS (Lattice 2012) 165 (2012)

[41] A. K. De, A. Harindranath and J. Maiti, arXiv:0803.1281 [hep-lat].
[42] W. Freeman *et al.* [MILC Collaboration], arXiv:1204.3866 [hep-lat].
[43] G. S. Bali *et al.*, Nucl. Phys. **B866**, 1 (2013)