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# Faking $B_s \rightarrow \mu^+ \mu^-$

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## Abstract

Recent evidence from observation of the flavor-changing neutral current decay  $B_s^0 \rightarrow \mu^+ \mu^-$  by the LHCb collaboration  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.2_{-1.2}^{+1.5}) \times 10^{-9}$  is consistent with the latest standard model prediction  $\mathcal{B}_{SM}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.23 \pm 0.27) \times 10^{-9}$ . While new physics can still affect this decay amplitude, its contribution is certainly not the dominant one. We analyze branching ratios of the decays  $B_s^0 \rightarrow \mu^+ \mu^- X$ , with  $X = \gamma$  or  $\nu \bar{\nu}$ , which can mimic  $B_s^0 \rightarrow \mu^+ \mu^-$  on portions of the parameter space where  $X$  is soft. Since detection efficiency of  $\gamma$  or  $\nu \bar{\nu}$  in modern charm-physics experiments is quite low in that region, those processes could be misidentified as  $B_s^0 \rightarrow \mu^+ \mu^-$ . We perform a model-independent standard model calculation of those processes incorporating heavy quark and chiral symmetries of QCD concentrating on the soft- $X$  region. We show that misidentification of the considered processes can only boost  $B_s^0 \rightarrow \mu^+ \mu^-$  rate by less than a percent.

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## I. INTRODUCTION

The rare leptonic decay of the  $B_s^0$  into a dimuon pair,  $B_s^0 \rightarrow \mu^+\mu^-$ , is an example of a flavor-changing neutral current (FCNC) process. Studies of such decay processes not only play an important role in determining electroweak and strong interaction parameters of the standard model (SM) of particle physics, but also serve as sensitive probes of possible physics beyond the standard model [1]. While recent evidence for observation of  $B_s^0 \rightarrow \mu^+\mu^-$  from LHC-b collaboration [2], as well as an earlier result from CDF [3] preclude any spectacular new physics (NP) effect, there is still room for NP to influence this decay. It is then important to have a firm evaluation of  $\mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-)$  in the SM [4, 5] and a firm understanding that the experimentally-observed branching ratio

$$\begin{aligned}\mathcal{B}_{LHCb}(B_s^0 \rightarrow \mu^+\mu^-) &= (3.2_{-1.2}^{+1.5}) \times 10^{-9} \\ \mathcal{B}_{CDF}(B_s^0 \rightarrow \mu^+\mu^-) &= (1.8_{-0.9}^{+1.1}) \times 10^{-8}\end{aligned}\tag{1}$$

actually corresponds to the  $B_s^0 \rightarrow \mu^+\mu^-$  transition. Parenthetically, we also note that due to a sizable width difference in the  $B_s$  system, additional care should be taken when comparing experimental measurement of the branching ratio of the  $B_s$  decay to a CP-eigenstate  $\mu^+\mu^-$  to available theoretical predictions [6].

It is well known that the  $B_s^0 \rightarrow \mu^+\mu^-$  decay is helicity suppressed in the SM by  $m_\mu^2/m_{B_s}^2$  due to the left handed nature of weak interactions [7]. This effect arises from the necessary spin flip on the outgoing back-to-back lepton pair in order to conserve angular momentum since the initial state meson is spinless.

This suppression is *absent* in  $B_s^0$  decays where the muon pair is produced with one or more additional particles in the final state that can carry away a unit of angular momentum, such as  $B_s^0 \rightarrow \mu^+\mu^-\gamma$  or  $B_s^0 \rightarrow \mu^+\mu^-\nu_\mu\bar{\nu}_\mu$ . This means that, in general, those processes could have sizable total branching ratios, comparable to that of  $B_s^0 \rightarrow \mu^+\mu^-$ , despite being suppressed by other small parameters (such as  $\alpha_{EM}$  for  $B_s^0 \rightarrow \mu^+\mu^-\gamma$ ) [8]. If, in addition, the final state photon or  $\nu\bar{\nu}$  is undetected, while the invariant mass of  $\mu^+\mu^-$  pair is close to  $m_{B_s^0}$ , then the

experimentally-measured branching ratio would correspond to

$$\mathcal{B}_{exp}(B_s^0 \rightarrow \mu^+ \mu^-) = \mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) \left[ 1 + \sum_X \frac{\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- X)|_{m(\mu^+ \mu^-) \approx m_{B_s}}}{\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-)} \right], \quad (2)$$

where  $X$  is an undetected particle or a group of particles. The contribution of  $B_s^0 \rightarrow \mu^+ \mu^- X$  would depend on how well  $X$  could be detected in a particular experiment, as well as on whether  $B_s^0 \rightarrow \mu^+ \mu^- X$  has any kind of resonance enhancement that is not well modeled by background models chosen by a particular experiment in a given window of  $m(\mu^+ \mu^-)$ , as well as the size of that window. For example, for  $X = \gamma$ , most current searches use di-lepton energy cuts that would correspond to an allowable soft photon of up to 60 MeV<sup>1</sup>. For  $B \rightarrow \ell \nu_\ell$  transition and  $X = \gamma$  similar effects were discussed in [9–11], and for  $X$  being light particles – in [12]. In the following we shall concentrate on the amplitudes that are non-vanishing in the  $m_\mu \rightarrow 0$  limit.

## II. $B_s^0 \rightarrow \mu^+ \mu^- \gamma$ TRANSITION

Due to higher backgrounds in hadron collider experiments soft photons emitted in  $B_s^0 \rightarrow \ell^+ \ell^- \gamma$  could be hard to detect, so this background could be quite important. These decays were previously analyzed in [8, 13, 14], where multiple models have been considered. The analysis presented in [8] was mainly geared towards kinematical regimes where the photon is sufficiently hard to be detected; in fact, low-energy cut-offs were introduced on photon energies. Similarly, perturbative QCD-related approaches of [13] cannot adequately describe the soft photon region. In this paper, we apply a model-independent approach that incorporates both heavy quark symmetry for hadrons containing a heavy quark with mass  $m_Q \gg \Lambda_{QCD}$ , and chiral  $SU(3)_L \times SU(3)_R$  symmetries in the  $m_q \rightarrow 0$  limit [15, 16] suited for this kinematical region. We organize our calculations in terms of an expansion in  $1/m_b$  and examine the contribution of terms up to leading order in  $\mathcal{O}(1/m_b)$ .

Similarly to  $B \rightarrow \ell \nu_\ell \gamma$  [10], the decay amplitude for  $B_s \rightarrow \mu^+ \mu^- \gamma$  transition can be broken into two generic parts containing internal bremsstrahlung (IB) and structure dependent (SD) contributions. The bremsstrahlung contributions are still helicity suppressed, while the SD

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<sup>1</sup> This cut would almost exclude the possibility of missing a  $\pi^0$  or heavier neutral particles.

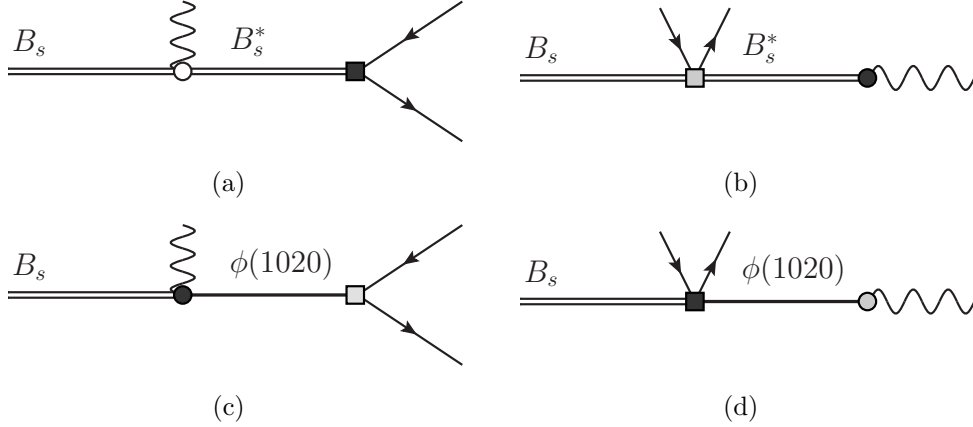


FIG. 1: Resonant contributions to  $B_s \rightarrow \mu^+ \mu^- \gamma$ . The dark grey vertices represent the  $b \rightarrow s$  effective Hamiltonians in Eqs. (12, 17). Light grey vertices are SM neutral currents while the white circle in (a) is defined in Eq. (6).

contribution contain the electromagnetic coupling  $\alpha$  but are not suppressed by the lepton mass. Phenomenologically, the origin of that can be understood as follows. When the soft photon is emitted from the  $B_s$  meson, heavy intermediate states including the  $J^P = 1^-$   $B_s^*$  vector meson state become possible. This lifts helicity suppression since the lepton pair couples directly to the spin 1 meson. In the kinematic regime where the photon is soft, we expect that significant contribution comes only from the vector  $B_s^*$  resonance for reasons analogous to the  $B^*$  pole dominance in  $B \rightarrow \pi \ell \nu$  at near zero pionic recoil energies [17]: in the large  $m_b$  limit the  $B_s^*$  and  $B_s$  become degenerate and the residual mass splitting is  $m_{B^*} - m_B \sim \mathcal{O}(1/m_b)$  [18]. Therefore, excitation of the  $B_s^*$  does not require much energy. There are two diagrams containing an intermediate  $B_s^*$  as seen in Fig. 1. In the kinematic region of interest where  $E_\gamma < 60$  MeV, Fig. 1 (b) is neglected. This can be easily seen considering it is a contribution from an off-shell intermediate  $B_s^*$  that decays to an on-shell soft photon in a vector-meson-dominance fashion via  $b \rightarrow s \gamma$  transition. Since the photon is on mass shell, the  $B_s$  propagator makes it  $(1/M_{B_0}^2)$  suppressed. In the effective field theory language this diagram corresponds to a higher-dimensional operator whose contribution is suppressed by powers of  $M_B$ . Similarly, a contribution of Fig. 1 (c) is also  $(1/M_{B_0}^2)$ , so it too will be neglected in what follows.

The calculation of soft photon effects should carefully deal with soft divergences. Those are canceled between one-loop radiative corrections to  $B_s \rightarrow \mu^+ \mu^-$  and  $B_s \rightarrow \mu^+ \mu^- \gamma$ .

We employ Heavy Meson Chiral Perturbation Theory (HM $\chi$ PT) to calculate Fig. 1 (a).

The heavy meson superfield  $H_a$  contains both the  $B_s^0$  and  $B_s^{*0}$  bosons,

$$H_a = \frac{1+\not{v}}{2}(B_{a\mu}^* \gamma^\mu - B_a \gamma_5), \quad \bar{H}_a = \gamma^0 (H_a)^\dagger \gamma^0, \quad (3)$$

where the indices  $a$  and  $b$  reflect the light quark flavor indices. The light mesons are introduced through the matrix  $Q^\xi = \frac{1}{2}(\xi^\dagger Q \xi + \xi Q \xi^\dagger)$  where the field  $\xi = \exp(i\Pi/f)$  is defined in terms of a  $3 \times 3$  unitary matrix containing the octet of pseudo-Goldstone bosons

$$\Pi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \quad (4)$$

To evaluate diagram Fig. 1 (a) we need an amplitude for a  $B \rightarrow B^* \gamma$  transition as

$$\mathcal{M}_{[B_s \rightarrow B_s^* \gamma_s \rightarrow \mu^+ \mu^- \gamma_s]} = \mathcal{M}_{B_s^* \rightarrow \mu^+ \mu^-}^\mu \times \frac{g_{\mu\alpha}}{M_{B_s^*}^2} \times \mathcal{M}_{B_s \rightarrow B_s^* \gamma}^\alpha, \quad (5)$$

The amplitude for  $B \rightarrow B^* \gamma$  is conventionally parameterized as

$$\mathcal{M}_{B_s \rightarrow B_s^* \gamma} = -ie\mu\eta_\alpha^* v_\beta k_\mu \epsilon_\nu^* \epsilon^{\mu\nu\alpha\beta}, \quad (6)$$

where  $k$  is the 4-momentum of the photon,  $v$  the velocity of the decaying heavy meson,  $\eta$  is the vector meson polarization, and  $\epsilon$  is the photon polarization. The strength of the transition is described by the magnetic moment,  $\mu$ , which receives contributions from the photon coupling to both heavy and the light quark components of the electromagnetic current [19],

$$\mu = \mu_b + \mu_\ell. \quad (7)$$

The bottom quark contribution is fixed by heavy quark symmetry to be  $\mu_b = Q_b/m_b = -1/(3m_b)$ , while the light quark contribution can be computed, to one loop, in HM $\chi$ PT. The relevant effective Lagrangian is [19, 20]

$$\mathcal{L}_\beta = \frac{\beta e}{4} \text{Tr}(\bar{H}_a H_b \sigma^{\mu\nu} F_{\mu\nu} Q_{ba}^\xi) + \frac{ig}{2} \text{Tr}(\bar{H}_a H_b \gamma_\mu \gamma_5 (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)_{ba}), \quad (8)$$

where  $\text{Tr}$  is a trace over the Dirac indices, and  $\beta$  is a coupling constant parameterizing a

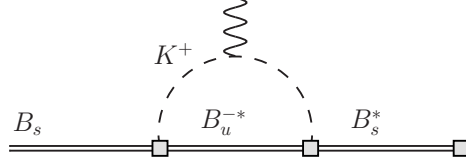


FIG. 2: One loop corrections to  $\mu$ . The double lines denote the heavy mesons  $B$  and  $B^*$  while the single line denotes the goldstone bosons

local contribution to the light quark magnetic moment. We include the most important one-loop correction, which is shown in Fig. 2.

The effective magnetic moment for the  $B_s \rightarrow B_s^* \gamma$  transition is then

$$\mu_{B_s \gamma} = -\frac{1}{3m_b} - \frac{1}{3}\beta + g^2 \frac{m_K}{4\pi f_K^2}, \quad (9)$$

where  $g$  is the  $\chi$ PT coupling constant, and  $m_K, f_K$  are the mass and decay constant of the Kaon respectively. The constants  $\beta$  and  $g$  can be extracted from a combination of the experimental  $D^{*+}$  branching ratios,  $\mathcal{B}(D^{*+} \rightarrow D^+ \gamma) = 0.016 \pm 0.004$  and  $\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+) = 0.677 \pm 0.005$ , and the total width, where the newest preliminary result from BaBar collaboration is reported to be  $\Gamma_{D^{*+}} = 83.5 \pm 1.7 \pm 1.2$  KeV [21]. The decay widths for these processes using the method above are given by

$$\Gamma(D^{*+} \rightarrow D^+ \gamma) = \frac{\alpha_{EM}}{3} \left( \frac{2}{3m_c} - \frac{1}{3}\beta + g^2 \frac{m_\pi}{4\pi f_\pi^2} \right)^2 |\vec{k}|^3, \quad (10)$$

$$\Gamma(D^{*+} \rightarrow D^0 \pi^+) = \frac{g^2}{6\pi f_\pi^2} |\vec{p}_\pi|^3. \quad (11)$$

This yields the approximate values of the coupling constants,  $g \approx 0.552$  and  $\beta \approx 7.29 \text{ GeV}^{-1}$ . With Eq.9 this gives us  $|\mu_{eff}| \approx 1.13 \text{ GeV}^{-1}$ .

To complete evaluation of Fig. 1 (a) in Eq. (5), we evaluate the  $B_s^* \rightarrow \mu^+ \mu^-$  transition. The effective Hamiltonian describing the weak  $b \rightarrow s \ell^+ \ell^-$  transition is

$$\begin{aligned} \mathcal{H}_{b \rightarrow s \bar{\ell} \ell} = & \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{8\pi^2} \left[ \bar{s} \gamma^\mu (1 - \gamma_5) b \cdot \bar{\ell} \left[ C_{9V}^{eff}(\mu, q^2) \gamma_\mu + C_{10A}(\mu^2) \gamma_\mu \gamma_5 \right] \ell \right. \\ & \left. - 2im_b \frac{C_{7\gamma}(\mu^2)}{q^2} q_\nu \cdot \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b \cdot \bar{\ell} \gamma_\mu \ell \right], \end{aligned} \quad (12)$$

where  $q_\nu = (p_{\ell^+} + p_{\ell^-})_\nu$  is the momentum of the lepton pair and  $C_i$  are scale-dependent Wilson coefficients. The matrix element for  $B_s^* \rightarrow \mu\bar{\mu}$  is then

$$\begin{aligned} \mathcal{M}_{B_s^* \rightarrow \mu^+ \mu^-} = & i \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{8\pi^2} f_{B_s} M_{B_s} \left[ \eta_\mu^* \bar{u}(p_{\mu^+}) [C_9 \gamma^\mu + C_{10} \gamma^\mu \gamma_5] v(p_{\mu^-}) \right. \\ & \left. - 2m_b \frac{C_7}{q^2} (\bar{u}(p_{\mu^+}) \gamma_\mu v(p_{\mu^-})) q_\nu (i\epsilon^{\mu\nu\alpha\beta} v_\alpha \eta_\beta + v^\mu \eta^\nu - v^\nu \eta^\mu) \right], \end{aligned} \quad (13)$$

where  $\eta^\mu$  and  $v^\mu$  are the polarization and 4-velocity of the vector meson respectively. We defined  $\langle 0 | \bar{s}_L \gamma^\mu b_L | B_s^* \rangle = \eta^\mu f_{B_s^*}/2$ , and  $\langle 0 | \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b | B_s^* \rangle = M_B f_{B_s} [i\epsilon^{\mu\nu\alpha\beta} v_\alpha \eta_\beta + v^\mu \eta^\nu - v^\nu \eta^\mu]$ , with  $f_{B_s^*} = M_{B_s} f_{B_s}$  [22]. This gives for the amplitude of Fig. 1 (a)

$$\begin{aligned} \mathcal{M}_{[B_s \rightarrow B_s^* \gamma_s \rightarrow \mu^+ \mu^- \gamma_s]} = & \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^3}{8\pi^2} \mu_{eff} \frac{f_{B_s}}{q^2 - M_{B_s^*}^2} (\epsilon^{\mu\nu\alpha\beta} \epsilon_\mu^* k_\alpha q_\beta) \\ & \times \left[ (2C_7 m_b - C_9 M_{B_s^*}) [\bar{u}_{p_1} \gamma_\nu v_{p_2}] - C_{10} M_{B_s^*} [\bar{u}_{p_1} \gamma_\nu \gamma_5 v_{p_2}] \right]. \end{aligned} \quad (14)$$

The other contribution that is leading the  $M_{B_s} \rightarrow \infty$  limit is given in Fig. 1 (c)

$$\mathcal{M}_{[B_s^0 \rightarrow \mu\bar{\mu}\phi \rightarrow \mu\bar{\mu}\gamma_s]} = \mathcal{M}_{B_s^0 \rightarrow \mu\bar{\mu}\phi} \times \frac{g_{\mu\nu}}{m_\phi^2} \times \mathcal{M}_{\phi \rightarrow \gamma_s}, \quad (15)$$

Employing vector-meson dominance, and using the definition of the vector meson decay constant  $\langle 0 | \bar{s} \gamma^\mu s | \phi \rangle = f_\phi m_\phi \eta_\phi^\mu$ , where  $\eta_\phi^\mu$  is the polarization of the  $\phi$  meson, and  $\langle \gamma | \bar{s} (-ieQ_s / A) s | \phi \rangle = (-ieQ_s) \epsilon_\mu^* \langle 0 | \bar{s} \gamma^\mu s | \phi \rangle$ ,

$$\mathcal{M}_{\phi \rightarrow \gamma_s}^\mu = \frac{1}{3} e f_\phi m_\phi \epsilon_\mu^*. \quad (16)$$

Again, we calculate  $\mathcal{M}_{B_s^0 \rightarrow \mu\bar{\mu}\phi}$  using (HM $\chi$ PT). For the short distance contributions we use the effective Hamiltonian describing  $b \rightarrow \bar{s} \ell \ell$  transitions in Eq. 12, as well as the effective Hamiltonian for  $b \rightarrow s \gamma$ ,

$$\mathcal{H}_{b \rightarrow s \gamma} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e}{8\pi^2} m_b C_{7\gamma}(\mu^2) \cdot \bar{s} \sigma^{\mu\nu} (1 + \gamma_5) b \cdot F_{\mu\nu}. \quad (17)$$

In order to bosonize the quark currents found in Eqs. (12) and (17) we introduce the light



vector octet to the  $\text{HM}\chi\text{pT}$  [22],

$$\rho_\mu \equiv i \frac{g_V}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \quad (18)$$

The bosonized currents  $\bar{s}\gamma^\mu(1 - \gamma_5)b$  and  $\bar{s}\sigma^{\mu\nu}(1 + \gamma_5)b$  are, respectively,

$$\begin{aligned} L_{1a}^\mu &= \alpha_1 \langle \gamma_5 H_b (\rho^\mu)_{bc} \xi_{ca}^\dagger \rangle, \\ L_{1a}^{\mu\nu} &= i\alpha_1 \left\{ g^{\mu\alpha} g^{\nu\beta} - \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \right\} \langle \gamma_5 H_b [\gamma_\alpha (\rho_\beta)_{bc} - \gamma_\beta (\rho_\alpha)_{bc}] \xi_{ca}^\dagger \rangle. \end{aligned} \quad (19)$$

A numerical value of  $\alpha_1 = -0.07 \text{ GeV}^{1/2}$  [22] will be used for our calculations. Keeping only the gauge invariant portion, the amplitude for the decay with an intermediate  $\phi(1020)$  is

$$\begin{aligned} \mathcal{M}_{[B_s^0 \rightarrow \mu \bar{\mu} \phi \rightarrow \mu \bar{\mu} \gamma_s]} &= G_F V_{tb} V_{ts}^* \frac{e^3 f_\phi g_\phi \alpha_1 C_7 m_b}{24\pi^2 \sqrt{M_{B_s}} m_\phi (p_1 \cdot p_2)} \epsilon_\mu^* \left( (k \cdot (p_1 + p_2)) [\bar{u}_{p_1} \gamma^\mu v_{p_2}] \right. \\ &\quad \left. - (p_1 + p_2)^\mu [\bar{u}_{p_1} \not{k} v_{p_2}] + i \epsilon^{\mu\nu\alpha\beta} k_\alpha (p_1 + p_2)_\beta [\bar{u}_{p_1} \gamma_\nu v_{p_2}] \right). \end{aligned} \quad (20)$$

We checked that other contributions to the decay are smaller than the ones considered above. We considered the bremsstrahlung diagrams where a soft photon is emitted from one of the outgoing leptons. These diagrams will result in an infrared divergence in the soft region, which has been shown to cancel with the 1-loop QED vertex corrections [23]. The vertex corrections, as well as the bremsstrahlung contributions, will remain suppressed by a power of the lepton mass. Therefore the remaining non-divergent contributions from both the bremsstrahlung and vertex corrections to final states with either an electron or a muon would not be significant.

The only contribution to the amplitude from the effective Hamiltonian describing the weak transition in Eq.(12) ends up being the  $\mathcal{O}_{10}$  operator. This comes from obtaining the matrix elements for the pseudoscalar meson,

$$\langle 0 | (\bar{s}\gamma^\mu(1 - \gamma_5)b) | B_s \rangle = -i f_B P_B^\mu, \quad (21)$$

$$\langle 0 | (\bar{s}\sigma^{\mu\nu}(1 + \gamma_5)b) | B_s \rangle = 0, \quad (22)$$

where  $f_B$  is the decay constant of the B meson. With these definitions and using the conservation of the vector current we can arrive at an expression for the amplitude

$$\mathcal{M}_{Brem} = ie \frac{\alpha_{EM} G_F}{2\sqrt{2}} V_{tb} V_{ts}^* f_B C_{10} m_\mu \left[ \bar{\mu} \left( \frac{\not{\epsilon} \not{P}_B}{p_{\mu^-} \cdot k} - \frac{\not{P}_B \not{\epsilon}}{p_{\mu^+} \cdot k} \right) \gamma_5 \mu \right], \quad (23)$$

where  $\epsilon^\mu$  and  $k$  are the polarization and momentum of the photon respectively. Just as we would expect from the helicity structure involved, the amplitude for the bremsstrahlung contribution is proportional to the lepton mass. So in the limit  $m_\ell \rightarrow 0$ , this contribution should be small compared to the non helicity-suppressed contributions.

Each amplitude from Eqs. (14, 20, 23) individually satisfy the Ward identity and are thus independently gauge invariant. Putting everything together, the distribution of the decay width as a function of the kinematic variable  $s = (P_{B_s} - k)^2/M_{B_s}^2 = q^2/M_{B_s}^2$ , in the limit  $m_\ell \rightarrow 0$ ,

$$\frac{d\Gamma}{ds} = \frac{d\Gamma}{ds} \Big|_{B_s^*} + \frac{d\Gamma}{ds} \Big|_{\phi B_s^*} + \frac{d\Gamma}{ds} \Big|_{\phi}, \quad (24)$$

where the decay distributions are given for the two different resonance amplitudes and their interference.

$$\begin{aligned} \frac{d\Gamma}{ds} \Big|_{B_s^*} &= X_{CKM} M_{B_s}^3 f_{B_s}^2 \mu_{eff}^2 \left[ (|C_9|^2 + |C_{10}|^2) x_{B_s^*} + 4 C_7^2 x_b - 4 C_7 C_9 x_b x_{B_s^*} \right] \frac{s(1-s)^3}{(s - x_{B_s^*}^2)^2}, \\ \frac{d\Gamma}{ds} \Big|_{\phi} &= X_{CKM} \left[ \frac{16 C_7^2 f_\phi^2 g_\phi^2 m_b^2 \alpha_1^2}{9 m_\phi^2} \right] \frac{(1-s)^3}{s}, \\ \frac{d\Gamma}{ds} \Big|_{\phi B_s^*} &= X_{CKM} \left[ \frac{4\sqrt{2} f_{B_s} f_\phi g_\phi M_{B_s}^{3/2} m_b \alpha_1 \mu_{eff}}{3 m_\phi} (C_7 C_9 x_{B_s^*} - 2 C_7^2 x_b) \right] \frac{(1-s)^3}{s - x_{B_s^*}^2}, \end{aligned} \quad (25)$$

where we have defined  $X_{CKM} = (G_F^2 |V_{tb} V_{ts}^*|^2 M_{B_s}^2 \alpha_{EM}^3)/(768\pi^4)$ ,  $x_b \equiv m_b/M_{B_s}$ , and  $x_{B_s^*} \equiv M_{B_s^*}/M_{B_s}$ . We use the Wilson coefficients  $C_i(\lambda)$  choosing the scale at  $\lambda \simeq m_b \simeq 5 GeV$ , with  $C_7 = 0.312$ ,  $C_9 = -4.21$  and  $C_{10} = 4.64$  [8][24]. The CKM matrix elements are  $|V_{tb} V_{ts}^*| = (4.7 \pm 0.8) \times 10^{-2}$  [25]. With the most recent lattice calculation of  $f_{B_s}$  is  $\approx 228$  MeV [26]. Note that, when integrated over the endpoint window the last two terms in Eq. (24) are much smaller than the first one. The interference contribution is destructive

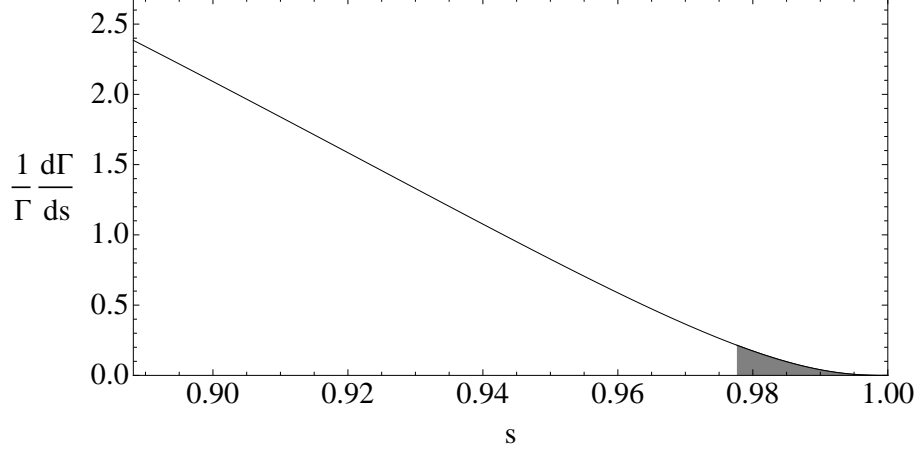


FIG. 3: Normalized differential spectrum in  $s$ . The grey shaded region corresponds to the contribution from a soft photon energy cut at  $E_\gamma \sim 60$  MeV.

and is

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^- \gamma_{E < 60})_{\phi B_s^*} = -5.0 \times 10^{-17}, \quad (26)$$

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^- \gamma_{E < 300})_{\phi B_s^*} = -1.1 \times 10^{-14}, \quad (27)$$

which are both much smaller than the  $B_s^*$  contribution alone.

The normalized differential spectrum in  $s$  is shown in Fig.(3). The photon energy is related to the invariant mass as  $E_\gamma = (1 - s)M_B/2$ , so we can integrate the differential spectrum over the required corresponding kinematic region in photon energy to obtain the decay width.

Integrating Eq.(24) over the kinematic region corresponding to a soft photon cut of  $E_\gamma \sim 60, 300$  MeV we get the respective branching ratios

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^- \gamma_{E < 60}) = 1.6 \times 10^{-12}, \quad (28)$$

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^- \gamma_{E < 300}) = 1.1 \times 10^{-10}, \quad (29)$$

which are quite too low to affect experimental determination of the branching ratio  $B_s \rightarrow \mu^+ \mu^-$ , agreeing with the estimates of Ref. [4] where  $\mathcal{B}_{\mathcal{SM}}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.23 \pm 0.27) \times 10^{-9}$ .

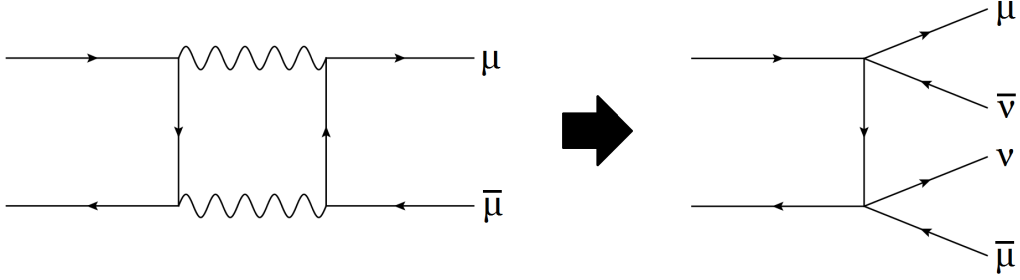


FIG. 4:  $B_s \rightarrow \mu^+ \mu^- \bar{\nu} \nu$

### III. $B_s^0 \rightarrow \mu^+ \mu^- \nu_\mu \bar{\nu}_\mu$ TRANSITION

Because of the Glashow-Illiopolous-Maiani (GIM) mechanism, the SM loop diagram for the helicity-suppressed  $B_s^0 \rightarrow \mu^+ \mu^-$  decay is dominated by the intermediate top quark despite being suppressed by the CKM factors  $|V_{tb}V_{ts}^*|^2$ . A transition similar to the ones described above, which on a portion of the available phase space looks like  $B_s^0 \rightarrow \mu^+ \mu^-$  is the tree-level decay  $B_s^0 \rightarrow \mu^+ \mu^- \nu \bar{\nu}$ . The dominant tree-level contribution for this process is depicted in Fig. 4. This decay can have a contribution to the background, which appears only below  $q^2 = M_{B_s}^2$  and, if numerically significant, can affect the extraction of  $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ . This process is neither loop-dominated nor is it helicity suppressed. It nevertheless has a kinematic phase space suppression due to the four-particle final state. For the  $B_s$  meson decay, an intermediate charm quark will give the largest contribution since the intermediate top quark diagram will be suppressed by the mass of the top quark. Also, the up quark contribution is suppressed by  $V_{ub}V_{us}^* \approx \lambda^4$  whereas the charm contribution is only suppressed by  $V_{cb}V_{cs}^* \approx \lambda^2$ , where  $\lambda \approx 0.22$ .

The transition amplitude for this process is simple,

$$\mathcal{M}_{B_s \rightarrow \mu^+ \mu^- \nu \bar{\nu}} = \frac{G_F^2}{2} V_{cb} V_{cs}^* \langle 0 | \bar{s} \gamma_\beta (1 - \gamma_5) \frac{i(\not{p}_c + m_c)}{p_c^2 - m_c^2} \gamma_\alpha (1 - \gamma_5) b | B_s \rangle L_1^\alpha L_2^\beta, \quad (30)$$

where  $L^\alpha = \bar{\mu} \gamma^\alpha (1 - \gamma_5) \nu_\mu$ . In the rest frame of the decaying meson we can reduce the phase space integral's dependence to five independent Lorentz invariants. In the same fashion as

in [27] we define these invariants as

$$\begin{aligned} S_{12} &= (p_{\mu^-} + p_{\mu^+})^2, \quad S_{13} = (p_{\mu^-} + p_{\bar{\nu}})^2, \quad S_{34} = (p_{\bar{\nu}} + p_{\nu})^2, \\ S_{123} &= (p_{\mu^-} + p_{\mu^+} + p_{\bar{\nu}})^2, \quad S_{134} = (p_{\mu^-} + p_{\bar{\nu}} + p_{\nu})^2. \end{aligned} \quad (31)$$

Our width then becomes

$$d\Gamma = \frac{(2\pi)^4}{2M} \int \left( \frac{\pi^2}{2M^2} \right) \frac{|\mathcal{M}_{B_s \rightarrow \mu^+ \mu^- \nu \bar{\nu}}|^2}{[-\Delta_4(p_{\mu^-}, p_{\mu^+}, p_{\bar{\nu}}, p_{\nu})]^{1/2}} dS_{12} dS_{123} dS_{13} dS_{134}, \quad (32)$$

where  $\Delta_4$  is the symmetric Gram determinant

$$\Delta_4(q, r, s, t) = \begin{vmatrix} q^2 & q \cdot r & q \cdot s & q \cdot t \\ r \cdot q & r^2 & r \cdot s & r \cdot t \\ s \cdot q & s \cdot r & s^2 & s \cdot t \\ t \cdot q & t \cdot r & t \cdot s & t^2 \end{vmatrix}. \quad (33)$$

In order to avoid the divergence of  $1/(-\Delta_4)^{1/2}$  on the boundary, suitable variable changes can be made thereby making the singularity integrable. We define

$$\begin{aligned} S_{134} &= \frac{1}{2a} \left[ -b + \sin(\tilde{S}_{134})(b^2 - 4ac)^{1/2} \right], \\ S_{13} &= 4(-a)^{1/2} \tilde{S}_{13} + m_\ell^2, \end{aligned} \quad (34)$$

where  $a, b$  and  $c$  are the parameters solved by

$$-\Delta_4(p_{\mu^-}, p_{\mu^+}, p_{\bar{\nu}}, p_{\nu}) = aS_{134}^2 + bS_{134} + c. \quad (35)$$

The limits of integration are calculated in [27], resulting in our partial width

$$\frac{d\Gamma}{dS_{12}} = \frac{2}{(4\pi)^6 M^3} \int_{S_{12}}^{M^2} dS_{123} \int_0^\xi dS_{34} \int_{m_\ell^2/S_{12}}^1 d\tilde{S}_{13} \int_{-\pi/2}^{\pi/2} d\tilde{S}_{134} |\mathcal{M}_{B_s \rightarrow \mu^+ \mu^- \nu \bar{\nu}}|^2, \quad (36)$$

where  $\xi = (M^2 - S_{123})(S_{123} - S_{12})/S_{123}$ . We define the cut on missing energy as  $S_{12}^{cut}(E_{cut}) = M^2 - 2M(E_{cut})$  which gives us a lower limit on  $S_{12}$  for the final integral in order to obtain the decay width. The branching ratios for this contribution can then be calculated using numerical

phase-space integration for various cuts including the one that corresponds to the invariant mass range seen at the LHCb [2].

$$\begin{aligned} BR[B_s \rightarrow \mu^+ \mu^- \nu \bar{\nu}]_{E_{cut}=60 MeV} &= 1.6 \times 10^{-25} \\ BR[B_s \rightarrow \mu^+ \mu^- \nu \bar{\nu}]_{E_{cut}=300 MeV} &= 1.4 \times 10^{-18}. \end{aligned} \quad (37)$$

As we can see, the due to enormous phase space suppression (we are only interested in a small sliver of the available four-particle final state), the possible contribution from this decay is unimportant for experimental analyses.

#### IV. CONCLUSIONS

Recent evidence of observation of the flavor-changing neutral current decay  $B_s^0 \rightarrow \mu^+ \mu^-$  by the LHCb collaboration  $\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^-) = (3.2_{-1.2}^{+1.5}) \times 10^{-9}$  is consistent with the latest standard model predictions. We analyzed branching ratios of the decays  $B_s^0 \rightarrow \mu^+ \mu^- X$ , with  $X = \gamma$  or  $\nu \bar{\nu}$ , which can mimic  $B_s^0 \rightarrow \mu^+ \mu^-$  on portions of the parameter space where  $X$  is soft. We performed a model-independent calculation of those processes incorporating heavy quark and chiral symmetries of QCD. Our calculations concentrated on the contributions that are not helicity suppressed by powers of  $m_\mu$  and leads to a correction to the SM prediction of approximately 3% at a photon energy cut of 300 MeV and less than 1% at a cut of 60 MeV from soft photon contributions to the decay  $B_s \rightarrow \mu^+ \mu^-$ . The possible contamination from  $B_s \rightarrow \mu^+ \mu^- \nu \bar{\nu}$  is even smaller, at the sub percent level.

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