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Abstract

Based on experimental data by Bell Collaboration, we present a phenomenological analysis of the angular distributions in $\tau^- \to K_S \pi^- \nu_\tau$ decay. Our study shows that the angular analysis could lead to some interesting observables, and the future experimental investigation of these observables might be very helpful to reveal the nature of the scalar components of the decay. New physics contributions from two-Higgs-doublet model to this decay have also been examined.

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1 Introduction

The τ lepton is the only known lepton massive enough to decay into hadrons and its hadronic decays provide a very useful laboratory to test the low energy dynamics of the standard model (SM) [1, 2]. With the increased experimental sensitivities achieved already or in the future, some interesting limits on possible new physics contributions to the τ decay amplitudes could be expected; for instance, in the decays $\tau^\pm \to K_S\pi^\pm\nu_\tau$, upper limits on CP violation parameter from multi-Higgs-doublet models have been obtained by Belle Collaboration [3].

The decay $\tau \to K\pi\nu_\tau$, which has the largest branching ratio of all Cabibbo-suppressed decays, could be a powerful probe of the strange sector of the weak charged current [4, 5, 6]. Early investigations have established that the dominant contribution to the decay spectrum is from the $K^*(892)$ meson [7], while the scalar or tensor contributions, which are expected theoretically [6, 8], are not excluded experimentally [9, 10]. Recently, a precise measurement of $\tau^- \to K_S\pi^-\nu_\tau$ decay has been done by Belle Collaboration [11] with high-statistics data, which showed that the $K^*(892)$ alone is not sufficient to describe the $K_S\pi$ invariant mass spectrum and other states such as scalar ($K_0^*(800)$, $K_0^*(1430)$, ...) or more vector resonances ($K^*(1410)$, $K^*(1680)$, ...) have to be included. Thus three models $K_0^*(800) + K^*(892) + K^*(1410)$, $K_0^*(800)+K^*(892)+K_0^*(1430)$, and $K_0^*(800)+K^*(892)+K^*(1680)$ were introduced to fit the data, and the best description of the decay spectrum is achieved in the first two models [11]. One can expect that these two models will lead to the different angular distribution behavior since they have different scalar and vector components. This angular analysis has not been done yet experimentally.

Motivated by the above new experimental data, in this paper we present a phenomenological analysis of the angular distributions in the $\tau^- \to K_S\pi^-\nu_\tau$ decay. We hope that this angular analysis would reveal the difference from the above models, which could further increase our understanding of the scalar contributions in this decay.

The paper is organized as follows. In section 2, we will discuss the decay distributions of $\tau^- \to K_S\pi^-\nu_\tau$, and some interesting observables will be introduced. In Section 3, a phenomenological analysis of the angular distribution is carried out, and new physics contributions from two-Higgs-doublet model to this decay will be examined. Finally, we summarize our results in Section 4.

2 Decay distributions

In the SM, the general invariant amplitude of $\tau^- \to K_S\pi^-\nu_\tau$ can be decomposed as a product of a leptonic current and a hadronic current [6]

$$\mathcal{M} = \frac{G_F \sin\theta_C}{\sqrt{2}} M_\mu J^\mu. \tag{1}$$

Here G_F is the Fermi coupling constant and θ_C is the Cabibbo angle. The leptonic current is given by

$$M_\mu = \bar{u}(p_{\nu_\tau})\gamma_\mu(1-\gamma_5)u(p_\tau), \tag{2}$$

and the hadronic current can be parameterized by two form factors as

$$J^\mu = \langle K_S \pi^- | \bar{s}\gamma^\mu u | 0 \rangle = F_V(s) \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) q_\nu + F_S(s) Q^\mu \tag{3}$$

with

$$Q^\mu = p_K^\mu + p_\pi^\mu, \qquad q_\mu = p_K^\mu - p_\pi^\mu, \qquad s = Q^2,$$

where F_V is the vector form factor, corresponding to the $J^P = 1^-$ component of the strange weak charged currents, and F_S is the scalar one, corresponding to the $J^P = 0^+$ component. In the limit of SU(3) symmetry, $m_K^2 = m_\pi^2$, the vector current is conserved and F_S is zero. Now the differentiate decay rate, in terms of s, the $K_S\pi$ invariant mass squared, and θ, the angle between the three-momentum of K_S and the three momentum of τ^- in the $K_S\pi^-$ rest frame, can be written as

$$\begin{aligned} \frac{d^2\Gamma}{ds\; d\cos\theta} \quad = \quad & \frac{G_F^2 \sin^2\theta_C}{2^6\pi^3\sqrt{s}} \frac{(m_\tau^2 - s)^2}{m_\tau^3} P(s) \left[\left(\frac{m_\tau^2}{s} \cos^2\theta + \sin^2\theta \right) P^2(s) |F_V|^2 \right. \\ & \left. + \frac{m_\tau^2}{4} |F_S|^2 - \frac{m_\tau^2}{\sqrt{s}} P(s) \mathrm{Re}(F_V F_S^*) \cos\theta \right] \end{aligned} \tag{4}$$

with

$$P(s) = \frac{1}{2\sqrt{s}} \sqrt{(s + m_K^2 - m_\pi^2)^2 - 4 s m_K^2},$$

and the phase space is given by

$$(m_\pi + m_K)^2 \leq s \leq m_\tau^2, \quad -1 \leq \cos\theta \leq 1. \tag{5}$$

After integrating over $\cos\theta$ in eq. (4), one can get

$$\frac{d\Gamma}{ds} = \frac{G_F^2 \sin^2\theta_C}{2^6\pi^3\sqrt{s}} \frac{(m_\tau^2 - s)^2}{m_\tau^3} P(s) \left[\left(\frac{2m_\tau^2}{3s} + \frac{4}{3} \right) P^2(s) |F_V|^2 + \frac{m_\tau^2}{2} |F_S|^2 \right]. \tag{6}$$

It is generally believed that this decay spectrum is dominated by the vector contributions ($K^*(892)$ resonances) [7]. Recent experimental results from Bell Collaboration [11] have shown that the scalar contribution is necessary to fit the data, although it provides only a small contribution to the decay rate. It is found in [11] that two different combinations $K_0^*(800) + K^*(892) + K^*(1410)$ and $K_0^*(800) + K^*(892) + K_0^*(1430)$ can both describe the spectrum very well. In order to further understand the scalar form factor, one may carry out the angular analysis. From eq. (4) and only focusing on the angular part, the distribution can be written as

$$\frac{d\Gamma}{d\cos\theta} = I_0 + I_1 \cos\theta + I_2 \cos^2\theta, \tag{7}$$

with

$$I_0 = \int_{s_{\min}}^{s_{\max}} ds \frac{G_F^2 \sin^2\theta_C}{2^6\pi^3\sqrt{s}} \frac{(m_\tau^2 - s)^2}{m_\tau^3} P(s) \left(\frac{m_\tau^2}{4} |F_S|^2 + P^2(s) |F_V|^2 \right), \tag{8}$$

$$I_1 = \int_{s_{\min}}^{s_{\max}} ds \frac{G_F^2 \sin^2\theta_C}{2^6\pi^3\sqrt{s}} \frac{(m_\tau^2 - s)^2}{m_\tau^3} P^2(s) \frac{-m_\tau^2}{\sqrt{s}} \mathrm{Re}(F_V F_S^*), \tag{9}$$

$$I_2 = \int_{s_{\min}}^{s_{\max}} ds \frac{G_F^2 \sin^2\theta_C}{2^6\pi^3\sqrt{s}} \frac{(m_\tau^2 - s)^2}{m_\tau^3} P^3(s) \frac{m_\tau^2 - s}{s} |F_V|^2. \tag{10}$$

Here $(s_{\min}, s_{\max})$ denotes the range of the integration over s, which could be the full phase space shown in eq. (5) or some kinematical cuts on s. Using above equations, one can further get

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \mathcal{G} + \mathcal{A}\cos\theta + \frac{3}{2}(1-2\mathcal{G})\cos^2\theta, \tag{11}$$

with

$$\Gamma = 2(I_0 + \frac{1}{3}I_2) \tag{12}$$

is the decay rate. The constant term in (11) can be expressed as

$$\mathcal{G} = \frac{I_0}{\Gamma}, \tag{13}$$

and the linear term in $\cos\theta$

$$\mathcal{A} = \frac{I_1}{\Gamma} \tag{14}$$

will give a vanishing contribution to the decay rate after integrating $\cos\theta$ in the full phase space. However, this term can induce an interesting observable, called as the forward-backward asymmetry. Eq. (14) gives the integrated and normalized asymmetry. One can also define the differential asymmetry as [12, 13]

$$A_{\rm FB}(s) = \frac{\displaystyle\int_0^1 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta - \int_{-1}^0 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta}{\displaystyle\int_0^1 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta + \int_{-1}^0 \left(\frac{d^2\Gamma}{ds\ d\cos\theta}\right) d\cos\theta}. \tag{15}$$

Thus together with eq. (4), we have

$$A_{\rm FB}(s) = \frac{-\dfrac{P(s)}{\sqrt{s}}{\rm Re}(F_V F_S^*)}{\dfrac{2}{3s}\left(1+\dfrac{2s}{m_\tau^2}\right)P^2(s)|F_V|^2 + \dfrac{1}{2}|F_S|^2}. \tag{16}$$

Note that the forward-backward asymmetries are generated from the interference between the scalar part and vector part amplitudes of the decay. In the case that the scalar contribution is not large, study of these asymmetries may be very important for us to extract the information on the scalar form factor.

3 Phenomenological analysis

In order to evaluate the quantities $\mathcal{G}$, $\mathcal{A}$, and $A_{\rm FB}$ defined in the previous section, we need the information on the hadronic form factors F_V and F_S. Theoretically, due to the non-perturbative Quantum Chromodynamics (QCD) effects at low energies, there is no model independent way at the present. Therefore, various methods have been employed to study these form factors of $\tau \to K\pi\nu_\tau$ decays, such as meson dominance models [6, 14] and chiral lagrangian including the resonances [15, 16, 17, 13]. Phenomenologically, as shown in [11, 6], these form factors could be parameterized by resonances contributions, which read

$$F_V = \frac{1}{\sqrt{2}(1+\beta+\chi)}\left[\mathrm{BW}_{K^*(892)}(s) + \beta\mathrm{BW}_{K^*(1410)}(s) + \chi\mathrm{BW}_{K^*(1680)}(s)\right] \tag{17}$$

and

$$F_S = \frac{1}{\sqrt{2}}(m_K^2 - m_\pi^2)\left[\frac{\kappa}{M^2_{K_0^*(800)}}\mathrm{BW}_{K_0^*(800)}(s) + \frac{\gamma}{M^2_{K_0^*(1430)}}\mathrm{BW}_{K_0^*(1430)}(s)\right], \tag{18}$$

where

$$\mathrm{BW}_R(s) = \frac{M_R^2}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)}$$

and

$$\Gamma_R(s) = \Gamma_{0R}\frac{M_R^2}{s}\left(\frac{P(s)}{P(M_R^2)}\right)^{2\ell+1}$$

is the s-dependent total width of the resonance. $\ell = 1(0)$ for $K\pi$ in the $P(S)$-wave state and Γ_{0R} is the width of the resonance at its peak. β, χ, κ, and γ in eqs.(17) and (18) are parameters for the fractions of the resonances, which would be determined by fitting the data of the hadronic invariant mass distribution.

Actually these parameters have been determined in Ref. [11] for two models with different combinations of the resonances, both of which can provide a good description of the decay spectrum. For the model I: $K_0^*(800) + K^*(892) + K^*(1410)$,

$$|\beta| = 0.075 \pm 0.006, \quad \arg(\beta) = 1.44 \pm 0.15, \quad \kappa = 1.57 \pm 0.23. \tag{19}$$

For the model II: $K_0^*(800) + K^*(892) + K_0^*(1430)$, the parameters could have two solutions from the data fit, which read (hereafter we call them as Model II-1 and Model II-2, respectively)

$$\texttt{Model II-1}: \quad \kappa = 1.27 \pm 0.22, \quad |\gamma| = 0.954 \pm 0.081, \quad \arg(\gamma) = 0.62 \pm 0.34, \tag{20}$$

$$\texttt{Model II-2}: \quad \kappa = 2.28 \pm 0.47, \quad |\gamma| = 1.92 \pm 0.20, \quad \arg(\gamma) = 4.03 \pm 0.09. \tag{21}$$

It is obvious that there is no contribution from $K^*(1680)$ in these two models, χ in eq.(17) is set to zero.

Now we can calculate the quantity $\mathcal{G}$ and the forward-backward asymmetry $\mathcal{A}$. The results have been shown in Tables 1 and 2, respectively. It is found that, when these two quantities are evaluated for the full phase space, all the models give the consistent results. Model II-1 and II-2 are consistent for $\mathcal{G}$ even if we calculate it for different cuts on s. However, in the large s region, $s \geq (1.2\ \mathrm{GeV})^2$, it is expected that one might distinguish Model I and Model II from the values of $\mathcal{G}$. The asymmetry $\mathcal{A}$ shown in Table 2 is more interesting since its values will show the difference from these three cases for s above 1 GeV^2. The similar conclusion can also be obtained from the differential asymmetry A_{FB}, which has been plotted in Figure 1. A_{FB}'s are almost consistent for three cases when s is below 1 GeV^2, they however behave differently for s above 1 GeV^2, in particular, A_{FB} from Model II-2 could change sign for large s.

$s_{\rm min} \sim s_{\rm max}$	Model I	Model II-1	Model II-2
$(m_K + m_\pi)^2 \sim (0.8\ \text{GeV})^2$	0.326±0.021	0.320±0.021	0.343±0.039
$(0.8\ \text{GeV})^2 \sim (1.0\ \text{GeV})^2$	0.257±0.001	0.257±0.001	0.257±0.002
$(1.0\ \text{GeV})^2 \sim (1.2\ \text{GeV})^2$	0.333±0.005	0.340±0.007	0.339±0.008
$(1.2\ \text{GeV})^2 \sim (1.4\ \text{GeV})^2$	0.398±0.003	0.436±0.006	0.437±0.009
$(1.4\ \text{GeV})^2 \sim m_\tau^2$	0.440±0.002	0.479±0.002	0.479±0.005
$(m_K + m_\pi)^2 \sim m_\tau^2$	0.269±0.002	0.270±0.003	0.271±0.004

Table 1: The values of $\mathcal{G}$ defined in eq. (13) are evaluated for different cuts on s in the above models [(19), (20), and (21)]. The last line is for the full phase space.

$s_{\rm min} \sim s_{\rm max}$	Model I	Model II-1	Model II-2
$(m_K + m_\pi)^2 \sim (0.8\ \text{GeV})^2$	-0.229±0.006	-0.300±0.030	-0.217±0.022
$(0.8\ \text{GeV})^2 \sim (1.0\ \text{GeV})^2$	-0.107±0.015	-0.108±0.016	-0.105±0.032
$(1.0\ \text{GeV})^2 \sim (1.2\ \text{GeV})^2$	-0.332±0.039	-0.182±0.073	-0.413±0.075
$(1.2\ \text{GeV})^2 \sim (1.4\ \text{GeV})^2$	-0.378±0.044	-0.292±0.116	-0.042±0.110
$(1.4\ \text{GeV})^2 \sim m_\tau^2$	-0.297±0.037	-0.481±0.044	0.471±0.019
$(m_K + m_\pi)^2 \sim m_\tau^2$	-0.133±0.018	-0.125±0.021	-0.123±0.037

Table 2: The integrated and normalized forward-backward asymmetry $\mathcal{A}$ are evaluated for different cuts on s in the above models [(19), (20), and (21)]. The last line is for the full phase space.

Early studies tell us that the decay $\tau \to K\pi\nu_\tau$ is dominated by vector $K^*(892)$ contributions. The above different models [(19), (20), and (21)] from Belle experiment have parameterized different scalar contents, therefore it is not surprising that analysis of the angular distribution of the decay will reveal some difference. This means that the future angular analysis from the high statistics experiment might be useful to distinguish the above models, and would help us to further understand the scalar components of the decay.

We would like to give some remarks here.

- The purpose of this paper is to show that the angular analysis of the decay $\tau^- \to K_S\pi^-\nu_\tau$ may help us to reveal the nature of the scalar form factors. In order to illustrate our analysis, we simply take the form factors adopted in the Belle experiment [11]. It is not surprising that other different descriptions of the form factors, in particular, for the scalar form factors, could lead to different results from those in the present work.

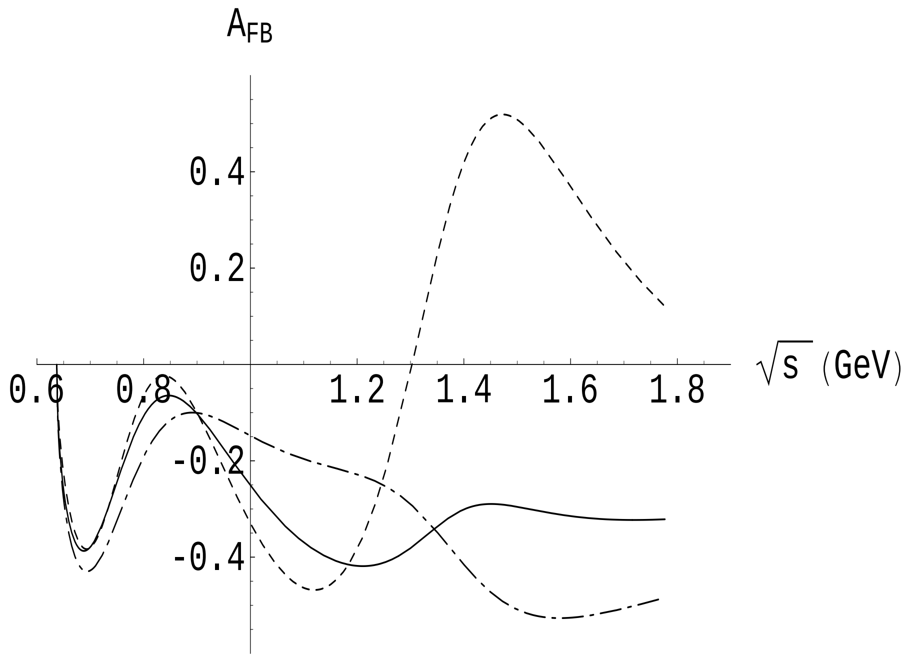

Figure 1: The differential forward-backward asymmetry $A_{\rm FB}$ is plotted as the function of $\sqrt{s}$. The solid line is for Model I, the dashed-dotted line is for Model II-1, and the dashed line is for Model II-2.

- Actually, some physically better motivated descriptions of the scalar form factors, which both satisfy constraints by analyticity and unitarity and provide a good description of the experimental data, have been obtained in a series of seminal papers [18]. Note that our F_S is equivalent to $(m_K^2 - m_\pi^2)/s \cdot F_0^{K\pi}$ (here $F_0^{K\pi}$ is the scalar form factor) in these papers due to different notations. Thus the different behavior of the differential forward-backward asymmetry $A_{\rm FB}$ for the small s region can be expected, to be compared with those shown in Fig. 1, in which $A_{\rm FB}$'s from the Belle data are consistent with one another for s below 1 GeV^2. A detailed quantitative comparison is beyond the scope of the present paper, which is left for future work.

- For comparison, we suggest our experimental colleagues should also include these physically better motivated form factors in the future experimental analysis.

In Ref. [13], the differential forward-backward asymmetry $A_{\rm FB}$ has been studied in the two-Higgs-doublet model (2HDM) of type II with large tanβ [19]. In this model, the charged Higgs exchange

$$\mathcal{L}^{H^\pm} = \frac{G_F}{\sqrt{2}} \sin\theta_C \frac{m_s m_\tau \tan^2\beta}{m_{H^\pm}^2} \bar{\nu_\tau}(1+\gamma_5)\tau \bar{s}(1-\gamma_5)u, \qquad (22)$$

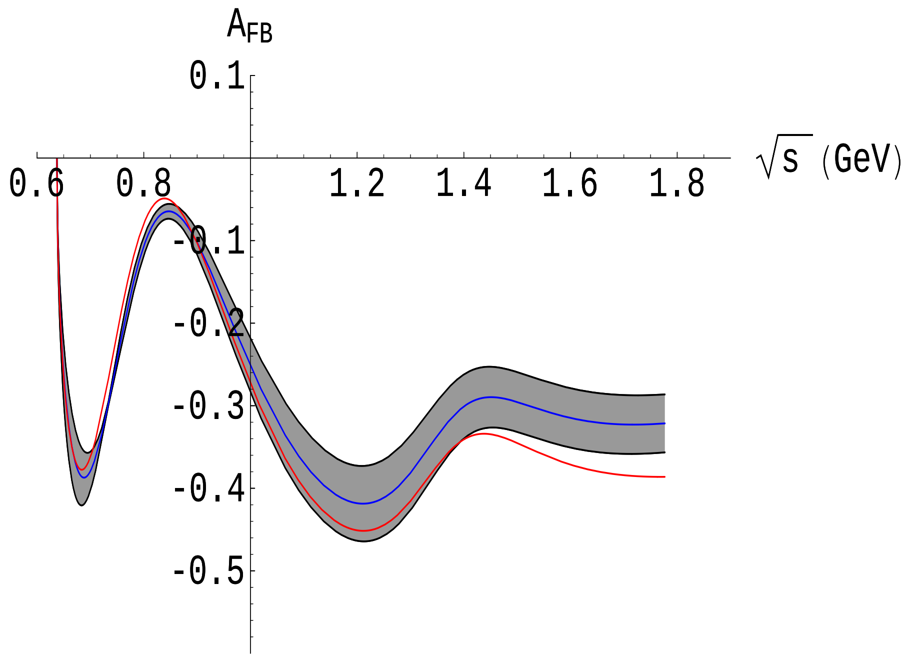

Figure 2: The differential forward-backward asymmetry $A_{\rm FB}$ is plotted as the function of $\sqrt{s}$. The red line is from $F_S^{\rm New}$, the blue line is for Model I and grey region denotes the uncertainty from the parameters in Model I.

can lead to a tree level contribution to $\tau \to K\pi\nu_\tau$, which thus gives a new contribution to the scalar form factor F_S,

$$F_S^{\rm New} = F_S + \frac{1}{\sqrt{2}} \frac{m_K^2 \tan^2\beta}{M_{H^\pm}^2} \frac{m_s}{m_s + m_u}. \tag{23}$$

In general, this new contribution will be strongly suppressed by the large charged Higgs mass $M_{H^\pm}$, which however may be substantially compensated by large tanβ. In terms of $F_S^{\rm New}$, together with eq. (16), one can evaluate the new contribution to $A_{\rm FB}$, which has been plotted in Figure 2. We take F_S in eq. (23) from Model I, and $\tan\beta/M_{H^\pm} = 0.4$ GeV^{-1}[20, 21]. In Figure 2, we include the uncertainty from the present experimental fit for $A_{\rm FB}$, which shows that the uncertainty could obscure the new physics signal, thus new physics searches through $A_{\rm FB}$ might be interesting in the future, however, might be not very significant at present.

4 Summary

Belle Collaboration reported a measurement of the decay spectrum of $\tau^- \to K_S\pi^-\nu_\tau$, and found two models, $K_0^*(800) + K^*(892) + K^*(1410)$ and $K_0^*(800) + K^*(892) + K_0^*(1430)$, can both describe the spectrum very well. Based on these data, we carry out a phenomenological analysis of the angular distributions in this decay. We find that the $\cos\theta$-dependence of the normalized spectrum $1/\Gamma d\Gamma/d\cos\theta$, see eq.(11), offers good opportunities to test the models. Therefore, the future experimental study of the angular distribution, in particular, the analysis of the forward-backward asymmetries, may be very useful to distinguish or rule out the above models, which would be helpful to reveal the nature of the scalar form factor of the decay. Possible new physics contributions from 2HDM to the $A_{\rm FB}$ have also been analyzed. we found that the present experimental uncertainty may obscure the new physics signal.

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