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C. Oswald,² P. Urquijo,² J. Dingfelder,² I. Adachi,⁹ H. Aihara,⁵³ K. Arinstein,³ D. M. Asner,⁴² T. Aushev,¹⁷
A. M. Bakich,⁴⁷ K. Belous,¹⁶ V. Bhardwaj,³ B. Bhuyan,¹² A. Bondar,³ G. Bouvinci,⁵⁸ A. Bozek,²⁸
M. Bračko,²⁷,¹⁸ T. E. Browder,⁸ P. Chang,³⁷ V. Chekelian,²⁸ A. Chen,³⁵ P. Chen,³⁷ B. G. Cheon,⁷
Z. Doležal,⁴ Z. Drásal,⁴ A. Drutskoy,¹⁷,³⁰ D. Dutta,¹² S. Eidelman,³ S. Esen,⁵ H. Farhat,⁵⁸ J. E. Fast,⁴²
V. Gaur,⁴⁸ N. Gabyshev,³ S. Ganguly,⁵⁸ R. Gillard,⁵⁸ Y. M. Goh,⁷ B. Golob,²⁵,¹⁸ J. Haba,⁹ K. Hayasaka,³³
J. B. Kim,²² K. T. Kim,²² M. J. Kim,²³ Y. J. Kim,²¹ K. Kinoshita,⁵ J. Klucar,¹⁸ B. R. Ko,²² S. Korpar,²⁷,¹⁸
R. T. Kouzes,⁴² P. Križan,²⁵,¹⁸ P. Krokovny,³ B. Kronenbitter,²⁰ T. Kuhr,²⁰ T. Kumita,⁵⁵ Y.-J. Kwon,⁶⁰
N. Muramatsu,⁴³ Y. Nagasaka,¹⁰ E. Nakano,⁴¹ M. Nakao,⁹ E. Nedelkovska,²⁸ N. K. Nisar,⁴⁹ S. Nishida,⁹
K. Prothmann,²⁹,⁴⁹ M. Ritter,²⁸ M. Röhrken,²⁰ M. Rozanska,³⁸ S. Ryu,⁴⁵ H. Sahoo,⁸ T. Saito,⁵² Y. Sakai,⁹
S. Sandilya,⁴⁸ L. Santelj,¹⁸ T. Sanuki,⁵² Y. Sato,⁵² O. Schneider,²⁴ G. Schnell,¹¹,¹¹ C. Schwanda,¹⁵
J.-G. Shiu,³⁷ B. Shwartz,³ A. Sibidanov,⁴⁷ F. Simon,²⁸,⁴⁹ P. Smerkol,¹⁸ Y.-S. Sohn,⁶⁰ A. Sokolov,¹⁶
E. Solovieva,¹⁷ M. Starič,¹⁸ T. Sumiyoshi,²⁵ G. Tatischevili,⁴² Y. Teramoto,¹³ K. Trabelsi,⁹ T. Tsuboyama,⁹
M. Uchida,⁵⁴ S. Uchihara,⁹ T. Uglov,¹⁷,³¹ Y. Unno,⁷ S. Uno,⁹ C. Van Hulse,¹ P. Vanhoefer,²⁸ G. Varner,⁸
E. Won,²² H. Yamamoto,⁵² Y. Yamashita,³⁹ C. C. Zhang,¹⁴ Z. P. Zhang,⁴⁴ V. Zillich,³ and A. Zupanc²⁰
(The Belle Collaboration)

¹University of the Basque Country UPV/EHU, 48080 Bilbao
²University of Bonn, 53115 Bonn
³Budker Institute of Nuclear Physics SB RAS and Novosibirsk State University, Novosibirsk 630090
⁴Faculty of Mathematics and Physics, Charles University, 121 16 Prague
⁵University of Cincinnati, Cincinnati, Ohio 45221
⁶Gyeongsang National University, Chinju 660-701
⁷Hanyang University, Seoul 133-791
⁸University of Hawaii, Honolulu, Hawaii 96822
⁹High Energy Accelerator Research Organization (KEK), Tsukuba 305-0801
¹⁰Hiroshima Institute of Technology, Hiroshima 731-5193
¹¹Ikerbasque, 48011 Bilbao
¹²Indian Institute of Technology Guwahati, Assam 781039
¹³Indian Institute of Technology Madras, Chennai 600036
¹⁴Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049
¹⁵Institute of High Energy Physics, Vienna 1050
¹⁶Institute for High Energy Physics, Protvino 142281
¹⁷Institute for Theoretical and Experimental Physics, Moscow 117218
¹⁸J. Stefan Institute, 1000 Ljubljana
¹⁹Kanagawa University, Yokohama 221-8686
²⁰Institut für Experimentelle Kernphysik, Karlsruher Institut für Technologie, 76131 Karlsruhe
²¹Korea Institute of Science and Technology Information, Daegu 305-806
²²Korea University, Seoul 136-713
²³Kyungpook National University, Daegu 702-701
²⁴École Polytechnique Fédérale de Lausanne (EPFL), Lausanne 1015
²⁵Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana
²⁶Luther College, Decorah, Iowa 52101
²⁷University of Maribor, 2000 Maribor
²⁸Max-Planck-Institut für Physik, 80805 München
²⁹School of Physics, University of Melbourne, Victoria 3010
³⁰Moscow Physical Engineering Institute, Moscow 115409
We report a measurement of the inclusive semileptonic $B_s^0$ branching fraction in a 121 fb$^{-1}$ data sample collected near the $\Upsilon(5S)$ resonance with the Belle detector at the KEKB asymmetric energy $e^+e^-$ collider. Events containing $B_s^{0(*)}B_{s}^{0(*)}$ pairs are selected by reconstructing a tag side $D_s^+$ and identifying a signal side lepton $\ell^+$ ($\ell = e, \mu$) that is required to have the same-sign charge to ensure that both originate from different $B_s^0$ mesons. The $B_s^0 \rightarrow X^- \ell^+\nu_\ell$ branching fraction is extracted from the ratio of the measured yields of $D_s^+$ mesons and $D_s^+\ell^+\nu_\ell$ pairs and the known production and branching fractions. The inclusive semileptonic branching fraction is measured to be $[10.6 \pm 0.5{\text{(stat)}} \pm 0.7{\text{(syst)}}] \%$.

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I. INTRODUCTION

Semileptonic decays of $b$-flavored mesons constitute a very important class of decays for determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $[1]$, $V_{ub}$ and $V_{cb}$, and for understanding the origin of $CP$ violation in the Standard Model (SM). Although semileptonic $B^0$ and $B^+$ meson decays have been precisely measured by experiments running at the $\Upsilon(4S)$ resonance, and have been well studied in theory, experimental information on the decay of the $B_s^0$ meson is relatively limited. The interest in the physics of the $B_s^0$ has intensified in recent years, motivated by studies of the dilepton production asymmetry in $b\bar{b}$ production $[2]$. Semileptonic $B_s^0$ decays are used as a normalization mode for various searches for new physics at hadron colliders $[3]$, and in the future with the next generation $B$ factories. Semileptonic $B_s^0$ decays also provide an analogous approach to studying the CKM matrix elements and testing theoretical predictions, as meson decays that involve a spectator strange quark can be predicted more accurately than analogous decays with a spectator up or down quark.

An important expectation from heavy quark theory that is exploited in studies of $B^0_s$ decays is the equality relation, based on $SU(3)$ symmetry, between the semileptonic decay widths $[4, 5]$:

$$\Gamma_{S\ell}(B_s^0) = \Gamma_{S\ell}(B^+) = \Gamma_{S\ell}(B^0). \quad (1)$$

The presence of the heavier spectator strange quark introduces, however, some amount of $SU(3)$ symmetry breaking, as observed in decays of open charm mesons $[6]$. Theoretical predictions based on heavy quark symmetry in Refs. $[4, 5]$ find that Eq. 1 should hold for $B_s$ decays to the percent level, which must be tested in experiment. The BaBar collaboration has determined the branching fraction $\mathcal{B}(B_s \rightarrow X\ell\nu) = [9.5^{+2.5}_{-2.0}(\text{stat})^{+1.1}_{-1.9}(\text{syst})]\%$ in a dataset obtained from an energy scan above the $\Upsilon(4S)$ resonance by measuring the inclusive yields of $\phi$ mesons...
and $\phi \ell$ pairs that are more abundant in $B^0_s$ decays [7]. The semileptonic $B_s$ width has been studied in part by the D0 and LHCb collaborations, which measured the exclusive decay modes $B_s \rightarrow D^{\ast}_{s+}\ell \nu$ and $B_s \rightarrow D_{1+}\ell \nu$ [8, 9]. In this paper, we report a measurement of the $B^0_s \rightarrow X^- \ell^+ \nu_\ell$ branching fractions for $\ell = e$ and $\mu$ separately and their weighted average. The measurements are the most precise to date.

II. DATA SAMPLE, DETECTOR AND SIMULATION

The data used in this analysis were collected with the Belle detector at the KEKB asymmetric energy $e^+e^-$ collider [10]. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) comprised of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect $K^0_L$ mesons and to identify muons (KLM). The detector is described in detail elsewhere [11].

The results in this paper are based on a 121 fb$^{-1}$ data sample collected near the $\Upsilon(5S)$ resonance ($\sqrt{s} = 10.87$ GeV), which contains $(7.1 \pm 1.3) \times 10^6 B_s^{0(*)}B_s^{0(*)}$ pairs [12]. An additional 63 fb$^{-1}$ data sample taken at $\sqrt{s} = 10.52$ GeV, below the energy threshold for $b$-flavored meson production (off-resonance) is used to subtract background arising from the continuum $e^+e^- \rightarrow q\bar{q}$ process.

We use Monte Carlo (MC) techniques to separately simulate the production of $B_{u,d}$ ($B^+$, $B^0$) and $B^0_s$ mesons at the $\Upsilon(5S)$ resonance. Events are generated with the EVTGEN event generator [13], and then processed through the detector simulation implemented in GEANT3 [14]. The simulated samples of $B$-pair events are equivalent to six times the integrated luminosity of the data. For the simulation of signal semileptonic $B^0_s$ decays, the lack of exclusive measurements of this system forces us to rely on prior knowledge in the $B_{u,d}$ systems and employ a variety of phenomenological models. First, we assume the composition of the $B^0_s$ semileptonic decay width is somewhat analogous to that of the $B^0$ system [15–18]. We include the following $B^0_s \rightarrow X_c \ell \nu$ decay modes in the simulation, with their nominal branching fractions in parentheses: $X_c = D_s(2.1\%)$, $D_s^0(4.9\%)$, $D_s^0(2317)(0.4\%)$, $D_{s1}(2460)(0.4\%)$, $D_{s1}(2536)(0.7\%)$, and $D_{s2}(2573)(0.7\%)$. To simulate these decay modes, we use the ISGW2 quark model [19] for all modes, and an additional model based on heavy quark effective theory (HQET) [20] for the $B_s \rightarrow D^{\ast+}_{s} \ell \nu$ modes. The form factors for the $B_s \rightarrow D^{\ast+}_{s} \ell \nu$ modes in the HQET parametrization are taken to be the same as in $B \rightarrow D^{(*)} \ell \nu$ decays, and the values taken from the Heavy Flavor Averaging Group [21]. QED final state radiation in semileptonic decays is added using the PHOTOS package [22].

III. MEASUREMENT OVERVIEW

Only one fifth of the mesons containing a $b$-quark produced near the $\Upsilon(5S)$ resonance are $B^0_s$ mesons; the remainder are $B_{u,d}$ mesons. In this analysis, the relative abundance of $B^0_s$ mesons is enhanced by reconstructing, or tagging, the CKM-favored $B^0 \rightarrow D^+_s \ell \nu$ transition [23], where $B(B^0_s \rightarrow D^+_s X) = (93 \pm 25)\%$ [24]. The signal signature is a lepton ($e^+, \mu^+$) from the decay of the other $B^0_s$ in the event. To ensure that this lepton does not originate from the same $B^0_s$ meson as the reconstructed $D^+_s$ meson, $D^+_s \ell^+ \nu_\ell$ pairs are selected wherein the $D^+_s$ and $\ell^+$ have the same electric charge. The quantity obtained in the measurement is the ratio

$$R = \frac{N_{D^+_s\ell^+}}{N_{D^+_s}}$$

with $\ell = e, \mu$, where $N_{D^+_s\ell^+}$ and $N_{D^+_s\ell^+}$ are the efficiency-corrected yields of $D^+_s$ and $D^+_s \ell^+ \nu_\ell$ pairs from $B(s)$ decays. The ratio is proportional to the inclusive semileptonic branching fraction $B(B^0_s \rightarrow X^-* \ell \nu_\ell)$, plus dilution terms due to background $B_{u,d}$ decays. The yields from $B_{u,d}$ decays are approximately 30% and 15% in the $D^+_s$ and $D^+_s \ell^+ \nu_\ell$ samples respectively, estimated using measured values of the $B_{u,d}$ and $B^0_s$ production fractions near the $\Upsilon(5S)$ resonance, their branching fractions to $D^+_s$ and $D^+_s \ell^+ \nu_\ell$ final states, and their mixing probabilities.

IV. EVENT SELECTION

A. $D_s$ selection

 Charged particle tracks are required to originate from a region close to the interaction point by applying the following selections on the impact parameters along the $z$ axis (opposite the positron beam) and in the perpendicular $r-\phi$ plane: $|dz| < 2 \text{ cm}$ and $dr < 0.5 \text{ cm}$. In addition, we demand at least one associated hit in the SVD detector. For pion and kaon candidates, the Cherenkov light yield from the ACC, the time of flight information from the TOF, and the specific ionization $dE/dx$ from the CDC are required to be consistent with the appropriate mass hypotheses.

Candidate $D^+_s$ mesons are reconstructed in the cleanest decay mode $D^+_s \rightarrow \phi \pi^+$, with the $\phi$ resonance reconstructed via $\phi \rightarrow K^+K^-$. The reconstructed $\phi$ and $D^+_s$ masses are required to lie within $\pm 8 \text{ MeV}$ and $\pm 65 \text{ MeV}$ of the nominal $\phi$ and $D^+_s$ masses [18]. The corresponding $\phi$ selection efficiency is 99%. To suppress misreconstructed $D^+_s$ mesons, we require $|\cos \theta_h| > 0.5$. The
helicity angle $\theta_h$ is defined as the angle between the reconstructed $D_s^+$ momentum and the $K^-$ momentum in the $\phi$ rest frame. Non-resonant $D_s^+ \rightarrow K^0\pi^+$ decays (such as from $S$-wave processes) passing the selection criteria are treated as signal. Multiple $D_s^+$ candidates per event are allowed. Correctly reconstructed $D_s^+$ mesons from continuum background are produced directly in processes of the type $e^+e^- \rightarrow \bar{\ell}\ell \rightarrow D_s^+X$, and typically have high momenta $p^*(D_s^+)$ in the center-of-mass (CM) frame of the $e^+e^-$ beams with a maximum of $p_{\text{max}}^*(D_s^+) = \sqrt{s}/4 - m(D_s^+)^2$ [25]. The maximum CM momentum of $D_s^+$ mesons produced in $B^0_s$ decays is half that of direct production, due to restricted decay phase space. Therefore, to suppress events from the continuum background we require

$$x(D_s^+) = \frac{p^*(D_s^+)}{p_{\text{max}}^*(D_s^+)} = \frac{p^*(D_s^+)}{\sqrt{s}/4 - m(D_s^+)^2} < 0.5.$$ (3)

### B. Lepton selection

Each $D_s^+$ candidate is combined with an electron or muon having the same-sign charge. Electron candidates are identified using the ratio of the energy detected in the ECL to the track momentum, the ECL shower shape, position matching between the track and ECL cluster, the energy loss in the CDC, and the response of the ACC counters. Muons are identified based on their penetration range and transverse scattering in the KLM detector. The polar acceptance regions are $18^\circ < \theta < 150^\circ$ and $25^\circ < \theta < 145^\circ$ for electrons and muons, respectively. Leptons are reconstructed with a minimum lepton momentum in the lab frame $p(\ell^\pm)$ of 0.6 GeV corresponding to the acceptance threshold of the detector. Lepton candidates are rejected if they are likely to have originated from $J/\psi$ decays, using the mass criterion $|m(\ell^+\ell^-) - m(J/\psi)| < 5$ MeV, where $h^-$ is any charged track with a mass hypothesis based on the signal candidate lepton. Electrons that appear to originate from Dalitz $\pi^0$ decays and from converted photons are removed by requiring $|m(\ell^+\ell^-\gamma) - m(\pi^0)| < 32$ MeV and $|m(\ell^+\ell^-)| < 100$ MeV, respectively, where $h^-$ is defined as above and $\gamma$ is any detected photon. The lepton identification efficiencies multiplied by the geometrical acceptance are 75% (electrons) and 68% (muons). The probabilities that a selected lepton candidate is a misidentified charged kaon or pion are 6% and 19% for electrons and muons, respectively.

The lepton detection efficiencies and misidentification probabilities in the MC simulation are calibrated to data. The calibration factors for the detection efficiencies are obtained from the study of $\gamma\gamma \rightarrow \ell^+\ell^-$ and $J/\psi \rightarrow \ell^+\ell^-$. The misidentification probabilities are determined from $D^{*+} \rightarrow D^0\pi^+$, $D^0 \rightarrow K^-\pi^+$ decays by studying the electron and muon likelihood of the $K^-$ and $\pi^+$ tracks from the $D^0$. The pion from the $D^{*+}$, $\pi_{\text{slow}}^+$, has a momentum of only a few hundred MeV as it is produced just above the kinematic threshold.

### V. FIT RESULTS

The number of $D_s^+$ mesons in data is determined from fits to the $KK\pi$ mass distribution. The signal shape used in the fit is modeled as two Gaussian functions with a common mean; the combinatorial background is modeled by a linear function. The fit parameters are the normalizations of signal ($N_{\text{sig}}$) and background ($N_{\text{bkg}}$), the slope of the linear function ($b$) and the parameters of the two Gaussian functions: the common mean ($\mu_{\text{gs}}$), the width of one Gaussian ($\sigma_1$), the ratio of the widths ($r_\sigma = \sigma_2/\sigma_1$) and the ratio of the normalizations ($r_N$).

For the measurement of $N_{D_s^+}$, the fits to $m_{KK\pi}$ are performed in 20 equal bins of the normalized $D_s^+$ momentum in the full range $[0, 1]$, including the control region $x(D_s^+) > 0.5$. A binned approach is used to accommodate $x(D_s^+)$ dependence on the signal and background shape parameters ($\mu_{\text{gs}}, \sigma_1, b$).

The fit results for the parameters $r_\sigma$ and $r_N$ are found to be independent of $x(D_s^+)$. Figure 1 shows the $KK\pi$ mass fits for $\Upsilon(5S)$ data in the signal region ($x(D_s^+) < 0.5$) and Fig. 2(a) the obtained $D_s^+$ momentum spectra for $\Upsilon(5S)$ data and off-resonance data. The off-resonance data is scaled with a factor $S_{\text{cont}} = (L_{\Upsilon(5S)}/s_{\Upsilon(5S)})/(L_{\text{off}}/s_{\text{off}}) = 1.81 \pm 0.02$ to account for the difference in integrated luminosities and the dependence of the quark pair production cross section on the center-of-mass energy $\sqrt{s}$.

The total $N_{D_s^+}$ is obtained by integrating over the region $x(D_s^+) < 0.5$ and subtracting the continuum background given by the scaled off-resonance distribution. A total of $[12.42 \pm 0.08(\text{stat})] \times 10^4 D_s^+$ mesons are reconstructed, where $[2.7 \pm 0.1(\text{stat})] \times 10^4$ of these are from continuum processes. This approach is validated by taking the difference between $\Upsilon(5S)$ and off-resonance data in the control region $x(D_s^+) > 0.5$, where only events from the continuum can contribute. The difference is found to be $-872 \pm 1778$, consistent with the expectation of zero.

For the $N_{D_s^{*+}}$ measurements, the $KK\pi$ mass fits are performed in nine bins of lepton momentum in the range 0.6 GeV $< p(\ell^+) < 3.1$ GeV, where the lower and upper thresholds are chosen due to the detection sensitivity to electrons and muons, and to the semileptonic decay kinematic endpoint, respectively (see Figs. 2(b) and 2(c)). The $D_s^{*+}\ell^+$ samples do not contain enough events to determine all seven fit parameters. Therefore $r_\sigma$ and $r_N$ are fixed to the values obtained in the $N_{D_s^+}$ measurement. The remaining parameters, other than $N_{\text{sig}}$ and $N_{\text{bkg}}$, are determined from a fit to the total $D_s^+\ell^+$ sample without the binning in $p(\ell^+)$ as shown in Fig. 3. The $\chi^2$/ndf of the fits over the full lepton momentum range are found to be 37/45 and 58/45 for electrons and muons, respectively. Compared to the $D_s^+$ sample, the...
FIG. 1: The invariant $K K \pi$ mass spectra collected near the $\Upsilon(5S)$ resonance in bins of normalised $D^+_s$ momentum, $x(D^+_s)$, in the signal region ($x(D^+_s) < 0.5$). The fits are used to determine the total number of $D^+_s$ mesons from $b$-flavored mesons in the $\Upsilon(5S)$ sample.

continuum background in the $D^+_s \ell^+$ sample is suppressed due to the same-sign lepton requirement. The remaining continuum background is subtracted using scaled off-resonance data. The shape difference of the continuum lepton momentum spectra at the $\Upsilon(5S)$ and in the off-resonance samples is determined from MC simulation and the effect is corrected by a bin-by-bin re-weighting before the subtraction.

A $\chi^2$ fit to the lepton momentum spectrum is performed with two components: the prompt lepton signal and the remaining $B_{u,d,s}$ backgrounds, which is the sum of secondary leptons (not coming directly from $B_{u,d,s}$ decays) and misidentified lepton candidates. The shapes of the signal and $B_{u,d,s}$ backgrounds are derived from MC simulation. Figures 2(b) and 2(c) show the fit result, and the goodness of the fits is listed in Table I. The numbers of prompt leptons obtained in the fit are corrected for efficiency and geometrical acceptance. The results are extrapolated from the experimental momentum threshold of $p(\ell^+) > 0.6$ GeV to the full phase space region using MC simulation, where the uncertainty on this acceptance is included in the systematic uncertainties. The signal acceptance in the region $p(\ell^+) > 0.6$ GeV is 91% for electrons and 92% for muons. Finally, we find $[4.26 \pm 0.19 \text{(stat)}] \times 10^3$ and $[4.76 \pm 0.23 \text{(stat)}] \times 10^3$ prompt signal electrons and muons, respectively. To determine $R$ (Eq. 2), we additionally account for the difference in $D^+_s$ reconstruction efficiencies between the inclusive $D^+_s$ and the signal samples $D^+_s \ell^+$. These efficiencies take into account the possibility of more than one
The knowledge of the composition of the fit component spectrum add uncertainties of 0.4% and 1%, respectively. The scale factor $S$ for the off-resonance data and the corrections of the off-resonance lepton momentum spectrum add uncertainties of 0.4% and 1%, respectively. The knowledge of the composition of the fit component containing the combined background from secondary leptons and and misidentified lepton candidates is limited are discussed in turn below, and are given as relative uncertainties. They are also summarized in Tab. II.

Numerous potential systematic uncertainties that relate to the reconstruction of the $D^{+}_s$ ultimately cancel in the ratio; these include uncertainties associated with kaon and pion reconstruction. The uncertainty on the calibration of the electron (muon) identification is 0.7% (1.4%). The uncertainty on the lepton misidentification is below 0.1%. Another 0.4% uncertainty is added for the reconstruction efficiency of the lepton track. The statistical uncertainty of the efficiencies $\epsilon_{D^{+}_s e^+}(KK\pi)$ and $\epsilon_{D^{+}_s \mu^+}(KK\pi)$ is 0.8%.

Uncertainties in the modeling of the $KK\pi$ mass shape cancel in the ratio $R$. The shape parameters fixed in the $N_{D^{+}_s \ell^+}$ fits are each varied by one standard deviation and the variations on the fit results are added in quadrature to determine the systematic uncertainty. This results in an uncertainty of 2.0% (2.2%) for electrons (muons).

The scale factor $S_{\text{cont}}$ for the off-resonance data and the correction of the off-resonance lepton momentum spectrum add uncertainties of 0.4% and 1%, respectively.
TABLE II: Overview of the relative systematic uncertainties of the ratio $R$.

<table>
<thead>
<tr>
<th>Uncertainty [%]</th>
<th>$\epsilon$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton identification</td>
<td>0.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Fake lepton rate</td>
<td>$&lt;0.1$</td>
<td>$&lt;0.1$</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$D^+_s$ reconstruction efficiencies</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Fitting procedure</td>
<td>Shape error in $KK\tau$ mass fits</td>
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</tr>
<tr>
<td>Background modelling</td>
<td>Continuum scale factor $S_{cont}$</td>
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</tr>
<tr>
<td></td>
<td>Kinematic smearing of $p(\ell)$ from continuum</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Secondary and and fake $\ell$ bkg. composition</td>
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</tr>
<tr>
<td>Signal modelling</td>
<td>Shape of the prompt lepton spectrum</td>
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</tr>
<tr>
<td></td>
<td>Composition of the semileptonic width</td>
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</tr>
<tr>
<td>Total</td>
<td></td>
<td>3.0</td>
</tr>
<tr>
<td>Total correlated</td>
<td></td>
<td>2.7</td>
</tr>
</tbody>
</table>

by the precision of the measurements of $B^0_u$ branching fractions, which is estimated to be of the order of 30%. Hence, the yields of secondary leptons from $D_{u,d,s}$, from $\tau$ and from other decays, as well as the rate of misidentified leptons are scaled by $\pm 30\%$ and the variation of $R$ is taken as systematic uncertainty, giving 1.0% (1.5%) for electrons (muons).

For the signal model, since most of the exclusive modes have not been measured, the shape uncertainty is estimated as the full difference between the result obtained with HQET and with the ISGW2 model where applicable. For electrons (muons), the obtained uncertainty is 0.7% (0.6%). Since the background from $B_{u,d}$ decays is expected to be approximately 15% of the measured semileptonic yield and the semileptonic width of $B_{u,d}$ decays has been studied in more detail, the shape uncertainties are found to be negligible compared to $B^0_u$ decays.

The uncertainty due to the composition of the semileptonic width is evaluated by varying the normalization of each mode by $\pm 30\%$ and adding the uncertainties in quadrature. The resulting uncertainties on $R$ are 1.0% and 1.1% for electrons and muons, respectively. Due to the inclusiveness of the analysis, the total uncertainty on the signal lepton acceptance is only 0.3%.

The total systematic uncertainty on $R$ is calculated by summing the above uncertainties in quadrature. It is found to be 3.0% (2.7%) for electrons and 3.5% (2.8%) for muons, where the values in parentheses are the fully correlated errors between both modes. Taking these correlations into account, the total systematic uncertainty on the combined value of $R$ is 3.0%.

VII. EXTRACTION OF THE BRANCHING FRACTION

The extraction of the $B^0 \to X^{-}\ell^+\nu_\ell$ branching fraction is based on a prediction of the measured ratio $R$ and includes the estimation of the background from $B_{u,d}$ decays. This approach is based on the calculation of the number of same-sign lepton pairs $\ell^+\ell^+$ in $\Upsilon(5S)$ decays discussed in Refs. [26, 27]. The measured yields $N_c$ (where $\zeta = D^+_s, D^+_t, \ell^+\ell^+$) contain a contribution from $B^0_u$ decays $N_c(B^0_uB^{0\ast\ell_\mu}(\pi))$ and from $B_{u,d}$ background $N_c(B^0_{u,d}B^{0\ast\ell_\mu}(\pi))$:

$$R = \frac{N_D^{\ast\ell_\mu}(B^0_uB^{0\ast\ell_\mu}) + N_D^{\ast\ell_\mu}(B^{0\ast}_{u,d}B^{0\ast}_{u,d}(\pi))}{N_D^{\ast\ell_\mu}(B^0_uB^{0\ast\ell_\mu}) + N_D^{\ast\ell_\mu}(B^{0\ast}_{u,d}B^{0\ast}_{u,d}(\pi))}. \quad (4)$$

The total number of produced $b$-quark pairs, $N_{bb'}$, cancels in the ratio. Pairs of $bb'$ quarks produced near the $\Upsilon(5S)$ resonance hadronize in pairs of $B_{u,d}$ mesons with a probability of $f_{ud} = f_u + f_d$, where $f_u = B(\Upsilon(5S) \to B^0\bar{B}^0)/2 = (36.1 \pm 3.2\%)$ and $f_d = B(\Upsilon(5S) \to B^0\bar{B}^0)/2 = (38.5 \pm 4.2\%)$ [28]. $B^0_u$ pairs are formed with a probability of $f_u = (19.9 \pm 3.0\%)$ [18]. The remaining contribution to the $\Upsilon(5S) \to bb'$ decay width are bottomonium resonances, but no subsequent decays of these resonances to $D^+_s$ mesons have been observed so far. The contribution from bottomonium is assumed to be negligible in the ratio $R$, and neglected in the calculations.

The production of $B_{u,d}$ mesons near the $\Upsilon(5S)$ center-of-mass energy is divided in three classes [28]: two-body decays $B^{0\ast}_{u,d}B^{0\ast}_{u,d}$, three-body decays with an additional pion $B^{0\ast}_{u,d}B^{0\ast}_{u,d}\pi$ and the initial state radiation (ISR) process $e^+e^- \to \gamma_{ISR} \Upsilon(4S) \to \gamma_{ISR}B_{u,d}\bar{B}_{u,d}$. The fractions of the different two-body production mechanisms are given by the parameters $F_{BB}, F_{B\bar{B}}$ and $F_{B^0\bar{B}^0}$, and their sum is denoted by $F_2$. The fraction of three-body decays is $(f_{ud} - F_2) \cdot F_3$, where $F_3'' = F_{BB} + F_{B\bar{B}} + F_{B^0\bar{B}^0}$. The remainder $(f_{ud} - F_2) \cdot (1 - F_3'')$ is attributed to the ISR process. From isospin symmetry, one can deduce that one-third of the three-body decay modes are $B^{+\ast\ast}B^{-\ast\ast}\pi^0$ and $B^0(B^{0\ast})\pi^0$, with the remainder being $B^{+\ast\ast}B^{0\ast\ast}\pi^0$ or $B^{-\ast\ast}B^{0\ast\ast}\pi^0$.

The mixing probability $\chi_q^{(C)}$ of a pair of $B^0_q$ mesons ($q=d,s$) depends on $x_q = \Delta m_{B^0_q}/\Gamma_{B^0_q}$ and the $C$ eigenstate in which the pair is produced:

$$\chi_q^{(C)} = \frac{x_q^2(3 + x_q^2)}{2(1 + x_q^2)^2} \quad \text{and} \quad \chi_q^{(-)} = \frac{x_q^2}{2(1 + x_q^2)}.$$  \quad (5)

In contrast to $B^0$ mesons, where $x_s = 0.770 \pm 0.008$ [18], $x_d = 26.49 \pm 0.29$ [18] is so large for $B^0_d$ mesons that the difference between even and odd $C$ eigenstates can be neglected. We use the approximation $\chi_s = (1 - \chi_s) = 0.500 \pm 0.001$. For $B^0$ produced together with a charged $B$ meson, the mixing probability is the same as for $C = -1$. With this information, the contributions $N$ to the
yields from each $b$-flavored meson production mode can be calculated. The factor of two takes into account the possibility that the reconstructed $D_s^+$ meson can stem from either of the two $b$-flavored mesons.

\[ N_{D_s^+}(B_s^{(*)} \bar{B}_s^{(*)})/N_{B_s} = 2 \cdot f_s \cdot B(B_s^0 \to D_s^0 X), \]

\[ N_{D_s^+}(B_{u,d}^{(*)} \bar{B}_{u,d}^{(*)}(\pi))/N_{B_s} = 2 \cdot f_d \cdot B(B_s^0 \to D_s^0 X) + 2 \cdot f_u \cdot B(B_s^+ \to D_s^+ X), \]

\[ N_{D_s^+ \ell^+}(B_s^{0(*)} \bar{B}_s^{0(*)})/N_{B_s} = 2 \cdot f_s \cdot B(B_s^0 \to X^- \ell^+ \nu_\ell) \cdot (1 - \chi_s) \cdot B(B_s^0 \to D_s^0 X), \]

\[ N_{D_s^+ \ell^+}(B_{u,d}^{0(*)} \bar{B}_{u,d}^{0(*)}(\pi))/N_{B_s} = 2 \cdot f_d \cdot B(B_s^0 \to D_s^0 X) \cdot 2 \cdot (1 - \chi_d) \cdot B(B_s^0 \to D_s^0 X) + N_{B_s} \]

Equation 4 is solved for $B(B_s^0 \to X^- \ell^+ \nu_\ell)$, which is the only unknown quantity.

\[ B(B_s^0 \to X^- \ell^+ \nu_\ell) = \left[ \frac{\mathcal{N}_{D_s^+} (B_s^{(*)} \bar{B}_s^{(*)}) + \mathcal{N}_{D_s^+} (B_{u,d}^{(*)} \bar{B}_{u,d}(\pi))}{2 \cdot f_s \cdot (1 - \chi_s) \cdot B(B_s^0 \to D_s^0 X) \cdot N_{B_s}} \right] \cdot R - \mathcal{N}_{D_s^+ \ell^+}(B_{u,d}^{0(*)} \bar{B}_{u,d}^{0(*)}(\pi)) \]

VIII. RESULTS AND DISCUSSION

We obtain the following values for the semileptonic branching fraction $B(B_s^0 \to X^- \ell^+ \nu_\ell)$:

<table>
<thead>
<tr>
<th>( \ell )</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>$[10.1 \pm 0.6(\text{stat}) \pm 0.7(\text{syst})]%$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$[11.3 \pm 0.7(\text{stat}) \pm 0.8(\text{syst})]%$</td>
</tr>
<tr>
<td>$e, \mu$</td>
<td>$[10.6 \pm 0.5(\text{stat}) \pm 0.7(\text{syst})]%$</td>
</tr>
</tbody>
</table>

The last branching fraction is the combination of the electron and muon mode measurements. Our result is consistent with the measurement in Ref. [7] and substantially improves on both the statistical and systematic precision.

Table IV summarizes the uncertainties of the branching fractions. The dominant uncertainty arises from the external parameters. This is typical for almost any $B_s$
The relative systematic uncertainty $[\Delta B/B]$ is given for the combined measurement. Parameter values are taken from Ref. [18] unless otherwise stated.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>$[\Delta B/B]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_s$</td>
<td>$B(Y(5S) \to B^{\pm} X)/2$</td>
<td>$(36.1 \pm 3.2)%$ [28]</td>
</tr>
<tr>
<td>$f_0$</td>
<td>$B(Y(5S) \to B^{0} X)/2$</td>
<td>$(38.5 \pm 4.2)%$ [28]</td>
</tr>
<tr>
<td>$f_s$</td>
<td>$(19.9 \pm 3.0)%$</td>
<td></td>
</tr>
<tr>
<td>$B(B_s \to D_{s}^{ \pm} X)$</td>
<td>$(93 \pm 25)%$ [24]</td>
<td>4.4</td>
</tr>
<tr>
<td>$B(B^+ \to D_{s}^{+} X)$</td>
<td>$(7.9 \pm 1.4)%$</td>
<td></td>
</tr>
<tr>
<td>$B(B^0 \to D_{s}^{0} X)$</td>
<td>$(10.3 \pm 2.1)%$</td>
<td></td>
</tr>
<tr>
<td>$B(B^0 \to D_{s}^{0} X)$</td>
<td>$(1.5 \pm 0.8)%$</td>
<td></td>
</tr>
<tr>
<td>$B(B^+ \to D_{s}^{-} X)$</td>
<td>$(1.1 \pm 0.4)%$</td>
<td></td>
</tr>
<tr>
<td>$B(B^0 \to X_{\ell}^{\pm} \nu_\ell)$</td>
<td>$(10.33 \pm 0.28)%$</td>
<td>0.4</td>
</tr>
<tr>
<td>$B(B^+ \to X_{\ell}^{\pm} \nu_\ell)$</td>
<td>$(10.99 \pm 0.28)%$</td>
<td>0.1</td>
</tr>
<tr>
<td>$F_{\bar{s} \bar{s}}$</td>
<td>$(38.1 \pm 3.4)%$</td>
<td></td>
</tr>
<tr>
<td>$F_{s \bar{s}}$</td>
<td>$(13.7 \pm 1.6)%$</td>
<td></td>
</tr>
<tr>
<td>$F_{\bar{s} \bar{s}} / F_{s \bar{s}}$</td>
<td>$(5.5 \pm 1.6)%$</td>
<td></td>
</tr>
<tr>
<td>$F_{\bar{s} \bar{s}} / F_{s \bar{s}}$</td>
<td>$(5.9 \pm 7.8)%$</td>
<td></td>
</tr>
<tr>
<td>$F_{\bar{s} \bar{s}} / F_{s \bar{s}}$</td>
<td>$(41.6 \pm 12.1)%$</td>
<td></td>
</tr>
<tr>
<td>$\delta_{s \bar{s}}$</td>
<td>$(0.2 \pm 0.8)%$</td>
<td></td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>$0.77 \pm 0.008$</td>
<td></td>
</tr>
<tr>
<td>$\chi_s$</td>
<td>$0.500 \pm 0.001$</td>
<td></td>
</tr>
</tbody>
</table>

We measured the inclusive semileptonic $B_s^0$ branching fraction $B(B_s^0 \to X_{\ell}^{\pm} \nu_\ell) = [10.6 \pm 0.5(stat) \pm 0.7(syst)]\%$. This is the most precise measurement to date and in agreement with the previous measurement [7] and theoretical expectations [4, 5].

### IX. SUMMARY

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### X. ACKNOWLEDGMENTS
[12] This number was obtained by the Belle Collaboration with the method described in A. Drutskoy et al. [Belle Collaboration], Phys. Rev. Lett. 98, 052001 (2007).
[23] Throughout this paper, the inclusion of the charge conjugate mode decay is implied unless otherwise stated.
[24] We interpret the average $\mathcal{B}(B_{s}^{0} \rightarrow D_{s}^{\pm} X) = (93 \pm 25)\%$ from Ref. [16] as a multiplicity (i.e., the value can be greater than one), not as a branching fraction $\mathcal{B}(B_{s}^{0} \rightarrow D_{s}^{\pm} X)$.
[25] Throughout this paper, the convention $\epsilon = 1$ is used.
[26] R. Louvot, “Study of $B_{s}^{0}$-meson production and measurement of $B_{s}^{0}$ decays into a $D_{s}^{(*)}$ and a light meson in $e^{+}e^{-}$ collisions at $\sqrt{s} = 10.87$ GeV,” PhD thesis #5213, EPFL (2012).