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Charged black holes in nonlinear massive gravity

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We find static charged black hole solutions in nonlinear massive gravity. In the parameter space of two gravitational potential parameters (α, β) we show that below the Compton wavelength the black hole solutions reduce to that of Reissner-Nordström via the Vainshtein mechanism in the weak field limit. In the simplest case with $\alpha = \beta = 0$ the solution exhibits the vDVZ discontinuity but ordinary General Relativity is recovered deep inside the horizon due to the existence of electric charge, though this case is observationally excluded. For $\alpha \neq 0$ and $\beta = 0$, the post-Newtonian parameter of the charged black hole evolves to that of General Relativity via the Vainshtein mechanism within a macroscopic distance; however, a logarithmic correction to the metric factor of the time coordinate is obtained. When α and β are both nonzero, there exist two branches of solutions depending on the positivity of β . When $\beta < 0$, the strong coupling of the scalar graviton weakens gravity at distances smaller than the Vainshtein radius. However, when $\beta > 0$ the metric factors exhibit only small corrections compared to the solutions obtained in General Relativity, and under a particular choice of $\beta = \alpha^2/6$ the standard Reissner-Nordström-de Sitter solution is recovered.

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I. INTRODUCTION

The question on whether there exists a consistent covariant theory for massive gravity, where the graviton acquires a mass and leads to a modification of General Relativity, was initiated by Fierz and Pauli (FP) [1]. It was observed that at quadratic order the FP mass term is the only ghost-free term describing a gravity theory with five degrees of freedom [2]. However, it is not possible to recover linearized Einstein gravity in the limit of vanishing graviton mass, due to the existence of the van Dam-Veltman-Zakharov (vDVZ) discontinuity arising from the coupling between the longitudinal mode of the graviton and the trace of the energy momentum tensor [3, 4]. It was later observed that this troublesome mode could be suppressed at macroscopic length scales due to a nonlinear effect: the so-called Vainshtein mechanism [5]. However, these nonlinear terms, which are responsible for the suppression of vDVZ discontinuity, lead inevitably to the existence of the Boulware-Deser (BD) ghost [6], making the theory unstable [7–10].

Although for many years it was believed that the theory of massive gravity always contains BD ghosts, a family of its nonlinear extension was recently constructed by de Rham, Gabadadze and Tolley (dRGT) [11, 12]. This is a two parameter family of nonlinear generalization of the FP theory, where the BD ghosts are removed in the decoupling limit to all orders in perturbation theory through a systematic construction of a covariant nonlinear action [13–16] (see [17] for a review). As a consequence, the theoretical and phenomenological advantages of the dRGT theory led to a wide investigation in the literature. For example, cosmological implications of the dRGT theory are discussed in [18–39]; black holes and

spherically symmetric solutions were analyzed in [40–49]; and the theory’s connections to bi-metric gravity models were studied in [50–63].

Among these phenomenological studies, is the search for observationally suitable spherically symmetry solutions. Theories of massive gravity can be strongly constrained due to the vDVZ discontinuity appearing in the post-Newtonian parameters. Recently, a class of black hole solutions in the theory of “ghost-free” massive gravity was analyzed by Koyama, Niz and Tasinato (KNT) [40, 41]. Their result shows that the behavior of linearized solutions in General Relativity can only be reproduced below the Vainshtein distance in a certain region of parameter space. An exact Schwarzschild-de Sitter (SdS) solution was found for a group of specially selected parameters [43].

In this paper we present a family of static, electrically charged black hole solutions in the theory of ghost-free massive gravity. The solutions possess a Vainshtein radius, below which the linearized solutions of Einstein gravity are approximately recovered in the weak charge limit. Electromagnetism is classically conformally invariant, and thus it does not contribute to the trace of energy momentum tensor, which would strongly couple to the longitudinal mode of gravitons in massive gravity. However, the existence of an electric charge gives rise to a classical contribution to the metric factors. As a consequence, the behavior of the longitudinal model can indeed be affected by an electromagnetic field.

The paper is organized as follows. In Section II, we review the model of nonlinear massive gravity and present the equations of motion for gravitational fields in a spherically symmetric background. In section III we investigate in detail a stellar background with a static electric

field. In section IV we analyze the charged black hole solutions both analytically and numerically, and we show that the Vainshtein effect can be made manifest in a certain parameter space in the weak field limit. Finally, section V summarizes our results.

II. GHOST-FREE MASSIVE GRAVITY

Massive gravity has an effective field theory description given by Einstein gravity plus the covariant FP mass term. For the dRGT model Lagrangian the potentially pathological term can be absorbed by total derivative terms, leading to equations of motion that are at most second order in time derivatives [12].

A. The dRGT action

The gravitational action is:

$$S = \int d^4x \sqrt{-g} \frac{1}{16\pi G} \left[R + m^2 \mathcal{U}(g, \phi^a) \right], \quad (1)$$

where R is the Ricci scalar, and \mathcal{U} is a potential for the graviton which modifies the gravitational sector. Specifically, \mathcal{U} is given by

$$\mathcal{U}(g, \phi^a) = \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4, \quad (2)$$

in which α_3 and α_4 are dimensionless parameters. Moreover, \mathcal{U}_2 , \mathcal{U}_3 and \mathcal{U}_4 are defined as

$$\mathcal{U}_2 \equiv [\mathcal{K}]^2 - [\mathcal{K}^2], \quad (3)$$

$$\mathcal{U}_3 \equiv [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3], \quad (4)$$

$$\mathcal{U}_4 \equiv [\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 8[\mathcal{K}][\mathcal{K}^3] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4], \quad (5)$$

with

$$\mathcal{K}_\nu^\mu = \delta_\nu^\mu - \sqrt{g^{\mu\sigma} \eta_{ab} \partial_\sigma \phi^a \partial_\nu \phi^b}, \quad (6)$$

where the rectangular brackets denote the traces, namely $[\mathcal{K}] = \mathcal{K}^\mu_\mu$. Finally, in the above relation the four-form fields ϕ^a are the Stückelberg scalars introduced to restore general covariance [7].

B. Generalized Einstein Equations

For convenience we choose the unitary gauge $\phi^a = x^\mu \delta_\mu^a$ and thus the tensor $g_{\mu\nu}$ is the observable describing the five degrees of freedom of the massive graviton. In addition, we regroup the two parameters α_3 and α_4 of the graviton potential (2) introducing two new parameters, α and β , as

$$\alpha_3 = \frac{\alpha - 1}{3}, \quad \alpha_4 = \frac{\beta}{2} + \frac{1 - \alpha}{12}, \quad (7)$$

to simplify the background equations of motion.

Varying the action with respect to $g_{\mu\nu}$ leads to the modified Einstein equations:

$$G_{\mu\nu} + m^2 X_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (8)$$

where $X_{\mu\nu}$ arises from the graviton potential¹

$$\begin{aligned} X_{\mu\nu} = & \mathcal{K}_{\mu\nu} - \mathcal{K} g_{\mu\nu} \\ & - \alpha \left\{ \mathcal{K}_{\mu\nu}^2 - \mathcal{K} \mathcal{K}_{\mu\nu} + \frac{[\mathcal{K}]^2 - [\mathcal{K}^2]}{2} g_{\mu\nu} \right\} \\ & + 6\beta \left\{ \mathcal{K}_{\mu\nu}^3 - \mathcal{K} \mathcal{K}_{\mu\nu}^2 + \frac{1}{2} \mathcal{K}_{\mu\nu} \{ [\mathcal{K}]^2 - [\mathcal{K}^2] \} \right\} \\ & - \beta g_{\mu\nu} \{ [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \}. \end{aligned} \quad (9)$$

In addition to the generalized Einstein equations, the Bianchi identities lead to the constraint:

$$\nabla^\mu X_{\mu\nu} = 0. \quad (10)$$

III. GENERAL ANALYSIS ON SPHERICALLY SYMMETRIC CHARGED BACKGROUND

Having derived the equations of motion we now study the dynamics of a gravitational system described by massive gravity under a fixed background symmetry. Following [40], we consider the most general form of the metric respecting spherical symmetry,

$$ds^2 = -N^2(r) dt^2 + \frac{dr^2}{F^2(r)} + 2D(r) dt dr + \frac{r^2 d\Omega_2^2}{H^2(r)}, \quad (11)$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\varphi^2$.

Notice that the identity $g_{0r} R_{00} - g_{00} R_{0r} = 0$ yields an algebraic constraint. By substituting the modified Einstein equations (8) into this identity, one obtains

$$g_{0r} (8\pi G T_{00} - m^2 X_{00}) - g_{00} (8\pi G T_{0r} - m^2 X_{0r}) = 0. \quad (12)$$

The part related to the electromagnetic field $g_{0r} T_{00} - g_{00} T_{0r}$ vanishes automatically in the system under consideration by using (15) which will be introduced in the following subsection. Consequently, the solution to (12) is the same as the neutral case as studied in [40]. The part related to the graviton potential leads to:

$$\begin{aligned} 0 = & g_{0r} m^2 X_{00} - g_{00} m^2 X_{0r} \\ = & \frac{m^2 D(r) (2H(r) - 3) \sqrt{\frac{N(r)^2}{F(r)^2} + D(r)^2}}{\left[\frac{1}{F(r)^2} + N(r)^2 + 2\sqrt{\frac{N(r)^2}{F(r)^2} + D(r)^2} \right]^{1/2}}, \end{aligned} \quad (13)$$

in our case. As a consequence, there are two solution branches: either $D(r) = 0$, or $H(r) = 3/2$ (in general

¹ Note that the Einstein equation shown in [47] contains a typo of an extra 1/2 factor, however the equations of motion which give rise to solutions were based on the method of varying the action with respect to the metric factor directly.

a constant metric factor $H(r)$ with value depending on specific backgrounds [47]), with the latter arising from the fact that the effective energy density is proportional to g_{tt} . In the following we will focus on the more interesting first case $D(r) = 0$, which corresponds to a diagonal metric.

A. Charged spherical symmetric background in General Relativity

We consider a generic Maxwell field $F_{\mu\nu}$ in curved spacetime, with standard Lagrangian. The Maxwell equations are

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = -\sqrt{-g}J^\nu, \quad (14)$$

where J^ν is the current density. For a static electric charge Q in the gravitational system, the components of the Maxwell field are:

$$E_r = F_{0r} = E(r), \quad E_\theta = E_\varphi = 0, \quad \vec{B} = 0. \quad (15)$$

With vanishing source term, the inhomogeneous Maxwell law gives

$$\partial_r(\sqrt{-g}F^{0r}) = 0, \quad (16)$$

yielding the solution:

$$E(r) = \frac{QNH^2}{4\pi r^2 F}, \quad (17)$$

where Q is an integration constant which is typically interpreted as the electric charge. The factors N , F , and H are the fields introduced in the background metric (11). Varying the electromagnetic Lagrangian with respect to the metric gives the energy momentum tensor:

$$T_{\mu\nu} = F_{\mu\sigma}F_\nu^\sigma - \frac{F^{\sigma\rho}F_{\sigma\rho}}{4}g_{\mu\nu}. \quad (18)$$

It is well known that a static and spherically symmetric solution under standard General Relativity is described by the Reissner-Nordström (RN) solution:

$$ds^2 = -\left(1 + \frac{r_Q^2}{r^2} - \frac{r_S}{r}\right)dt^2 + \frac{dr^2}{1 + \frac{r_Q^2}{r^2} - \frac{r_S}{r}} + r^2 d\Omega^2 \quad (19)$$

with

$$r_Q \equiv \sqrt{\frac{GQ^2}{4\pi}}, \quad r_S \equiv 2GM \quad (20)$$

being respectively the length scale associated with the electric charge Q and the Schwarzschild radius determined by the mass of the spherical object M . The above solution exhibits a singularity at $r = 0$, in which the metric coefficient becomes zero and invariants like the Ricci and Kretschmann scalars diverge, however it is usually shielded by a horizon at $r = \left(r_S + \sqrt{r_S^2 - 4r_Q^2}\right)/2$, in which the metric coefficients become zero in the above

coordinates but the invariants remain finite, thus satisfying the cosmic censorship and no-hair conjectures. Note however that negative mass or highly charged solutions do not have a horizon, and thus a naked singularity appears.

B. Equations of motion for spherically symmetric charged background

Going beyond the above General Relativity solution, in the case of nonlinear massive gravity, we must consider the effects of the graviton potential. Thus one can combine the generalized Einstein equations (8), the solution to the Maxwell field (17), and the energy momentum tensor (18), and obtain three main equations of motion.

We consider the spherically symmetric metric Ansatz (11) with $D(r) = 0$. The background equations of motion are derived from the generalized Einstein equations (8). After some algebra, we extract the equations of motion. For the “00” component of generalized Einstein equation:

$$\begin{aligned} \frac{GQ^2H^6}{4\pi r^2} &= (1 + m^2r^2)H^4 + 2m^2r^2(F - 3)H^3 \\ &- H^2[2r(F\dot{F} - 3m^2r^2) + 3m^2r^2F + F^2] \\ &+ 2rFH[F(r\ddot{H} + 3\dot{H}) + r\dot{F}\dot{H}] - 5r^2F^2\dot{H}^2 \\ &+ m^2r^2(H - 1)H^2[2H(1 - \alpha + 3\beta) - 6\beta \\ &+ F(1 - \alpha + 6\beta - 3H(1 - \alpha + 2\beta))]. \end{aligned} \quad (21)$$

The “ rr ” component of generalized Einstein equation takes the form:

$$\begin{aligned} \frac{GQ^2NH^6}{4\pi r^2} &= (1 + m^2r^2)NH^4 + 2m^2r^2(1 - 3N)H^3 \\ &+ H^2[N(6m^2r^2 - F^2) - r(2F^2\dot{N} + 3m^2r^2)] \\ &+ 2rF^2H\dot{H}(r\dot{N} + N) - r^2F^2N\dot{H}^2 \\ &+ m^2r^2(H - 1)H^2[1 - \alpha + 6\beta(1 - N) \\ &- H(3(1 - \alpha) - 2N(1 - \alpha + 3\beta) + 6\beta)]. \end{aligned} \quad (22)$$

Further, the generalized Einstein equation along the solid angle is given by,

$$\begin{aligned} \frac{GQ^2NH^6}{4\pi r^3} &= m^2rH^3[(3 - F)N - 1] + 2rF^2N\dot{H}^2 \\ &- FH[rN\dot{F}\dot{H} + F(rN\ddot{H} + r\dot{N}\dot{H} + 2N\dot{H})] \\ &+ H^2[rF\dot{F}\dot{N} + NF\dot{F} + F^2(r\dot{N} + \dot{N}) \\ &+ m^2rF(3N - 1) - 6m^2rN + 3m^2r] \\ &+ m^2rH^2\{(1 - \alpha)[4N - 3 + H(2 - 3N) \\ &+ F(2 - 3N + H(2N - 1))] \\ &+ 2(F - 1)(H - 1)(N - 1)(2 - 2\alpha + 3\beta)\}. \end{aligned} \quad (23)$$

The equations of motion may also be obtained by varying the action with respect to the metric fields N , F and H , respectively. In addition, there is another constraint

equation from the Bianchi identity (10):

$$\begin{aligned}
0 = & \frac{1}{rNH} \left\{ F \left\{ H[2 - 3r\dot{N} - 2N(3 + r\dot{H})] \right. \right. \\
& + 2(3N - 1)r\dot{H} + 2H^2(r\dot{N} + N) \left. \right\} - 2H^2[1 + N(H - 3)] \\
& + (1 - \alpha) \left\{ 2H^2[2 - 3N + H(2N - 1)] \right. \\
& + F[r\dot{N}H^3 + 2r\dot{H}(2 - 3N) + H^2(2 - 4N - 4r\dot{N}) \\
& + H(6N - 4 - 2r\dot{H} + 3r\dot{N} + 4r\dot{H}N)] \left. \right\} \\
& + 2(2\alpha - 3\beta - 2)(H - 1) \left\{ 2H^2(N - 1) \right. \\
& \left. \left. + F[r\dot{N}H^2 + 2r\dot{H}(N - 1) - H(2N - 2 + r\dot{N})] \right\} \right\}, \quad (24)
\end{aligned}$$

where in the above equations the dot denotes a derivative with respect to the radial coordinate r .

Note that when we take $Q = 0$ the above equations of motion reduce to the case of the strong interactive spherical system governed by nonlinear massive gravity, discussed in [41]. Our expressions are in agreement with theirs, expect that the radial metric factor is F^2 in our case but becomes F in their convention. The relations between the parameter spaces (α_3, α_4) and (α, β) are already given in (7). Equation (24) obtained from the Bianchi identity constraint is not an independent equation after we choose $D(r) = 0$.

C. The linearized treatment of nonlinear massive gravity

Following the idea developed by KNT [40, 41], we study the solutions to such a gravitational system in the weak field limit. We can expand the metric factors around a Minkowski background as

$$\begin{aligned}
N(r) &= 1 + n(r), \\
F(r) &= 1 + f(r), \\
H(r) &= 1 + h(r), \quad (25)
\end{aligned}$$

and then investigate the linear perturbations. However, we need to be aware of the fact that the factors n and f can be treated as linear perturbations as in General Relativity, while h could, in principle, take large values since this factor corresponds to the strong interactive nature of the scalar mode of graviton in solar system. Therefore, we need to keep higher orders in h and truncate equations of motion to leading order of n and f . We demonstrate this behavior both in analytical and numerical calculations in the following.

Before expanding the background equations perturbatively, we rescale the radial coordinate by introducing a new metric variable

$$\rho \equiv \frac{r}{H}, \quad (26)$$

and correspondingly introduce a new metric factor

$$1 + \tilde{f} = \frac{1 + f}{1 + h + \rho h'}, \quad (27)$$

where the prime denotes a derivative with respect to ρ . As a consequence, the linearized metric can be expressed as

$$ds^2 = -[1 + 2n(\rho)]dt^2 + [1 - 2\tilde{f}(\rho)]d\rho^2 + \rho^2 d\Omega^2 \quad (28)$$

which is asymptotic to Minkowski background when n and \tilde{f} become negligible.

Apart from the usual curvature invariants, nonlinear massive gravity presents a new basic invariant incorporating both the metric and the Stückelberg scalars, namely $I^{ab} \equiv g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b$. Under unitary gauge it seems that this invariant encounters a divergence on the event horizon if one takes the Minkowskian asymptotic at large scales [65], that is, one obtains singularities in the place where General Relativity had simple horizons. The authors of [47] argue that a black hole solution to nonlinear massive gravity might be viable only when the invariant I^{ab} is non-singular. Though its divergence does not bring any manifest problem to observable variables at background level, it may be a problem for fluctuations passing through the horizon. In the present paper, we focus on the dynamics of background solutions outside the horizon and thus we do not address this issue.

D. The post-Newtonian parameter and the vDVZ discontinuity

To study the nontrivial effects of massive gravity we consider the metric factor h as a perturbation mode. By expanding the equations of motion (21), (22) and (24) presented in the Appendix, consistent with the linearized background (28), we get

$$(2 + m^2 \rho^2) \tilde{f} + 2\rho \tilde{f}' + m^2 \rho^2 (3h + \rho h') = -\frac{GQ^2}{4\pi\rho^2}, \quad (29)$$

$$2\rho n' + 2\tilde{f} + m^2 \rho^2 (2h - n) = -\frac{GQ^2}{4\pi\rho^2} (1 + n), \quad (30)$$

$$\rho n' + 2\tilde{f} = 0, \quad (31)$$

up to leading order.

Under the above approximation the metric factors are:

$$\begin{aligned}
n(\rho) \simeq & -\frac{4GM e^{-m\rho}}{3\rho} + \frac{GQ^2}{8\pi\rho^2} \\
& + \frac{GmQ^2}{16\pi\rho} [e^{m\rho} \text{Ei}(-m\rho) - e^{-m\rho} \text{Ei}(m\rho)], \quad (32)
\end{aligned}$$

$$\begin{aligned}
\tilde{f}(\rho) \simeq & -\frac{2GM e^{-m\rho} (1 + m\rho)}{3\rho} + \frac{GQ^2}{8\pi\rho^2} \\
& + \frac{GmQ^2}{32\pi\rho} \times [(1 - m\rho) e^{m\rho} \text{Ei}(-m\rho) \\
& - (1 + m\rho) e^{-m\rho} \text{Ei}(m\rho)], \quad (33)
\end{aligned}$$

and

$$h(\rho) \simeq -\frac{2GM e^{-m\rho}}{3m^2\rho^3}(1+m\rho+m^2\rho^2) + \frac{GQ^2}{16\pi\rho^2} + \frac{GQ^2}{32\pi m\rho^3} \times [(1-m\rho+m^2\rho^2)e^{m\rho}\text{Ei}(-m\rho) - (1+m\rho+m^2\rho^2)e^{-m\rho}\text{Ei}(m\rho)] , \quad (34)$$

where ‘‘Ei’’ is the exponential integral function defined by $\text{Ei}(x) \equiv \int_{-\infty}^x e^t d\ln t$.

Let us now examine the post-Newtonian parameters for nonlinear massive gravity. The post-Newtonian parameters are strongly constrained by solar system observations (see [64] for a detailed introduction and a comprehensive review), and therefore may be used to place strong constraints on a given theory. The first post-Newtonian parameter γ is defined as the ratio of \tilde{f} and n :

$$\gamma \equiv \frac{\tilde{f}}{n} , \quad (35)$$

in the weak field limit. In the case of General Relativity $\gamma = 1$, as we can immediately read from the RN solution (19). However, in the regime of length scales much smaller than the Compton wavelength ($\rho_m = 1/m$) in nonlinear massive gravity, this parameter can be approximately expressed as

$$\gamma \simeq \frac{32\pi M\rho(1+m\rho) - 6Q^2 e^{m\rho}}{64\pi M\rho - 6Q^2 e^{m\rho}} , \quad (36)$$

where we have made the reasonable assumption that the solar-system mass M is much larger than graviton mass.

It is straightforward to see that when the spherical system does not carry an electric charge ($Q = 0$), we obtain $\gamma \simeq (1+m\rho)/2$, and thus in the massless limit it reduces to $\gamma = 1/2$, which is in stark disagreement with the value in General Relativity. This behavior exactly manifests the vDVZ discontinuity, and thus we face the difficulty of explaining solar-system observations at present. However, when the contribution of Q is taken into account, the post-Newtonian parameter could approach to 1 when ρ is much smaller than the length scale

$$\rho_G = \frac{3Q^2}{32\pi M} . \quad (37)$$

This result implies that the vDVZ discontinuity exists in a charged spherical system governed by nonlinear massive gravity, but the explicit behavior is different from an electro-neutral one.

Finally, from the solutions (32), (33) and (34) we find that the weak-field limit is a good approximation at large distances. However, when the radial coordinate ρ decreases below a critical value

$$\rho_V \equiv \left(\frac{GM}{m^2}\right)^{1/3} , \quad (38)$$

the absolute value of the factor h will increase exponentially and become much larger than unity. This critical

radius is the so-called Vainshtein radius. We perform a detailed analysis on the perturbation equations by keeping all nonlinear order terms of h in the next section.

IV. CHARGED BLACK HOLES AND VAINSHTEIN MECHANISM

Due to the famous Vainshtein mechanism [5] the scalar degree of freedom in massive gravity becomes strongly coupled in the limit of small graviton mass, and thus the linearized treatment performed in equations (29), (30) and (31) is no longer reliable. This is observed by following the evolution of metric factor h below the Vainshtein radius, where the absolute value of h becomes much greater than unity. Consequently, although we can keep treating n and \tilde{f} as small perturbations in this regime, higher order terms in h should be taken into account.

A. Perturbation equations with nonlinear corrections

Keeping leading order in n and \tilde{f} , the perturbed equations of motion including nonlinear corrections of h are given by

$$2\tilde{f} + 2\rho\tilde{f}' + m^2\rho^2 \left[(1 - 2\alpha h + 6\beta h^2)[(1 + \tilde{f})\rho h' + (1 + h)\tilde{f}] + 3h(1 - \alpha h + 2\beta h^2) \right] + \frac{GQ^2}{4\pi\rho^2} = 0 , \quad (39)$$

$$2\tilde{f} + 2\rho n' - m^2\rho^2 \left[n - 2(1 + n + \alpha n)h + (\alpha + \alpha n + 6\beta n)h^2 \right] + \frac{GQ^2(1 + n)}{4\pi\rho^2} = 0 , \quad (40)$$

$$\rho n'[1 - 2\alpha h + 6\beta h^2] + 2\tilde{f}[1 - \alpha h] = 0 , \quad (41)$$

of which the first two correspond to the (00) and (rr) components of the generalized Einstein equations and the last one arises from the perturbed Bianchi constraint. Note that the second derivative of h does not appear in equation (39), since when we transform from r to ρ variable, \tilde{F} acquires a term with the second derivative of h which exactly cancels the one in \tilde{H} .

We first solve equation (39) by neglecting all high-order terms proportional to m^4 , G^2M^2 , m^2GM , G^2Q^4 , and m^2GQ^2 . Therefore, the metric factor \tilde{f} can be expressed in terms of h :

$$\tilde{f} \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} - \frac{m^2\rho^2}{2}(h - \alpha h^2 + 2\beta h^3) . \quad (42)$$

Inserting the expression (42) into the perturbation equation (40), we obtain the radial derivative of n as a function of h :

$$n' \simeq -\frac{GQ^2}{4\pi\rho^3} + \frac{GM}{\rho^2} - \frac{m^2\rho}{2}(h - 2\beta h^3) , \quad (43)$$

and thus the metric factor n can be acquired by performing integration. The key to solving for the metric factors n and f is to extract the solution for h . Therefore, we combine the expressions (42), (43), and the perturbed Bianchi constraint (41), and then we derive the polynomial equation:

$$\frac{GM}{\rho} (1 - 6\beta h^2) - \frac{GQ^2}{4\pi\rho^2} (\alpha h - 6\beta h^2) = m^2 \rho^2 \left[-\frac{3}{2}h + 3\alpha h^2 - (\alpha^2 + 4\beta)h^3 + 6\beta^2 h^5 \right], \quad (44)$$

in which all nonlinear terms of h have been taken into account.

After having obtained equations (42), (43) and (44), we are able to calculate the explicit forms of the metric factors n , \tilde{f} and h under different parameter choices of (α, β) . In the following subsections, we investigate these solutions further.

B. Case I: $\alpha = \beta = 0$

Let us first consider the special subclass with $\alpha = \beta = 0$. In this case all higher order terms of h vanish automatically, and thus our task reduces to solving the linearized equations (29), (30) and (31). Therefore, the corresponding solutions of metric factors are already given in equations (32), (33) and (34).

The post-Newtonian parameter takes the value of $1/2$ in the regime of length scales of interest. Although this parameter increases to 1 at very small values of radial coordinate due to the effect of electric charge, the corresponding length scale is deeply inside the horizon of the black hole. As a consequence, the standard General Relativity result can not be recovered in the spherical system governed by massive gravity with such a parameter choice, and therefore this case is already observationally ruled out.

C. Case II: $\alpha \neq 0$ and $\beta = 0$

We now proceed to the case with a vanishing β but non-vanishing α . In principle, one can solve the h -equation (41) exactly, however the resulting expression is quite complicated, hiding the underlying physics. Therefore, instead of finding an exact solution to h we solve the perturbed Bianchi equation approximately by keeping dominant terms in h . Different from the electro-neutral case of [41], both the solar-system mass M and its electric charge Q contribute to the L.H.S of equation (44). Therefore, we need to solve (44) by assuming its L.H.S is dominated by M and Q respectively.

We first consider that the contribution of the electric charge Q becomes dominant in the L.H.S of (44) when $\rho \ll \rho_V$. In this limit we solve equation (44) and expand

h up to order $\mathcal{O}(\rho^2)$:

$$h \simeq -\frac{\rho_Q^2}{\alpha^{1/2}\rho^2} + \frac{3}{2\alpha} - \frac{\rho_V^3\rho}{2\alpha\rho_Q^4}, \quad (45)$$

where we have introduced a new parameter for a critical length scale

$$\rho_Q \equiv \left(\frac{GQ^2}{4\pi m^2} \right)^{1/4}. \quad (46)$$

One can see that the approximate expression for h is valid only when $\rho < \rho_Q$. Comparing with the non-charged system analyzed in [41], we find that the leading term of h in a charged black hole is proportional to ρ^{-2} instead of ρ^{-1} . Moreover, the suppression scale is ρ_Q rather than the Vainshtein radius ρ_V . However, since for a group of canonical parameters accommodating with astronomical data ρ_V is usually much bigger than ρ_Q , we deduce that the contribution of the third term in the R.H.S. of equation (45) is considerable in a wide regime of length scales. Then, from equations (42) and (43) we can derive the expressions for n and \tilde{f} through a Taylor expansion

$$n \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{m^2\rho_Q^2}{2\alpha^{1/2}} \ln(m\rho), \quad (47)$$

$$\tilde{f} \simeq \frac{GQ^2}{4\pi\rho^2} - \frac{GM}{\rho} - \frac{m^2\rho_Q^2}{\alpha^{1/2}} + \frac{GM\rho}{2\alpha^{1/2}\rho_Q^2}, \quad (48)$$

up to order $\mathcal{O}(\rho^2)$. By observing (47) and (48), when $2GM < \rho < \rho_Q$, the corrections of massive gravity to the General Relativity results are quite small. When ρ is larger than ρ_Q but smaller than ρ_V , the higher order corrections are suppressed by ρ_V and the formulae are in agreement with Ref. [41].

When the radial coordinate ρ evolves to the regime which is close to ρ_Q but still smaller than ρ_V , the main contribution of the L.H.S of (44) comes from the mass term M . In this case, we only keep the leading order in h , obtaining

$$h \simeq -\frac{\rho_V}{\alpha^{2/3}\rho}, \quad (49)$$

$$n \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{GM\rho}{2\alpha^{2/3}\rho_V^2}, \quad (50)$$

$$\tilde{f} \simeq \frac{GQ^2}{8\pi\rho^2} - \frac{GM}{\rho} + \frac{GM}{2\alpha^{1/3}\rho_V} + \frac{GM\rho}{2\alpha^{2/3}\rho_V^2}. \quad (51)$$

Comparing our results with the analysis of [41] we find that the above three solutions are consistent with those in the spherical system without a charge. This implies that there must exist an intermediate regime along the radial coordinate in which the behaviors of the metric factors of charged black holes are the same as those of neutral black holes.

In order to provide a more transparent picture of the dynamics of the charged black hole described by nonlinear massive gravity with the above parameter choice

($\alpha \neq 0$ and $\beta = 0$), we numerically evolve the perturbation equations. In particular, without loss of generality we consider $\alpha = 1$, and we choose the graviton mass $m = 10^{-20}M_P$ and the solar-system mass $M = 10^6M_P$, setting also the Planck unit $M_P^2 = 1/G = 1$.² Correspondingly, this group of parameters yield the Compton wavelength $\rho_m = 10^{20}$, which is much larger than the Vainshtein radius $\rho_V = 2.15 \times 10^{15}$. Moreover, we consider the source term of the matter field to be a weak charge $Q_W = 1.77 \times 10^3$ or a strong charge $Q_S = 3.54 \times 10^6$, respectively.

In Fig 1 we depict the absolute value of the metric factor h as a function of the radial coordinate ρ . From the above parameter choices we find that the suppression scales associated with the electric charges are given by $\rho_{QW} = 2.24 \times 10^{11}$ (which is represented as a purple dashed line for the weak charge Q_W) and $\rho_{QS} = 10^{13}$ (which is represented as a pink dashed line for the strong charge Q_S). The wide sparse shadow regime denotes the space between the inner horizon and outer horizon in the case of the weakly charged black hole, and the narrow dark shadow regime denotes the space between the two horizons in the case of the strongly charged black hole.

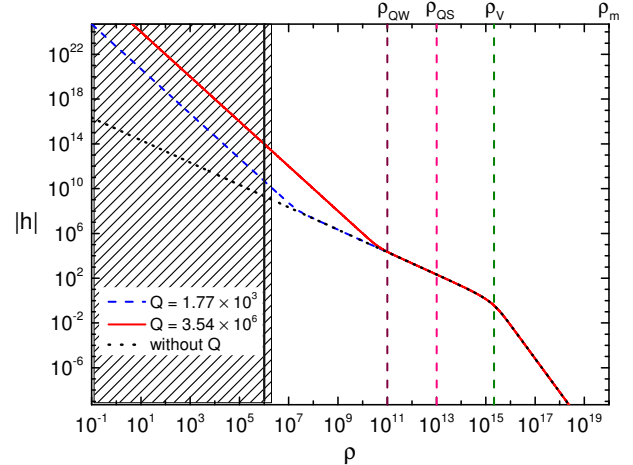


Figure 1. Plot of the evolutions of the absolute value of metric factor h as functions of radial coordinate ρ in a charged spherical system described by nonlinear massive gravity. The model parameters are taken as: $\alpha = 1$ and $\beta = 0$. In the numerical calculation, we take $m = 10^{-20}$ and $M = 10^6$. The corresponding Compton wavelength $\rho_m = 10^{20}$ and the Vainshtein radius $\rho_V = 2.15 \times 10^{15}$ are denoted on the top of the figure. The values of weak and strong electric charges are provided in the plot. All dimensional parameters are in Planck units.

From Fig. 1 we that there exist three different slopes for the metric factor h along the radial coordinate ρ in a charged spherical system. When ρ is greater than the Vainshtein radius ρ_V but less than the Compton wavelength ρ_m , $|h|$ scales approximately as ρ^{-3} and its value is much smaller than 1, and therefore the weak field limit at large distances is valid. When ρ evolves to be smaller than ρ_V , $|h|$ becomes much larger than unity quickly and scales as ρ^{-1} . This behavior is in precise agreement with (49). When ρ decreases to a regime which is much shorter than ρ_Q , we observe that the slope of $|h|$ changes again which gives rise to $|h| \sim \rho^{-2}$. This transition on $|h|$ -slope implies that the contribution of electric charge Q becomes dominant in determining the dynamics of scalar graviton. Moreover, the length scale for the slope transition on h depends on the value of ρ_Q and thus it is determined by the combination of Q and m . For a fixed graviton mass m , the value of ρ_Q in a strongly charged spherical system (denoted by the purple dashed line) is much larger than that in the case of weak charge (denoted by the pink dashed line). Correspondingly, the transition of $|h|$ -slope in the case of strong charge occurs at a larger distance as shown in the red solid curve, while the transition of $|h|$ -slope in the case of weak charge takes place at a smaller distance as shown in the blue dashed curve.

In Fig. 2 we depict the ratios n'/n'_{GR} and \tilde{f}/\tilde{f}_{GR} as functions of radial coordinate ρ . From the upper graph

² We mention that the parameter choices for m and M are not realistic, however they do correspond to the same qualitative behavior with the realistic data, with the advantage that the corresponding figures are much more transparent and compact.

we observe that the correction to the metric factor n from the electric charge Q is very small, since the three curves (a red solid line representing for strongly charged case, a blue dashed line representing the weakly charged case, and a black dotted line representing the non-charged case) almost coincide. Additionally, when ρ is outside the Vainshtein radius, the ratio of n' in nonlinear massive gravity and n' in General Relativity takes the value of $4/3$. This numerical result is in agreement with the analytic result obtained in the previous section when one compares the first term in the R.H.S of equation (32) and the second term in the R.H.S of equation (47). However, this ratio tends to 1 when ρ lies in the regime of $\rho < \rho_V$ due to the Vainshtein effect. As a consequence, the metric factor n of nonlinear massive gravity roughly recovers the General Relativity result inside the Vainshtein radius.

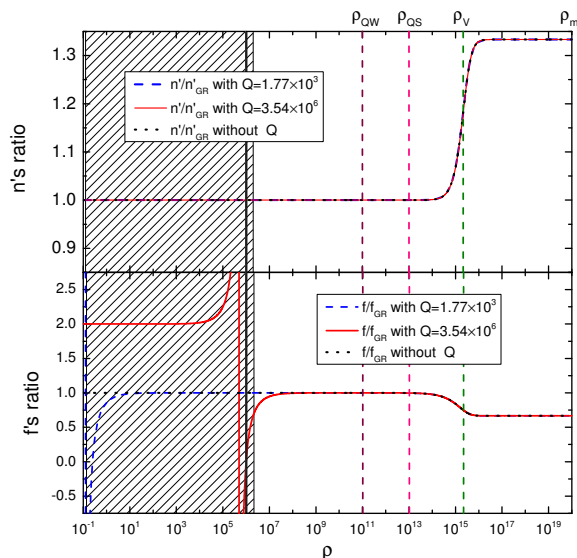


Figure 2. Plot of the evolutions of the ratios n'/n'_{GR} and \tilde{f}/\tilde{f}_{GR} as functions of radial coordinate ρ in a charged spherical system described by nonlinear massive gravity. In the numerical calculation, the parameters are chosen to be the same as those provided in Fig. 1. The “ f ” in the lower panel represents for the metric factor \tilde{f} in the main text.

From the evolutions of f in charged gravitational backgrounds, depicted in the lower graph of Fig. 2, we can see that they are different from the non-charged one when ρ approaches to the inner horizons. This yields the non-trivial modification to the post-Newtonian parameter at small length scales as shown in Fig. 3. However, one needs to be aware of the fact that in the regime of small length scales the perturbative treatment of n and \tilde{f} is not valid and the definition of the horizons is not clear. In the figures we depict this regime by the shadowed region, and we would like to emphasize that an exact description of this regime would require a complete non-perturbative

analysis. Such an analysis lies beyond the scope of the present work, and it is left for a future investigation.

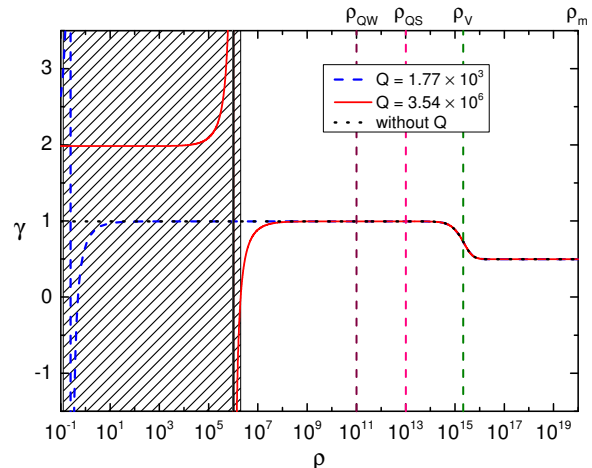


Figure 3. Plot of the evolutions of the post-Newtonian parameter γ as functions of radial coordinate ρ in a charged spherical system described by nonlinear massive gravity. In numerical calculation, the parameters are chosen to be the same as those provided in Fig. 1.

D. Case III: $\beta \neq 0$

The case where both α, β parameters are non vanishing can be divided in two subcases: $\beta < 0$ and $\beta > 0$. Generally, we are unable to obtain the exact solution of the metric factor h from equation (44). Therefore, we can only solve the equations of motion semi-analytically by keeping the leading order terms and then compare with numerical computations. However, it is interesting to notice that, in the case of $\beta > 0$ there exists a class of exactly analytic solutions for a special family of parameter choice $\beta = \alpha^2/6$, which was also observed in Schwarzschild-like solutions in massive gravity [43, 44, 47]³. In the following we will analyze these cases in both analytical and numerical ways, respectively.

³ An RN solution was constructed in the de Sitter (dS) background in [43, 47]. In the present work we solve the exact equations of motion of a charged spherical system in massive gravity and we obtain an exact form of RN-dS which will be shown in later context.

1. $\beta < 0$

Let us first consider the subcase $\beta < 0$. By solving h numerically we find that there is only one branch of solution which is real between the Schwarzschild radius and infinity. It evolves from a very large value to a small constant, as moving from the Schwarzschild radius to the Compton wavelength. Around the Vainshtein radius it is of order $\mathcal{O}(1)$. A strong charge suppresses the value of h slightly when it is close to and inside the Schwarzschild radius. Beyond this range the charge does not cause an obvious effect. This can be seen in Fig. 4, where the three curves almost coincide.

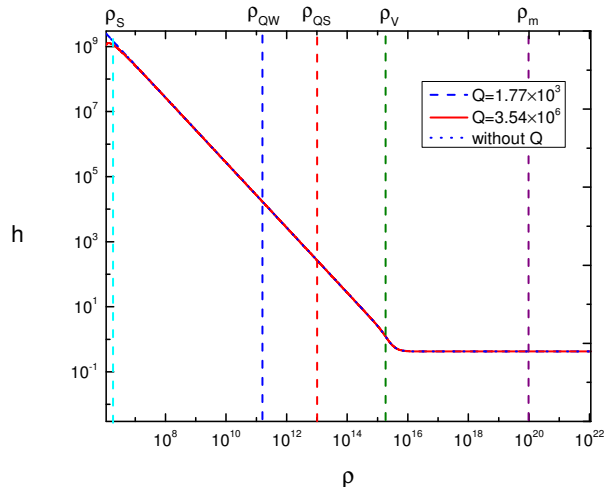


Figure 4. Plot of the evolutions of metric factor h as functions of radial coordinate ρ in a charged spherical system described by nonlinear massive gravity. The parameters of the massive gravity model are taken as: $\alpha = 1$ and $\beta = -1/2$. Moreover, m and M are the same as those provided in Fig 1.

As shown in this figure $h \gg 1$ in the range $r_S < \rho < \rho_Q$. Thus, in the semi-analytical calculation, we keep the leading order terms of the equations of motion for the metric factors, and then we acquire

$$h \simeq \frac{1}{\beta^{1/3}} \left(\frac{\rho_Q^4}{\rho^4} - \frac{\rho_V^3}{\rho^3} \right)^{1/3}, \quad (52)$$

$$n' \simeq -\frac{m^2}{2\beta^{1/3}} \left(\frac{\rho_Q^4}{\rho} - \rho_V^3 \right)^{1/3}, \quad (53)$$

$$\begin{aligned} \tilde{f} \simeq & -\frac{GQ^2}{8\pi\rho^2} + \frac{\alpha m^2}{2\beta^{2/3}} \left(\frac{\rho_Q^4}{\rho} - \rho_V^3 \right)^{2/3} \\ & - \frac{m^2}{2\beta^{1/3}} \left(\rho_Q^4 - \rho\rho_V^3 \right)^{1/3} \rho^{2/3}. \end{aligned} \quad (54)$$

From the above semi-analytic results we find that the corrections to \tilde{f} and n are so dramatic that the usual Schwarzschild-like gravitational potential (in form of $1/\rho$)

is exactly canceled. This result agrees with the conclusion of [41] in which a neutral spherical system was considered.

Finally, for the gravitational potential associated with the electric charge, the RN-like factor $\frac{Q^2}{\rho^2}$ in g_{tt} also disappears, but the dominant term in the small radius regime roughly takes the form of $\rho^{2/3}$. For the g_{rr} component the sign in front of $\frac{Q^2}{\rho^2}$ changes from positive to negative compared to the General Relativity result. These new features suggest that the charged spherical system described by nonlinear massive gravity is much more smooth near the origin than that described by General Relativity. However, since below the Vainshtein radius the difference from General Relativity is quite significant in this case, the corresponding parameter space is likely ruled out by local, solar system experiments.

2. $\beta > 0$

We now consider the subcase $\beta > 0$. Apart from the previous solution in which h is large-valued below the Vainshtein radius, there exists a second solution in which the metric factor h always takes a small value outside the Schwarzschild radius. This was first observed in [46] within a spherically symmetric neutral background. Similarly to the previous subcase we solve the equations of motion numerically, obtaining the solution for the metric factor h as shown in Fig. 5. In addition, we extract the

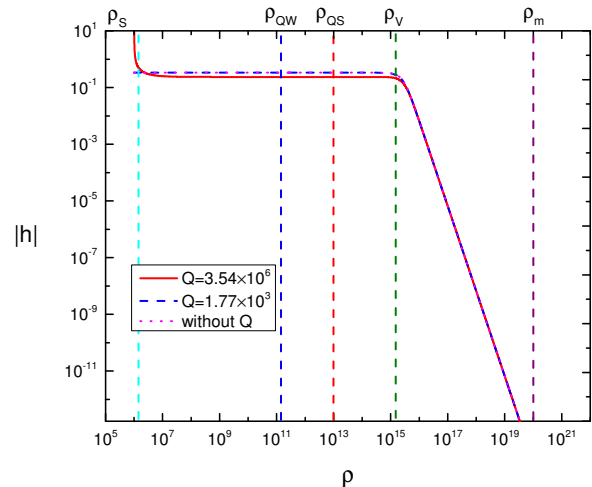


Figure 5. Plot of the evolutions of metric factor h as functions of radial coordinate ρ in a charged spherical system described by nonlinear massive gravity. The parameters of the massive gravity model are taken as: $\alpha = 1$ and $\beta = 3$. Other parameters are the same as those used in Fig. 4.

evolutions of \tilde{f} and n' along the radial coordinate ρ . In

order to provide a clearer picture of the difference of the dynamics in massive gravity from that of General Relativity, in Fig. 6 we plot the ratios n'/n'_{GR} and \tilde{f}/\tilde{f}_{GR} , respectively.

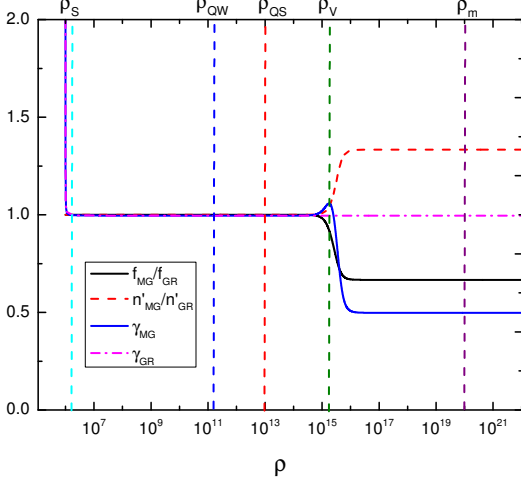


Figure 6. Plot of the evolutions of the ratios n'/n'_{GR} , \tilde{f}/\tilde{f}_{GR} , and the quotient $\gamma \equiv \frac{\tilde{f}'}{n'}$ as functions of radial coordinate ρ in a charged spherical system described by nonlinear massive gravity. The parameters of the massive gravity model are taken as: $\alpha = 1$, $\beta = 3$. Other parameters are the same as those used in Fig. 4. The ‘f’ in the plot represents the metric factor \tilde{f} in the main text.

From Fig. 5 we can see that in a strongly charged spherical system the absolute value $|h|$ can become larger than 1 inside the Schwarzschild radius. As we move away from the Schwarzschild radius h evolves to a constant, which coincides with the value when the charge is weak. After crossing the Vainshtein radius h approaches 0 rapidly. By comparing \tilde{f} and n' with the results of General Relativity, we can clearly see the Vainshtein mechanism from Fig. 6.

Since $|h|$ is always smaller than unity outside the Schwarzschild radius, we can solve its equation of motion by neglecting the terms proportional to h^3 and h^5 . Correspondingly, the approximate solution is given by

$$h \simeq \frac{3\rho^4 - 2\alpha\rho_Q^4}{12(\alpha\rho^4 - 2\beta\rho_Q^4 + 2\beta\rho\rho_V^3)} \times \left\{ 1 - \left[1 + \frac{48\rho\rho_V^3(\alpha\rho^4 - 2\beta\rho_Q^4 + 2\beta\rho\rho_V^3)}{(3\rho^4 - 2\alpha\rho_Q^4)^2} \right]^{1/2} \right\}, \quad (55)$$

which proves to be in good agreement with the numerical solution. The corresponding expressions for \tilde{f} and n' are obtained from equations (42) and (43), however they are quite complicated and thus we do not present

them explicitly. Furthermore, we would like to point out that in this case the dynamical features of the metric factors \tilde{f} and n are quite similar to the case of $\alpha \neq 0$ and $\beta = 0$, as can be seen by comparing Fig. 6 and Fig. 2. This implies that the solution obtained in the $\beta = 0$ case might be dynamically stable in the parameter space of the nonlinear massive gravity model.

3. $\beta = \alpha^2/6$: An exact analytic solution

In last subsection we examine a family of solutions in the model of nonlinear massive gravity under the particular parameter choice $\beta = \alpha^2/6$. This special parameter choice was first noticed in [43], where the exact SdS and RN-dS solutions were obtained with an arbitrary cosmological constant term. This relation was also applied by the authors of [47], who constructed a special family of black hole solutions on a fixed dS background.

We insert the relation $\beta = \alpha^2/6$ into the nonlinear equation of motion (44) and we find that there exists a special solution for h , namely

$$h = \frac{1}{\alpha}, \quad (56)$$

which implies a constant metric factor $H = (1 + \alpha)/\alpha$. Then we substitute the solution (56) into the exact background equation of motion (8) and the Bianchi constraint (24). It is easy to verify that the constraint equation (24) is automatically satisfied. Working with the r coordinate directly we see that the main equation (8) yields the following exact solution:

$$N^2(r) = F^2(r) = \frac{(1 + \alpha)^2}{\alpha^2} + \frac{GQ^2(1 + \alpha)^4}{4\pi\alpha^4 r^2} - \frac{r_M}{r} - \frac{m^2 r^2}{3\alpha}, \quad (57)$$

where r_M is an integration constant.

We can perform the following coordinate rescaling:

$$t \rightarrow \frac{\alpha}{1 + \alpha} t, \quad r \rightarrow \frac{1 + \alpha}{\alpha} r, \quad (58)$$

and introduce two coefficients

$$\tilde{r}_S \equiv \frac{\alpha^3 r_M}{(1 + \alpha)^3}, \quad r_\Lambda \equiv \frac{\sqrt{3\alpha}}{m}, \quad (59)$$

which are related to the Schwarzschild radius and the de Sitter radius, respectively. We get the exact form of the RN-dS-like solution as

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2, \quad (60)$$

with

$$A(r) = 1 + \frac{r_Q^2}{r^2} - \frac{\tilde{r}_S}{r} - \frac{r^2}{r_\Lambda^2}, \quad (61)$$

where the forms of r_Q , \tilde{r}_S and r_Λ are provided in (20) and (59), respectively. The above solution can recover

the standard RN result in General Relativity, and \tilde{r}_S coincides to the usual Schwarzschild radius r_S when we take $m = 0$. Furthermore, our result is in agreement with the one obtained in [43], however we did not introduce an additional cosmological constant in order to see whether and how a pure massive gravity model can yield a dS background by itself.

Moreover, the coefficient r_Λ is determined by the combination of the graviton mass m and the model parameter α . When α is of order $\mathcal{O}(1)$ we can apparently observe that the property of dS background is completely determined by the graviton mass. If we apply this feature in a cosmological setup, the existence of a tiny graviton mass can drive a late-time acceleration of the universe and thus might explain the present cosmological observations. However, if we tune α to an extremely large value then the effect of the graviton mass can be decreased and the background dynamics approach those of General Relativity. Mathematically this effect can be seen by the way \mathcal{K} appearing in the graviton potential \mathcal{U} is roughly proportional to $1/\alpha$ and thus it yields the amplitude of the effective energy-momentum tensor $X_{\mu\nu}$ in form of $1/\alpha$. Eventually, the behavior of nonlinear massive gravity approach standard Einstein gravity in the limit of $\alpha \gg 1$.

V. CONCLUSIONS

In the present work, we theoretically investigated the solutions of a spherically symmetric system involving a static electric field in the context of nonlinear massive gravity. Due to properties of the graviton potential in the dRGT model, the BD ghost which historically plagues all massive gravity theories can be removed. However, inherited from other massive gravity models, the longitudinal mode of gravitons is strongly coupled and thus greatly affect the gravitational potential at macroscopic scales. Therefore, this model is expected to be constrained by solar system observations.

Depending on the different dynamics of our solutions, the solution parameter space can be roughly categorized into three parts:

- The first class corresponds to $\alpha = \beta = 0$ and thus the graviton's potential takes a fixed form. The solution in this subclass is well described at the perturbative level, but the vDVZ discontinuity cannot be avoided. However, the post-Newtonian parameter in this class fails to agree with General Relativity and thus the corresponding parameter choice is observationally ruled out.
- In the second subclass, we keep $\beta = 0$ but we allow α to be an arbitrary constant. The corresponding solution shows that General Relativity can be recovered between the outer horizon of the black hole and the Vainshtein radius by virtue of the Vainshtein mechanism. This scenario is similar to the

case of the neutral black hole in massive gravity. However, the existence of an electric charge could increase the value of the metric factor h within a newly defined radius ρ_Q and thus the detailed evolutions of time-like and space-like metric components behave differently from those of a neutral black hole. Namely, the metric factor n obtains a logarithmic correction when the radius is close to the outer horizon.

- The third subclass of parameter choice is the most general in the parameter space, which requires both α and β to be non-vanishing. In this case the dynamics of solutions behave dramatically different depending on the positivity of β . When $\beta < 0$ the strongly coupled scalar graviton greatly decreases the strength of gravity at small length scales, and thus the usual Schwarzschild-like gravitational potential totally disappears which severely challenges all astronomical observations. However, if β is positive General Relativity can be recovered again through the Vainshtein mechanism. This behavior is similar to the solution in the second subclass with $\beta = 0$. Therefore, the solution in this case, together with the solution in the second subclass, might provide a certain parameter space for nonlinear massive gravity to conform with current solar system observations.
- Finally, there exists a particular parameter choice in the last subclass which suggests $\beta = \alpha^2/6$. Under this condition the background equations of motion can be solved exactly and yield a solution which is identical to the RN-dS form in which only the dS radius r_Λ contains the model parameter α . The exact solution with such a special parameter choice can recover the standard result in General Relativity in the limit of either a vanishing graviton mass or an extremely large value of the parameter α .

Note that, the structure of solutions does not only depend on the sign of β , but also on a diverse structure in the parameter space of α and β . The authors of [42] analyzed the whole parameter space of a neutral spherical system under nonlinear massive gravity and found that each region showed a completely different behavior with respect to the inner solutions (near the body) and the asymptotic solutions, and thus affect the Vainshtein mechanism. It would be interesting to perform a global analysis on the parameter space in our case too.

Lastly, we would like to mention that in the present work we only focus on the analysis of background solutions of a charged spherical system in the dRGT model, without studying the perturbations. At the background level the parameter space of model parameter has already shown rich behaviors and a sizable regime has already been ruled out by observations. Due to the Vainshtein mechanism there exists an island in the parameter space

which is consistent with astronomical data at the background level. Moreover, this situation could dramatically change if the perturbations were taken into account and we expect the model parameters appearing in the nonlinear massive gravity would be further constrained. However, we leave such a study for future investigation.

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