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We compute the light-cone momentum distributions of pions in the nucleon in chiral effective theory using both pseudovector and pseudoscalar pion–nucleon couplings. For the pseudovector coupling we identify δ-function contributions associated with end-point singularities arising from the pion–nucleon rainbow diagrams, as well as from pion bubble and tadpole diagrams which are not present in the pseudoscalar model. Gauge invariance is demonstrated, to all orders in the pion mass, with the inclusion of Kroll-Ruderman couplings involving operator insertions at the πNN vertex. The results pave the way for phenomenological applications of pion cloud models that are manifestly consistent with the chiral symmetry properties of QCD.

PACS numbers:

I. INTRODUCTION

Contributions from the meson cloud of the nucleon to deep-inelastic scattering (DIS) were first discussed in the early 1970s by Drell, Levy and Yan [1] and Sullivan [2]. It was of little interest, however, until in the early 1980s a possible enhancement of the pion cloud of a bound nucleon was suggested as an explanation of the nuclear EMC effect [3, 4]. While that issue is still of interest, the modern importance of the pion cloud in high-energy scattering originates in the work of Thomas [5], who showed (within a particular chiral quark model) that the pion cloud contribution provided a natural explanation for the observed excess of non-strange over strange sea quarks in the nucleon. This calculation also predicted an excess of $\bar{d}$ over $\bar{u}$ quarks in the nucleon sea, which was confirmed by the violation of the Gottfried sum rule observed in the NMC experiment at CERN [6], and the $\bar{d}/\bar{u}$ ratio measured in the Drell-Yan reaction in pd and pp scattering by the E866/NuSea collaboration at Fermilab [7].

Although this excess of $\bar{d}$ over $\bar{u}$, as well as the $s-\bar{s}$ asymmetry [8], was motivated by the physics of spontaneous chiral symmetry breaking in QCD, the early calculations were based on models whose connections to QCD were not manifest. It was later realized [9], however, that because of their origin in chiral loops, these contributions had a nonanalytic dependence on quark mass that could only be generated by a Goldstone mechanism, which placed these effects on a more rigorous theoretical footing. In fact, it is a model-independent consequence of spontaneous chiral symmetry breaking in QCD that $\bar{d}-\bar{u}$ and $s-\bar{s}$ are nonzero. The only open question is how large these asymmetries actually are.

This issue has become even more important in the last few years because of the widespread interest in the so-called five-quark components of baryon wave functions [10]. In particular, there is some evidence that the Roper resonance may not be a simple excitation of a single valence quark in the nucleon but rather involves a large five-quark contribution [11–15]. This naturally leads to questions about such components in the wave function of the nucleon, and there have been suggestions that these may be sizable. Even though the pentaquark now appears to be defunct, legitimate questions about five-quark components in baryon spectroscopy remain, especially within models such as the chiral quark soliton model [16].

In the context of low energy tests of the Standard Model this issue is also very topical, with a potential asymmetry between the s and $\bar{s}$ parton distributions potentially yielding a large correction to the value of $\sin^2\theta_W$ derived from the NuTeV measurement [17, 18]. Model-dependent estimates of the five-quark component of the nucleon wave function involving three light quarks plus $s\bar{s}$ pairs were recently used to investigate this correction [19–22].

In all of these examples, the contribution from the meson cloud of the nucleon has been a crucial factor in reconciling the physics results. Indeed, because of the model-independent constraints imposed through chiral symmetry on the nonanalytic behavior [9, 23, 24], the presence of meson cloud contributions to the various observables is firmly established, justifying the extensive theoretical attention that has been paid to this issue over the past decade. (We should note, however, that in chiral perturbation theory observables in general receive contributions from pions as well as local short-distance operators, or counter-terms, and it is possible under renormalization to move strength between them [25]; the nonanalytic contributions, on the other hand, remain model independent.) A further motivation relates to the fact that lattice QCD is now providing considerable information on the moments of parton distribution functions

[26], but at larger quark masses than occur in nature. The extrapolation of those moments as a function of quark mass to the physical point is critically dependent on knowing the correct nonanalytic behavior [27], and that in turn is uniquely determined by the pion cloud of the nucleon.

Given the phenomenological importance of the meson cloud, it is unfortunate that the literature contains a number of sometimes contradictory results for meson cloud contributions to DIS. In particular, the majority of calculations, from the original work of Drell *et al.* [1] and Sullivan [2], to the early studies of the nonanalytic behavior [9], and essentially all model analyses of pion cloud effects in DIS in between [28], have used pseudoscalar (PS) coupling, which is, by itself, inconsistent with chiral symmetry [29–31]. The restoration of chiral symmetry can be achieved through the addition of a scalar "σ" field; however, determining its practical consequences for DIS is problematic because of uncertainties in identifying the nature of low-mass scalars in meson spectroscopy [32].

More recently, the matrix elements of nonsinglet twist-2 operators were computed by Chen and Ji [33, 34] and Arndt and Savage [35] using lowest order, heavy baryon chiral perturbation theory. By construction this theory uses pseudovector (PV) πN couplings, which are manifestly invariant under chiral transformations [31, 36, 37]. The twist-2 matrix elements are related through the operator product expansion to moments of parton distributions measured in inclusive DIS. At lowest order in the low-energy expansion (minimum number of derivatives of the pion field $\boldsymbol{\pi}$), the effective chiral Lagrangian for the interaction of pions and nucleons, consistent with chiral symmetry, can be written as [31, 36–38]

$$\mathcal{L}_{\pi N} \;=\; \frac{g_A}{2f_\pi}\,\bar\psi_N\gamma^\mu\gamma_5\,\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\,\psi_N \;-\; \frac{1}{(2f_\pi)^2}\,\bar\psi_N\gamma^\mu\,\boldsymbol{\tau}\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\,\psi_N, \tag{1}$$

where ψ_N is the nucleon field, $f_\pi = 93$ MeV is the pion decay constant, and $g_A = 1.267$ is the nucleon axial vector charge. The first term in the Lagrangian (1) gives rise to the well-known "rainbow" diagram in which a pion is emitted and reabsorbed by the nucleon at different space-time points. The second is the so-called Weinberg-Tomozawa term [31, 39], in which two pion fields couple to the nucleon at the same point, and gives the leading contribution to S-wave pion–nucleon scattering [40]. It also generates the pion tadpole or bubble diagrams, which in the presence of external fields generally give non-vanishing contributions to nucleon matrix elements. This term does not appear in the PS theory, which leads to some of the differences between the moments of twist-2 parton distributions from pion loops computed in the PS [9] and PV [33–35] theories.

In this paper we present a detailed analysis of the light-cone momentum distributions of pions, and the corresponding recoil baryons, inside a physical nucleon within the chirally symmetric effective field theory defined by the Lagrangian (1). We further contrast the results with those obtained in the PS theory, conventionally defined by the Lagrangian

$$\mathcal{L}_{\pi N}^{\rm PS} \;=\; -g_{\pi NN}\,\bar\psi_N\,i\gamma_5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi}\,\psi_N, \tag{2}$$

where the πNN coupling constant $g_{\pi NN}$ is related to g_A and f_π by the Goldberger-Treiman relation [41],

$$\frac{g_{\pi NN}}{M} \;=\; \frac{g_A}{f_\pi}\,, \tag{3}$$

with M the nucleon mass. Unlike the earlier chiral effective theory calculations which only computed the light-cone distributions of pions [34] or considered the nonanalytic behavior of their moments to lowest order in the pion mass m_π [33, 35], we compute the complete set of diagrams relevant for DIS from nucleons dressed by pions resulting from the Lagrangians (1) and (2), without taking the heavy baryon limit. In particular, we demonstrate explicitly the consistency of the computed distribution functions with electromagnetic gauge invariance. This requires consideration of the Kroll-Ruderman terms [42], which although entering at higher orders in m_π, are nonetheless essential for ensuring conservation of charge to all orders in m_π [43].

Formally, the light-cone distribution functions can be defined by considering the vertex renormalization constant $Z_1^{-1} - 1$ for the physical $\gamma^* NN$ vertex. If the contributions from the pion cloud are not large, then $Z_1 \approx 1$ and one has $Z_1^{-1} - 1 \approx 1 - Z_1$. The light-cone distributions $f_i(y)$ associated with a particular contribution i can then be defined as [44]

$$(1 - Z_1)_i \;=\; \int_0^1 dy\, f_i(y), \tag{4}$$

where $y = k_+/p_+$ is the momentum fraction of the physical nucleon carried by the pion, with k and p the four-momenta of the pion and physical nucleon, respectively. Note that in this work we define the light-front "+" and "−" components of a four-vector v^μ as $v^\pm = v^0 \pm v^z$. It will be most natural to perform the calculation in light-front coordinates; however, as demonstrated in Ref. [45] for the nucleon self-energy, the results for the model-independent, long-distance physics associated with the pion cloud are reproduced in any formalism, be it instant form, covariant, or light-front.

Because the tadpole or bubble diagrams in the PV theory involve pions emitted and absorbed at the same point, these can only contribute at zero light-cone momentum fraction, $y = 0$. A novel feature of the PV theory, however, is the appearance of δ-function contributions arising also from the rainbow diagram at $y = 0$. Inclusion of these singular terms is in fact vital for preserving gauge invariance in the PV theory. In the following we will elucidate the origin of the δ-function terms and discuss their possible physical implications.

In the rest of this paper, we derive in Sec. II the light-cone momentum distribution functions for pions in a physical nucleon, including both the pion rainbow and bubble contributions. The analogous nucleon momentum distributions are discussed in Sec. III, where the Kroll-Ruderman couplings required by gauge invariance are introduced. The complete set of pion and nucleon distributions allows us for the first time to explicitly verify the symmetry relations between them. The application of the results to inclusive DIS is described in Sec. IV within the convolution approach, where we discuss the effect of the pion loop corrections, including the δ-function contributions, on various sum rules. Finally, we summarize our findings and their implications in Sec. V.

II. PION LIGHT-CONE MOMENTUM DISTRIBUTIONS

Assuming isospin symmetry, the contributions from pion loops to nucleon matrix elements cancel for isoscalar combinations. In this work we therefore consider only isovector light-cone momentum distributions, $f_i^{(iv)}(y) \equiv f_i^p(y) - f_i^n(y)$. The distributions are evaluated by taking the "+" components of the operators. We begin by considering the contributions from diagrams involving a direct coupling to the pion, as illustrated in Fig. 1.

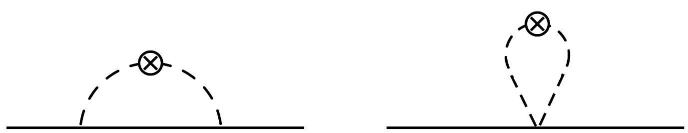

FIG. 1: Contributions to the pion light-cone momentum distributions in the nucleon, from (a) the pion rainbow, and (b) the pion bubble diagrams.

The isovector light-cone momentum distribution function for the pion rainbow diagram in Fig. 1(a) is given by

$$f_\pi^{(iv)}(y) = 4M \left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \bar{u}(p)\, (\not{k}\gamma_5) \frac{i(\not{p} - \not{k} + M)}{D_N} (\gamma_5 \not{k}) u(p) \frac{i}{D_\pi} \frac{i}{D_\pi}\, 2k^+ \delta(k^+ - yp^+), \tag{5}$$

where the pion and nucleon propagators are given by

$$D_\pi \equiv k^2 - m_\pi^2 + i\varepsilon, \qquad D_N \equiv (p-k)^2 - M^2 + i\varepsilon, \tag{6}$$

respectively. In Eq. (5) and throughout this work the spinors $u(p)$ are normalized such that $\bar{u}(p)\, u(p) = 1$. (Note that for the pion coupling diagrams, the isovector combination $p - n$ is equivalent to the notation "$\pi^+ - \pi^-$" [34].) Using the Dirac equation, the distribution function in Eq. (5) can be decomposed into several terms,

$$f_\pi^{(iv)}(y) = -4i \left(\frac{g_A}{2f_\pi}\right)^2 \int \frac{d^4k}{(2\pi)^4} \left[\frac{4M^2\, p\cdot k}{D_\pi^2 D_N} + \frac{2M^2}{D_\pi^2} + \frac{p\cdot k}{D_\pi^2}\right] 2y\, \delta\left(y - \frac{k^+}{p^+}\right). \tag{7}$$

To evaluate the expression in Eq. (7), we proceed to first integrate over the k^- component of the pion momentum. Without loss of generality, we work in a frame where $p_\perp = 0$. For the first term in (7) we use Cauchy's integral formula and take the single nucleon pole by closing the contour in the upper half-plane,

$$D_N = (p^+ - k^+)\left(p^- - k^- - \frac{k_\perp^2 + M^2 + i\varepsilon}{p^+ - k^+}\right) \longrightarrow 0. \tag{8}$$

Note that the arc contribution at infinity vanishes for this term. A straightforward calculation then yields the contribution which we associate with the on-shell part of the nucleon propagator,

$$f^{(\mathrm{on})}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp^2\, \frac{y(k_\perp^2 + y^2 M^2)}{[k_\perp^2 + y^2 M^2 + (1-y) m_\pi^2]^2}, \tag{9}$$

where for convenience we have factored out all isospin factors. This term corresponds to the usual pion distribution function for the "Sullivan process" [2], which is computed from a PS πN coupling. Performing the $k_\perp$ integration with an ultraviolet cut-off Λ gives

$$f^{(\rm on)}(y) \;=\; -\frac{g_A^2 M^2}{(4\pi f_\pi)^2}\, y \left[\frac{m_\pi^2(1-y)}{m_\pi^2(1-y)+M^2y^2} \;+\; \log\left(\frac{m_\pi^2(1-y)+M^2y^2}{\Lambda^2}\right)\right] + \mathcal{O}\left(\frac{M^2}{\Lambda^2}\right), \tag{10}$$

where only the leading term in Λ has been kept.

In the second and third terms in Eq. (7), the cancellation of the nucleon propagators D_N leaves tadpole-like contributions involving pion propagators only. This will have important consequences for the structure and interpretation of the distribution functions. Integrating the second term in (7) proportional to y/D_π^2 over k^- gives a result which is zero everywhere except at $k^+ = yp^+ = 0$,

$$\int dk^- \frac{1}{D_\pi^2} \;=\; \frac{2\pi i}{k_\perp^2 + m_\pi^2}\, \delta(k^+). \tag{11}$$

After multiplication by y, this term therefore vanishes.

Evaluation of the third term in Eq. (7) proportional to $(y\, p\cdot k)/D_\pi^2$ requires particular care. Since $p\cdot k = (p^+k^- + p^-k^+)/2$, integration over k^- involves terms such as in (11) which vanish, as well as terms of the type $\int dk^-\,(k^-/D_\pi^2)$. To evaluate the latter, one can make use of the identity

$$\frac{k^-}{D_\pi^2} \;=\; -\frac{\partial}{\partial k^+}\frac{1}{D_\pi} \tag{12}$$

to write

$$\int dk^- \frac{k^+k^-}{D_\pi^2} \;=\; \int dk^- \frac{1}{D_\pi} \;=\; 2\pi i \,\log\left(\frac{k_\perp^2 + m_\pi^2}{\mu^2}\right)\delta(k^+), \tag{13}$$

where μ is a mass parameter. This gives rise to a δ-function contribution (again defined without isospin factors)

$$f^{(\delta)}(y) \;=\; \frac{g_A^2}{4(4\pi f_\pi)^2}\int dk_\perp^2\, \log\left(\frac{k_\perp^2+m_\pi^2}{\mu^2}\right)\delta(y), \tag{14}$$

with the total pion distribution function then given by

$$f_\pi^{(iv)}(y) \;=\; 4f^{(\rm on)}(y) + 4f^{(\delta)}(y). \tag{15}$$

When integrated over $k_\perp$ from 0 to Λ, the function $f^{(\delta)}(y)$ becomes

$$f^{(\delta)}(y) \;=\; -\frac{g_A^2}{4(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2\;\delta(y) \;+\; \text{terms involving } \Lambda, \tag{16}$$

where we have isolated the leading nonanalytic term in m_π^2, whose coefficient is independent of the short-distance physics parametrized by Λ.

Thus the pion rainbow diagram has contributions from the regular $y>0$ region as well as the ultrasoft $y=0$ point. In the PS formulation of pion–nucleon interactions, where the pion light-cone momentum distribution is given by just the first term in Eq. (7), the $f^{(\delta)}$ term in (15) does not arise, and the total PS pion distribution is given by [1]

$$f_{\pi(\rm PS)}^{(iv)}(y) \;=\; 4f^{(\rm on)}(y). \tag{17}$$

In addition to the pion rainbow diagram in Fig. 1(a), there is also a contribution from the term in the Lagrangian (1) involving two pion fields, which gives rise to the bubble diagram in Fig. 1(b). A straightforward calculation of the resulting contribution to the pion light-cone momentum distribution gives

$$f_{\pi(\rm bub)}^{(iv)}(y) \;=\; \frac{M}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\,\bar u(p)\,(-i\not{k})\,u(p)\,\frac{i}{D_\pi}\frac{i}{D_\pi}\,2k^+\delta(k^+-yp^+) \tag{18}$$

$$\;=\; \frac{i}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\,\frac{p\cdot k}{D_\pi^2}\,2y\,\delta\left(y-\frac{k^+}{p^+}\right). \tag{19}$$

A similar calculation to that for the $f^{(\delta)}(y)$ term above gives the final pion bubble contribution as

$$f^{(iv)}_{\pi(\text{bub})}(y) \;=\; -\frac{1}{(4\pi f_\pi)^2}\int dk_\perp^2\, \log\left(\frac{k_\perp^2+m_\pi^2}{\mu^2}\right)\delta(y) \;\equiv\; 2f^{(\text{bub})}(y), \tag{20}$$

where for convenience we have defined the distribution $f^{(\text{bub})}(y)$ without the isospin factor 2. Integrating over $k_\perp$ from 0 to Λ then yields a result for $f^{(\text{bub})}(y)$ similar to that for the $f^{(\delta)}(y)$ term in Eq. (14),

$$f^{(\text{bub})}(y) \;=\; \frac{1}{2(4\pi f_\pi)^2} m_\pi^2 \log m_\pi^2\;\delta(y) \;+\; \text{terms involving } \Lambda \tag{21}$$

$$\;=\; -\frac{2}{g_A^2}\, f^{(\delta)}(y). \tag{22}$$

Finally, combining the results in this section, the total isovector pion distribution in the nucleon, including both the pion rainbow and bubble diagrams in Fig. 1, can be written as

$$\begin{aligned} f^{(iv)}_{\pi}(y) \;+\; f^{(iv)}_{\pi(\text{bub})}(y) \;&=\; -\frac{4g_A^2M^2}{(4\pi f_\pi)^2}\, y\left[\frac{m_\pi^2(1-y)}{m_\pi^2(1-y)+M^2y^2} \;+\; \log\left(m_\pi^2(1-y)+M^2y^2\right)\right] \\ &\quad+\; \frac{(1-g_A^2)}{(4\pi f_\pi)^2} m_\pi^2\log m_\pi^2\;\delta(y) \;+\; \text{terms involving } \Lambda, \end{aligned} \tag{23}$$

where we have explicitly separated the long-distance contributions from the short-distance effects involving the cut-off Λ. In particular, the total term proportional to the δ-function is found to vanish in the limit $g_A \to 1$, as one would have in a PS pion–nucleon theory [33, 34]. Note that a similar behavior for the $\delta(y)$ term is observed for the pion cloud contribution to the isovector form factor [46, 47].

III. NUCLEON LIGHT-CONE DISTRIBUTIONS

For the nucleon rainbow diagram in Fig. 2(a), the distribution function for the neutron is $2\times$ that of the proton, so that for the isovector distribution one has $f^{(iv)}_N(y) = -f_p(y)$. Specifically, the isovector nucleon distribution function is given by

$$\begin{aligned} f^{(iv)}_N(y) \;&=\; -M\left(\frac{g_A}{2f_\pi}\right)^2\int\frac{d^4k}{(2\pi)^4}\,\bar{u}(p)\,(\not{k}\gamma_5)\,\frac{i(\not{p}-\not{k}+M)}{D_N}\,\gamma^+\,\frac{i(\not{p}-\not{k}+M)}{D_N}\,(\gamma_5\not{k})\,u(p) \\ &\quad\times\frac{i}{D_\pi}\,\delta(k^+-yp^+), \end{aligned} \tag{24}$$

which, using the Dirac equation, can be written as

$$f^{(iv)}_N(y) \;=\; i\left(\frac{g_A}{2f_\pi}\right)^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{4M^2(k^2-2y\,p\cdot k)}{D_\pi D_N^2} \;-\; \frac{4M^2y}{D_\pi D_N} \;-\; \frac{1}{D_\pi}\right]\delta\left(y-\frac{k^+}{p^+}\right). \tag{25}$$

The first term ($\sim 1/D_\pi D_N^2$) in Eq. (25) corresponds to the on-shell nucleon contribution, and is proportional to the function $f^{(\text{on})}(y)$ in Eq. (9). The second term ($\sim 1/D_\pi D_N$) arises from the off-shell components of the nucleon propagator, while the third term involving the single pion propagator ($\sim 1/D_\pi$) contributes only at $y=0$, and is proportional to $f^{(\delta)}(y)$. The total isovector nucleon light-cone distribution function arising from the nucleon rainbow diagram is then given by

$$f^{(iv)}_N(y) \;=\; -f^{(\text{on})}(y) - f^{(\text{off})}(y) + f^{(\delta)}(y), \tag{26}$$

where

$$f^{(\text{off})}(y) \;=\; -\frac{g_A^2M^2}{(4\pi f_\pi)^2}\int dk_\perp^2\,\frac{y}{k_\perp^2+y^2M^2+(1-y)m_\pi^2} \tag{27}$$

$$\;=\; \frac{g_A^2M^2}{(4\pi f_\pi)^2}\, y\log\left(\frac{m_\pi^2(1-y)+M^2y^2}{\Lambda^2}\right) + \mathcal{O}\left(\frac{M^2}{\Lambda^2}\right), \tag{28}$$

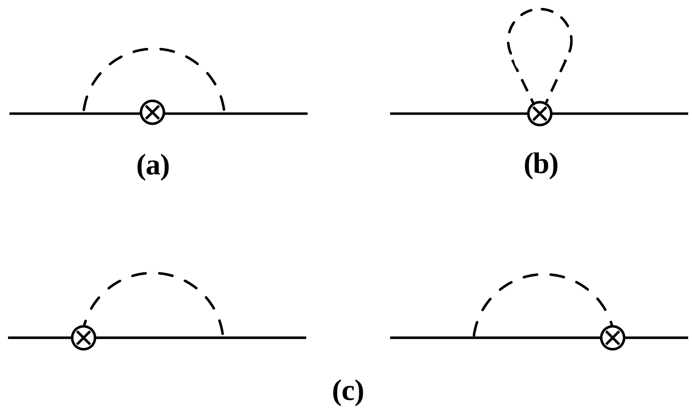

FIG. 2: Contributions to the nucleon light-cone momentum distributions in the nucleon, from (a) the nucleon rainbow, and (b) the tadpole diagrams, as well as (c) the contributions from the Kroll-Ruderman coupling required by gauge invariance.

with the $k_\perp$ integration taken up to the cut-off scale Λ.

For the pseudoscalar model the nucleon light-cone momentum distribution is again just given by the on-shell term in Eq. (26),

$$f^{(iv)}_{N(\mathrm{PS})}(y) \;=\; -f^{(\mathrm{on})}(y). \tag{29}$$

This result was also obtained in the infinite momentum frame calculation of Drell, Levy and Yan [1].

On the other hand, the PV theory contains, in addition to the off-shell and δ-function pieces, also the contribution from the operator insertion at the $NN\pi\pi$ vertex, Fig. 2(b). The distribution function associated with this diagram can be written

$$f^{(iv)}_{N(\mathrm{tad})}(y) \;=\; -\frac{M}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\,\bar{u}(p)\,\gamma^+\,u(p)\,\frac{i}{D_\pi}\,\delta(k^+ - yp^+) \tag{30}$$

$$\;=\; -\frac{i}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{D_\pi}\,\delta\left(y-\frac{k^+}{p^+}\right). \tag{31}$$

Performing the k^- integration and again using the relation in Eq. (13) then gives

$$f^{(iv)}_{N(\mathrm{tad})}(y) \;=\; -2f^{(\mathrm{bub})}(y). \tag{32}$$

Comparing the pion bubble and tadpole contributions, Eqs. (20) and (32), one finds that their sum vanishes,

$$f^{(iv)}_{\pi(\mathrm{bub})}(y) \;+\; f^{(iv)}_{N(\mathrm{tad})}(y) \;=\; 0. \tag{33}$$

These contributions themselves therefore have no net effect on the sum of the pion and nucleon light-cone distributions, and hence on the total nucleon charge, as will be discussed in Sec. IV below.

Finally, the light-cone momentum distribution associated with the Kroll-Ruderman diagrams in Fig. 2(c), which arises from the derivative coupling in the PV theory, is given by

$$f^{(iv)}_{\mathrm{KR}}(y) \;=\; 4M\left(\frac{g_A}{2f_\pi}\right)^2\int\frac{d^4k}{(2\pi)^4}\,\bar{u}(p)\,\Bigg[\not{k}\gamma_5\frac{i(\not{p}-\not{k}+M)}{D_N}i\gamma^+\gamma_5 + i\gamma_5\gamma^+\frac{i(\not{p}-\not{k}+M)}{D_N}\gamma_5\not{k}\Bigg]\,u(p)\,\frac{i}{D_\pi}\,\delta\left(y-\frac{k^+}{p^+}\right) \tag{34}$$

$$\;=\; 4i\left(\frac{g_A}{2f_\pi}\right)^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{4M^2y}{D_\pi D_N} \;+\; \frac{2}{D_\pi}\right]\delta\left(y-\frac{k^+}{p^+}\right). \tag{35}$$

After k^- integration, one then obtains

$$f^{(iv)}_{\mathrm{KR}}(y) \;=\; 4f^{(\mathrm{off})}(y) \;-\; 8f^{(\delta)}(y). \tag{36}$$

One observes then that the sum of the pion (Eq. (15)) and KR (Eq. (36)) distributions is proportional to the nucleon light-cone distribution (Eq. (26)),

$$f_{\pi}^{(iv)}(y) \; + \; f_{\rm KR}^{(iv)}(y) \;=\; 4f^{(\rm on)}(y) \; + \; 4f^{(\rm off)}(y) \; - \; 4f^{(\delta)}(y) \;=\; -4f_N^{(iv)}(y), \tag{37}$$

which, as will be discussed in the next section, is a necessary condition for the gauge invariance of the theory.

IV. SUM RULES

The effect of pion loops on observables such as parton distribution functions can be computed by considering DIS from a nucleon viewed in the infinite momentum frame [1]. Here the total contribution to the nucleon isovector quark distribution $q_N^{(iv)}$ from the pionic corrections in Figs. 1 and 2 can be written in convolution form (for a review see *e.g.* Ref. [28]),

$$q_N^{(iv)}(x) \;=\; Z_2\, q_{N_0}^{(iv)}(x) \; + \; \sum_i \left(f_i^{(iv)} \otimes q_i^{(iv)} \right)(x), \tag{38}$$

where $q_{N_0}^{(iv)}$ is the isovector quark distribution in the bare nucleon state N_0, and the sum over i includes the nucleon and pion rainbow, bubble, tadpole and KR contributions. The symbol $\otimes$ denotes convolution; for the pion rainbow contribution, for instance, one has $(f_\pi^{(iv)} \otimes q_\pi^{(iv)})(x) = \int_x^1 (dy/y)\, f_\pi^{(iv)}(y)\, q_\pi^{(iv)}(x/y)$, where $q_\pi^{(iv)}$ is the isovector quark distribution in the pion, and similar for the other terms. While the shapes of the bare quark distribution functions, and especially for the KR and tadpole terms, are not known *a priori*, charge conservation is nevertheless ensured as long as the distributions are normalized according to

$$\int_0^1 dx\, q_i^{(iv)}(x) \;=\; 1, \tag{39}$$

for all $i = N, \pi, {\rm KR}, \pi N$.

The wave function renormalization factor Z_2 in Eq. (38) can be evaluated from the nucleon self-energy Σ by [48–51],

$$Z_2^{-1} - 1 \;=\; \left.\frac{\partial \Sigma}{\partial p_0}\right|_{p_0 = M}, \tag{40}$$

where the derivative is taken at the nucleon pole, $p_0 = M$. For weak pion fields, explicit evaluation gives

$$Z_2^{-1} - 1 \;\approx\; 1 - Z_2 \;=\; 3 \int_0^1 dy\, \left(f^{(\rm on)}(y) + f^{(\rm off)}(y) - f^{(\delta)}(y) \right), \tag{41}$$

where the factor 3 arises from the sum of the proton and neutron intermediate state contributions, for either a proton or neutron initial state. In fact, Eq. (41) is required by the Ward-Takahashi identity which relates the wave function and vertex renormalization factors by $Z_2 = 3(1 - Z_1^N)$, where Z_1^N is the lowest moment of the nucleon distribution function f_N for a *proton* initial state.

Integrating the convolution expressions in Eq. (38) over all x and using the relation in Eq. (37) one obtains, for a nonsinglet quark distribution $q_N^{(iv)}(x)$ normalized to unity, the quark number sum rule,

$$\int_0^1 dx\, q_N^{(iv)}(x) \;=\; 1 \;=\; Z_2 \; + \; 3 \int_0^1 dy\, \left(f^{(\rm on)}(y) + f^{(\rm off)}(y) - f^{(\delta)}(y) \right), \tag{42}$$

with the contributions from the $f_{N({\rm tad})}^{(iv)}$ and $f_{\pi({\rm bub})}^{(iv)}$ terms canceling as in Eq. (33). From the expression for Z_2 in Eq. (41) one can verify that the baryon number of the nucleon is not modified by pion loops.

Inclusion of the $y = 0$ contributions is vital for formal sum rules to be satisfied; if these were to be evaluated for $y > 0$ only, the $\delta(y)$ terms would be missed. Indeed, as was illustrated in Ref. [52] in the case of the Bukhardt–Cottingham sum rule for the twist-3 polarized parton distribution $g_2(x)$, if such $\delta(x)$ terms are present then sum rules based on the formal operator product expansion appear to be violated when evaluated for $x > 0$ only. The same was observed for the corresponding sum rule for the chirally odd twist-3 parton distribution $h_2(x)$. In the context of dispersion relations, $\delta(x)$ terms reflect the presence of subtractions. A deep-inelastic scatterin experiment to test such sum rules would thus find them to be violated, since $x = 0$ is never measured. In the convolution calculation the

$\delta(y)$ terms would thus imply a $\delta(x)$ contribution to the quark distribution $q_N^{(iv)}(x)$, which would lead to the violation of both the Gottfried [53] and Adler sum rules [54], if these are applied to nonzero x only.

The appearance of the $\delta(x)$ terms in the isovector quark distributions is possibly an artifact of taking chiral perturbation theory too literally in this context. For example, although a bubble diagram in the chiral effective theory inevitably leads to an s-independent contribution to the forward Compton amplitude — and hence a subtraction in the corresponding dispersion relation — it may no longer be justified to assume a point-like πN four-point coupling that gives rise to the bubble when $s - M^2 \gg m_\pi^2$. While a $\delta(x)$ contribution formally appears in the chiral effective theory framework, this contribution will presumably soften into a peak near $x = 0$ in QCD. Nevertheless, even if the Adler sum rule is in the end satisfied in QCD, the presence of $\delta(x)$ terms in the effective theory indicates that in order to confirm this sum rule it is essential to include the small-x regime. Of course, none of these considerations apply to the case of the PS pion-nucleon coupling, which contains nonzero contributions only at $y > 0$.

V. CONCLUSION

In this paper we have presented a detailed derivation of the isovector light-cone momentum distributions of pions in the nucleon for both the pseudovector and pseudoscalar πN interactions. In the PV theory the direct coupling to the pion gives rise to a pion rainbow distribution, $f_\pi^{(iv)}$, and a distribution from the pion bubble, $f_{\pi(\text{bub})}^{(iv)}$, with the latter involving emission and absorption of a pion at the same point. Consequently the bubble contribution to the momentum distribution exists only at zero light-cone momentum, $y = 0$. We found, however, that the pion rainbow diagram also has a δ-function component, on top of the regular terms at $y > 0$. The $\delta(y)$ contributions arise because the PV πNN vertex is proportional to the momentum of the pion, which leads to a more singular integrand in the $k^- \to \infty$ limit than in the case of the momentum-independent PS coupling.

In addition to the pion distribution, we have also computed the isovector light-cone momentum distribution of the corresponding recoil nucleon dressed by the pion, including contributions from the rainbow diagram with nucleon coupling, $f_N^{(iv)}$, and a tadpole coupling at the $\pi\pi NN$ vertex, $f_{N(\text{tad})}^{(iv)}$, as well as the Kroll-Ruderman terms $f_{\text{KR}}^{(iv)}$ which arise at the same order in the chiral expansion of the PV Lagrangian. The latter are in fact essential for preserving the gauge invariance of the PV theory. The nucleon rainbow distribution $f_N^{(iv)}$ contains pieces that are associated with the off-shell components of the nucleon propagator, in addition to the on-shell components, and a δ-function term analogous to that in the pion light-cone distribution $f_\pi^{(iv)}$. In contrast, the nucleon distribution in the PS model only contains the on-shell contributions. The KR distribution $f_{\text{KR}}^{(iv)}$ has contributions from the off-shell components of the nucleon and from the singular δ-function terms. The singular nucleon tadpole distribution is related to the pion tadpole by $f_{N(\text{tad})}^{(iv)} = -f_{\pi(\text{bub})}^{(iv)}$ for all y.

Combined, the sum of the pion rainbow and KR distributions in the PV theory is shown to be proportional to the nucleon rainbow distribution,

$$f_\pi^{(iv)}(y) \ + \ f_{\text{KR}}^{(iv)}(y) \ = \ -4 f_N^{(iv)}(y), \tag{43}$$

which is a necessary condition for ensuring gauge invariance. Within the convolution model, we have demonstrated that this in fact gives the correct number sum rule: the sum of all the pion cloud contributions does not alter the valence quark number of the nucleon, although it does of course affect the shape. This is also true in the PS formulation.

Our results pave the way for future phenomenological applications of pion cloud models that are manifestly consistent with the chiral symmetry properties of QCD. Most calculations in the literature of the effects of the pion cloud in deep-inelastic scattering have been made using the PS πN coupling [1, 2, 28], for which chiral symmetry is not manifest without the addition of a scalar field. In fact, the absence of the δ-function contributions in the PS version of the rainbow diagrams leads to the coefficient of the leading nonanalytic term in the chiral expansion of the quark distribution function moments that differs by a factor 4/3 from the PV result [44]. A detailed study of the phenomenological consequences of the distributions derived here in the PV theory will be the subject of an upcoming study [55].

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