Unification of gauge couplings in the standard model with extra vectorlike families

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Unification of Gauge Couplings in the Standard Model with Extra Vector-like Families

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Abstract

We discuss gauge coupling unification in models with additional 1 to 4 complete vector-like families, and derive simple rules for masses of vector-like fermions required for exact gauge couplings unification. These mass rules and the classification scheme are generalized to arbitrary extension of the standard model. We focus on scenarios with 3 or more vector-like families in which the values of gauge couplings at the electroweak scale are highly insensitive to the grand unification scale, the unified gauge coupling, and the masses of vector-like fermions. Their observed values can be mostly understood from infrared fixed point behavior. With respect to sensitivity to fundamental parameters the model with 3 extra vector-like families stands out. It requires vector-like fermions with masses of order 1 TeV – 100 TeV, and thus at least part of the spectrum may be within the reach of the LHC. The constraints on proton lifetime can be easily satisfied in these models since the best motivated grand unification scale is at $\sim 10^{16}$ GeV. The Higgs quartic coupling remains positive all the way to the GUT scale, and thus the electroweak minimum of the Higgs potential is stable.

I. INTRODUCTION

Models for new physics at the TeV scale are typically motivated by the hierarchy problem. They strive to explain the hierarchy between the electroweak (EW) scale and the Planck scale, or at least remove the incredible fine tuning required in the standard model (SM) for having such a hierarchy. However, the SM is stubbornly surviving the first tests at the LHC and there are no traces of new physics yet. In addition, the mass of the recently discovered Higgs-like particle suggests that the SM can be a consistent theory all the way to the Planck scale. This gives more weight to speculations that there is no mechanism, no new physics, that stabilizes the hierarchy, or that the EW scale is selected based on anthropic reasoning.

However, even when we ignore the hierarchy problem, the SM is still not very satisfactory. The three gauge couplings, all couplings of the Higgs boson to fermions, the Higgs mass, and the Higgs quartic coupling are free parameters. This motivates us to explore extensions of the standard model in which at least some of these parameters could be understood.

We have recently showed that extending the standard model by three complete vector-like families (SM+3VFs) with masses of order 1 TeV - 100 TeV allows for unification of gauge couplings [1]. Predictions for gauge couplings at the EW scale are highly insensitive to fundamental parameters: the grand unification scale, the unified gauge coupling, and the masses of vector-like fermions. Their observed values can be mostly understood from infrared fixed point behavior.

In this paper we discuss gauge coupling unification in detail in models with additional 1 to 4 complete vector-like families (VFs), and derive simple rules for masses of vector-like fermions required for exact gauge couplings unification. We then focus on scenarios with 3 or more vector-like families which lead to insensitive unification of gauge couplings. Requiring the smallest splitting between masses of vector-like fermions we show that the best motivated GUT scale is at $\sim 10^{16}$ GeV. We provide examples of the spectrum as a function of the GUT scale which can be as large as the Planck scale. We discuss constraints from proton decay and show that predictions from the best motivated region are close to current limits. However, due to insensitivity of predicted EW scale values of gauge couplings to GUT scale parameters no sharp predictions can be made without knowing the spectrum of vector-like fermions.

The focus on complete families follows from the fact that quantum numbers of quarks and leptons in the SM nicely fill representations of a GUT symmetry, **10** and $\bar{\mathbf{5}}$ of $SU(5)$ or **16** of $SO(10)$. This provides a support for the idea of grand unification and the unification of gauge couplings [2]. Additional complete families represent some of the simplest extensions of the SM that can be embedded into simple GUTs.[1] Consequently, there are many studies exploring various features of vector-like families (mostly in supersymmetric models), see for example Refs. [4–8].

In addition, vector-like fermions, not necessarily coming in complete GUT multiplets, are often introduced on purely phenomenological grounds, to explain various discrepancies between observations and SM predictions. Examples include discrepancies in precision EW Z-pole observables [9–12], and the muon g-2 anomaly [13]. However, with arbitrary new particles there are many possibilities for gauge coupling unification.[2] Therefore, we generalize the mass rules and the method to classify scenarios consistent with gauge coupling unification to an arbitrary extension of the standard model.

The method to classify scenarios consistent with gauge coupling unification in terms of physical masses of extra particles starts with finding the mass scales that represent "average" masses of all particles charged under given gauge symmetry required for gauge coupling unification (they are defined precisely in the next section and are referred to as crossing scales). These crossing scales are easy to obtain and they immediately give us information about the required spectrum. First of all, if they do not exist between the EW scale and the GUT scale, the gauge coupling unification in a given model is not possible, no matter what the splitting between masses of extra particles is. Second, the splitting between crossing scales represents the minimum necessary splitting in the spectrum required. Third, from the mass formulas that define crossing scales in terms of masses of extra particles one can immediately see the basic features of the spectrum required, and the spectrum can be calculated. In addition, these formulas also indicate the freedom one has in imposing further relations between masses of extra particles. This might be useful when searching for models that

[1] This does not mean that the masses of vector-like fermions needed for gauge coupling unification necessarily result from a simple unified boundary condition. By simple GUTs we mean that there is no additional mechanism required to keep particles in incomplete GUT multiplets significantly below the GUT scale, or to split their masses over many orders of magnitudes that would to large extent ameliorate the motivation for GUTs.

[2] For examples of recent studies investigating effects of extra particles on gauge coupling unification in models without supersymmetry, see Refs. [14, 15].

relate masses of particles at a given scale. The mass rules given in terms of particle masses can be evolved to arbitrary scale, *e.g.* the GUT scale, which would provide the boundary conditions that need to be satisfied. However, the RG evolution of the mass rules depends on additional assumptions one has to make about the origin of the masses and the scale at which these masses are generated.[3]

This paper is organized as follows. In Sec. II we discuss RG evolution of gauge couplings in models with extra VFs. We start with the discussion of IR fixed point predictions for gauge couplings, then add threshold corrections from universal mass of vector-like fermions, and finally we add effects from splitting masses of vector-like fermions. We discuss sensitivity of predicted values of gage couplings to fundamental parameters. Finally, we derive simple mass rules that have to be satisfied in order to get exact gauge coupling unification. We generalize the method to classify all solutions consistent with gauge coupling unification to arbitrary extension of the SM. In Sec. III we discuss constraints from proton decay, stability of the EW minimum of the Higgs potential, and discuss possible origin of masses of vector-like fermions. We give few concluding remarks in Sec. IV.

II. RENORMALIZATION GROUP EVOLUTION OF GAUGE COUPLINGS

The one-loop renormalization group equations (RGEs) for three gauge couplings, $\alpha_i = g_i^2/4\pi$, are given by:

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi}\, b_i, \tag{1}$$

where $t = \ln Q/Q_0$ with Q representing the energy scale at which gauge couplings are evaluated. The beta function coefficients, b_i, in the SM with n_f families are given by

$$b_i = \left(\frac{1}{10} + \frac{4}{3}n_f,\; -\frac{43}{6} + \frac{4}{3}n_f,\; -11 + \frac{4}{3}n_f\right). \tag{2}$$

For $n_f = 3$ we get the usual SM result, $b_i = (41/10,\ -19/6,\ -7)$. With extra N pairs of complete VFs we have $n_f = 3 + 2 \times N$ (a vector-like partner contributes in the same way).

[3] The study of gauge coupling unification is to large extent unaffected by these assumptions, only the physical masses of particles matter in the leading order. If the masses originate from Yukawa couplings to extra scalars that get vacuum expectation values at an intermediate scale, these may contribute to the RG evolution of gauge couplings at 2-loop level. However unless the extra couplings are large these effects would be negligible.

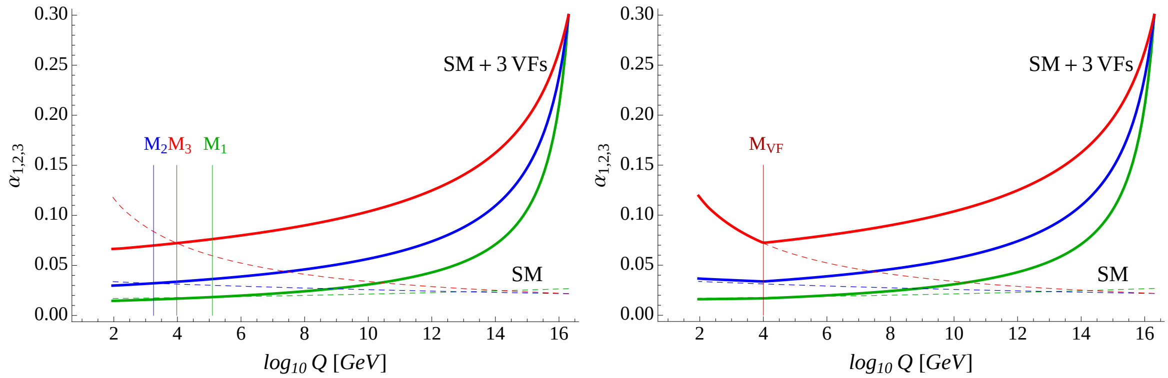

FIG. 1: RG evolution of gauge couplings: α_3 (top solid line), α_2 (middle solid line), and α_1 (bottom solid line) in the SM extended by three vector-like families for $\alpha_G = 0.3$ at $M_G = 2 \times 10^{16}$ GeV. Dashed lines in the same order show running of gauge couplings in the SM. Masses of 3 VFs are neglected in the left plot, and fixed to 10 TeV (indicated by M_{VF}) in the right plot. The crossing points in the evolution of gauge couplings in the SM+3VFs and the SM indicated in the left plot define the common threshold scales, $M_{1,2,3}$, for masses of particles charged under given symmetry required for exact gauge coupling unification.

For example, in SM+3VFs we find $b_i = (121/10,\ 29/6,\ +1)$ which indicates that all three gauge couplings are asymptotically divergent (this result obviously holds for 3 or more pairs of VFs).

The evolution of gauge couplings in the SM and an example of the evolution in the SM+3VFs case are showed in Fig. 1. The numerical analysis closely follows that of Ref. [1]. For the SM evolution we use the Z-scale central values of $\alpha_{EM}^{-1}(M_Z) = 127.916$, $\sin^2\theta_W = 0.2313$, and $\alpha_3(M_Z) = 0.1184$, together with the top quark mass $m_t = 173.2$ GeV which can be found in Ref. [3]. The α_{EM} and $\sin^2\theta_W$ are related to $\alpha_{1,2}(M_Z)$ through

$$\sin^2\theta_W \;=\; \frac{\alpha'}{\alpha_2 + \alpha'}, \quad \text{and} \quad \alpha_{EM} \;=\; \alpha_2 \sin^2\theta_W, \tag{3}$$

where, assuming the $SU(5)$ normalization of hypercharge, $\alpha' \equiv (3/5)\alpha_1$. We set the Higgs boson mass to $m_h = 126$ GeV [16, 17]. The example of the RG evolution of gauge couplings in the SM+3VFs starts with unified gauge coupling $\alpha_G = 0.3$ at $M_G = 2 \times 10^{16}$ GeV. The crossing points in the evolutions of gauge couplings in these two cases, which will be important for the discussion of threshold corrections, are indicated in the left plot by $M_{1,2,3}$. In all numerical results we use full two loop RGEs [18], we integrate out all particles with masses above M_Z at their mass scale, and include one-loop matching corrections for m_t and

m_h [19]. We assume that Yukawa couplings of vector-like fermions are negligible, and we also neglect Yukawa couplings of all fermions in the SM except the top quark.

The results of the numerical analysis we present can be understood from approximate analytic formulas. The one-loop RGEs can be solved, and we can express gauge couplings at the EW scale in terms of the GUT scale, and values of gauge couplings at M_G:

$$\alpha_i^{-1}(M_Z) \;=\; \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z} \;+\; \alpha_i^{-1}(M_G), \tag{4}$$

Assuming gauge coupling unification, $\alpha_i(M_G) = \alpha_G$, and neglecting threshold corrections both at the EW scale and the GUT scale, we can express one gauge coupling in terms of the other two. For example:

$$\alpha_3(M_Z) \;=\; \frac{b_1 - b_2}{(b_1 - b_3)s_W^2 + 3/5(b_3 - b_2)c_W^2}\,\alpha_{EM}(M_Z), \tag{5}$$

where $s_W^2 \equiv \sin^2\theta_W(M_Z)$, and $c_W^2 \equiv \cos^2\theta_W(M_Z)$. For the measured values of α_{EM} and s_W^2 the SU(5) embedding of the SM predicts $\alpha_3(M_Z) \simeq 0.07$ which is about 40% below the experimental value.

Adding complete chiral or vector-like families at the EW scale does not change at all the one-loop prediction given in Eq. (5) since complete families contribute equally to all three beta function coefficients, see Eq. (2). Furthermore, the scale of unification (more precisely the scales where any two couplings meet) does not change at one-loop, only the value of the unified gauge coupling increases. With increasing the number of extra families, at some point, the couplings become non-perturbative before they meet, and eventually reach the Landau pole. Further increase of the number of families lowers the energy scale at which the Landau pole occurs.

However, the SM extended with sufficient number of complete vector-like families so that all couplings are asymptotically divergent offers a new possibility. Vector-like families introduce additional scale to the problem associated with masses of vector-like fermions, M_{VF}, and they contribute to the RG evolution of gauge couplings only above this energy scale. This allows us to consider models with a large (but still perturbative) unified gauge coupling at a high scale, higher than the scale at which the Landau pole would occur if the VFs were at the EW scale. Consequently, in the RG evolution to lower energies gauge couplings run to the (trivial) infrared (IR) fixed point. Thus, at lower energies the values of gauge couplings are determined only by the particle content of the theory and how far from the GUT scale we measure them. Since the exact value of α_G becomes irrelevant, instead of one prediction of the conventional unification, Eq. (5), we have two predictions for ratios of gauge couplings. At the M_{VF} scale the vector-like fermions are integrated out, and below this scale gauge couplings run according to the usual RG equations of the standard model. In a way, the two parameters of the conventional unification, M_G and α_G, are replaced by M_G and M_{VF}. The discrepancies of IR fixed point predictions from observed values can be explained by threshold effects of extra vector-like fermions.

A. IR fixed point predictions for gauge couplings

The IR fixed point predictions were discussed in detail in Ref. [1]. In models with asymptotically divergent couplings, these can be easily obtained if the 1-loop RGEs are good approximations. Assuming large enough unification scale and large (but still perturbative) unified gauge coupling, the first term in Eq. (4) dominates, and the ratios of gauge couplings are given by ratios of beta function coefficients,

$$\frac{\alpha_i(M_Z)}{\alpha_j(M_Z)} \simeq \frac{b_j}{b_i}. \tag{6}$$

This can be translated into the prediction for $\sin^2\theta_W$:

$$\sin^2\theta_W \equiv \frac{\alpha'}{\alpha_2+\alpha'} = \frac{b_2}{b_2+b'}, \tag{7}$$

where $b' \equiv (5/3)b_1$. Numerically we find $\sin^2\theta_W = 0.193$ in the case of SM+3VFs, which is identical to the value obtained assuming 9 chiral families [20, 21]. Similarly, in SM+4VFs we find $\sin^2\theta_W = 0.234$.

In the case of SM+3VFs, the one-loop RGE for α_3 given in Eq. (1) is not a good approximation because of the accidentally small b_3 coefficient. The two-loop contribution to the beta function is well approximated by the term proportional to α_3^3,

$$\frac{d\alpha_3}{dt} = \beta(\alpha_3) \simeq \frac{\alpha_3^2}{2\pi}\, b_3 + \frac{\alpha_3^3}{8\pi^2}\, B_3, \tag{8}$$

where $B_3 = -102 + (76/3)n_f = 126$ for SM+3VFs [18]. Thus the two loop contribution is larger than the one-loop contribution for $\alpha_3 \gtrsim 0.1$.[4]

[4] This is a consequence of a very small 1-loop beta function coefficient and it is not an indication of non-perturbativity. The coupling is still perturbative, the dominant 3-loop contribution to $\beta(\alpha_3)$, proportional to α_3^4, represents a $\sim 5\%$ correction to 1 + 2-loop beta function for $\alpha_3 \simeq 0.1$ [22].

The RGE for α_3 can be solved by adding the 1-loop contribution as an expansion in $\epsilon = 4\pi b_3/B_3$ to the solution obtained from the 2-loop contribution only [23], [1]. Alternatively, we can solve the full RGE given in Eq. (8) and find:

$$\alpha_3^{-1}(M_Z) - \frac{1}{\epsilon}\ln\left(1+\frac{\epsilon}{\alpha_3(M_Z)}\right) = \frac{b_3}{2\pi}\ln\frac{M_G}{M_Z} + \alpha_G^{-1} - \frac{1}{\epsilon}\ln\left(1+\frac{\epsilon}{\alpha_G}\right). \tag{9}$$

Neglecting α_G^{-1}, we obtain the second prediction:

$$\frac{\alpha_3(M_Z)}{1-\frac{\alpha_3(M_Z)}{\epsilon}\ln(1+\frac{\epsilon}{\alpha_3(M_Z)})} = \frac{b_2+b'}{b_3}\alpha_{EM}(M_Z). \tag{10}$$

Numerically, for $\alpha_{EM}(M_Z) = 1/127.916$, it predicts $\alpha_3(M_Z) \simeq 0.072$ in the case of SM+3VFs.

The beta function coefficients for α_3 in SM+4VFs scenario are $b_3 = 11/3$ and $B_3 = 530/3$. The 1-loop term in the RG equation (8) dominates for $\alpha_3 < 0.26$ in this case.

The proximity of predictions from the IR fixed point, Eqs. (7) and (10), to observed values is certainly intriguing. Although they are not a perfect match to measured values, the discrepancies can be easily accommodated by taking into account threshold corrections from vector-like fermions that should be integrated out at the M_{VF} scale.

B. Mass scale of vector-like fermions and sensitivity to fundamental parameters

The existence of a scale associated with masses of vector-like fermions is strongly suggested by the overlay of the RG evolution of gauge couplings in the SM and those in the SM+3VFs assuming unified gauge coupling at a high scale given in Fig. 1. All three gauge couplings in these two scenarios cross at comparable scales suggesting a common threshold at which particles from VFs are integrated out. Indeed, for the example given in Fig. 1, fixing all the masses of 3 VFs to 10 TeV, showed in Fig. 1 (right), the EW scale values of gauge couplings are predicted within 8% from measured values. In the next subsection we will show that the measured values of gauge couplings can be precisely reproduced by splitting the masses of vector-like fermions. First however, we would like to discuss general features of this result assuming the common mass of VFs.

The fairly good agreement of predicted values of gauge couplings from 3 VFs at ~ 10 TeV with observed values does not rely on the specific choice of the GUT scale and the value of the unified gauge coupling. The EW scale values of gauge couplings are highly insensitive to these parameters which can be understood from IR fixed point behavior.

The low sensitivity of predicted values of gauge couplings to fundamental parameters is demonstrated in Fig. 2 (left). It shows a large region of the GUT scale, M_G, and the universal mass of fermions from 3 vector-like families, M_{VF}, (bright red region) from which the values of gauge couplings at the EW scale are simultaneously predicted within 10% from the measured values. It also shows the best fit which predicts all couplings within 6%. The GUT scale is the best motivated between 10^{15} GeV and 10^{17} GeV with the best fit close to 10^{16} GeV. For completeness, a similar plot for α_{EM}, s_W^2, and α_3 is presented in Fig. 2 (right). However, as we will see from the discussion of threshold corrections in the next subsection, the plot on the left for α_1, α_2, and α_3 is more indicative about the best motivated values of M_G and M_{VF}.

In order to understand the sensitivity of the EW scale values of gauge couplings to fundamental parameters quantitatively, it is instructive to estimate separate contributions to $\alpha_{1,2,3}(M_Z)$ from α_G, M_G and M_{VF}. Note, that the values of $\alpha^{-1}_{1,2,3}(M_Z)$ are approximately 59, 30, and 8.4 respectively. From Eqs. (4) and (10) we see that $\alpha_G \gtrsim 0.3$ contributes less than $\sim 10\%$ to the EW scale values of gauge couplings. It is the least important parameter. Plots in Fig. 2 for any $\alpha_G > 0.3$ would look almost identical. Increasing α_G moves all the contours slightly to the right. The largest contribution to EW scale values of couplings originates from $1/(2\pi)\ln(M_G/M_Z) \simeq 5.2$ term multiplied by corresponding beta function coefficients.

The second largest contribution to gauge couplings come from masses of vector-like fermions. The IR fixed point predictions for the gauge couplings at the EW scale, obtained from Eq. (4) for $\alpha_{1,2}$ with $\alpha_{1,2}(M_G) = \alpha_G$, and from Eq. (9) for α_3, are modified by threshold corrections T_i:

$$\alpha_i(M_Z) \;\to\; \frac{\alpha_i(M_Z)}{1-\alpha_i(M_Z)T_i}, \tag{11}$$

that depend on masses of the extra vector-like fermions. These threshold effects are well

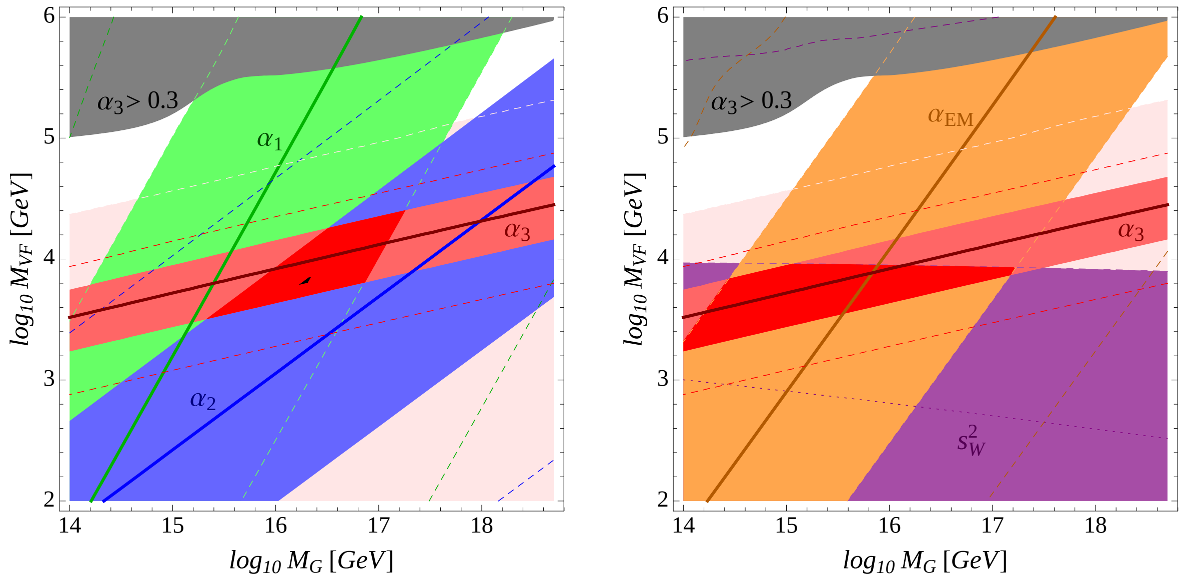

FIG. 2: Left: contours of constant values of predicted gauge couplings at M_Z, α_1 green, α_2 blue, and α_3 red, as functions of the GUT scale, M_G, and the universal mass of fermions from 3 vector-like families, M_{VF}, for fixed $\alpha_G = 0.3$. Solid lines represent the central experimental values of three gauge couplings, the shaded regions represent $\pm 10\%$ ranges, and the dashed lines in unshaded areas represent $\pm 20\%$ ranges. Lightly shaded area corresponds to $\pm 50\%$ range of α_3. In the bright red region all three gauge couplings are simultaneously predicted within 10% from the measured values, and the small black area in the red region represents the best fit with all three couplings within 6% from the measured values. The gray region corresponds to $\alpha_3(M_Z) > 0.3$; $\alpha_3(M_Z)$ becomes non-perturbative very fast with increasing M_{VF} from the value that corresponds to the boundary of this region. Right: the same as in the plot on the left but for α_{EM} orange, s_W^2 purple, and α_3 red. The dotted purple line represents s_W^2 being -5% from the central value.

approximated by the leading logarithmic corrections:[5]

$$T_i \;=\; \frac{1}{2\pi}\sum_f b_i^f \ln\frac{M_f}{M_Z}, \tag{12}$$

where b_i^f is the contribution of a given fermion f, with mass M_f, to the corresponding beta function coefficient [18]. For particles originating from vector-like families these contribu-

[5] These corrections correspond to removing one loop contributions of vector-like fermions from Eqs. (4) and Eq. (9) below their mass. It is an excellent approximation for $\alpha_{1,2}$ and sufficient approximation for α_3 since, for the IR value of α_3, the 1-loop term in the RG equation dominates.

TABLE I: Quantum numbers and contributions to beta function coefficients of particles from extra vector-like families. The names are chosen to mimic those of the standard model particles with the same quantum numbers. For each particle there is a corresponding vector-like partner, and its contributions to the beta function coefficients are identical. The b_1 coefficients correspond to SU(5) normalization of the hypercharge.

Particle	$SU(3)$	$\times$	$SU(2)$	$\times$	$U(1)$	b_3	b_2	b_1
Q	3		2		1/6	2/3	1	1/15
U	$\bar{3}$		1		-2/3	1/3	0	8/15
E	1		1		1	0	0	2/5
L	1		2		-1/2	0	1/3	1/5
D	$\bar{3}$		1		1/3	1/3	0	2/15

tions, summarized in Table I, are identical to contributions from fermions in the standard model. The contribution from the complete family is identical to all three beta function coefficients and equal to 4/3 for a chiral family, and 8/3 for a vector-like pair ($16 + \overline{16}$ in the SO(10) language).

The correction to α_3 of about +40% is crucial in order to reproduce the measured value. As can be seen in Figs. 1 and 2, it is indeed α_3 that determines $M_{VF} \simeq 10^4$ GeV and consequently $M_G \simeq 10^{16}$ GeV. The other two couplings are within 10% from measured values in much larger ranges of M_{VF} and would actually prefer smaller M_{VF} and M_G.

Out of the three parameters, the EW scale values of gauge couplings are the most sensitive to changes in M_{VF}. However, since M_{VF} is only responsible for at most $\sim 40\%$ of the EW scale values of couplings, the overall sensitivity is still very small. Most of the EW scale values of couplings originate from the IR fixed point. Since no precise cancellations between separate contributions are required, there are large ranges of fundamental parameters from which the predicted values of gauge couplings at the EW scale are close to observed values.

The standard model extended by 4 vector-like families (SM+4VFs) allows for insensitive unification of gauge couplings in a similar way as the SM+3VF. Predicted values of gauge couplings at M_Z as functions of the GUT scale and the universal mass of fermions from 4 vector-like families for fixed $\alpha_G = 0.3$ are showed in Fig. 3. There are however notable

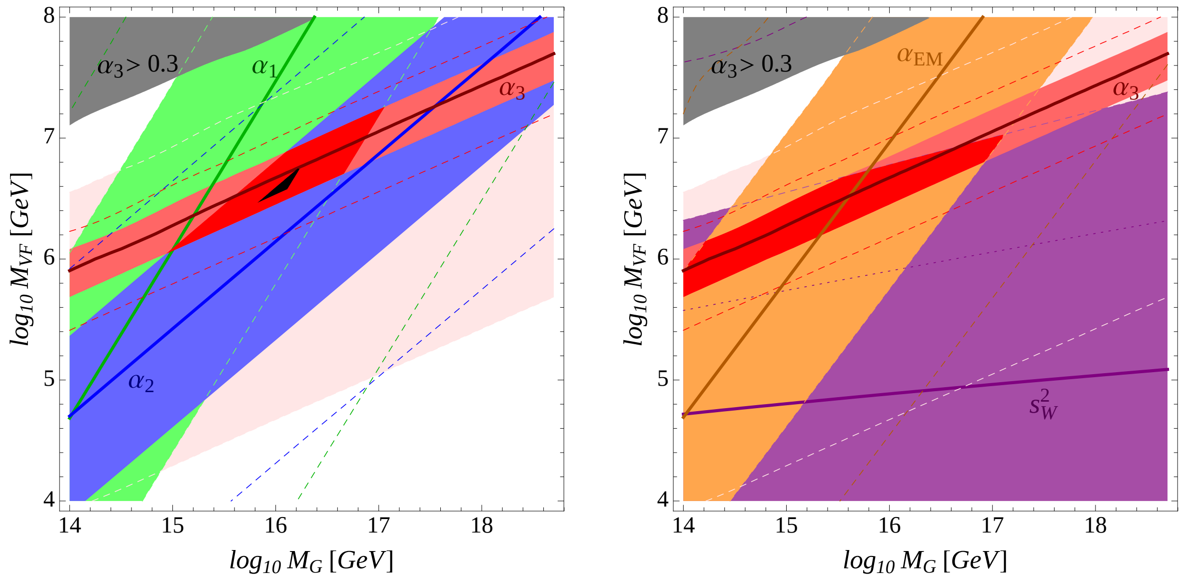

FIG. 3: The same as in Fig. 2 but for the SM extended by 4 vector-like families.

differences from the SM+3VFs case. First of all, the common mass of vector-like families moves to $\sim 10^6 - 10^7$ GeV. This is easily understood from the fact that more matter makes gauge couplings run faster and thus the VFs must stop contributing to RG evolution at a higher scale, otherwise the EW scale values of gauge couplings would be too small. Second of all, the 1-loop IR fixed point value of $\sin^2\theta_W$ is 0.234 which is larger than in the SM+3VFs case and actually very close to the measured value. Overall, this however does not make the predictions much better than in the SM+3VFs case since α_3 requires M_{VF} larger than the one needed to reach the measured value of $\sin^2\theta_W$. Finally, as a result of larger masses of VFs required in SM+4VFs scenario, the sensitivity of EW scale values of gauge couplings to fundamental parameters increased, which is visible in Fig. 3 as narrower 10% bands compared to those in Fig. 2 corresponding to the case of SM+3VFs.

It is easy to extrapolate to larger number of VFs. Increasing the number of VFs requires larger M_{VF} closer and closer to the GUT scale. The sensitivity of predicted values of gauge couplings to fundamental parameters is increasing and approaching the sensitivity in the SM.

In the SM extended by 1 or 2 vector-like families the predictive power is lost, since the unified gauge coupling is small and its specific value is crucial for predictions for gauge couplings at the EW scale in a similar way as in the SM. The difference form the SM is that the exact unification of gauge couplings is now possible with split masses of VFs. We will include these solutions as a curiosity in the next subsection.

C. Threshold effects of vector-like fermions

Let's now turn our attention to precise predictions for gauge couplings rather than ∼10% agreement. For this we need to consider threshold effects from splitting masses of VFs.

The necessity to split masses of particles from extra 3VFs is indicated in Fig. 1 (left) by slightly different scales at which the RG evolutions of gauge couplings in the SM and SM+3VFs cross. For the example in this figure the crossing scales for α_1, α_2, and α_3 are $M_1 \simeq 100$ TeV, $M_2 \simeq 1$ TeV, and $M_3 \simeq 10$ TeV. These scales determine threshold corrections $T_i = (4/\pi)\ln(M_i/M_Z)$, see Eq. (12), required for gauge coupling unification. Any spectrum that leads to required threshold corrections will reproduce the measured values of gauge couplings.

The crossing scales are increasing with increasing M_G and depend very little on α_G for $\alpha_G \gtrsim 0.3$. For different values of M_G they can be read out of Fig. 2 (left) as corresponding values of M_{VF} for which we obtain the measured value of given gauge coupling. Similarly, in the case of SM+4VFs the values of $M_{1,2,3}$ can be read out of Fig. 3 (left). For values of M_G not showed, or for other scenarios, the crossing scales can be easily calculated from RG equations as functions of M_G and α_G.

In general, for N pairs of vector-like families, once we know values of crossing scales $M_{1,2,3}$ for chosen GUT scale, the masses of fermions must satisfy:

$$\frac{4N}{3\pi}\ln\frac{M_3}{M_Z} = \frac{1}{\pi}\sum_{i=1}^{N}\left(b_3^Q\ln\frac{M_{Q_i}}{M_Z} + b_3^U\ln\frac{M_{U_i}}{M_Z} + b_3^D\ln\frac{M_{D_i}}{M_Z}\right), \tag{13}$$

$$\frac{4N}{3\pi}\ln\frac{M_2}{M_Z} = \frac{1}{\pi}\sum_{i=1}^{N}\left(b_2^Q\ln\frac{M_{Q_i}}{M_Z} + b_2^L\ln\frac{M_{L_i}}{M_Z}\right), \tag{14}$$

$$\frac{4N}{3\pi}\ln\frac{M_1}{M_Z} = \frac{1}{\pi}\sum_{i=1}^{N}\left(b_1^Q\ln\frac{M_{Q_i}}{M_Z} + b_1^U\ln\frac{M_{U_i}}{M_Z} + b_1^D\ln\frac{M_{D_i}}{M_Z} + b_1^L\ln\frac{M_{L_i}}{M_Z} + b_1^E\ln\frac{M_{E_i}}{M_Z}\right), \tag{15}$$

in order to get exact gauge coupling unification at given GUT scale. In the case of universal masses of particles with the same quantum numbers, *e.g.* $M_{Q_1} = M_{Q_2} = \cdots = M_{Q_N} \equiv M_Q$, these mass rules can be written in a simple form:

$$M_i^{4/3} = \prod_{F=Q,U,D,L,E} M_F^{b_i^F}, \qquad i = 1,2,3. \tag{16}$$

Inserting the beta function coefficients from Table I we find:

$$M_3^4 = M_Q^2 M_U M_D, \tag{17}$$

$$M_2^4 = M_Q^3 M_L, \tag{18}$$

$$M_1^{20} = M_Q M_U^8 M_D^2 M_L^3 M_E^6. \tag{19}$$

These formulas hold for any number of complete vector-like families, only the values of $M_{1,2,3}$ depend on the specific scenario. In the case of non-universal masses of particles with the same quantum numbers the above formulas are still valid with the replacement:

$$M_F \equiv \left(M_{F_1} M_{F_2} \dots M_{F_N}\right)^{1/N}, \qquad F = Q, U, D, L, E. \tag{20}$$

From Eqs. (17) - (20) we can immediately see that splitting fermions with the same quantum numbers does not help to find a solution if a solution does not exist with universal masses. Thus, it is sufficient to assume universal masses, M_F, of particles with the same quantum numbers, and Eqs. (17) - (19) classify possible solutions. In addition, for each solution with universal masses there are other solutions with split masses, and the only constrain is that their geometric mean is the universal mass needed for given solution.

There are many solutions available, since we have 5 different masses that have to satisfy three conditions (17) - (19) . However, it is not guaranteed that for given GUT scale there is a phenomenologically viable solution. Clearly, the crossing scales have to be above the EW scale, and even then the solution might require new fermions below experimental limits or some fermions above the GUT scale or the Planck scale.

Representative examples of the spectrum for various values of M_G in the case of SM+3VFs are given in Fig. 4. The value of α_G is fixed to 0.3, however the spectrum is not very sensitive to this choice as previously discussed. The spectrum showed is just an example, motivated by the smallest splitting between masses required, and it is not unique. A specific example with exact numerical values was also given in Ref. [1]. The GUT scale motivated by lowest splitting required between masses of vector-like fermions is at $\sim 10^{16}$ GeV in agreement with what is suggested in Fig. 2 (left), and the masses are split between ∼1 TeV and ∼100 TeV.

There is a lower bound on possible GUT scale at $\sim 10^{15}$ GeV. For smaller M_G the crossing scale M_2 is too small, see Fig. 2 (left), and thus a phenomenologically viable solution does not exist. With increasing M_G the splitting of fermion masses is increasing that can be inferred from larger splitting of crossing scales. The GUT scale can be as high as the Planck

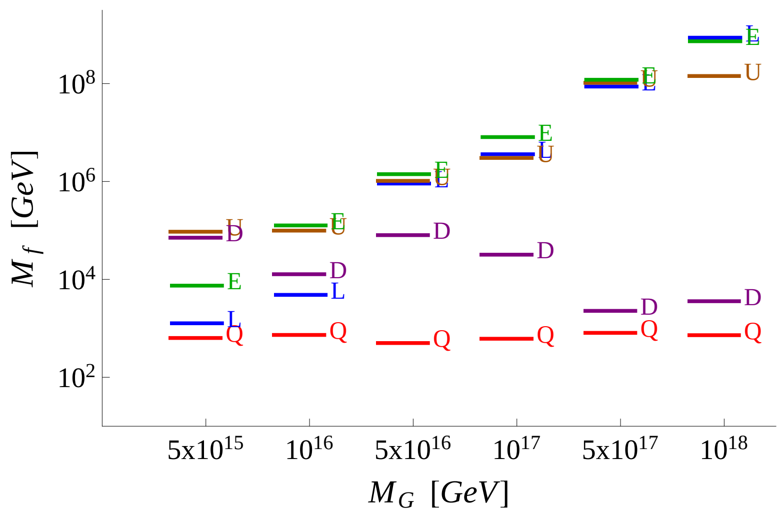

FIG. 4: Masses of vector-like fermions leading to exact gauge coupling unification as functions of the GUT scale in the case of SM+3VFs. The universal mass for particles with the same quantum numbers is assumed. The value of α_G is fixed to 0.3. Smaller values of M_G (not showed) can still be consistent with gauge coupling unification for smaller α_G. The spectrum showed is just an example, it is not unique.

scale. However, in that case the masses of vector-like fermions have to be split over 6 orders of magnitude.

Note that quark doublets, Q, are typically predicted at $\sim$1 TeV. Preference for Q being the lightest of vector-like fermions can be understood from $M_2 < M_{1,3}$. However there are also solutions with L being the lightest. Keep in mind however, that these masses represent geometric means of masses of particles with the same quantum numbers. Therefore, when considering split masses of fermions with the same quantum numbers any fermion can be the lightest one and as light as current experimental limits.

For SM+4VFs examples of the spectrum are given in Fig. 5. The main features are very similar to the case of SM+3VFs. The GUT scale motivated by lowest splitting required between masses of vector-like fermions is also at $\sim 10^{16}$ GeV in agreement with what is suggested in Fig. 3 (left), and about two orders of magnitude splitting of masses of vector-like fermions is required. The main difference from the SM+3VFs case is that the spectrum shifted to $10^6 - 10^8$ GeV.

For completeness we also include examples of the spectrum needed for exact gauge coupling unification in the case of SM+1VF in Fig. 6 and SM+2VFs in Fig. 7. For these cases

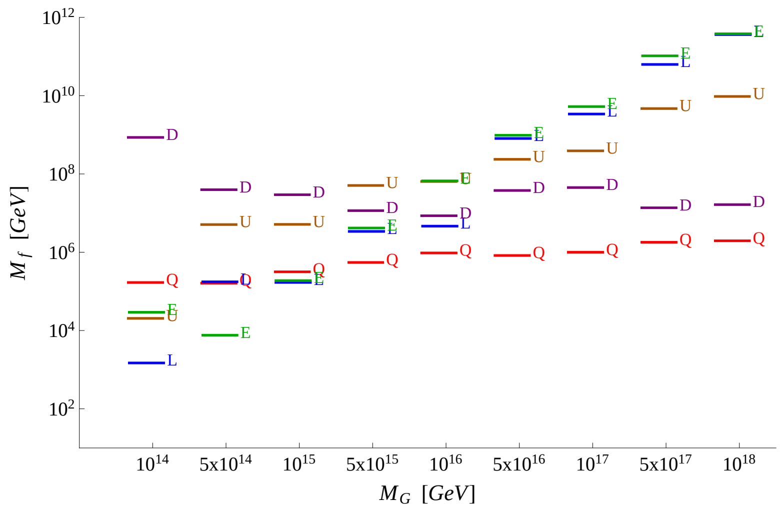

FIG. 5: The same as in Fig. 4 but in the case of SM+4VFs.

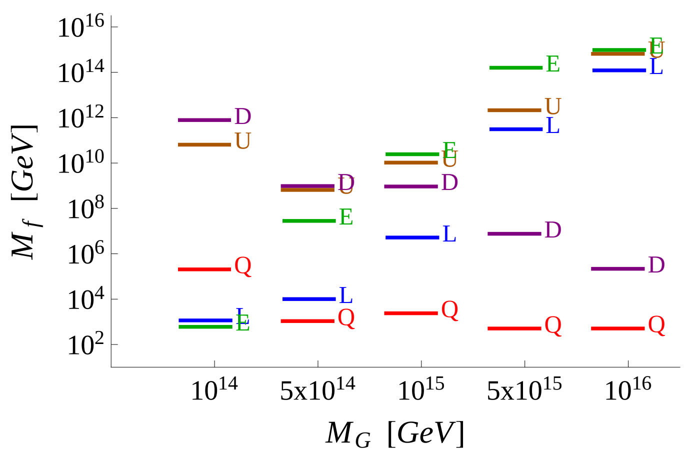

FIG. 6: The same as in Fig. 4 but in the case of SM+1VF. In this case values of α_G are optimized for given GUT scale, and are close to 0.03 for all M_G showed. For smaller or larger values of M_G the unification is not possible for any spectrum.

the EW scale values of gauge couplings are highly sensitive to α_G. Thus α_G in these examples is not fixed but rather optimized for given M_G. In both cases the exact unification can be achieved even in the region consistent with limits on proton lifetime. However the required splitting between masses of vector-like fermions is sizable.

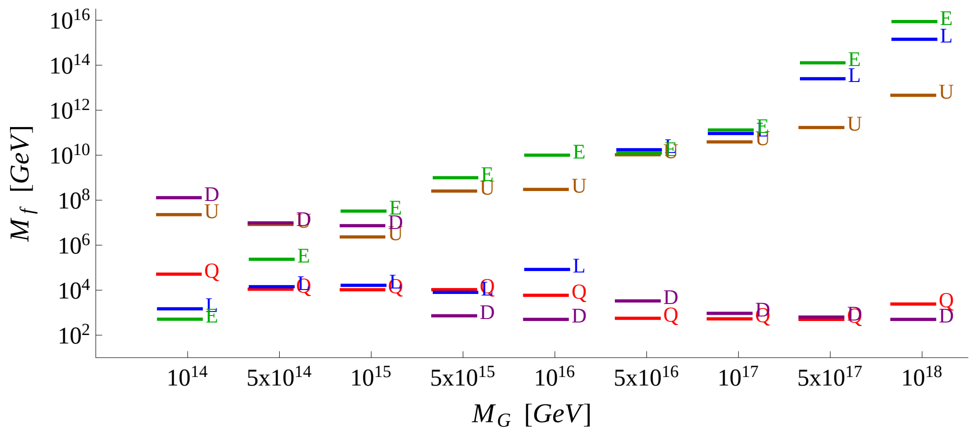

FIG. 7: The same as in Fig. 4 but in the case of SM+2VF. In this case values of α_G are optimized for given GUT scale, and vary between 0.042 and 0.048. Smaller values of M_G (not showed) can still be consistent with gauge coupling unification.

D. Generalization of mass rules to other extensions of the SM

The mass rules we have just derived can be generalized to any extension of the SM. The existence of crossing scales is a necessary condition for achieving gauge coupling unification in a given model. This follows from the fact that integrating out extra fields above the EW scale can only increase gauge couplings at the EW scale. Therefore, values of predicted couplings at the EW scale without considering the mass effect of extra matter fields have to be smaller than the measured values. The crossing scales depend only on α_G and M_G. Thus, requiring that the crossing scales exist leads to limits on possible values of the GUT scale and α_G.

For chosen α_G and M_G we can find the crossing scales $M_{1,2,3}$ for all three gauge couplings. If one loop RGEs are good approximations these crossing scales can be easily found by applying Eq. (4) separately between M_Z the M_i scales using the SM beta function coefficients, b_i^{SM}, starting with observed values of $\alpha_{i,exp}(M_Z)$, and between M_i and M_G scales using beta function coefficients in the given extension, b_i, assuming gauge couplings exactly unify. We get:

$$\ln\frac{M_i}{M_Z} = \frac{2\pi}{b_i - b_i^{SM}}\left(-\alpha_{i,exp}^{-1}(M_Z) + \alpha_G^{-1} + \frac{b_i}{2\pi}\ln\frac{M_G}{M_Z}\right). \tag{21}$$

The meaning of crossing scales is the same as in extensions of the SM with VFs, namely they represent the threshold corrections, $T_i = (4/\pi)\ln(M_i/M_Z)$, that masses of extra particles must generate in order to reproduce the measured values of gauge couplings starting from given α_G and M_G. The rest follows what we did for complete VFs. Once we know values of crossing scales $M_{1,2,3}$, in order to get exact gauge coupling unification, the masses of extra particles must satisfy:

$$M_i^{(b_i - b_i^{SM})} = \prod_F M_F^{b_i^F}, \qquad i = 1, 2, 3, \tag{22}$$

where the product is over all extra fermions (or scalars) charged under given gauge symmetry. For a vector-like pair of fermions the corresponding mass on the right-hand side appears twice. As in the case of complete VFs it is sufficient to consider the universal mass of all particles with the same quantum numbers. The universal mass that enters Eq. (22) represents their geometric mean.

For any model with arbitrary particle content, the crossing scales (21) as functions of α_G and M_G together with the mass rules (22) classify all the solutions consistent with gauge coupling unification in terms of physical masses of extra particles.

Let us illustrate the usefulness of crossing scales and the mass rules on one example. Let us ask if there is any spectrum of extra particles in the SM extended by one vector-like family that leads to exact gauge coupling unification for $M_G = 10^{16}$ GeV. This choice corresponds to one of the points in Fig. 6 and so we already have the answer we can compare with. However this answer is obtained by fairly complicated numerical procedure, that iteratively solves coupled differential equations with masses of extra vector-like fermions varied till the EW scale values of gauge couplings are precisely reproduced. Using our method, we can get the basic features of the required spectrum fast.

For $\alpha_G = 0.0286$ that corresponds to the given example in Fig. 6, the crossing scales $M_{1,2,3}$, easily calculated from Eq. (21), are 1×10^{13} GeV, 7×10^4 GeV, and 2×10^6 GeV. This immediately tells us that there will be more than 8 orders of magnitude splitting between masses required. Knowing the crossing scales we can easily see basic features of the spectrum that will work. From Eqs. (22), which in this case are the same as Eqs. (17) - (19), we see that M_Q, which heavily weighs on M_2, should be less than M_2, while everything with large hypercharge (especially E and U) should be above M_1 in order to find a solution. For a specific example, one can choose two masses, and calculate the rest of the spectrum from formulas Eqs. (17) - (19). Given large splitting between M_2 and M_1 in this case, it would be easiest to choose the masses of Q and U as a starting point. Once we have one solution, varying starting masses of Q and U, and calculating the rest of masses from Eqs. (17) - (19) will give us all possible solutions for given α_G and M_G. This procedure can be repeated for any α_G and M_G, or the solutions can be plotted as functions of these variables.

III. DISCUSSION

So far we have only considered constraints on the GUT scale and masses of vector-like fermions from gauge coupling unification. In order for this scenario to be easily embedded into simple grand unified theories, based on $SU(5)$ or $SO(10)$, the constraints on proton lifetime and the stability of EW minimum of the Higgs potential should be satisfied.

The most stringent limits on proton lifetime come from Super-Kamiokande. For the dominant decay mode from dimension 6 operators, the limit is $\tau(p \to \pi^0 e^+) > 1.4 \times 10^{34}$ yrs [24]. Assuming naively, that proton lifetime is $\tau_p \sim M_G^4/(\alpha_G^2 m_p^5)$, where m_p is the mass of the proton, this limit translates into the lower bound on the GUT scale: $M_G > 1.5 \times 10^{16}$ GeV for $\alpha_G = 0.3$ which we use in our examples. However, the prediction for proton lifetime is somewhat model dependent, see for example Refs. [2, 24, 25] and references therein, and so we do not impose the strict limit in the plots we present. In addition, the plots would look very similar for any large value of α_G but the limits would differ. The interested reader can easily impose the limit on any scenario by simple rescaling of the mentioned limit using the formula for proton lifetime.

It is interesting to note that the best motivated value of the GUT scale is in the $\sim 10^{16}$ GeV range which is basically at the current limit. It is however not possible to make precise predictions without knowing masses of vector-like fermions. For example, a scenario with the GUT scale larger by a factor of 3 results in $\sim$2 orders of magnitude enhancement of proton lifetime, but would only require modest changes in the spectrum of vector-like fermions in order to have exact gauge coupling unification. This inability to make precise predictions of GUT scale parameters is a direct consequence of the insensitivity of EW scale couplings to GUT scale boundary conditions.

The RG evolution of the top Yukawa and Higgs quartic couplings in the SM+3VFs for $M_G = 2 \times 10^{16}$ GeV and $\alpha_G = 0.3$ was given in Ref. [1]. The Higgs quartic coupling remains positive all the way to the GUT scale and thus the electroweak minimum of the Higgs potential is stable. This result holds in a large range of M_G and α_G especially in the best motivated region. Therefore, these scenarios represent some of the simplest possible extensions of the standard model that can be embedded into grand unified theories, with sufficiently long lived proton, and stable EW minimum of the Higgs potential.

We have not investigated the origin of masses of vector-like fermions needed for gauge coupling unification. This would require additional assumptions about the mechanism that generates them and the scale at which boundary conditions are set. The masses of vector-like fermions may be fundamental lagrangian parameters, or they can originate from Yukawa couplings to one or several additional scalars (singlets under SM gauge symmetry, but possibly charged under family symmetries) that acquire vacuum expectation values at any scale between the GUT scale and the M_{VF} scale. In addition, vector-like fermions can have non-zero Yukawa couplings to the SM Higgs doublet, which add another layer of complexity by contributing to the physical masses and possibly significantly affecting the RG evolution of other parameters that directly determine their masses. The study of gauge coupling unification is to large extent unaffected by these assumptions, only the physical masses of particles matter in the leading order. Fundamental lagrangian masses would not affect running of gauge couplings at all, and the Yukawa couplings to extra scalars may only contribute to the RG evolution of gauge couplings at 2-loop level. However, in any specific scenario, the mass rules (17) - (19) can be evolved to the GUT scale (or other relevant scale) and the freedom to choose some of the masses can be used to search for simple boundary conditions that are consistent with gauge coupling unification.

Finally, it is intriguing to consider a connection with the anthropic solution to the hierarchy problem, or the EW scale [26, 27]. Adding VFs to the SM makes this possibility more appealing, since in the SM special values of gauge couplings either at the EW scale or some high scale have to be selected. In scenarios that we discussed the EW scale values of gauge couplings close to the observed values are not very special, but rather quite generic outcome from large ranges of fundamental parameters. For example in the SM+3VFs case, as far as $M_{VF} < 25$ TeV, for any $\alpha_G \gtrsim 0.2$, and M_G anywhere between 10^{14} GeV and the Planck scale, the predicted values of gauge coupling at the EW scale are always within 50% of the measured values ($\alpha_{1,2}$ typically well within 20%). This is indicated by lightly shaded region in Fig. 2, and similar region is indicated in Fig. 3 for SM+4VFs. Furthermore, if the EW scale and M_{VF} have the same origin it would also explain the proximity of the QCD scale to the EW scale. The beta function of α_3 changes the sign at M_{VF}, and below this scale it starts running fast toward Λ_{QCD}.

IV. CONCLUSIONS

We have discussed gauge coupling unification in models with additional 1 to 4 complete vector-like families. In scenarios with 3 or more vector-like families the values of gauge couplings at the electroweak scale are highly insensitive to the grand unification scale, the unified gauge coupling, and the masses of vector-like fermions. Their observed values can be mostly understood from infrared fixed point behavior. Starting with a large (but still perturbative) unified gauge coupling at a high scale, the values of gauge couplings at lower energies are determined only by the particle content of the theory and how far from the GUT scale we measure them. Since the exact value of α_G becomes irrelevant, instead of one prediction of the conventional unification we have two predictions for ratios of gauge couplings. These predictions are modified at the M_{VF} scale, where the vector-like fermions are integrated out, and below this scale gauge couplings run according to the usual RG equations of the standard model.

Assuming first a common mass of vector-like fermions, M_{VF}, we showed predictions for three gauge couplings at the EW scale as functions of M_G and M_{VF}. We found that the observed values of gauge coupling are reproduced with good precision from a large range of parameters. Especially M_G can be varied over several orders of magnitude while having all three gauge couplings within 20% from observed values. The best fit, which predicts all three couplings within 6% from measured values, suggests $M_G \sim 10^{16}$ GeV, and $M_{VF} \simeq 10^4$ GeV in the case of SM+3VFs, and $M_{VF} \simeq 10^6 - 10^7$ GeV in the case of SM+4VFs.

The best motivated GUT scale, $\sim 10^{16}$ GeV, predicts proton lifetime close to current limits. However, due to insensitivity of predicted EW scale values of gauge couplings to GUT scale parameters no sharp predictions can be made without knowing the spectrum of vector-like fermions. In addition, it was previously showed that the Higgs quartic coupling remains positive all the way to the GUT scale, and thus the electroweak minimum of the Higgs potential is stable. This result holds in a large range of M_G and α_G especially in the best motivated region. Therefore, these scenarios represent some of the simplest possible extensions of the standard model that can be embedded into grand unified theories, with sufficiently long lived proton, and stable EW minimum of the Higgs potential.

The discrepancies of IR fixed point predictions from observed values can be explained by threshold effects of extra vector-like fermions. We showed examples of the spectrum for GUT scale varied between 10^{14} GeV and 10^{18} GeV. We derived simple rules for masses of vector-like fermions required for exact gauge couplings unification. In addition, we generalized the mass rules and the method of using crossing scales of evolutions of gauge couplings in the SM and given extension to classify scenarios consistent with gauge coupling unification to arbitrary extension of the standard model. The problem of finding all possible mass spectra in a given model consistent with gauge coupling unification is reduced to solving a set of simple algebraic equations that masses of extra particles have to satisfy.

With respect to sensitivity to fundamental parameters the model with 3 extra vector-like families stands out. In the best motivated region it requires vector-like fermions with masses of order 1 TeV – 100 TeV, and thus at least part of the spectrum may be within the reach of the LHC. Notably, quark doublets, Q, are typically predicted at $\sim$1 TeV. However, only geometric means of masses of particles with the same quantum numbers are constrained by gauge coupling unification. Therefore, when considering split masses of fermions with the same quantum numbers, any fermion can be the lightest one, and as light as current experimental limits. Besides direct production of these particles at the LHC, it may be also possible to observe their effects in a variety of processes. However, they typically affect standard model predictions only through mixing with light fermions which is highly model dependent. The discussion of gauge coupling unification, that we focused on here, is negligibly affected by such mixing.

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