Ultraviolet complete model of large N thermal QCD

Fang Chen, Long Chen, Keshav Dasgupta, Mohammed Mia, and Olivier Trottier

Phys. Rev. D **87**, 041901 — Published 19 February 2013

DOI: 10.1103/PhysRevD.87.041901

A UV complete model of Large N Thermal QCD

Fang Chen,[1,*] Long Chen,[2,†] Keshav Dasgupta,[2,‡] Mohammed Mia,[3,§] and Olivier Trottier[2,¶]

[1]*KITP, University of California, Santa Barbara, CA 93106-4030, USA*
[2]*Department of Physics, McGill University, Montréal, QC, H3A 2T8, Canada*
[3]*Department of Physics, Columbia University, 538 West 120th Street, New York 10027, USA*

Many recent works on large N holographic QCD in the planar limit have not considered UV completions, restricting exclusively towards analyzing the IR physics. Due to this, the UV problems like Landau poles and divergences of Wilson loops including instabilities at high temperatures have not been addressed. In some of our recent papers, we have discussed a possible UV completion, which is conformal in the UV and confining in the far IR, that avoids the Landau poles and the Wilson loop divergences. In this paper we give a general field theoretic considerations of this including the complete RG flow. We extend our UV complete model to study scenarios both above and below the deconfinement temperature and argue how phase transition in our model should be understood. Interestingly, because of the UV completion, subtle issues like instability due to negative specific heat do not appear.

PACS numbers: 98.80.Cq

1. INTRODUCTION

The gauge/gravity duality has so far proved to be a powerful technique to solve many strong coupling problems of large N gauge theories, and especially large N QCD, in the planar limit. The application of a *gravity* dual to understand strongly coupled gauge theory was, in retrospect, the next best thing to do. A simple way to see this would be to consider a particular gauge-theory defined on a $3+1$ dimensional *slice* at a certain energy scale Λ. Now imagine we stack up all the slices together, described at different energy scales, along an orthogonal direction (call it the "radial" direction r). This way we will get a *five* dimensional space that captures the full dynamics of a given gauge theory from the Ultra-Violet (UV), i.e large r, to the Infra-Red (IR), i.e small r. The "radial" direction would then obviously be the direction along which the energy would change, i.e the direction of the Renormalisation Group (RG) flow. For a Conformal Field Theory (CFT), the theory does not change along the radial direction[1] and therefore could as well be defined at the boundary of the five-dimensional space. The scale invariance of the underlying gauge theory will restrict the geometry of the five-dimensional space to the Anti-deSitter (AdS) space [1], although it would be interesting to argue that this is the unique choice[2].

However, for gauge theories with inherent RG flows, the situation will be different and it would be instructive to study the theories at various r (although we could also restrict ourselves to the boundary again). The example that we are interested in is large N QCD, which we expect to be asymptotically conformal[3] in the UV and confining in the far IR. Specific geometries that do the jobs for both zero and non-zero temperatures were presented in [5, 6] although the details of the gauge theories were not presented there. In this paper we will fill up some of the gaps left in [5, 6] and argue why we believe our choice of the gravity dual is better suited to study large N thermal QCD (see also [7] for another model that studies UV complete large N thermal QCD from a bottom-up five-dimensional point of view).

2. THE FIELD THEORY FROM THE GRAVITY DUAL

The gravity dual of a large N thermal QCD above the deconfinement temperature, described using only a *flavored* Klebanov-Strassler geometry [2] with a black-hole has few ultra-violet (UV) problems. For example, there are Landau poles coming from the flavor branes, and the Wilson loops are generically UV divergent [4]. All these issues could be resolved if we properly augment the Klebanov-Strassler geometry, which we will henceforth call the Ouyang-Klebanov-Strassler black-hole (OKS-BH) [3, 5, 6] geometry, with a suitable asymptotically

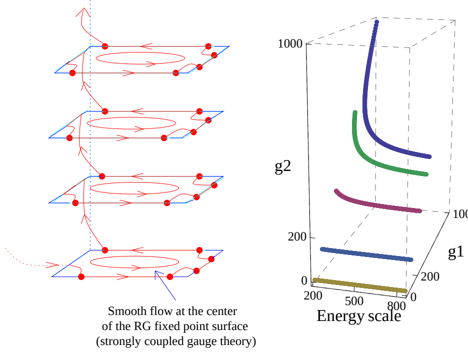
*email: ccfprince@gmail.com
†email: long.chen2@mail.mcgill.ca
‡email: keshav@hep.physics.mcgill.ca
§email: mm3994@columbia.edu
¶email: olivier.trottier@mail.mcgill.ca

[1] Assuming the usual behavior of the irrelevant operators.

[2] Furthermore, a Feynman diagram for any interaction between point-like particles, when *stacked up* as above, would look like an interaction between extended objects, i.e strings! This is basically the essence of using string (or gravity) duals to study gauge theories. It will be informative to make this more precise.

[3] It is important that we demand conformal behavior in the UV and not asymptotic freedom. This is because the 'tHooft coupling $\lambda \equiv g_{YM}^2 N$ approaches a constant in the limit $g_{YM}^2 \to 0$ and $N \to \infty$. This way, the theory is actually asymptotically free in terms of g_{YM}^2 but conformal in terms of λ. Furthermore, we will demand λ to be very large throughout the whole RG flow so that the gravity dual can be restricted to its classical limit.

Anti-de Sitter (AdS) space. In this paper we will start with a gauge theory interpretation of the background.



(a) RG flow in the far IR (b) RG flow in the far UV

FIG. 1: In the far IR, one may note that the cascading RG flow is never captured by the classical gravity theory. The classical supergravity description would capture only the smooth parts of the RG flow shown in (a) at the center of each slices. Note that the circles drawn on each slices should be taken as helices that become more and more straight as we go to the center of each of the slices. The vertical distances in (a) has no physical meaning and only refer to the slices described using appropriate Seiberg dual descriptions. On the other hand, in (b) the RG flows all tend to go to zero at some UV scales. This is where the theory becomes conformal (all scales are chosen with $\alpha' = 1$). Note that, only the strongly coupled parts of figure (b) are captured by the classical supergravity description.

2.1. Continuous RG Flow from UV to IR

The Beta functions are also easy to compute to first order in $g_s N_f$ from the gravity dual. Region 1, 2 and 3 correspond to small, intermediate and large r region in the dual geometry. In Region 1 the two couplings at a scale Λ run in the following way:

$$\Lambda \frac{\partial}{\partial \Lambda}\left[\frac{4\pi}{{g_1}^2} + \frac{4\pi}{{g_2}^2}\right] = \frac{N_f}{8}\left(\frac{6r^6 + 36a^2r^4}{r^6 + 9a^2r^4}\right)\Big|_{r=\Lambda} \tag{2.1}$$

$$\Lambda \frac{\partial}{\partial \Lambda}\left[\frac{4\pi}{{g_1}^2} - \frac{4\pi}{{g_2}^2}\right] = 3M\left[1 + \frac{3g_s N_f}{4\pi}\log(\Lambda^2 + 9a^2)\right]_{r=\Lambda}$$

where the RHS of both equations is evaluated at $r \equiv \Lambda$ in the gravity picture. The constant a appearing above is the bare resolution parameter that one may set to zero and the flow is depicted by the inner circles of Fig 1 (a)[4].

In this limit, the RG flow is clearly the NSVZ RG flow [12]. On the other hand, in region 2, where we still have two couplings, the RG flow is highly non-trivial. This can be derived from the gravity dual where we see that the three-form fluxes play an important role in the running of the couplings [11]:

$$\frac{8\pi^2}{{g_1}^2} = e^{-\Phi}\left[\pi - \frac{1}{2} + \frac{1}{2\pi}\int_{S^2} B_2\right] \tag{2.2}$$

$$\frac{8\pi^2}{{g_2}^2} = e^{-\Phi}\left[\pi + \frac{1}{2} - \frac{1}{2\pi}\int_{S^2} B_2\right] \tag{2.3}$$

Finally in region 3, the scenario is somewhat simpler. The two couplings flow approximately at the same rate and the flow is governed by the N_f D7 and anti-D7 pairs that we keep in region 3 to cancel the Landau poles. These seven-branes are responsible for restoring the $SU(N_f) \times SU(N_f)$ chiral symmetry above the deconfinement temperature (i.e when we insert a black-hole with a horizon radius r_h [5, 6]). The running of the coupling, which we call g_{YM}, is now:

$$\Lambda \frac{\partial g_{\rm YM}}{\partial \Lambda} = {g_{\rm YM}}^3 \sum_{n=1}^{\infty} \frac{\mathcal{D}_n}{\Lambda^{3n/2}} \tag{2.4}$$

where $\mathcal{D}_n$ are all independent of Λ and whose precise form will be derived in [11]. The RG flow in the UV region i.e. region 3 is shown in Fig 1(b).

The Beta functions discussed above can now be succinctly expressed as a continuous flow from UV to IR as shown in fig 2. We have represented this using a slightly unconventional way. We get the complete RG flow by gluing the three regions altogether and using S-duality to transmute strong coupling into weak coupling. Starting from IR regime, once a particular coupling gets strong, a S-duality is performed to reverse the sign of the beta function associated with that coupling. This appears as the sharp edges in the figure above. From the UV region this can be seen in the following way: The coupling starts as a constant in Region 3, when it gets to the transition point ($r_0 = 200$), it has a small plateau region continuing in region 2, then it flows down to Region 1. The RG flow continues after the transition point $r_{min} = 100$, but since the rate of change is fast more sharp corners appear in region 1. These are the points connected to their S-dual values. Eventually this reaches the smallest energy possible after which we expect linear confinement at low temperatures. More details on this construction will appear soon in [11].

2.2. Higgsing

In region 3, we have a $SU(N+M) \times SU(N+M)$ gauge group[5] which breaks down to $SU(N+M) \times SU(N)$

[4] This is however not so above the deconfinement temperature. As shown recently in [8], even if we demand a vanishing bare resolution parameter, it will get a contribution from the horizon radius r_h, such that $a \sim \mathcal{O}(r_h)$. Of course, on the gauge theory side, the branes are still wrapped on vanishing cycle.

[5] The UV gauge group is generated by wrapping D5 and anti-D5 branes on a vanishing cycle of a *resolved* conifold, removing the

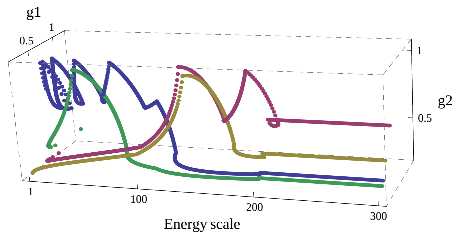


FIG. 2: A slightly unconventional way to represent the RG flow in our model. We get the complete RG flow by gluing the three regions altogether and using S-duality to transmute strong coupling into weak coupling. As before, all scales are chosen with $\alpha' \equiv 1$.

by the Higgs mechanism as we enter Region 1. We will study this mechanism in two versions: supersymmetric and non-supersymmetric. Since the purpose is to break the gauge group, we will ignore any fundamental matter fields in the following discussion.

The Higgsing process should simply be engineered by making some anti-D5 brane DOFs heavy, i.e by moving the anti-D5 branes away from the N D3 and the M wrapped D5 branes on the resolved sphere. In a supersymmetric theory, where the UV completion is done by a $\mathcal{N} = 2$ theory, this process would mean moving the anti-D5 brane DOFs along the Coulomb branch, which in turn implies that the anti-D5 branes' world-volume scalar multiplets, transforming under a certain subgroup of $SU(N+M)$, will be responsible for the Higgsing mechanism.

For the non-supersymmetric theory, this is rather easy to demonstrate. All we require is that the vev of the Higgs field ϕ should only transform under a certain subgroup of the first $SU(N+M)$ group. The Lagrangian is:

$$\mathcal{L} = -\frac{1}{2} D^\mu \phi_k D_\mu \phi_k - V(\phi) - \frac{1}{4} F_i^{a\mu\nu} F^a_{i\mu\nu} \tag{2.5}$$

where $i = 1, 2$ refers to each $SU(N+M)$ copy in the product gauge group. $D_\mu \phi_k = \partial_\mu \phi_k - i g_1 A^a_{1\mu} (T^a_1)_{kl} \phi_l$ with g_1, $A_{1\mu}$ and T_1 being the gauge coupling, gauge field and generators of the first $SU(N+M)$ group respectively.

The rest of the discussion should follow the standard Higgs mechanism once we demand that the potential $V(\phi)$ is minimized at $\langle \phi_i \rangle \equiv v_i$ and the field $\phi_i(x)$ is expressed as:

$$\phi_i(x) = v_i + H_i(x) \tag{2.6}$$

where $H_i(x)$ is a real scalar field. It is obvious now that the gauge field A^a will get massive if $T^a_{ij} v_j \neq 0$ and thus the gauge group is broken[6]. In our case, we only want to break M of the generators. How this is done depends on the details of the potentials and the specific values of N and M.

The supersymmetric case follows the same line of argument as above. The general Lagrangian now is:

$$\mathcal{L} = \int d^4\theta \; \mathcal{K} \bar{\Phi} e^V \Phi + \int d^2\theta \left[\mathcal{W} + \frac{1}{32\pi i} \tau \mathrm{tr}_f W^2_\alpha \right] + h.c. \tag{2.7}$$

where $\left(\Phi, W_\alpha = -\frac{1}{4} \bar{D}\bar{D} D_\alpha V\right)$ are the appropriate $\mathcal{N} = 1$ chiral and the vector multiplets with (ϕ_k, A^a_μ) being the complex scalar and the vector fields in their respective multiplets, $\mathcal{K}$ is a gauge invariant Kähler potential, $\mathcal{W}$ is a gauge invariant superpotential, $\tau \equiv \frac{\vartheta}{2\pi} + i \frac{4\pi}{g^2}$ is the complexified gauge coupling with ϑ-angle and tr_f is the trace in the fundamental representation. The FI terms don't appear because they are forbidden by the non-Abelian gauge invariance.

The scalar potential obtained by expanding the above Lagrangian in components is a sum of F^2 and D^2 terms. The F and the D terms are:

$$F_k = -\frac{\partial \bar{\mathcal{W}}}{\partial \phi_k}, \qquad D^a = \bar{\phi}_k \, (T^a_R)_{kl} \, \phi_l \tag{2.8}$$

where T^a_R denotes the generator in the R representation. To preserve supersymmetry we must have $F_k = 0$ and $D^a = 0$. Actually in the absence of FI terms whenever $F_k = 0$ has a solution, $D^a = 0$ always has a solution. So we assume we already have a solution that satisfies $F_k = 0$. This solution can break the gauge symmetry as in the non-supersymmetric case. This can be seen in the following way.

Write down the relevant kinetic terms of the scalar component of the Higgs multiplet, as:

$$\int d^4\theta \; \bar{\Phi} e^V \Phi = - \left| (\partial_\mu + i g A^a_\mu T^a) \phi \right|^2 + ... \tag{2.9}$$

which is exactly the same as in the non-supersymmetric case. If the F-term solution leads to M generators T^a such that $T^a_{ij} v_j \neq 0$, then the gauge group is broken from $SU(N+M)$ to $SU(N)$. How this happens again depends on the details of the N, M values and the form of the superpotential.

---

tachyons by fluxes and separating the D5 and the anti-D5 branes along the other sphere which is kept resolved at $r = 0$.

[6] The condition $T^a_{ij} v_j \neq 0$ may not be too stringent a requirement. This is because $T^a_{kj} v_j T^a_{ki} v_i$ are non-negative no matter what linear transformations we do. So it must be zero to preserve the symmetry. However generically one could also take linear combinations of various T that leave v unchanged. This can be done by diagonalizing the mass matrix $M^a_k M^b_k$, where $M^a_k \equiv i g_1 T^a_{kj} vj$, which would involve such linear combinations.

3. PHASE TRANSITION AND OTHER APPLICATIONS

In [5, 6, 8], the theory above the deconfinement temperature was studied in details. The high temperature phase was understood therein as the one coming from a black-hole with a horizon radius r_h where the temperature was related to r_h. The scenario at low temperatures were not discussed in details in [5, 6, 8]. Here, we will study these two phases and discuss their associate phase transition. More elaborations on this is presented in [15].

Before actually computing the phase transition, let us discuss the stability of the black hole geometry at high temperatures. A positive specific heat c_v implies stability while a negative specific heat implies instability, which in fact turned out to be the case of many models that study large N thermal QCD without a UV completion [9]. The specific heat can be obtained from the internal energy as: $E_{\rm int}$

$$c_v \;=\; \left(\frac{\partial E_{\rm int}}{\partial T}\right)_V, \quad {\rm T} = \frac{g'}{4\pi\sqrt{h}}\Big|_{r_h} \simeq \frac{r_h}{\pi L^2} \tag{3.10}$$

where $g = 1 - r_h^4/r^4$. We have introduced the AdS_5 length scale L in anticipation of the AdS cap, and the internal energy is given by the integral of the zeroth component of the stress tensor. To calculate the heat capacity, we have to know how much energy is encoded in the geometry. As $r \to \infty$, the space-time is approximately $AdS_5 \times T^{1,1}$, where $T^{1,1}$ is the internal space. The internal energy T_{00} of asymptotically AdS_5 space-time can be easily calculated using results from [10]. Thus, using the background above the deconfinement temperature given in [5, 6, 8], the internal energy, in terms of the string coupling g_s and the Newton's constant G_N, becomes:

$$E_{\rm int} = \int d^3x\sqrt{g}\; T_{00} = \frac{\pi^2 {r_h}^4}{{g_s}^2 G_N} \tag{3.11}$$

which gives the following value for the specific heat:

$$c_v \;=\; +\frac{4\pi^6 L^8}{{g_s}^2 G_N}{\rm T}^3 \tag{3.12}$$

This means that the heat capacity is positive for positive temperatures, showing that the model is stable at high temperatures. With a stable blackhole geometry we will now see how phase transitions are realized in our model. For more details please refer to [11, 15].

Phase transitions of $SU(N)$ gauge theory can be realized by spontaneous breaking of the center symmetry $\mathbf{Z}_N$. In the confined phase, $\mathbf{Z}_N$ symmetry is preserved and its associated order parameter, a temporal Wilson loop, is zero (i.e. $\langle W\rangle = 0$). In the deconfined phase, $\mathbf{Z}_N$ symmetry is spontaneously broken with $\langle W\rangle \neq 0$. In [6], we computed $\langle W\rangle$ using the gravity description and showed that OKS-BH geometry with large black holes give $\langle W\rangle \neq 0$ while the OKS geometry without black holes gives $\langle W\rangle = 0$. This indicates that the extremal geometry is dual to the confined phase while the non-extremal geometry corresponds to the deconfined phase.

Here we will obtain the critical temperature for confinement/deconfinement transition by computing the free enegy of extremal and non-extremal geometries and identifying it with the free energy of the gauge theory. We start with the on-shell type IIB supergravity action with appropriate Gibbons-Hawking boundary terms S_{GH}, counter-term, $S_{\rm counter}$, necessary to renormalize the action [10][5][15] as:

$$\mathcal{S} = \beta E_{\rm free} = S_{IIB} + S_{GH} + S_{\rm counter} \tag{3.13}$$

where $E_{\rm free}$ is the free energy, and $S_{\rm IIB}$ is the ten dimensional type IIB Euclidean supergravity action including localized sources [16][17]. Just like the case for AdS gravity discussed by Hawking and Page [14] and subsequently by Witten [13], the above action gives rise to both extremal and non-extremal metric and both geometries can incorporate non-zero temperature of the dual gauge theory in the following way: Wick rotate $t \to i\tau, \tau \in (0, \beta)$ and identify temperature T as $T = 1/\beta$. At a fixed temperature of the gauge theory, we have two geometries – extremal and non-extremal – and the geometry with smaller on-shell action will be preferred. The free energy of the gauge theory will then be given by the free energy of the geometry obtained through (3.13).

Denoting the on-shell value of the action for the extremal geometry with $\mathcal{S}_1$ and the non-extremal geometry with $\mathcal{S}_2$, we compute the action difference in the absence of D7 branes and localized sources, i.e. $N_f = 0$ and the axio-dilaton τ is a constant (i.e. without fundamental matter), as [15]:

$$\begin{aligned} \triangle\mathcal{S} \;&=\; \mathcal{S}_2 - \mathcal{S}_1 \\ &=\; \frac{g_s M^2 \beta_2 V_8}{2\kappa_{10}^2 N}\lim_{\mathcal{R}\to\infty}\left[\frac{r_h^4}{32}\log\left(\frac{\mathcal{R}}{r_h}\right) - \frac{5dr_h^4}{128}\right] \end{aligned} \tag{3.14}$$

where V_8 is the volume of $R^3 \times T^{1,1}$, $T^{1,1}$ being the base of the conifold with approximate radius $L = (g_s N)^{1/4}\sqrt{\alpha'}$, N, M are number of D3 and D5 branes in the dual gauge theory, $\mathcal{R}$ is the boundary value of r, and r_h is the black hole horizon radius. Here $d > 0$ is a constant independent of N, M, g_s and depends on the boundary values of derivatives of the metric [15]. In obtaining (3.14), we have only kept terms up to linear order in $g_s M^2/N$ which is valid for $N \gg g_s M^2$ and the exact form of $S_{\rm counter}, S_{\rm GH}$ is presented in [15]. Furthermore in the limit $\mathcal{R} \to \infty$ in (3.14), we are dropping $\mathcal{R}^{-n}, n > 0$ terms. The critical temperature is obtained by evaluating the critical horizon r_h^c for which $\triangle S(r_h^c) = 0$ and the result is [15]:

$$r_h^c = \mathcal{R}{\rm exp}\left(-\frac{5d}{4}\right), \; T_c = \frac{1 + \mathcal{O}\left(\frac{g_s M^2}{N}\right)}{\pi\,\exp\left(\frac{5d}{4}\right)(g_s N)^{1/4}\sqrt{\alpha'}} \tag{3.15}$$

where we have used the scaling $\mathcal{R} = L = (g_s N)^{1/4}\sqrt{\alpha'} \to \infty$. For $T > T_c$, $\triangle S < 0$, i.e the black hole geometry has lower free energy and thus preferred, while for $T < T_c$, $\triangle S > 0$, i.e the extremal geometry is preferred. For extremal geometry, one readily gets an entropy $s = -\frac{\partial E_{\rm free}}{\partial T} = 0$, while for the black hole $s \sim N^2 T^3 \left[1 + \frac{g_s M^2 b}{N}\log(LT)\right]$ at lowest order in $g_s M^2/N$ and $b > 0$ is a constant independent of N, M, g_s. Thus the black hole describes liberated degrees of freedom and we have phase transition. Note we do not have any D7-branes − i.e we do not have any matter in the fundamental representation[7] − the confinement to deconfinement phase transition for the gauge theory mimics the first order transition in pure glue theory and is described by a Hawking-Page transition in the dual geometry.

Observe that in deriving (3.14), we defined the boundary $r = \mathcal{R} \to \infty$, but *did not* explicitly add a UV geometry. By adding counter terms $S_{\rm counter}$ to the on-shell action, we subtracted the terms in $S_{IIB} + S_{GH}$ that diverge at the boundary $r = \mathcal{R}$, which is effectively choosing a particular UV completion. The UV completion resulting from our regularization already gives us a first order phase transition with an exact result for the critical temperature and thus is already insightful. Furthermore, since confinement is an IR phenomenon, the critical temperature may not be extremely sensitive to the details of the UV completion and thus the T_c in (3.15) can even be relevant for the UV complete geometry.

[7] The field theory has bi-fundamental fields A_i, B_j and in the far IR can be equivalently described by pure glue $SU(M)$ theory. If T_c is very small, the confined phase consists of glue balls and the deconfined phase consists of free gluons of $SU(M)$. If T_c is large, the deconfined phase is best described by A_i, B_j fields.

4. CONCLUSION AND DISCUSSIONS

In this paper we have managed to tie up some of the loose ends of our earlier works [5, 6, 8] related to the gauge theory description of the UV complete geometry predicted in the gravity side. The RG flow from UV to IR at zero temperature shows how the conformal behavior in the far UV ties up with the confining dynamics in the far IR. The intermediate-energy physics is more involved and will be elucidated in our upcoming work [11] where we will also discuss how to evaluate the spectrum of the theory. As an interesting outcome of the UV completion, we could see how the stability of our background could be justified. Furthermore phase transition and related IR issues appear naturally in our set-up. If we ignore the flavor branes, our gravity description gives us a first-order phase transition. Further details on this has appeared in [15]. In the presence of the flavor branes, the physics is slightly more involved and will be discussed in [11].

Acknowledgement: We would like to thank M. Gyulassy for helpful discussions. The work of L. C and K. D is supported in part by the NSERC grant, the work of F. C is supported in part by the Schulich grant, the work of O. T is supported in part by the FQRNT grant and the work of M. M is supported in part by the Office of the Nuclear Science of the US DOE grant number DE-FGO2-93ER40764.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200].

[2] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [hep-th/0007191]; M. J. Strassler, hep-th/0505153.

[3] P. Ouyang, Nucl. Phys. B **699**, 207 (2004) [hep-th/0311084].

[4] R. McNees, R. C. Myers and A. Sinha, JHEP **0811**, 056 (2008) [arXiv:0807.5127 [hep-th]]; C. -S. Chu and D. Giataganas, JHEP **0812**, 103 (2008) [arXiv:0810.5729 [hep-th]].

[5] M. Mia, K. Dasgupta, C. Gale, S. Jeon, Nucl. Phys. **B839**, 187-293 (2010). [arXiv:0902.1540 [hep-th]].

[6] M. Mia, K. Dasgupta, C. Gale and S. Jeon, Phys. Rev. D **82**, 026004 (2010) [arXiv:1004.0387 [hep-th]]; Phys. Lett. B **694**, 460 (2011) [arXiv:1006.0055 [hep-th]].

[7] U. Gursoy and E. Kiritsis, JHEP **0802**, 032 (2008) [arXiv:0707.1324 [hep-th]]; U. Gursoy, E. Kiritsis and F. Nitti, JHEP **0802**, 019 (2008) [arXiv:0707.1349 [hep-th]]; U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti, Phys. Rev. Lett. **101**, 181601 (2008) [arXiv:0804.0899 [hep-th]]; JHEP **0905**, 033 (2009) [arXiv:0812.0792 [hep-th]]; U. Gursoy, E. Kiritsis, L. Mazzanti, G. Michalogiorgakis and F. Nitti, Lect. Notes Phys. **828**, 79 (2011) [arXiv:1006.5461 [hep-th]].

[8] M. Mia, F. Chen, K. Dasgupta, P. Franche and S. Vaidya, Phys. Rev. D **86**, 086002 (2012), arXiv:1202.5321 [hep-th].

[9] E. Caceres and S. Young, arXiv:1205.2397 [hep-th].

[10] M. C. N. Cheng and K. Skenderis, JHEP **0508**, 107 (2005) [hep-th/0506123]; K. Skenderis, Class. Quant. Grav. **19**, 5849 (2002) [hep-th/0209067].

[11] F. Chen, L. Chen, K. Dasgupta, M. Mia and O. Trottier, *To Appear.*

[12] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **229**, 407 (1983).

[13] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131]; Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[14] S. W. Hawking and D. N. Page, Commun. Math. Phys. **87**, 577 (1983).

[15] M. Mia and F. Chen, JHEP **1301**, 083 (2013) [arXiv:1210.3365 [hep-th]].

[16] K. Dasgupta, G. Rajesh and S. Sethi, JHEP **9908**, 023 (1999) [arXiv:hep-th/9908088].

[17] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D **66**, 106006 (2002) [arXiv:hep-th/0105097].