Erratum: Order-v^{4} corrections to S-wave quarkonium decay [Phys. Rev. D 66, 094011 (2002)]

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In this erratum, I correct several errors in Ref. [1].

In Eqs. (6.25), (6.26), (6.27), (6.28b), and (6.30) of Ref. [1], the denominator factor 81 should be written as $27(D-1)$, where D is the space-time dimension. The factor $D-1$ arises from the projection of bilinears of p onto S-wave states. Here p is one-half the relative momentum of the quark and antiquark in the quark-antiquark rest frame. Specifically, the S-wave projections yield

$$
p_i p_j p_i' p_j' \to \frac{\boldsymbol{p}^2}{D-1}\delta_{ij}\frac{\boldsymbol{p}^2}{D-1}\delta_{ij} = \frac{\boldsymbol{p}^4}{D-1},
\tag{1}
$$

where $\boldsymbol{p}^2 = \boldsymbol{p'}^2$, but I distinguish the orientations of the momenta $\boldsymbol{p}$ in the incoming state and the momentum $\boldsymbol{p'}$ in the outgoing state in order to project each onto an S-wave angular-momentum state. The denominator factors $27(D-1)$ affect the $\overline{\text{MS}}$ renormalization of the NRQCD matrix element. In $\overline{\text{MS}}$ renormalization, one constructs the counterterm for a UV-divergent subdiagram by subtracting the poles in $\epsilon = (4-D)/2$ that appear in that subdiagram, plus some associated constants. In order to insure that the counterterm contribution removes the contribution that is proportional to the UV divergence in the divergent subdiagram, one must compute in D dimensions all factors that are external to the divergent subdiagram. These external factors include factors that derive from the external momenta, such as the projections onto specific orbital-angular-momentum states. A failure to follow this procedure at the one-loop level leads to a shift of the result by a constant term. However, at the level of two or more loops, it leads to a breakdown in the consistency of the renormalization program, with the consequence that uncanceled poles appear in short-distance coefficients [2]. It follows from these corrections that the term $-833/972$ in Eq. (6.31c) should be replaced with $-805/972$ and that the term 7.62 in the last line of Table IV should be replaced with 8.22.

Eq. (6.3) of Ref. [1] should read

$$
\begin{aligned}
f(p) &= \left[\frac{E(p)}{m}\right]^{3(D-2)-D}\\
&= \left[\frac{E(p)}{m}\right]^2\left[1 - 4\epsilon \log\frac{E(p)}{m}\right] + O(\epsilon^2).
\end{aligned}
\tag{2}
$$

This correction does not affect the term of order ϵ^0 in Eq. (6.3). Hence, as is explained just after Eq. (6.3), it does not affect the result of the calculation.

Eq. (6.21) of Ref. [1] should read

$$
\begin{aligned}
\tilde{\mathcal{M}}_{\text{IR}} = &\frac{\boldsymbol{p}^4}{m^8}\frac{(N_c^2-1)(N_c^2-4)}{N_c^2}\frac{128\pi^3\alpha_s^3}{(3-2\epsilon)^3}\mu^{6\epsilon}\\
&\times\left\{\frac{(3-2\epsilon)(1-\epsilon)}{x^2} - \frac{2(2-\epsilon)y(1-y)}{x^2}\right.\\
&\quad + \frac{(3-2\epsilon)(1-\epsilon)}{(1-xy)^2} - \frac{2(2-\epsilon)x(1-x)(1-y)}{(1-xy)^4}\\
&\quad \left. + \frac{(3-2\epsilon)(1-\epsilon)}{[1-x(1-y)]^2} - \frac{2(2-\epsilon)x(1-x)y}{[1-x(1-y)]^4}\right\},
\end{aligned}
\tag{3}
$$

The terms that contain factors of x^2 in the denominator, the terms that contain factors of $(1-xy)^2$ in the denominator, and the terms that contain factors of $[1-x(1-y)]^2$ in the denominator are related to each other by cyclic permutations

of the energy fractions x_1, x_2, and x_3 and give equal contributions to the integral over the final-state phase space. This error in Eq. (6.21) of Ref. [1] was not propagated into the remainder of the calculation.

In Eq. (2.16) of Ref. [1], $0.82n_f$ should be $0.81n_f$. An exact expression for $F_{ee}(^3S_1)$ through order $\alpha^2\alpha_s^2$ is [3]

$$
\begin{aligned}
F_{ee}(^3S_1) = {} & \frac{2\pi Q^2\alpha^2}{3}\bigg\{1 - 4C_F\frac{\alpha_s(m)}{\pi} + \bigg[\frac{103}{27} - \frac{511\pi^2}{162} \\
& - \frac{224\pi^2\ln(2)}{27} - \frac{250\,\zeta(3)}{9} + \frac{22}{27}n_f \\
& + \frac{140\pi^2}{27}\ln\bigg(\frac{2m}{\mu_\Lambda}\bigg)\bigg(\frac{\alpha_s}{\pi}\bigg)^2\bigg]\bigg\},
\end{aligned}
\tag{4}
$$

where $\zeta(z)$ is the Riemann zeta function.

There are several additional errors of a minor nature in Ref. [1]. Just after Eq. (2.10), $m^2F_1(^3S_1)$ should be $F_1(^3S_1)$. Just after Eq. (2.14), $m^2F_{\gamma\gamma}(^1S_0)$ should be $F_{\gamma\gamma}(^1S_0)$. In Eq. (5.5c), 58/54 should be 29/27. In Eq. (6.13), $k_1 + k_2 + k_3$ should be $k_1^0 + k_2^0 + k_3^0$.

Acknowledgments

[1] G. T. Bodwin and A. Petrelli, Phys. Rev. D **66**, 094011 (2002) [hep-ph/0205210].
[2] G. T. Bodwin, U-R. Kim, and J. Lee, JHEP **1211**, 020 (2012) [arXiv:1208.5301 [hep-ph]].
[3] M. Beneke, A. Signer, and V. A. Smirnov, Phys. Rev. Lett. **80**, 2535 (1998) [hep-ph/9712302].