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# Transverse momentum-weighted Siverts asymmetry in semi-inclusive deep inelastic scattering at next-to-leading order

Zhong-Bo Kang,<sup>1,\*</sup> Ivan Vitev,<sup>1,†</sup> and Hongxi Xing<sup>2,3,‡</sup>

<sup>1</sup>*Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

<sup>2</sup>*Interdisciplinary Center for Theoretical Study and Department of Modern Physics,  
University of Science and Technology of China, Hefei 230026, China*

<sup>3</sup>*Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

We study the next-to-leading order perturbative QCD corrections to the transverse momentum-weighted Siverts asymmetry in semi-inclusive hadron production in lepton-proton deep inelastic scattering. The corresponding differential cross section is evaluated as a convolution of a twist-three quark-gluon correlation function, often referred to as Qiu-Sterman function, the usual unpolarized fragmentation function, and a hard coefficient function. By studying the collinear divergence structure, we identify the evolution kernel for the Qiu-Sterman function. The hard coefficient function, which is finite and free of any divergence, is evaluated at one-loop order.

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## I. INTRODUCTION

In recent years, transverse spin physics has attracted tremendous attention from both the experimental and theoretical communities. As transverse spin can correlate with the transverse momentum of the partons inside a polarized proton, transverse spin observables, such as single transverse spin asymmetries (SSAs), are sensitive probes of parton's transverse motion and a path to three-dimensional proton tomography [1]. Significant theoretical progress has been made in studying the single transverse spin asymmetries in the past several years. Two QCD mechanisms for generating SSAs have been proposed and applied extensively to phenomenology: the transverse momentum dependent (TMD) factorization approach [2–6] and the collinear twist-3 factorization approach [7–11]. They were shown to be closely related and, thus, provide a unified picture for the SSAs [12].

One of the most studied asymmetries is the so-called Siverts effect [13]. At the partonic level, it corresponds to an azimuthal correlation  $\sim S_{\perp} \cdot (P \times k_{\perp})$ , with  $S_{\perp}$  and  $P$  being the spin and momentum vector of the polarized proton and  $k_{\perp}$  being the transverse momentum of the parton. Such correlation is encoded in the so-called Siverts function, if one uses the TMD factorization formalism; or the twist-3 quark-gluon correlation function, often referred to as Qiu-Sterman function, within the collinear twist-3 factorization formalism. The evolution equations for either the Siverts function [14–17] or the Qiu-Sterman function [18–25] have been derived recently, which enhances the accuracy of the phenomenological applications.

A natural step forward, as a follow-up to the derivation of evolution equations, will be the computation of the next-to-leading order (NLO) corrections to the transverse spin-dependent cross sections. Although tremendous progress has been made in the evaluation of NLO perturbative QCD (pQCD) corrections to the spin-averaged cross sections, similar efforts on the transverse spin-dependent cross sections are still rather limited. This is largely due to the complexity of such type of calculations. So far, the only NLO correction in this direction is performed for Drell-Yan production [20]. NLO corrections to other processes will provide process-dependent corrections to the hard-part coefficient functions, and can also be used to extract the universal behavior of the evolution kernel for the relevant spin-dependent parton distributions and/or fragmentation functions. A NLO calculation for a particular physical process, thus, provides a direct test of QCD factorization for the associated observables.

In this paper we follow Ref. [20] on the Drell-Yan production and perform a NLO calculation for the  $P_{h\perp}$ -weighted Siverts asymmetry in semi-inclusive deep inelastic scattering (SIDIS). Here,  $P_{h\perp}$  is the transverse momentum of the final-state hadron. Since the transverse momentum is being integrated out, our result is presented within the collinear factorization formalism in terms of twist-3 Qiu-Sterman function, NLO hard-part coefficient function, and the usual unpolarized fragmentation function, as we demonstrate in detail below. We expect that our result will lead to an improved extraction of Qiu-Sterman function at the NLO level from the SIDIS data. A similar opportunity has been recently discussed for the inclusive DIS process [26]. The rest of our paper is organized as follows. In Sec.

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\* zkang@lanl.gov

† ivitev@lanl.gov

‡ xinghx@iopp.cnu.edu.cn

II we introduce the notation for the semi-inclusive hadron production in deep inelastic scattering and present the  $P_{h\perp}$ -weighted Siverts asymmetry at leading order. In Sec. III we present the NLO pQCD corrections. We first give the result for virtual corrections and then study the real corrections. We then combine the real and virtual parts to obtain the final cross section. We show that all the soft divergences cancel out between real and virtual corrections. The remaining collinear divergence is absorbed in the redefinition of the unpolarized fragmentation function, and the twist-3 Qiu-Sterman function. This provides an alternative way to derive the evolution equation for the Qiu-Sterman function. We conclude our paper in Sec. IV.

## II. TRANSVERSE MOMENTUM-WEIGHTED SIVERTS ASYMMETRY AT LEADING ORDER

We start this section by specifying our notation and the kinematics of SIDIS. We consider the scattering of an unpolarized lepton  $e$  with momentum  $\ell$ , on a transversely polarized proton  $p$  with momentum  $P$  and transverse spin vector  $S_\perp$ ,

$$e(\ell) + p(P, S_\perp) \rightarrow e(\ell') + h(P_h) + X, \quad (1)$$

where  $h$  represents the observed final-state meson with momentum  $P_h$ . In the approximation of one-photon exchange, we define the virtual photon momentum  $q = \ell - \ell'$  and its invariant mass  $Q^2 = -q^2$ . The usual SIDIS variables are defined as follows:

$$S = (P + \ell)^2, \quad x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot \ell} = \frac{Q^2}{x_B S}, \quad z_h = \frac{P \cdot P_h}{P \cdot q}. \quad (2)$$

The differential cross section that includes the so-called Siverts effect, the  $\sin(\phi_h - \phi_s)$  module, can be written as the following form [27]

$$\frac{d\sigma_{\text{Siverts}}}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0 \left[ F_{UU} + \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h - \phi_s)} \right], \quad (3)$$

where  $\sigma_0 = \frac{2\pi\alpha_{\text{em}}^2}{Q^2} \frac{1+(1-y)^2}{y}$ ,  $F_{UU}$  and  $F_{UT}^{\sin(\phi_h - \phi_s)}$  are the spin-averaged and transverse spin-dependent structure functions, respectively.  $\phi_h$  and  $\phi_s$  are the azimuthal angles for the final-state hadron momentum  $P_h$  and spin vector  $S_\perp$ . In this paper, we define all our angles in the so-called *hadron frame* [28]. The spin-averaged differential cross section  $d\sigma/dx_B dy dz_h d^2 P_{h\perp} \equiv \sigma_0 F_{UU}$ , and the  $P_{h\perp}$ -integrated spin-averaged cross section is defined as

$$\frac{d\sigma}{dx_B dy dz_h} \equiv \int d^2 P_{h\perp} \frac{d\sigma}{dx_B dy dz_h d^2 P_{h\perp}}. \quad (4)$$

At the same time, the transverse spin-dependent differential cross section  $d\Delta\sigma(S_\perp)/dx_B dy dz_h d^2 P_{h\perp} \equiv \sigma_0 \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h - \phi_s)}$ , and the transverse momentum-weighted transverse spin-dependent cross section is given by [5]

$$\frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} \equiv \int d^2 P_{h\perp} \epsilon^{\alpha\beta} S_\perp^\alpha P_{h\perp}^\beta \frac{d\Delta\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}}, \quad (5)$$

where  $\epsilon^{\alpha\beta}$  is a two-dimensional anti-symmetric tensor with  $\epsilon^{12} = 1$ , and  $\epsilon^{\alpha\beta} S_\perp^\alpha P_{h\perp}^\beta = P_{h\perp} \sin(\phi_h - \phi_s)$ . Thus, the  $P_{h\perp}$ -weighted Siverts asymmetry is given by

$$A_{UT}^{P_{h\perp} \sin(\phi_h - \phi_s)}(x_B, y, z_h) \equiv \frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} \bigg/ \frac{d\sigma}{dx_B dy dz_h}. \quad (6)$$

There are multiple studies on the higher order pQCD corrections to the spin-averaged cross sections, see, for example, Ref. [29]. As a warm-up exercise, we also calculate this cross section to NLO order, and our findings are consistent with Ref. [29]. Since this result is well-known, we only give the final expression here (in the  $\overline{\text{MS}}$  scheme)

$$\begin{aligned} \frac{d\sigma}{dx_B dy dz_h} &= \sigma_0 \sum_q e_q^2 \int \frac{dx}{x} \frac{dz}{z} q(x, \mu^2) D_{q \rightarrow h}(z, \mu^2) \delta(1 - \hat{x}) \delta(1 - \hat{z}) + \sigma_0 \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dx}{x} \frac{dz}{z} q(x, \mu^2) D_{q \rightarrow h}(z, \mu^2) \\ &\times \left\{ \ln \left( \frac{Q^2}{\mu^2} \right) [P_{qq}(\hat{x}) \delta(1 - \hat{z}) + P_{qq}(\hat{z}) \delta(1 - \hat{x})] + C_F \left[ \frac{1 + (1 - \hat{x} - \hat{z})^2}{(1 - \hat{x})_+ (1 - \hat{z})_+} - 8\delta(1 - \hat{x}) \delta(1 - \hat{z}) \right] \right. \\ &+ \delta(1 - \hat{z}) C_F \left[ (1 + \hat{x}^2) \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ - \frac{1 + \hat{x}^2}{1 - \hat{x}} \ln \hat{x} + (1 - \hat{x}) \right] \\ &\left. + \delta(1 - \hat{x}) C_F \left[ (1 + \hat{z}^2) \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ + \frac{1 + \hat{z}^2}{1 - \hat{z}} \ln \hat{z} + (1 - \hat{z}) \right] \right\}, \quad (7) \end{aligned}$$

where  $\hat{x} = x_B/x$  and  $\hat{z} = z_h/z$  and  $P_{qq}(x)$  is the usual spin-averaged quark-to-quark splitting kernel

$$P_{qq}(x) = C_F \left[ \frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]. \quad (8)$$

Let us now concentrate on the NLO correction to the transverse spin-dependent differential cross section. This will allow us to study the  $P_{h\perp}$ -weighted Siverts asymmetry, as in Eq. (6), at next-to-leading order. We will start with the leading order calculation for the transverse spin-dependent cross section. At this order, the final-state hadron is produced through the hadronization of the quark, which comes from the virtual-photon quark scattering. In order to obtain a non-vanishing  $P_{h\perp}$ -weighted transverse spin-dependent cross section  $\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle$ , we have to include the final-state multiple interactions, as shown in Fig. 1, to provide the required phase [8]. We work in the covariant

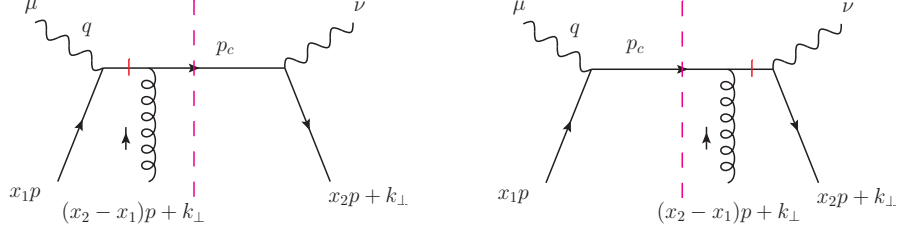


FIG. 1. Leading order Feynman diagrams. Left: gluon to the left of the cut. Right: gluon to the right of the cut.

gauge. For the gluon on the left of the  $t = \infty$  cut, shown in Fig. 1(left), we have the  $P_{h\perp}$ -weighted cross section as

$$\begin{aligned} \left. \frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} \right|_{\text{Fig. 1(left)}} &\propto \int d^2 P_{h\perp} \epsilon^{\alpha\beta} S_\perp^\alpha P_{h\perp}^\beta \int dz D_{q \rightarrow h}(z) \int dx_1 dx_2 d^2 k_\perp T_{q,A}(x_1, x_2, k_\perp) \\ &\times [-g^{\mu\nu} H_{\mu\nu}(x_1, x_2, k_\perp)] \delta^2(P_{h\perp} - z k_\perp). \end{aligned} \quad (9)$$

Here (and throughout the paper), for simplicity we only consider the so-called metric contribution [29–31]. This means that we contract our hadronic tensor  $H_{\mu\nu}$  with  $-g^{\mu\nu}$ . The twist-3 correlation function  $T_{q,A}(x_1, x_2, k_\perp)$  is defined as

$$T_{q,A}(x_1, x_2, k_\perp) = \int \frac{dy_1^-}{2\pi} \frac{dy_2^-}{2\pi} \frac{d^2 y_\perp}{(2\pi)^2} e^{i x_1 P^+ y_1^-} e^{i(x_2 - x_1) P^+ y_2^-} e^{i k_\perp \cdot y_\perp} \frac{1}{2} \langle PS | \bar{\psi}_q(0) \gamma^+ A^+(y_2^-, y_\perp) \psi_q(y_1^-) | PS \rangle. \quad (10)$$

One can take advantage of  $\delta^2(P_{h\perp} - z k_\perp)$  to integrate out  $d^2 P_{h\perp}$ , thus  $P_{h\perp}^\beta = z k_\perp^\beta$ . We then use  $k_\perp^\beta$  to convert  $A^+$  to the  $F^{+\beta}$  field strength through integration by parts [33]. At the same time, one realizes that the Feynman diagram with the gluon to the right of the cut (Fig. 1(right)) gives no contribution to the  $P_{h\perp}$ -weighted Siverts asymmetry. This is because with our choice of frame for this diagram the final-state hadron momentum is given by  $P_h = z p_c = z(x_1 p + q)$ , thus  $P_{h\perp} = 0$  and then the  $P_{h\perp}$ -weighted spin-dependent cross section  $\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle$  vanishes. This conclusion also holds true in the virtual diagram calculation.

The required phase to generate a Siverts asymmetry comes from a pole in the propagator, which is represented by a short-bar in Fig. 1,

$$\frac{1}{(p_c - (x_2 - x_1)P)^2 + i\epsilon} = \frac{1}{2P \cdot p_c} \frac{1}{x_1 - x_2 + i\epsilon} \rightarrow \frac{1}{2P \cdot p_c} (-i\pi) \delta(x_1 - x_2). \quad (11)$$

With this phase, we have

$$g_s \int d^2 k_\perp i \epsilon^{\alpha\beta} S_\perp^\alpha k_\perp^\beta T_{q,A}(x_1, x_2, k_\perp) = \frac{1}{2\pi} T_{q,F}(x_1, x_2), \quad (12)$$

where  $T_{q,F}(x_1, x_2)$  is the well-known Qiu-Sterman function, with the following definition:

$$T_{q,F}(x_1, x_2) = \int \frac{dy_1^-}{4\pi} \frac{dy_2^-}{4\pi} e^{i x_1 P^+ y_1^- + i(x_2 - x_1) P^+ y_2^-} \frac{1}{2} \langle PS | \bar{\psi}_q(0) \gamma^+ \epsilon^{\alpha\beta} S_{\perp\alpha} F^+_{\beta}(y_2^-) \psi_q(y_1^-) | PS \rangle. \quad (13)$$

Finally, the  $P_{h\perp}$ -weighted Siverts asymmetry at LO has the following form [5]

$$\frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} = -\frac{z_h \sigma_0}{2} \sum_q e_q^2 \int \frac{dx dz}{x z} T_{q,F}(x, x) D_{q \rightarrow h}(z) \delta(1 - \hat{x}) \delta(1 - \hat{z}), \quad (14)$$

where we recall that  $\hat{x} = x_B/x$  and  $\hat{z} = z_h/z$ . Since we will use dimensional regularization for our NLO calculation in the next section, we also need the LO result in  $n = 4 - 2\epsilon$  dimension. We find that the only change is the appearance of  $1 - \epsilon$  in the overall normalization of  $\sigma_0$ , i.e. in  $n = 4 - 2\epsilon$  dimension we have  $\sigma_0$  in Eq. (14) defined as

$$\sigma_0 = \frac{2\pi\alpha_{em}^2}{Q^2} \frac{1 + (1-y)^2}{y} (1 - \epsilon). \quad (15)$$

### III. TRANSVERSE MOMENTUM-WEIGHTED SIVERS ASYMMETRY AT NLO

In this section we present the NLO pQCD corrections to the transverse spin-dependent differential cross section. We first give the result for virtual corrections, and then study the real corrections. We then combine the real and virtual corrections to obtain the final expression. We show that all the soft divergences cancel out between real and virtual diagrams. The remaining collinear divergence can be absorbed by the redefinition of the unpolarized fragmentation function, and the twist-3 Qiu-Sterman function. This provides an alternative way to derive the evolution equation for the Qiu-Sterman function.

#### A. Virtual corrections

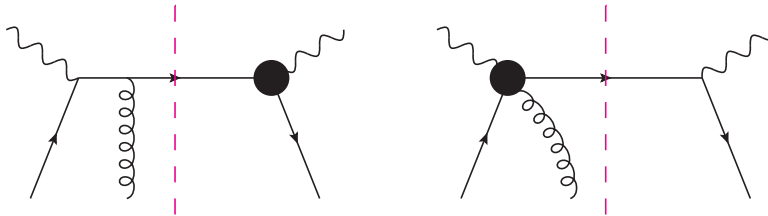


FIG. 2. The generic Feynman diagrams for the virtual corrections to the  $P_{h\perp}$ -weighted cross section.

We first study the virtual corrections. The relevant generic Feynman diagrams are shown in Fig. 2. Here, we only include the diagrams which have the gluon attached to the left of the cut. This is because the diagrams with the gluon to the right of the cut, just like in the LO calculation, give no contribution to the  $P_{h\perp}$ -weighted Sivers asymmetry because of the same  $\delta^2(P_{h\perp})$ . The blob in Fig. 2(left) is given by Fig. 3. These diagrams are pretty easy to compute since they are the same as the usual virtual corrections in the unpolarized cross section. The result is given by [29]

$$\begin{aligned} \left. \frac{d\langle P_{h\perp} \Delta\sigma(S_{\perp}) \rangle}{dx_B dy dz_h} \right|_{\text{Fig. 2(left)}} &= -\frac{z_h \sigma_0}{2} \frac{\alpha_s}{4\pi} \sum_q e_q^2 \int \frac{dx}{x} \frac{dz}{z} T_{q,F}(x, x) D_{q \rightarrow h}(z) \delta(1 - \hat{x}) \delta(1 - \hat{z}) \\ &\times C_F \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1 - \epsilon)} \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right] \end{aligned} \quad (16)$$

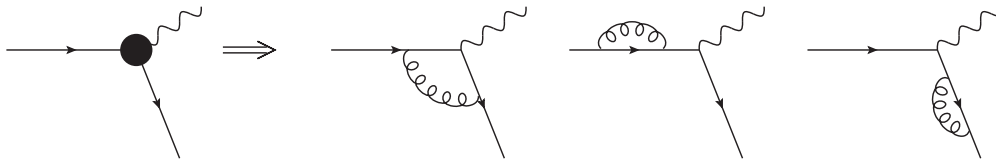


FIG. 3. One-loop virtual corrections for the  $P_{h\perp}$ -weighted cross section: shown here are the corrections to the quark-photon-quark vertex, corresponding to the blob in Fig. 2(left).

On the other hand, the blob in Fig. 2(right) is much more complicated and the explicit diagrams are given in Fig. 4. The calculation is lengthy and contains significant amount of tensor reduction and integration. The diagrams contain three types of color factors: (a) and (e) have color factors  $C_F$ ; (b), (c) and (f) have color factors  $-1/2N_c = C_F - N_c/2$ ;

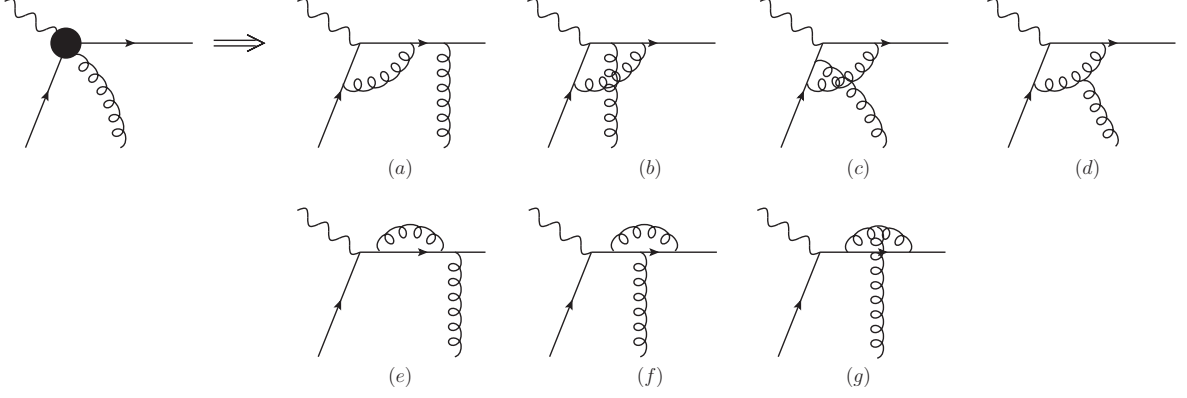


FIG. 4. One-loop virtual corrections for the  $P_{h\perp}$ -weighted cross section: shown here are the corrections to the quark-photon-quark vertex with gluon attachment, corresponding to the blob in Fig. 2(right).

(d) and (g) have color factors  $N_c/2$ . We find that the terms associated with  $N_c/2$  cancel out and only the color structure proportional to  $C_F$  remains. The final result is

$$\begin{aligned} \left. \frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} \right|_{\text{Fig. 2(right)}} &= -\frac{z_h \sigma_0 \alpha_s}{2} \frac{\alpha_s}{4\pi} \sum_q e_q^2 \int \frac{dx}{x} \frac{dz}{z} T_{q,F}(x, x) D_{q \rightarrow h}(z) \delta(1 - \hat{x}) \delta(1 - \hat{z}) \\ &\quad \times C_F \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1 - \epsilon)} \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right] \end{aligned} \quad (17)$$

Thus, the virtual correction is given by the sum of both diagrams in Fig. 2:

$$\begin{aligned} \left. \frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} \right|_{\text{virtual}} &= -\frac{z_h \sigma_0 \alpha_s}{2} \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int \frac{dx}{x} \frac{dz}{z} T_{q,F}(x, x) D_{q \rightarrow h}(z) \delta(1 - \hat{x}) \delta(1 - \hat{z}) \\ &\quad \times C_F \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1 - \epsilon)} \left[ -\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right] \end{aligned} \quad (18)$$

## B. Real corrections

Let us now study the real corrections. We point out that we only consider contributions associated with twist-3 Qiu-Sterman function. We do not discuss in the current study contributions entering with a tri-gluon correlation function [10]. We also ignore contributions of the ‘‘axial’’ twist-three quark-gluon correlation function considered in [32]. Both contributions lead to additional terms in the evolution of Qiu-Sterman function, as demonstrated already in [18, 21, 23], but from different approaches. To calculate the real corrections to the transverse spin-dependent cross section, we need to perform the usual  $k_\perp$ -expansion (also referred to as collinear expansion). The techniques for  $k_\perp$ -expansion are well established in the literature, see e.g. Refs. [8–11]. The  $P_{h\perp}$ -weighted cross section can be written as follows:

$$\begin{aligned} \left. \frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} \right|_{\text{Real}} &\propto \int d^2 P_{h\perp} \epsilon^{\alpha\beta} S_\perp^\alpha P_{h\perp}^\beta \int dz D_{q \rightarrow h}(z) \int dx_1 dx_2 d^2 k_\perp T_{q,A}(x_1, x_2, k_\perp) \\ &\quad \times k_\perp^\rho \frac{\partial}{\partial k_\perp^\rho} [-g^{\mu\nu} H_{\mu\nu}(x_1, x_2, k_\perp)] \end{aligned} \quad (19)$$

Again, we need a phase to generate the Sivers asymmetry, which also comes from the pole in the propagators. For the SIDIS process we can have both soft-pole and hard-pole contributions. Soft-pole contributions come from the Feynman diagrams shown in Fig. 5, with the soft-pole marked by a short bar in the diagram. For gluon to the left of the cut, it arises from

$$\frac{1}{(p_c - (x_2 - x_1)P - k_\perp)^2 + i\epsilon} = \frac{1}{2P \cdot p_c} \frac{1}{x_1 - x_2 - v_1 \cdot k_\perp + i\epsilon} \rightarrow -\frac{x}{\hat{u}} (-i\pi) \delta(x_1 - x_2 + v_1 \cdot k_\perp), \quad (20)$$

where the Mandelstam variables are defined as

$$\hat{s} = (xP + q)^2, \quad \hat{t} = (p_c - q)^2, \quad \hat{u} = (xP - p_c)^2. \quad (21)$$

Here,  $p_c$  is the momentum of the final-state quark which fragments into the observed hadron and  $v_1^\mu = 2xp_c^\mu/\hat{u}$ . When  $k_\perp \rightarrow 0$ ,  $x_1 = x_2$ , i.e. the attached gluon momentum becomes zero. This clarifies the name ‘‘soft-pole’’ contribution.

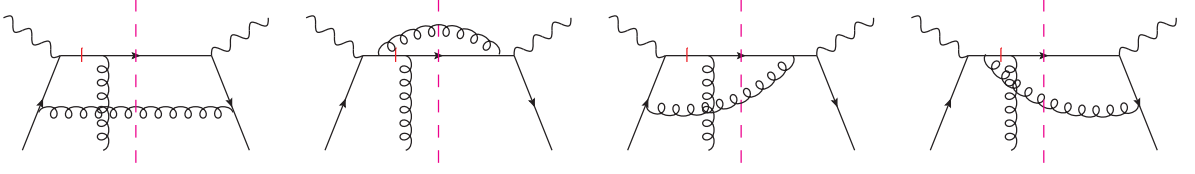


FIG. 5. Feynman diagrams for soft-pole contributions to the  $P_{h\perp}$ -weighted transverse spin-dependent cross section. The short bars indicate the propagators where the soft pole arises. The ‘‘mirror’’ diagrams (those with gluon attached to the right of the cut) are not shown, but are included in the calculations.

On the other hand, the hard-pole contributions come from the Feynman diagrams shown in Fig. 6, with the hard-pole marked by a short-bar in the diagram. For the gluon to the left of the cut, the phase arises as follows:

$$\frac{1}{(x_1 P + q)^2 + i\epsilon} \rightarrow \frac{1}{2P \cdot q} \frac{1}{x_1 - x_B + i\epsilon} \rightarrow \frac{x}{\hat{s} + Q^2} (-i\pi) \delta(x_1 - x_B), \quad (22)$$

which sets  $x_1 = x_B = xQ^2/(\hat{s} + Q^2)$ . On the other hand, the on-shell condition for the unobserved gluon leads to another  $\delta$ -function,  $\delta(x_2 - x - v_2 \cdot k_\perp)$  with  $v_2^\mu = -2xp_c^\mu/\hat{t}$ . Thus when  $k_\perp \rightarrow 0$ , the attached gluon momentum  $\sim x_2 - x_1 = x - x_B = x\hat{s}/(\hat{s} + Q^2)$ , which is finite. This clarifies the name ‘‘hard-pole’’ contribution.

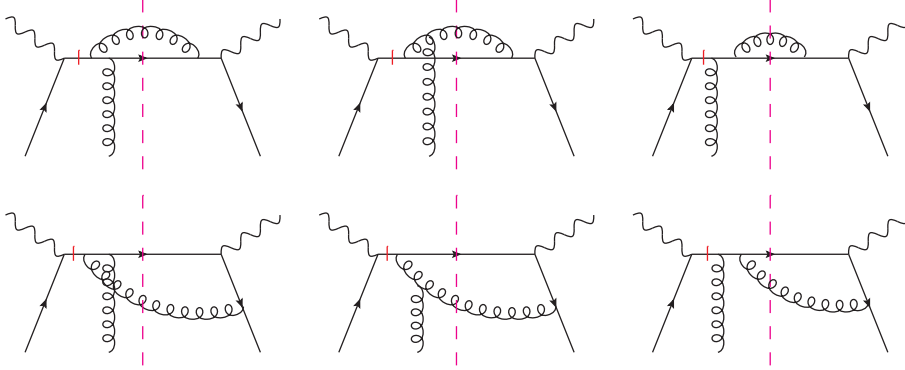


FIG. 6. Feynman diagrams for hard-pole contributions to the  $P_{h\perp}$ -weighted transverse spin-dependent cross section. The short bars indicate the propagators where the hard pole arises. The ‘‘mirror’’ diagrams (those with gluon attached to the right of the cut) are not shown, but are included in the calculations.

The collinear expansion for both soft-pole and hard-pole contributions are straightforward. After such an expansion, we will have a linear  $P_{h\perp}^\rho$ -dependence:

$$\frac{\partial}{\partial k_\perp^\rho} [-g^{\mu\nu} H_{\mu\nu}(x_1, x_2, k_\perp)] \propto p_{c\perp}^\rho = \frac{P_{h\perp}^\rho}{z}. \quad (23)$$

This linear  $P_{h\perp}^\rho$  will combine with  $P_{h\perp}^\beta$  in Eq. (19) to become  $P_{h\perp}^2$ ,

$$P_{h\perp}^\beta P_{h\perp}^\rho \rightarrow \frac{1}{2(1-\epsilon)} P_{h\perp}^2 g_\perp^{\beta\rho}, \quad (24)$$

where  $g_\perp^{\beta\rho}$  is the metric tensor in the transverse components. We can further express  $P_{h\perp}^2$  in terms of Mandelstam variables,

$$P_{h\perp}^2 = z^2 \frac{\hat{s}\hat{t}}{(\hat{s} + Q^2)^2}. \quad (25)$$

Finally our result for the real corrections is given by

$$\begin{aligned} \left. \frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} \right|_{\text{Real}} &= -\frac{1}{2} \sigma_0 \sum_q e_q^2 \int \frac{dx}{x} dz D_{q \rightarrow h}(z) \frac{\alpha_s}{2\pi} \left( \frac{4\pi\mu^2}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \frac{1}{1-\epsilon} \hat{z}^{-\epsilon} (1-\hat{z})^{-\epsilon} \hat{x}^\epsilon (1-\hat{x})^{-\epsilon} \\ &\times \left[ x \frac{d}{dx} T_{q,F}(x, x) D_q^s + T_{q,F}(x, x) N_q^s + T_{q,F}(x, x_B) N_q^h \right], \end{aligned} \quad (26)$$

where we have used the expression for the two-body phase space integral in  $n = 4 - 2\epsilon$  dimension

$$\begin{aligned} dPS^{(2)} &= \frac{d^{n-1} p_c}{(2\pi)^{n-1} 2p_c^0} \frac{d^{n-1} p_d}{(2\pi)^{n-1} 2p_d^0} (2\pi)^n \delta^n(xP + q - p_c - p_d), \\ &= \frac{dz_h}{z} \frac{1}{8\pi} \left( \frac{4\pi}{Q^2} \right)^\epsilon \frac{1}{\Gamma(1-\epsilon)} \hat{z}^{-\epsilon} (1-\hat{z})^{-\epsilon} \hat{x}^\epsilon (1-\hat{x})^{-\epsilon}. \end{aligned} \quad (27)$$

The hard-part coefficient functions in Eq. (26) are

$$D_q^s = \frac{1}{2N_c} \frac{\hat{s}}{\hat{s} + Q^2} \left[ (1-\epsilon) \left( -\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}} \right) + \frac{2Q^2 \hat{u}}{\hat{s}\hat{t}} + 2\epsilon \right], \quad (28)$$

$$N_q^s = \frac{1}{2N_c} \frac{1}{\hat{s}\hat{t}(\hat{s} + Q^2)} \left[ (\hat{s} + Q^2)^3 + Q^2(\hat{u}^2 - \hat{s}^2) + \hat{s}(\hat{s} + \hat{u})^2 - \epsilon(Q^2 + \hat{u})((Q^2 + \hat{u})(Q^2 + \hat{s}) + 2Q^2\hat{s}) \right], \quad (29)$$

$$N_q^h = \left[ \frac{\hat{u}}{\hat{t} + \hat{u}} C_F + \frac{1}{2N_c} \right] \frac{1}{\hat{s}\hat{t}(\hat{s} + Q^2)} \left[ (\hat{t} + \hat{u})^3 - Q^2 \hat{u}^2 + \epsilon \hat{t}(Q^2 + \hat{u})(\hat{t} + \hat{u}) \right]. \quad (30)$$

### C. Combining the real and virtual corrections

To combine real and virtual corrections and demonstrate how the divergences cancel out, we need to perform the  $\epsilon$ -expansion for the real corrections in Eq. (26). To proceed, we realize that

$$\hat{s} = \frac{1-\hat{x}}{\hat{x}} Q^2, \quad \hat{t} = -\frac{1-\hat{z}}{\hat{x}} Q^2, \quad \hat{u} = -\frac{\hat{z}}{\hat{x}} Q^2. \quad (31)$$

Let us define the following common factor:

$$I = \frac{1}{1-\epsilon} \hat{z}^{-\epsilon} (1-\hat{z})^{-\epsilon} \hat{x}^\epsilon (1-\hat{x})^{-\epsilon}. \quad (32)$$

We carry out the  $\epsilon$ -expansion for the  $I \times (D_q^s, N_q^s, N_q^h)$  terms. Using the following formulas [29]:

$$\hat{z}^{-\epsilon} (1-\hat{z})^{-\epsilon-1} = -\frac{1}{\epsilon} \delta(1-\hat{z}) + \frac{1}{(1-\hat{z})_+} - \epsilon \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ - \epsilon \frac{\ln \hat{z}}{1-\hat{z}}, \quad (33)$$

$$\hat{x}^\epsilon (1-\hat{x})^{-\epsilon-1} = -\frac{1}{\epsilon} \delta(1-\hat{x}) + \frac{1}{(1-\hat{x})_+} - \epsilon \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ + \epsilon \frac{\ln \hat{x}}{1-\hat{x}}, \quad (34)$$

$$\hat{z}^{-\epsilon} (1-\hat{z})^{-\epsilon} = 1 - \epsilon \ln \hat{z} - \epsilon \ln(1-\hat{z}), \quad (35)$$

$$\hat{x}^\epsilon (1-\hat{x})^{-\epsilon} = 1 + \epsilon \ln \hat{x} - \epsilon \ln(1-\hat{x}), \quad (36)$$

we present the expanded results for these hard-part coefficient functions. For the so-called derivative term  $D_q^s$  we have

$$I \times D_q^s = \frac{1}{2N_c} \left[ -\frac{1}{\epsilon} (1 + \hat{x}^2) \delta(1-\hat{z}) + (1-\hat{z}) + \frac{(1-\hat{x})^2 + 2\hat{x}\hat{z}}{(1-\hat{z})_+} - (1 + \hat{x}^2) \ln \frac{\hat{x}}{1-\hat{x}} \delta(1-\hat{z}) - 2\hat{x}\delta(1-\hat{z}) \right]. \quad (37)$$

For the first divergent piece we further perform integration by parts to convert  $x \frac{d}{dx} T_{q,F}(x, x)$  to the function  $T_{q,F}(x, x)$  itself:

$$\frac{1}{2N_c} \int_{x_B}^1 \frac{dx}{x} x \frac{d}{dx} T_{q,F}(x, x) (1 + \hat{x}^2) = \frac{1}{2N_c} \int_{x_B}^1 \frac{dx}{x} T_{q,F}(x, x) [2\hat{x}^2 - 2\delta(1-\hat{x})]. \quad (38)$$



Thus, the term associated with  $D_q^s$  leads to the following expression:

$$\int \frac{dx}{x} \left( -\frac{1}{\epsilon} \right) \delta(1-\hat{z}) T_{q,F}(x, x) \frac{1}{2N_c} [2\hat{x}^2 - 2\delta(1-\hat{x})] \\ + x \frac{dx}{x} T_{q,F}(x, x) \frac{1}{2N_c} \left[ (1-\hat{z}) + \frac{(1-\hat{x})^2 + 2\hat{x}\hat{z}}{(1-\hat{z})_+} - (1+\hat{x}^2) \ln \frac{\hat{x}}{1-\hat{x}} \delta(1-\hat{z}) - 2\hat{x}\delta(1-\hat{z}) \right]. \quad (39)$$

Likewise, we have

$$I \times N_q^s = \frac{1}{2N_c} \left\{ -\frac{2}{\epsilon^2} \delta(1-\hat{x})\delta(1-\hat{z}) - \frac{2}{\epsilon} \delta(1-\hat{x})\delta(1-\hat{z}) + \frac{1}{\epsilon} \frac{1+\hat{z}^2}{(1-\hat{z})_+} \delta(1-\hat{x}) \right. \\ \left. - \frac{1}{\epsilon} \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1-\hat{x})_+} \delta(1-\hat{z}) - 2\delta(1-\hat{x})\delta(1-\hat{z}) \right. \\ \left. + \delta(1-\hat{z}) \left[ -(1-\hat{x})(1+2\hat{x}) \ln \frac{\hat{x}}{1-\hat{x}} - 2 \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ + \frac{2}{(1-\hat{x})_+} - 2(1-\hat{x}) + 2 \frac{\ln \hat{x}}{1-\hat{x}} \right] \right. \\ \left. + \delta(1-\hat{x}) \left[ (1+\hat{z}) \ln \hat{z}(1-\hat{z}) - 2 \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + \frac{2\hat{z}}{(1-\hat{z})_+} - 2 \frac{\ln \hat{z}}{1-\hat{z}} \right] \right. \\ \left. + \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1-\hat{x})_+(1-\hat{z})_+} + \frac{1+\hat{z}}{(1-\hat{x})_+} - 2(1-\hat{x}) \right\}, \quad (40)$$

$$I \times N_q^h = \left[ \hat{z} C_F + \frac{1}{2N_c} \right] \left\{ \frac{2}{\epsilon^2} \delta(1-\hat{x})\delta(1-\hat{z}) + \frac{2}{\epsilon} \delta(1-\hat{x})\delta(1-\hat{z}) - \frac{1}{\epsilon} \frac{1+\hat{z}^2}{(1-\hat{z})_+} \delta(1-\hat{x}) \right. \\ \left. - \frac{1}{\epsilon} \frac{1+\hat{x}}{(1-\hat{x})_+} \delta(1-\hat{z}) + 2\delta(1-\hat{x})\delta(1-\hat{z}) + \frac{1+\hat{x}\hat{z}^2}{(1-\hat{x})_+(1-\hat{z})_+} \right. \\ \left. + \delta(1-\hat{z}) \left[ \ln \frac{\hat{x}}{1-\hat{x}} + 2 \left( \frac{\ln(1-\hat{x})}{1-\hat{x}} \right)_+ - 2 \frac{\ln \hat{x}}{1-\hat{x}} - \frac{1+\hat{x}}{(1-\hat{x})_+} \right] \right. \\ \left. + \delta(1-\hat{x}) \left[ -(1+\hat{z}) \ln \hat{z}(1-\hat{z}) + 2 \left( \frac{\ln(1-\hat{z})}{1-\hat{z}} \right)_+ + 2 \frac{\ln \hat{z}}{1-\hat{z}} - \frac{2\hat{z}}{(1-\hat{z})_+} \right] \right\}. \quad (41)$$

Now let us combine the results for the real corrections in Eqs. (39), (40), and (41) with the virtual correction in Eq. (18). First, for the double-pole  $1/\epsilon^2$  term, which represents a soft-collinear divergence, we find that they cancel out between real and virtual diagrams. Specifically, the term  $\sim 1/2N_c$  in the soft-pole contribution  $N_q^s$  cancel the corresponding term in the hard-pole contribution  $N_q^h$ , which leaves the remaining  $1/\epsilon^2$  term in  $N_q^h$  with a color factor  $C_F$ . This remaining term exactly cancels the  $1/\epsilon^2$  term in the virtual diagrams.

Now we turn our attention to the  $1/\epsilon$  term. By adding the corresponding terms in real and virtual diagrams, we end up with the following expression:

$$\hat{z} \left( -\frac{1}{\epsilon} \right) [\delta(1-\hat{x}) T_{q,F}(x, x) P_{qq}(\hat{z}) + \delta(1-\hat{z}) P_{qg \rightarrow qg} \otimes T_{q,F}(x, x\hat{x})], \quad (42)$$

where  $P_{qq}(\hat{z})$  is the usual quark-to-quark splitting kernel, as in Eq. (8). Thus, the first term, the part containing  $\hat{z}$ , is just the collinear QCD correction to the bare leading order fragmentation function  $D_{q \rightarrow h}^{(0)}(z_h)$  (after including the pre-factor  $(4\pi\mu^2/Q^2)^\epsilon/\Gamma[1-\epsilon]$ ):

$$D_{q \rightarrow h}(z_h) = D_{q \rightarrow h}^{(0)}(z_h) + \frac{\alpha_s}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left( -\frac{1}{\epsilon} \right) D_{q \rightarrow h}(z) P_{qq}(\hat{z}), \quad (43)$$

where we have adopted  $\overline{\text{MS}}$ -scheme, and

$$\frac{1}{\epsilon} = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi. \quad (44)$$

On the other hand, the second term  $-\frac{1}{\epsilon}P_{qg \rightarrow qg} \otimes T_{q,F}(x, x\hat{x})$  is given by

$$\begin{aligned} \left(-\frac{1}{\epsilon}\right) P_{qg \rightarrow qg} \otimes T_{q,F}(x, x\hat{x}) &= \left(-\frac{1}{\epsilon}\right) \left\{ T_{q,F}(x, x) C_F \left[ \frac{1 + \hat{x}^2}{(1 - \hat{x})_+} + \frac{3}{2} \delta(1 - \hat{x}) \right] - N_c \delta(1 - \hat{x}) T_{q,F}(x, x) \right. \\ &\quad \left. + \frac{N_c}{2} \left[ \frac{1 + \hat{x}}{(1 - \hat{x})_+} T_{q,F}(x, x\hat{x}) - \frac{1 + \hat{x}^2}{(1 - \hat{x})_+} T_{q,F}(x, x) \right] \right\}, \end{aligned} \quad (45)$$

from which we immediately obtain the collinear QCD correction to the leading order bare Qiu-Sterman function  $T_{q,F}^{(0)}(x_B, x_B)$  as follows:

$$\begin{aligned} T_{q,F}(x_B, x_B) &= T_{q,F}^{(0)}(x_B, x_B) + \frac{\alpha_s}{2\pi} \int_{x_B}^1 \frac{dx}{x} \left(-\frac{1}{\epsilon}\right) \left\{ T_{q,F}(x, x) C_F \left[ \frac{1 + \hat{x}^2}{(1 - \hat{x})_+} + \frac{3}{2} \delta(1 - \hat{x}) \right] - N_c \delta(1 - \hat{x}) T_{q,F}(x, x) \right. \\ &\quad \left. + \frac{N_c}{2} \left[ \frac{1 + \hat{x}}{(1 - \hat{x})_+} T_{q,F}(x, x\hat{x}) - \frac{1 + \hat{x}^2}{(1 - \hat{x})_+} T_{q,F}(x, x) \right] \right\}. \end{aligned} \quad (46)$$

From this equation, we can obtain the evolution equation for the twist-3 Qiu-Sterman function as follows

$$\begin{aligned} \frac{\partial}{\partial \ln \mu^2} T_{q,F}(x_B, x_B, \mu^2) &= \frac{\alpha_s}{2\pi} \int_{x_B}^1 \frac{dx}{x} \left\{ T_{q,F}(x, x, \mu^2) C_F \left[ \frac{1 + \hat{x}^2}{(1 - \hat{x})_+} + \frac{3}{2} \delta(1 - \hat{x}) \right] - N_c \delta(1 - \hat{x}) T_{q,F}(x, x, \mu^2) \right. \\ &\quad \left. + \frac{N_c}{2} \left[ \frac{1 + \hat{x}}{(1 - \hat{x})_+} T_{q,F}(x, x\hat{x}, \mu^2) - \frac{1 + \hat{x}^2}{(1 - \hat{x})_+} T_{q,F}(x, x, \mu^2) \right] \right\}. \end{aligned} \quad (47)$$

This result agrees with earlier findings from different approaches [21, 23–25]. In particular, we are able to reproduce the term  $-N_c T_{q,F}(x, x, \mu^2)$ , which was missing in some early works [18–20]. This piece arises as follows: there is a boundary term  $\propto \frac{2}{\epsilon} \frac{1}{2N_c} \delta(1 - \hat{x})$  from  $D_q^s$ , as in Eqs. (38) and (39), and this term cancels the same term with opposite sign in  $N_q^s$  in Eq. (40). On the other hand, the hard-pole contribution  $N_q^h$  contains the following term:

$$\left[ \hat{z} C_F + \frac{1}{2N_c} \right] \frac{2}{\epsilon} \delta(1 - \hat{x}) \delta(1 - \hat{z}) = \left( C_F + \frac{1}{2N_c} \right) \frac{2}{\epsilon} \delta(1 - \hat{x}) \delta(1 - \hat{z}) = \left( -\frac{1}{\epsilon} \right) [-N_c \delta(1 - \hat{x}) \delta(1 - \hat{z})], \quad (48)$$

which gives exactly the  $-N_c T_{q,F}(x, x, \mu^2)$  term to the evolution of the Qiu-Sterman function  $T_{q,F}(x, x, \mu^2)$ .

Now, let us turn to the finite NLO corrections to the hard-part coefficient function. After  $\overline{\text{MS}}$  subtraction of the collinear divergences into the fragmentation function  $D_{q \rightarrow h}(z, \mu^2)$  and the twist-3 Qiu-Sterman function  $T_{q,F}(x, x, \mu^2)$ , we obtain the NLO correction for both soft-pole and hard-pole contributions to the  $P_{h\perp}$ -weighted transverse spin-

dependent cross section:

$$\begin{aligned}
\frac{d\langle P_{h\perp} \Delta\sigma(S_\perp) \rangle}{dx_B dy dz_h} = & -\frac{z_h \sigma_0}{2} \sum_q e_q^2 \int_{x_B}^1 \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} T_{q,F}(x, x, \mu^2) D_{q \rightarrow h}(z, \mu^2) \delta(1 - \hat{x}) \delta(1 - \hat{z}) \\
& -\frac{z_h \sigma_0}{2} \frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_{x_B}^1 \frac{dx}{x} \int_{z_h}^1 \frac{dz}{z} D_{q \rightarrow h}(z, \mu^2) \left\{ \ln\left(\frac{Q^2}{\mu^2}\right) [\delta(1 - \hat{x}) T_{q,F}(x, x, \mu^2) P_{qq}(\hat{z}) \right. \\
& + \delta(1 - \hat{z}) P_{qg \rightarrow qg} \otimes T_{q,F}(x, x\hat{x}, \mu^2)] \\
& + x \frac{d}{dx} T_{q,F}(x, x, \mu^2) \frac{1}{2N_c} \left[ \frac{1 - \hat{z}}{\hat{z}} + \frac{(1 - \hat{x})^2 + 2\hat{x}\hat{z}}{\hat{z}(1 - \hat{z})_+} - \delta(1 - \hat{z}) \left( (1 + \hat{x}^2) \ln \frac{\hat{x}}{1 - \hat{x}} + 2\hat{x} \right) \right] \\
& + T_{q,F}(x, x, \mu^2) \delta(1 - \hat{z}) \frac{1}{2N_c} \left[ (2\hat{x}^2 - \hat{x} - 1) \ln \frac{\hat{x}}{1 - \hat{x}} - 2 \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ + \frac{2\hat{x}(2 - \hat{x})}{(1 - \hat{x})_+} + 2 \frac{\ln \hat{x}}{1 - \hat{x}} \right] \\
& + T_{q,F}(x, x, \mu^2) \delta(1 - \hat{x}) C_F \left[ -(1 + \hat{z}) \ln \hat{z}(1 - \hat{z}) + 2 \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ - \frac{2\hat{z}}{(1 - \hat{z})_+} + 2 \frac{\ln \hat{z}}{1 - \hat{z}} \right] \\
& + T_{q,F}(x, x, \mu^2) \frac{1}{2N_c \hat{z}} \left[ \frac{2\hat{x}^3 - 3\hat{x}^2 - 1}{(1 - \hat{x})_+(1 - \hat{z})_+} + \frac{1 + \hat{z}}{(1 - \hat{x})_+} - 2(1 - \hat{x}) \right] \\
& + T_{q,F}(x, x\hat{x}, \mu^2) \delta(1 - \hat{z}) \frac{N_c}{2} \left[ \ln \frac{\hat{x}}{1 - \hat{x}} + 2 \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ - 2 \frac{\ln \hat{x}}{1 - \hat{x}} - \frac{1 + \hat{x}}{(1 - \hat{x})_+} \right] \\
& \left. + T_{q,F}(x, x\hat{x}, \mu^2) \frac{1 + \hat{x}\hat{z}^2}{(1 - \hat{x})_+(1 - \hat{z})_+} \left( C_F + \frac{1}{2N_c \hat{z}} \right) - T_{q,F}(x, x, \mu^2) 6C_F \delta(1 - \hat{x}) \delta(1 - \hat{z}) \right\} \quad (49)
\end{aligned}$$

Just like the NLO correction to the spin-averaged cross section in Eq. (7), the logarithms containing the factorization scale together with the splitting functions determine the evolution of the twist-3 Qiu-Sterman function and the usual unpolarized quark-to-hadron fragmentation function. Eq. (49) is the main result of our paper: once combined with the NLO spin-averaged cross section in Eq. (7), one will be able to compute the  $P_{h\perp}$ -weighted Siverts asymmetry as defined in Eq. (6).

#### IV. CONCLUSIONS

We calculated the next-to-leading order perturbative QCD corrections to the transverse momentum-weighted Siverts asymmetry in semi-inclusive hadron production in lepton-proton deep inelastic scattering. Specifically, we demonstrated in detail how to evaluate at NLO the  $P_{h\perp}$ -weighted transverse spin-dependent differential cross section. We found that the result can be written as a convolution of a twist-3 quark-gluon correlation function (often referred as Qiu-Sterman function), the usual unpolarized fragmentation function and the hard coefficient function. In the course of this calculation we showed that the soft divergences cancel out between real and virtual contributions, and that the collinear divergences can be absorbed into the NLO twist-3 Qiu-Sterman function of the transversely polarized proton and the unpolarized quark-to-hadron fragmentation function. Such a procedure also provides an alternative way to identify the evolution equation for the twist-3 Qiu-Sterman function. We found that our evolution equation agrees with those derived previously from different approaches. Using our NLO results for both the spin-averaged and  $P_{h\perp}$ -weighted transverse spin-dependent differential cross section, we plan to study in the future the  $P_{h\perp}$ -weighted Siverts asymmetry. We anticipate that our findings will have important phenomenological applications relevant to the experimental programs at HERMES, COMPASS, and Jefferson Lab.

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10825523.

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- [1] D. Boer, M. Diehl, R. Milner, R. Venugopalan, W. Vogelsang, D. Kaplan, H. Montgomery and S. Vigdor *et al.*, arXiv:1108.1713 [nucl-th]; M. Anselmino, H. Avakian, D. Boer, F. Bradamante, M. Burkardt, J. P. Chen, E. Cisbani and M. Contalbrigo *et al.*, Eur. Phys. J. A **47**, 35 (2011) [arXiv:1101.4199 [hep-ex]].
- [2] X. d. Ji, J. p. Ma and F. Yuan, Phys. Rev. D **71**, 034005 (2005) [arXiv:hep-ph/0404183]; Phys. Lett. B **597**, 299 (2004) [arXiv:hep-ph/0405085].
- [3] S. J. Brodsky, D. S. Hwang and I. Schmidt, Phys. Lett. B **530**, 99 (2002) [arXiv:hep-ph/0201296]; Nucl. Phys. B **642**, 344 (2002) [arXiv:hep-ph/0206259].
- [4] P. J. Mulders and R. D. Tangerman, Nucl. Phys. B **461**, 197 (1996) [Erratum-ibid. B **484**, 538 (1997)] [arXiv:hep-ph/9510301]; A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders and M. Schlegel, JHEP **0702**, 093 (2007) [hep-ph/0611265]; D. Boer, P. J. Mulders and F. Pijlman, Nucl. Phys. B **667**, 201 (2003) [arXiv:hep-ph/0303034].
- [5] D. Boer and P. J. Mulders, Phys. Rev. D **57**, 5780 (1998) [hep-ph/9711485].
- [6] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia and A. Prokudin, Phys. Rev. D **79**, 054010 (2009) [arXiv:0901.3078 [hep-ph]]; M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin and C. Turk, Eur. Phys. J. A **39**, 89 (2009) [arXiv:0805.2677 [hep-ph]]; Z. -B. Kang and J. -W. Qiu, Phys. Rev. Lett. **103**, 172001 (2009) [arXiv:0903.3629 [hep-ph]]; Phys. Rev. D **81**, 054020 (2010) [arXiv:0912.1319 [hep-ph]]; J. C. Collins, A. V. Efremov, K. Goeke, M. Grosse Perdekamp, S. Menzel, B. Meredith, A. Metz and P. Schweitzer, Phys. Rev. D **73**, 094023 (2006) [hep-ph/0511272]; Z. -B. Kang and B. -W. Xiao, arXiv:1212.4809 [hep-ph].
- [7] A. V. Efremov and O. V. Teryaev, Sov. J. Nucl. Phys. **36**, 140 (1982) [Yad. Fiz. **36**, 242 (1982)]; A. V. Efremov and O. V. Teryaev, Phys. Lett. B **150**, 383 (1985);
- [8] J. W. Qiu and G. Sterman, Phys. Rev. Lett. **67**, 2264 (1991); Nucl. Phys. B **378**, 52 (1992); Phys. Rev. D **59**, 014004 (1999) [arXiv:hep-ph/9806356]; C. Kouvaris, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D **74**, 114013 (2006) [arXiv:hep-ph/0609238].
- [9] H. Eguchi, Y. Koike and K. Tanaka, Nucl. Phys. B **763**, 198 (2007) [arXiv:hep-ph/0610314]; Y. Koike and K. Tanaka, Phys. Lett. B **646**, 232 (2007) [Erratum-ibid. B **668**, 458 (2008)] [arXiv:hep-ph/0612117]; Phys. Rev. D **76**, 011502 (2007) [arXiv:hep-ph/0703169]; Z. -B. Kang, F. Yuan and J. Zhou, Phys. Lett. B **691**, 243 (2010) [arXiv:1002.0399 [hep-ph]]; Z. -B. Kang, A. Metz, J. -W. Qiu and J. Zhou, Phys. Rev. D **84**, 034046 (2011) [arXiv:1106.3514 [hep-ph]]; L. Gamberg and Z. -B. Kang, Phys. Lett. B **718**, 181 (2012) [arXiv:1208.1962 [hep-ph]].
- [10] Y. Koike and S. Yoshida, Phys. Rev. D **85**, 034030 (2012) [arXiv:1112.1161 [hep-ph]]; Phys. Rev. D **84**, 014026 (2011) [arXiv:1104.3943 [hep-ph]]; Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D **83**, 114014 (2011) [arXiv:1104.0798 [hep-ph]]; H. Beppu, Y. Koike, K. Tanaka and S. Yoshida, Phys. Rev. D **82**, 054005 (2010) [arXiv:1007.2034 [hep-ph]]; Z. -B. Kang and J. -W. Qiu, Phys. Rev. D **78**, 034005 (2008) [arXiv:0806.1970 [hep-ph]]; Z. -B. Kang, J. -W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D **78**, 114013 (2008) [arXiv:0810.3333 [hep-ph]].
- [11] Z. -T. Liang, A. Metz, D. Pitonyak, A. Schafer, Y. -K. Song and J. Zhou, Phys. Lett. B **712**, 235 (2012) [arXiv:1203.3956 [hep-ph]]; A. Metz, D. Pitonyak, A. Schafer and J. Zhou, arXiv:1210.6555 [hep-ph].
- [12] X. Ji, J. W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. Lett. **97**, 082002 (2006) [arXiv:hep-ph/0602239]; Phys. Rev. D **73**, 094017 (2006) [arXiv:hep-ph/0604023]; Phys. Lett. B **638**, 178 (2006) [arXiv:hep-ph/0604128]; Y. Koike, W. Vogelsang and F. Yuan, Phys. Lett. B **659**, 878 (2008) [arXiv:0711.0636 [hep-ph]]; A. Bacchetta, D. Boer, M. Diehl and P. J. Mulders, JHEP **0808**, 023 (2008) [arXiv:0803.0227 [hep-ph]]; D. Boer, Z. -B. Kang, W. Vogelsang and F. Yuan, Phys. Rev. Lett. **105**, 202001 (2010) [arXiv:1008.3543 [hep-ph]]; Z. -B. Kang, J. -W. Qiu, W. Vogelsang and F. Yuan, Phys. Rev. D **83**, 094001 (2011) [arXiv:1103.1591 [hep-ph]]; L. Gamberg and Z. -B. Kang, Phys. Lett. B **696**, 109 (2011) [arXiv:1009.1936 [hep-ph]].
- [13] D. W. Sivers, Phys. Rev. D **41**, 83 (1990); Phys. Rev. D **43**, 261 (1991).
- [14] Z. -B. Kang, B. -W. Xiao and F. Yuan, Phys. Rev. Lett. **107**, 152002 (2011) [arXiv:1106.0266 [hep-ph]].
- [15] S. M. Aybat and T. C. Rogers, Phys. Rev. D **83**, 114042 (2011) [arXiv:1101.5057 [hep-ph]]; S. M. Aybat, J. C. Collins, J. -W. Qiu and T. C. Rogers, Phys. Rev. D **85**, 034043 (2012) [arXiv:1110.6428 [hep-ph]]; S. M. Aybat, A. Prokudin and T. C. Rogers, Phys. Rev. Lett. **108**, 242003 (2012) [arXiv:1112.4423 [hep-ph]].
- [16] M. Anselmino, M. Boglione and S. Melis, Phys. Rev. D **86**, 014028 (2012) [arXiv:1204.1239 [hep-ph]].
- [17] M. G. Echevarria, A. Idilbi, A. Schafer and I. Scimemi, arXiv:1208.1281 [hep-ph].
- [18] Z. -B. Kang and J. -W. Qiu, Phys. Rev. D **79**, 016003 (2009) [arXiv:0811.3101 [hep-ph]].
- [19] J. Zhou, F. Yuan and Z. -T. Liang, Phys. Rev. D **79**, 114022 (2009) [arXiv:0812.4484 [hep-ph]].
- [20] W. Vogelsang and F. Yuan, Phys. Rev. D **79**, 094010 (2009) [arXiv:0904.0410 [hep-ph]].
- [21] V. M. Braun, A. N. Manashov and B. Pirnay, Phys. Rev. D **80**, 114002 (2009) [arXiv:0909.3410 [hep-ph]].
- [22] Z. -B. Kang, Phys. Rev. D **83**, 036006 (2011) [arXiv:1012.3419 [hep-ph]].
- [23] Z. -B. Kang and J. -W. Qiu, Phys. Lett. B **713**, 273 (2012) [arXiv:1205.1019 [hep-ph]]; arXiv:1210.4103 [hep-ph].
- [24] A. Schafer and J. Zhou, Phys. Rev. D **85**, 117501 (2012) [arXiv:1203.5293 [hep-ph]].
- [25] J. P. Ma and Q. Wang, Phys. Lett. B **715**, 157 (2012) [arXiv:1205.0611 [hep-ph]].
- [26] A. Metz, D. Pitonyak, A. Schafer, M. Schlegel, W. Vogelsang and J. Zhou, Phys. Rev. D **86**, 094039 (2012) [arXiv:1209.3138 [hep-ph]]; M. Schlegel, arXiv:1211.3579 [hep-ph].

- [27] Z. -B. Kang and A. Prokudin, Phys. Rev. D **85**, 074008 (2012) [arXiv:1201.5427 [hep-ph]].
- [28] R. B. Meng, F. I. Olness and D. E. Soper, Nucl. Phys. B **371**, 79 (1992); Y. Koike and J. Nagashima, Nucl. Phys. B **660**, 269 (2003).
- [29] G. Altarelli, R. K. Ellis and G. Martinelli, Nucl. Phys. B **157**, 461 (1979).
- [30] D. Graudenz, Nucl. Phys. B **432**, 351 (1994) [hep-ph/9406274].
- [31] A. Daleo, D. de Florian and R. Sassot, Phys. Rev. D **71**, 034013 (2005) [hep-ph/0411212].
- [32] Y. Koike, W. Vogelsang and F. Yuan, Phys. Lett. B **659**, 878 (2008) [arXiv:0711.0636 [hep-ph]].
- [33] See, for example: Z. -B. Kang and J. -W. Qiu, Phys. Rev. D **77**, 114027 (2008) [arXiv:0802.2904 [hep-ph]]; Z. -B. Kang, I. Vitev and H. Xing, Phys. Rev. D **85**, 054024 (2012) [arXiv:1112.6021 [hep-ph]]; H. Xing, Z. -B. Kang, I. Vitev and E. Wang, Phys. Rev. D **86**, 094010 (2012) [arXiv:1206.1826 [hep-ph]].