General form of s, t, and u symmetric polynomials and heavy quarkonium physics

Yan-Qing Ma

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General Form of s, t, u Symmetric Polynomial and Heavy Quarkonium physics

Yan-Qing Ma*

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

Induced by three gluons symmetry, Mandelstam variables s, t, u symmetric expressions are widely involved in collider physics, especially in heavy quarkonium physics. In this work we study general form of s, t, u symmetric polynomials, and find that they can be expressed as polynomials where the symmetry is manifest. The general form is then used to simplify expressions which asymptotically reduces the length of original expression to one-sixth. Based on the general form, we reproduce the exact differential cross section of J/ψ hadron production at leading order in v^2 up to four unknown constant numbers by simple analysis. Furthermore, we prove that differential cross section at higher order in v^2 is proportional to that at leading order. This proof explains the proportion relation at next-to-leading order in v^2 found in previous work and generalizes it to all order.

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*Electronic address: yqma@bnl.gov

I. INTRODUCTION

Quantum chromodynamics (QCD) is currently believed to be the fundamental theory of the strong interaction. Thanks to asymptotic freedom and factorization [1–6] properties, application of QCD to a physical process can be factorized into convolution of non-perturbative, but universal, long distance matrix elements with perturbative calculable infrared-safe [7] short distance coefficients. Short distance coefficient at each order in perturbative expansion can be typically understood as the differential cross section of two partons scattering to produce n partons. This $2 \to n$ parton level process is conveniently obtained from the process $0 \to 2+n$ through crossing. Although partons can be either gluon or (anti-)quark, in many cases these $2+n$ partons are mainly gluons which are the gauge bosons of QCD. Therefore, the identity property between gluons results in a large symmetry for these processes.

Considering the specific type of processes $0 \to g(k_1)+g(k_2)+g(k_3)+H(P)$, where k_i are momentum of each gluons, P is the total momentum of a "cluster" H which includes one or more partons. The "cluster" here means differential cross section of the process should be sensitive only to the total momentum P of the cluster, but insensitive to the detail within the cluster. Momentum conservation gives

$$k_1^\mu + k_2^\mu + k_3^\mu + P^\mu = 0\,, \tag{1}$$

with $k_i^2 = 0$ and $P^2 = M^2$, where M is the invariant mass of cluster H. The associated Mandelstam variables are given by

$$s = (k_1+k_2)^2 = (P+k_3)^2\,, \tag{2a}$$

$$t = (k_2-k_3)^2 = (P-k_1)^2\,, \tag{2b}$$

$$u = (k_3-k_1)^2 = (P-k_2)^2\,, \tag{2c}$$

with $s+t+u = M^2$. Symmetry between the three gluons implies that the following function is symmetric under the exchange of s, t and u:

$$F(M^2, s, t, u) = \overline{\sum}|A|^2, \tag{3}$$

where A is Feynman amplitude of the process and $\overline{\sum}$ means summation/average over spin and color of these three gluons. Before doing any integration, $F(M^2, s, t, u)$ is a fraction polynomial, and both its numerator and denominator are symmetric polynomials. As a result, to study the restriction introduced by gluons symmetry, it is equivalent to study the property of symmetric polynomial.

The above type of processes are widely involved in collider physics. The simplest example is studying two jets production in hadron colliders and calculation of four gluons scattering is need, where H is also a gluon. In heavy quarkonium physics, there are a lot of processes belong to this type, including heavy quarkonium decays to light hadrons [8–13] and heavy quarkonium production in hadron colliders (see, for example Refs. [14–21] and references therein), where H is a heavy quark anti-quark pair with a very small relative momentum. Hopefully, study the property of symmetric polynomial will introduce rigorous restriction for these processes.

Among others, there is a very interesting finding in heavy quarkonium physics recently that, at leading order (LO) in α_s and in the large transverse momentum p_T limit, relativistic correction term for J/ψ hadron production is proportional to the leading term [17]. The proportion behavior is nontrivial because there are more than one parameters even in the large transverse momentum limit. It is likely to have a symmetry to protect this behavior, which is one of the motivations to study the symmetry property induced by three identical gluons.

The rest of the paper is organized as follows. We study the general form of a s, t, u symmetric polynomial in Sec. II. In Sec. II A we devote to massless case and find the polynomial can be expressed in a form where symmetry is manifest. By explicitly constructing, we generalize the massless result to include also massive particles in Sec. II B. The general form is then used to simplify expression in Sec. III. Asymptotically, this simplification can reduce the length of a expression to one-sixth. In Sec. IV, we use the general form to reproduce some known results and explain the unexpected proportion behavior of relativistic correction of J/ψ hadron production [17]. The proportion behavior is also generalized to all order in v^2. Finally, we summarize the results in this work and give a outlook for future works in Sec. V.

II. GENERAL FORM OF s, t, u SYMMETRIC POLYNOMIAL

In this section, we study the general form of a polynomial $F_n(M^2, s, t, u)$ which is symmetric under the exchange of s, t and u. Because all terms in $F_n(M^2, s, t, u)$ should have the same mass dimensions, a explicit subscript n is attached to denote the mass dimensions $2n$. Define three symmetric combinations

$$S_1 := s + t + u \,, \tag{4a}$$

$$S_2 := -st - tu - us \,, \tag{4b}$$

$$S_3 := stu \,. \tag{4c}$$

We will find that $F_n(M^2, s, t, u)$ can be expressed as a polynomial $\hat{F}_n(S_1, S_2, S_3)$. This result is not hard to be understood because the relation $s + t + u = M^2$ has already suggested that only three variables are independent in F_n. What is not clear is that whether $\hat{F}_n$ is a polynomial. In fact, s, t, u symmetric variables has been used for specific processes for a long time [22–24], but a general proof is still missing. In the following, we first study the case where $M = 0$. Then the extension to massive case is straightforward. Main results in this section are Eqs. (18) and (19).

A. Massless case: $s + t + u = 0$

Define

$$f_n(s, t, u) := F_n(0, s, t, u) \,, \tag{5}$$

which is a symmetric homogeneous polynomial. The general expression of $f_n(s, t, u)$ is

$$f_n(s, t, u) = \sum_{i=0}^{\frac{n+1}{2}} x_i \left[s^{n-i}(t^i + u^i) + t^{n-i}(u^i + s^i) + u^{n-i}(s^i + t^i) \right] + stu \; \tilde{f}_{n-3}(s, t, u) \,, \tag{6}$$

where x_i are independent of s, t and u, $\tilde{f}_{n-3}(s, t, u)$ is a symmetric polynomial with power $n - 3$ (obviously, $\tilde{f}_{n-3}(s, t, u) = 0$ if $n < 3$). Therefore, we need only prove the following function is a polynomial in S_2 and S_3:

$$\bar{f}_n(s, t, u) = \sum_{i=0}^{\frac{n+1}{2}} x_i \left[s^{n-i}(t^i + u^i) + t^{n-i}(u^i + s^i) + u^{n-i}(s^i + t^i) \right] \,. \tag{7}$$

We prove it recursively.

- $n = 0$ or 1 is trivial.

- $n = 2$:

$$\bar{f}_2(s,t,u) = 2x_0(s^2+t^2+u^2) + 2x_1(st+tu+us) = 2(2x_0 - x_1)S_2\,. \tag{8}$$

- $n \geq 3$ and $n \in 2N+1$:

  In this case, we will show that $\bar{f}_n(s,t,u) \propto s$, and then using symmetry we can get $\bar{f}_n(s,t,u) \propto stu$, that is $\bar{f}_n(s,t,u) = S_3 f_{n-3}(s,t,u)$. Setting $s = 0$, one has $t = -u$, thus

$$\bar{f}_n(0,-u,u) = \sum_{i=0}^{\frac{n+1}{2}} x_i \left[t^{n-i}u^i + u^{n-i}t^i\right] = \sum_{i=0}^{\frac{n+1}{2}} x_i \left[(-1)^{n-i} + (-1)^i\right] u^n = 0\,. \tag{9}$$

  Recall the general expression of $\bar{f}_n(s,t,u)$ in Eq. (7), the above result means $\bar{f}_n(s,t,u) \propto s$.

- $n \geq 3$ and $n \in 2N$, but $n \notin 6N$:

  In this case, we will show that $\bar{f}_n(s,t,u) \propto S_2$. Solutions of

$$\begin{cases} S_1 = 0\,, \\ S_2 = 0\,, \end{cases} \tag{10}$$

  is

$$\begin{cases} s = e^{\pm i\frac{2\pi}{3}}u\,, \\ t = e^{\mp i\frac{2\pi}{3}}u\,. \end{cases} \tag{11}$$

  Thus, equivalently, we need to show that $\bar{f}_n(s,t,u)$ vanishes for solutions in Eq. (11):

$$\begin{aligned} &\bar{f}_n(e^{\pm i\frac{2\pi}{3}}u, e^{\mp i\frac{2\pi}{3}}u, u) \\ =&u^n \sum_{j=0}^{\frac{n}{2}} x_j \left[e^{\pm i\frac{2(n-j)\pi}{3}}(e^{\mp i\frac{2j\pi}{3}}+1) + e^{\mp i\frac{2(n-j)\pi}{3}}(e^{\pm i\frac{2j\pi}{3}}+1) + (e^{\pm i\frac{2j\pi}{3}} + e^{\mp i\frac{2j\pi}{3}})\right] \\ =&u^n \sum_{j=0}^{\frac{n}{2}} x_j \left[e^{\pm i\frac{2(n-2j)\pi}{3}} + e^{\pm i\frac{2(n-j)\pi}{3}} + e^{\pm i\frac{2j\pi}{3}} + c.c.\right] \\ =&2u^n \sum_{j=0}^{\frac{n}{2}} x_j \left[\cos\frac{2(n-2j)\pi}{3} + \cos\frac{2(n-j)\pi}{3} + \cos\frac{2j\pi}{3}\right]\,. \end{aligned} \tag{12}$$

  Because $\cos\frac{2j\pi}{3} = 1$ if $j \in 3N$ and $\cos\frac{2j\pi}{3} = -\frac{1}{2}$ if $j \notin 3N$, $\cos\frac{2(n-2j)\pi}{3} + \cos\frac{2(n-j)\pi}{3} + \cos\frac{2j\pi}{3} = 0$ for all j, that is $\bar{f}_n(e^{\pm i\frac{2\pi}{3}}u, e^{\mp i\frac{2\pi}{3}}u, u) = 0$.

- $n \geq 3$ and $n \in 6N$:

  Define

$$G_i(s,t,u) := s^{n-i}(t^i + u^i) + t^{n-i}(u^i + s^i) + u^{n-i}(s^i + t^i)\,. \tag{13}$$

  It is easy to find that

$$\begin{cases} G_j(0,-u,u) = 2(-1)^j u^n\,, \\ G_j(e^{\pm i\frac{2\pi}{3}}u, e^{\mp i\frac{2\pi}{3}}u, u) = 6e^{i\frac{2j\pi}{3}}u^n\,. \end{cases} \tag{14}$$

  Thus

$$\begin{cases} (-1)^{j_1}G_{j_1}(0,-u,u) - (-1)^{j_2}G_{j_2}(0,-u,u) = 0\,, \\ e^{-i\frac{2j_1\pi}{3}}G_{j_1}(e^{\pm i\frac{2\pi}{3}}u, e^{\mp i\frac{2\pi}{3}}u, u) - e^{-i\frac{2j_2\pi}{3}}G_{j_2}(e^{\pm i\frac{2\pi}{3}}u, e^{\mp i\frac{2\pi}{3}}u, u) = 0\,, \end{cases} \tag{15}$$

  which means

$$\begin{cases} (-1)^{j_1}G_{j_1}(s,t,u) - (-1)^{j_2}G_{j_2}(s,t,u) \propto S_3\,, \\ e^{-i\frac{2j_1\pi}{3}}G_{j_1}(s,t,u) - e^{-i\frac{2j_2\pi}{3}}G_{j_2}(s,t,u) \propto S_2\,. \end{cases} \tag{16}$$

  For any j_1, there exists a j_2 which guarantees the coefficient matrix of Eq. (16) to be non-zero. Therefore, solution of Eq. (16) gives

$$G_j(s,t,u) = S_2 A_j(s,t,u) + S_3 B_j(s,t,u)\,, \tag{17}$$

  where $A_j(s,t,u)$ and $B_j(s,t,u)$ are symmetric polynomial. Specifically, taking advantage of results in previous cases, we find $A_j(s,t,u) \propto S_2^2$ and $B_j(s,t,u) \propto S_3$. Using this argument recursively, one gets that $G_j(s,t,u)$ is a polynomial in S_2^3 and S_3^2. As a result, $\bar{f}_n(s,t,u)$ is a polynomial in S_2^3 and S_3^2.

Combine all possible cases above, we indeed proved that $f_n(s,t,u)$ is a polynomial in S_2 and S_3 for any n. More precisely, we find

$$f_n(s,t,u) = \hat{f}_n(S_2,S_3) = \begin{cases} X_{\frac{n}{6}}(S_2^3, S_3^2), n \in 6N\,, \\ S_2^2 S_3 X_{\frac{n-7}{6}}(S_2^3, S_3^2), n \in 6N+1\,, \\ S_2 X_{\frac{n-2}{6}}(S_2^3, S_3^2), n \in 6N+2\,, \\ S_3 X_{\frac{n-3}{6}}(S_2^3, S_3^2), n \in 6N+3\,, \\ S_2^2 X_{\frac{n-4}{6}}(S_2^3, S_3^2), n \in 6N+4\,, \\ S_2 S_3 X_{\frac{n-5}{6}}(S_2^3, S_3^2), n \in 6N+5\,, \end{cases} \tag{18}$$

where $X_i(x,y)$ is an arbitrary homogeneous polynomial in x and y with power i. Specially, $X_i(x,y)=0$ if $i<0$.

B. Massive case: $s+t+u=M^2$

When $S_1=M^2\neq 0$, we will show that $F_n(M^2,s,t,u)$ can be expressed as:

$$F_n(M^2,s,t,u)=\hat{F}_n(S_1,S_2,S_3)=\sum_{i=0}^{n} S_1^i\hat{f}_{n-i}(S_2,S_3)\,, \tag{19}$$

where general form of $\hat{f}_{n-i}(S_2,S_3)$ is explicit in Eq. (18). We prove Eq. (19) by construction.

First, $i=0$ term in Eq. (19) can be obtained by setting $S_1=M^2=0$ in $F_n(M^2,s,t,u)$ using the method discussed in Sec. II A. Then we find $F_n(M^2,s,t,u)-\hat{f}_n(S_2,S_3)$ is zero if one sets $S_1=0$, that is $F_n(M^2,s,t,u)-\hat{f}_n(S_2,S_3)\propto S_1$. Considering that $F_n(M^2,s,t,u)$, $\hat{f}_n(S_2,S_3)$ and S_1 are symmetric polynomials,

$$F_{n-1}(M^2,s,t,u)=\frac{F_n(M^2,s,t,u)-\hat{f}_n(S_2,S_3)}{S_1} \tag{20}$$

is also a symmetric polynomial. Applying this method repeatedly, result of Eq. (19) can be achieved.

III. SIMPLIFY EXPRESSIONS

General forms in the last section can be used to simplify expressions that are symmetric polynomials in s, t and u by expressing them in terms of manifest symmetric form. In large n limit, one finds from Eq. (18) that the symmetric form is a polynomial with power $\frac{n}{6}$, therefore, asymptotically this method can reduce the length of original expression to one-sixth. Also, the method can be easily realized in terms computer program. Here, we give two examples.

The first example is an ideal expression

$$f_{42}:=\sum_{i=1}^{42} s^i(t^{42-i}+u^{42-i})+t^i(u^{42-i}+s^{42-i})+u^i(s^{42-i}+t^{42-i})\,, \tag{21}$$

with $s+t+u=0$. Its explicit expression in terms of t and u is

$$
\begin{aligned}
f_{42} = 2(t^2+tu+u^2)^3\big[&t^{36}+18t^{35}u+342t^{34}u^2+4029t^{33}u^3+34542t^{32}u^4+229536t^{31}u^5\\
&+1229611t^{30}u^6+5455041t^{29}u^7+20431896t^{28}u^8+65541187t^{27}u^9+182032158t^{26}u^{10}\\
&+441440337t^{25}u^{11}+940899497t^{24}u^{12}+1771715799t^{23}u^{13}+2959077438t^{22}u^{14}\\
&+4396930001t^{21}u^{15}+5825638020t^{20}u^{16}+6892901679t^{19}u^{17}+7289748245t^{18}u^{18}\\
&+6892901679t^{17}u^{19}+5825638020t^{16}u^{20}+4396930001t^{15}u^{21}+2959077438t^{14}u^{22}\\
&+1771715799t^{13}u^{23}+940899497t^{12}u^{24}+441440337t^{11}u^{25}+182032158t^{10}u^{26}\\
&+65541187t^9u^{27}+20431896t^8u^{28}+5455041t^7u^{29}+1229611t^6u^{30}+229536t^5u^{31}\\
&+34542t^4u^{32}+4029t^3u^{33}+342t^2u^{34}+18tu^{35}+u^{36}\big]\,.
\end{aligned}
\tag{22}
$$

As f_{42} is symmetric under exchanging of s, t and u, we can express it in terms of S_2 and S_3. Specifically, we find result for terms within brackets of Eq. (22) is

$$
\big[\cdots\big] = S_2^{18}+171S_2^{15}S_3^2+3060S_2^{12}S_3^4+12376S_2^9S_3^6+12870S_2^6S_3^8+3003S_2^3S_3^{10}+91S_3^{12}\,,
\tag{23}
$$

which is much simpler than the original expression. Result in Eq. (23) has two meanings. Firstly, it tests the proof in Sec. II A to be true by explicit calculation. Secondly, it shows the asymptotic behavior that our method can reduce the length of original expression to one-sixth.

The second example is a massive one. We take the equation (A5d) of Ref. [15]

$$
\begin{aligned}
\overline{\sum}|\mathcal{A}(gg\to c\bar{c}[^3P_0^{[8]}]g)|^2 = \frac{5(4\pi\alpha_s)^3}{12M^3\big[sz^2(s-M^2)^4(sM^2+z^2)^4\big]}\Big\{&\\
&+s^2z^4(s^2-z^2)^4+M^2sz^2(s^2-z^2)^2(3s^2-2z^2)(2s^4-6s^2z^2+3z^4)\\
&+M^4\big[9s^{12}-84s^{10}z^2+265s^8z^4-382s^6z^6+276s^4z^8-88s^2z^{10}+9z^{12}\big]\\
&-M^6s\big[54s^{10}-357s^8z^2+844s^6z^4-898s^4z^6+439s^2z^8-81z^{10}\big]\\
&+M^8\big[153s^{10}-798s^8z^2+1415s^6z^4-1041s^4z^6+301s^2z^8-18z^{10}\big]\\
&-M^{10}s\big[270s^8-1089s^6z^2+1365s^4z^4-616s^2z^6+87z^8\big]\\
&+M^{12}\big[324s^8-951s^6z^2+769s^4z^4-189s^2z^6+9z^8\big]\\
&-9M^{14}s(6s^2-z^2)(5s^4-9s^2z^2+3z^4)3M^{16}s^2(51s^4-59s^2z^2+12z^4)\\
&-27M^{18}s^3(2s^2-z^2)+9M^{20}s^4\Big\}\,,
\end{aligned}
\tag{24}
$$

which is the squared amplitude of ${}^3P_0^{[8]}$ channel for J/ψ hadron production via gluon-gluon fusion. Even a new variable $z := \sqrt{tu}$ is introduced, the expression is still very long. Using our method, it can be expressed in a much compact form. Terms within braces of Eq. (24) can be expressed as

$$\begin{aligned}&\Big\{\cdots\Big\} = S_2^4 S_3^2 + S_1 S_2^2 S_3 \left(6S_2^3 - S_3^2\right) + S_1^2 \left(9S_2^6 + 4S_2^3 S_3^2 + 5S_3^4\right) + S_1^3 S_2 S_3 \left(27S_2^3 - 11S_3^2\right) \\ &2S_1^4 S_2^2 \left(9S_2^3 - 16S_3^2\right) - S_1^5 S_3 \left(3S_2^3 + 13S_3^2\right) + 9S_1^6 S_2 \left(S_2^3 - 2S_3^2\right) - 9S_1^7 S_2^2 S_3 \,.\end{aligned} \tag{25}$$

More compactly, ordering it according to the power of S_3, we have

$$\begin{aligned}\Big\{\cdots\Big\} =& \, 9S_1^2 S_2^4 \left(S_1^2 + S_2\right)^2 - 3S_1 S_2^2 S_3 \left(3S_1^6 + S_1^4 S_2 - 9S_1^2 S_2^2 - 2S_2^3\right) \\ &- S_2 S_3^2 \left(18S_1^6 + 32S_1^4 S_2 - 4S_1^2 S_2^2 - S_2^3\right) - S_1 S_3^3 \left(13S_1^4 + 11S_1^2 S_2 + S_2^2\right) + 5S_1^2 S_3^4 \,.\end{aligned} \tag{26}$$

Note although that the simplification in this example is not so significant as the previous one, which is because the power here is smaller.

IV. J/ψ HADRON PRODUCTION

A. J/ψ hadron production at leading order in v^2

A well known result in heavy quarkonium physics is the J/ψ hadron production in color-singlet model[25–28], one out of six Feynman diagrams at leading order in α_s is shown in Fig. 1. Based on results in Sec. II, we will reproduce the behavior of the exact result by simple analysis.

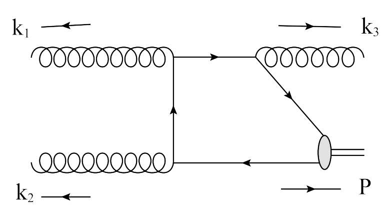

FIG. 1: One out of six Feynman diagrams for $g + g \to c\bar{c}[{}^3S_1^{[1]}] + g$, where all momentum are outgoing. The other five diagrams can be obtained by permutating the three gluons.

It is easy to find that denominator of the amplitude in Fig. 1 is proportional to $(s - M^2)(u - M^2)$, thus the summation of all six diagrams is proportional to

$[(s-M^2)(t-M^2)(u-M^2)]^{-1}$. In Feynman gauge, squaring the summed amplitude and summing/average over polarization, we get

$$F(^3S_1^{[1]}) \propto \frac{F_6(M^2,s,t,u)}{[(s-M^2)(t-M^2)(u-M^2)]^2}\,. \tag{27}$$

Note that, an additional dimensionless factor $\langle\mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle/M^3$ and a flux factor $1/2s$ are necessary to get the cross section of J/ψ production, but for our purpose it does not matter and we will neglect them. Before further discussion, we will prove that the behavior of this process to the differential cross section is $\frac{d\sigma}{dp_T^2} \propto M^4/p_T^8$ in large p_T limit, namely $s,\ t,\ u \gg M^2$. Note that, to demonstrate it, we will choose gauge other than Feynman gauge.

We begin with the amplitude in Fig. 1 (we label it with a "1" to denote that it is the first diagram. The other five diagrams will be labeled with 2, $\cdots$, 6.)

$$\begin{aligned}\mathcal{A}_1 =& \mathrm{Tr}\left[(-\frac{\not P}{2}+m)\gamma^\alpha(\frac{\not P}{2}+m)(-igT^{a_3}\gamma^{\mu_3})\frac{i(\not k_3+\frac{\not P}{2}+m)}{(k_3+\frac{P}{2})^2-m^2}(-igT^{a_1}\gamma^{\mu_1})\right.\\ &\left.\frac{i(-\not k_2-\frac{\not P}{2}+m)}{(-k_2-\frac{P}{2})^2-m^2}(-igT^{a_2}\gamma^{\mu_2})\right]\\ =& -i2g^3m\mathrm{Tr}\left[T^{a_1}T^{a_2}T^{a_3}\right]\mathrm{Tr}\left[\gamma^\alpha(\frac{\not P}{2}+m)\gamma^{\mu_3}\frac{\not k_3+\frac{\not P}{2}+m}{k_3\cdot P}\gamma^{\mu_1}\frac{-\not k_2-\frac{\not P}{2}+m}{k_2\cdot P}\gamma^{\mu_2}\right],\end{aligned} \tag{28}$$

where μ_i (a_i) is the spin index (color index) of the gluon with momentum k_i, α is the spin index of the pair $c\bar{c}[^3S_1^{[1]}]$ and $m = M/2$ is the mass of the charm quark. To get the above result, $P^2 = 4m^2$ and $P^\alpha = 0$ [1] have been used. Notice that, if one sets $m = 0$, trace of the gamma chain in the last line in Eq. (28) vanishes because there are odd number of gamma matrixes then, which implies $\mathcal{A}_1 \propto M^2$. Therefore, squaring the summation of all six amplitudes and contracting with summations of polarizations, we will get $\frac{d\sigma}{dp_T^2} \propto M^4/p_T^8$ if all denominators do not vanish when $M \to 0$. The only possible vanishing denominator comes from the summation of spin polarization of $c\bar{c}[^3S_1^{[1]}]$: $-g^{\alpha\alpha'} + \frac{P^\alpha P^{\alpha'}}{M^2}$, where the second term violates the above argument. Thus, we should prove that contracting with $\frac{P^\alpha P^{\alpha'}}{M^2}$ will at most give contributions at $O(M^4)$.

[1] This is possible because we will contract $\mathcal{A}_1$ with the summation of polariztion $-g^{\alpha\alpha'} + \frac{P^\alpha P^{\alpha'}}{M^2}$.

Contracting the trace of the gamma chain in the last line in Eq. (28) with $\frac{P^\alpha}{M}$, we get

$$\begin{aligned}\bar{\mathcal{A}}_1 &= \frac{1}{M}\mathrm{Tr}\left[\not{P}(\frac{\not{P}}{2}+m)\gamma^{\mu_3}\frac{\not{k}_3+\frac{\not{P}}{2}+m}{k_3\cdot P}\gamma^{\mu_1}\frac{-\not{k}_2-\frac{\not{P}}{2}+m}{k_2\cdot P}\gamma^{\mu_2}\right] \\ &= 2\ \mathrm{Tr}\left[\not{P}\gamma^{\mu_3}\frac{\not{k}_3+\frac{\not{P}}{2}}{2k_3\cdot P}\gamma^{\mu_1}\frac{-\not{k}_2-\frac{\not{P}}{2}}{2k_2\cdot P}\gamma^{\mu_2}\right]+O(M^2)\,.\end{aligned} \tag{29}$$

It is convenient to choose a axial gauge so that $P\cdot\epsilon(k_i)=0$ $(i=1,2,3)$, where $\epsilon(k_i)$ is the polarization vector of gluon with momentum k_i. This choice of gauge does not change our argument in the last paragraph because it does not introduce small denominators. In this gauge, terms more than two $\not{P}$ in Eq. (29) contribute to higher order in M^2, thus

$$\begin{aligned}\bar{\mathcal{A}}_1 &= -2\ \mathrm{Tr}\left[\not{P}\gamma^{\mu_3}\frac{\not{k}_3}{2k_3\cdot P}\gamma^{\mu_1}\frac{\not{k}_2}{2k_2\cdot P}\gamma^{\mu_2}\right]+O(M^2) \\ &= \mathrm{Tr}\left[\gamma^{\mu_3}\gamma^{\mu_1}\frac{\not{k}_2}{2k_2\cdot P}\gamma^{\mu_2}\right]+\mathrm{Tr}\left[\gamma^{\mu_3}\frac{\not{k}_3}{2k_3\cdot P}\gamma^{\mu_1}\gamma^{\mu_2}\right]+O(M^2) \\ &= \mathrm{Tr}\left[\gamma^{\mu_3}\gamma^{\mu_1}\frac{\not{k}_2}{u}\gamma^{\mu_2}\right]+\mathrm{Tr}\left[\gamma^{\mu_3}\frac{\not{k}_3}{s}\gamma^{\mu_1}\gamma^{\mu_2}\right]+O(M^2)\,.\end{aligned} \tag{30}$$

By circulating the three gluons, we get two other amplitudes which are proportional to $\mathrm{Tr}\,[T^{a_1}T^{a_2}T^{a_3}]$, with

$$\bar{\mathcal{A}}_2 = \mathrm{Tr}\left[\gamma^{\mu_1}\gamma^{\mu_2}\frac{\not{k}_3}{s}\gamma^{\mu_3}\right]+\mathrm{Tr}\left[\gamma^{\mu_1}\frac{\not{k}_1}{t}\gamma^{\mu_2}\gamma^{\mu_3}\right]+O(M^2)\,, \tag{31a}$$

$$\bar{\mathcal{A}}_3 = \mathrm{Tr}\left[\gamma^{\mu_2}\gamma^{\mu_3}\frac{\not{k}_1}{t}\gamma^{\mu_1}\right]+\mathrm{Tr}\left[\gamma^{\mu_2}\frac{\not{k}_2}{u}\gamma^{\mu_3}\gamma^{\mu_1}\right]+O(M^2)\,. \tag{31b}$$

Hence, using relation $\{\not{\epsilon}(k_i),\not{k}_i\}=0$, we have $\bar{\mathcal{A}}_1+\bar{\mathcal{A}}_2+\bar{\mathcal{A}}_3=O(M^2)$. Considering that the summation of the other three amplitudes has the same behavior, we find contracting with $\frac{P^\alpha P^{\alpha'}}{M^2}$ will give contributions at $O(M^6)$. Finally, we complete the proof that $\frac{d\sigma}{dp_T^2}\propto M^4/p_T^8$.

As $\frac{d\sigma}{dp_T^2}\propto M^4/p_T^8$ at large p_T limit, $F(^3S_1^{[1]})$ and $F_6(M^2,s,t,u)$ in Eq. (27) must be proportional to M^4. Using results in Sec. II, we get the general form

$$F_6(M^2,s,t,u)=M^4F_4(M^2,s,t,u)=S_1^2(x_0S_2^2+x_1S_1S_3+x_2S_1^2S_2+x_4S_1^4)\,, \tag{32}$$

where x_i $(i=0,1,2,4)$ are dimensionless numbers. The above simple analysis indeed reproduced the behavior of exact result (e.g. one can find it in [17]), which expressed in our form is

$$F(^3S_1^{[1]})\propto\frac{S_1^2\,(S_2^2-S_1S_3)}{[(s-M^2)(t-M^2)(u-M^2)]^2}\,. \tag{33}$$

B. Relativistic corrections for J/ψ hadron production

A more interesting application of results in Sec. II is to understand the relativistic correction behavior of J/ψ hadron production in large p_T limit. We will study in the limit $s,\ t,\ u \gg M^2$, where result at leading order in v^2 as shown in Sec. IV A is

$$F(^3S_1^{[1]}) \propto \frac{S_1^2 S_2^2}{S_3^2}\,. \tag{34}$$

We will demonstrate that, in large p_T limit, relativistic correction does not change this behavior, therefore, relativistic correction term is proportional to the leading term. The proof includes two steps. Firstly, we show that the denominator of summed amplitude is not changed by relativistic correction. Secondly, we show that the large p_T behavior $\frac{d\sigma}{dp_T^2} \propto M^4/p_T^8$ is also not changed by relativistic correction. Combining these two points and using the general form in Sec. II, it is straightforward to find that relativistic correction term is proportional to $\frac{S_1^2 S_2^2}{S_3^2}$.

Before two steps arguments, we briefly introduce the relativistic correction. In Sec. IV A, we use the color-singlet model to calculate the J/ψ hadron production, where a charm (anti-) quark pair with definite quantum number ($c\bar{c}[^3S_1^{[1]}]$) is produced in hard collision. Further more, we did the non-relativistic approximation that relative momentum between $c\bar{c}$ pair is zero there. The relativistic corrections in this paper refer to corrections by expanding the relative momentum to higher power. For definiteness, we denote the momentum of c and $\bar{c}$ as

$$\begin{cases} p_c = \frac{P}{2} + q\,, \\ p_{\bar{c}} = \frac{P}{2} - q\,, \end{cases} \tag{35}$$

where P is the total momentum of $c\bar{c}$ pair and q is half of the relative momentum. As the J/ψ is a S-wave quarkonium, only terms with even power of relative momentum in amplitude level contribute. In additional, projecting amplitude to S-wave is equivalent to contract coefficients of relative momentum with terms like $-g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2}$.

Now, we are ready for the first step. The only possible source that may change the denominator of summed amplitude is the expansion of relative momentum for denominator with finite relative momentum. As an example, we study the expansion of upper denominator in

Fig. 1

$$\frac{1}{(k_3+\frac{P}{2}+q)^2-m^2}=\frac{1}{k_3\cdot P}-\frac{2k_3^\mu}{(k_3\cdot P)^2}q_\mu+\cdots\,. \tag{36}$$

When contracting k_3^μ with $-g^{\mu\nu}+\frac{P^\mu P^\nu}{M^2}$ to get S-wave, $\frac{P^\mu P^\nu}{M^2}$ gives leading contribution while $-g^{\mu\nu}$ is suppressed as $M\to 0$. Notice that contracting k_3^μ with $\frac{P^\mu P^\nu}{M^2}$ cancels the denominator $(k_3\cdot P)^{-2}$ by one power exactly, therefore, we find that expanding this denominator to higher order in q does not change the denominator at large p_T limit. This argument can be easily extended to any denominator that expanding to any power of q. Thus the first step is achieved.

Let's then argue that the large p_T behavior $\frac{d\sigma}{dp_T^2}\propto M^4/p_T^8$ holds to all order of relativistic corrections, that is, terms of order M^2/p_T^6 vanish at the cross section level. For a finite relative momentum, the amplitude in Fig. 1 is

$$\mathcal{A}_1=\mathrm{Tr}\left[(-\frac{\not{P}}{2}+\not{q}+m)\gamma^\alpha(\frac{\not{P}}{2}+\not{q}+m)\Pi(P,q,m)\right]\,, \tag{37}$$

with

$$\Pi(P,q,m)=(-igT^{a_3}\gamma^{\mu_3})\frac{i(\not{k}_3+\frac{\not{P}}{2}+\not{q}+m)}{(k_3+\frac{P}{2}+q)^2-m^2}(-igT^{a_1}\gamma^{\mu_1})\frac{i(-\not{k}_2-\frac{\not{P}}{2}+\not{q}+m)}{(-k_2-\frac{P}{2}+q)^2-m^2}(-igT^{a_2}\gamma^{\mu_2})\,. \tag{38}$$

As we study the large p_T limit, both P and q have been boosted to a similar direction, say $\hat{P}$, thus they have the following decomposition

$$\begin{cases} P^\mu=\hat{P}^\mu+\lambda P_\perp^\mu+\lambda^2\frac{P^2-P_\perp^2}{2P^+}n^\mu\,,\\ q^\mu=\frac{\zeta}{2}\hat{P}^\mu+\lambda q_\perp^\mu+\lambda^2\frac{q^2-q_\perp^2}{2q^+}n^\mu\,,\end{cases} \tag{39}$$

where $\zeta=\frac{2q^+}{P^+}=\frac{2q\cdot n}{P\cdot n}$, n is a light like momentum which satisfies $(P\cdot n)^2/(n^0)^2\gg P^2$, and λ is used to denote the power counting of corresponding term, that is, term proportional to λ^i behaviors as $O(M^i)$. Using the on-shell relations $\left(\pm\frac{P}{2}+q\right)^2=m^2$ and the fact that

$$(\pm\frac{\not{P}}{2}+\not{q}+m)\not{n}(\pm\frac{\not{P}}{2}+\not{q}+m)=(\pm1+\zeta)P^+(\pm\frac{\not{P}}{2}+\not{q}+m)\,, \tag{40}$$

we can rewrite $\mathcal{A}_1$ as

$$\mathcal{A}_1=\frac{-1}{(1-\zeta^2)P^{+2}}\mathrm{Tr}\left[(-\frac{\not{P}}{2}+\not{q}+m)\not{n}\mathbf{1}(-\frac{\not{P}}{2}+\not{q}+m)\gamma^\alpha(\frac{\not{P}}{2}+\not{q}+m)\mathbf{1}\not{n}(\frac{\not{P}}{2}+\not{q}+m)\Pi(P,q,m)\right]\,, \tag{41}$$

where we also inserted two unit matrixes. Doing the Fierz transformation

$$\mathbf{1}_{ij}\mathbf{1}_{kl} = \frac{1}{4}\sum_{\lambda}\Gamma^{\lambda}_{il}\Gamma_{\lambda,kj}\,, \tag{42}$$

with $\Gamma^{\lambda} = \mathbf{1}, \gamma^5, \gamma^{\mu}, \gamma^5\gamma^{\mu}, \sigma^{\mu\nu}/\sqrt{2}$ and $\Gamma_{\lambda} = \Gamma^{\lambda\dagger}$, $\mathcal{A}_1$ becomes

$$\begin{aligned}
\mathcal{A}_1 =& \frac{-1}{4(1-\zeta^2)P^{+2}}\sum_{\lambda}\mathrm{Tr}\left[\Gamma_{\lambda}(-\frac{\not{P}}{2}+\not{q}+m)\gamma^{\alpha}(\frac{\not{P}}{2}+\not{q}+m)\right] \\
&\times \mathrm{Tr}\left[(-\frac{\not{P}}{2}+\not{q}+m)\not{n}\Gamma^{\lambda}\not{n}(\frac{\not{P}}{2}+\not{q}+m)\Pi(P,q,m)\right] \\
=& \frac{-1}{2(1-\zeta^2)P^{+2}}\sum_{\lambda=1,2,3}\mathrm{Tr}\left[\hat{\Gamma}^{\lambda}(-\frac{\not{P}}{2}+\not{q}+m)\gamma^{\alpha}(\frac{\not{P}}{2}+\not{q}+m)\right] \\
&\times \mathrm{Tr}\left[(-\frac{\not{P}}{2}+\not{q}+m)\hat{\Gamma}^{\lambda}(\frac{\not{P}}{2}+\not{q}+m)\Pi(P,q,m)\right]\,,
\end{aligned} \tag{43}$$

where relations

$$\Gamma_{\lambda}\otimes\not{n}\Gamma^{\lambda}\not{n} = 0\,,\ 0\,,\ 2\not{n}\otimes\not{n}\,,\ 2\gamma^5\not{n}\otimes\gamma^5\not{n}\,,\ \ 2\gamma^{\mu}_{\perp}\not{n}\otimes\gamma^{\mu}_{\perp}\not{n}\,,$$

have been used, and $\hat{\Gamma}^1 = \not{n}, \hat{\Gamma}^2 = \gamma^5\not{n}, \hat{\Gamma}^3 = \gamma^{\mu}_{\perp}\not{n}$. Note that we can think $\mathrm{Tr}\left[\hat{\Gamma}^{\lambda}(-\frac{\not{P}}{2}+\not{q}+m)\gamma^{\alpha}(\frac{\not{P}}{2}+\not{q}+m)\right]$ to be $O(M)$ effectively because squaring it will give a $O(M^2)$ result, thus we do leading power approximation for other terms and gives

$$\mathcal{A}_1 = \frac{1}{8P^{+2}}\sum_{\lambda=1,2,3}\mathrm{Tr}\left[\hat{\Gamma}^{\lambda}(-\frac{\not{P}}{2}+\not{q}+m)\gamma^{\alpha}(\frac{\not{P}}{2}+\not{q}+m)\right]\mathrm{Tr}\left[\hat{\not{P}}\hat{\Gamma}^{\lambda}\hat{\not{P}}\Pi(\hat{P},\frac{\zeta}{2}\hat{P},0)\right] + O(M^2)\,, \tag{44}$$

with

$$\Pi(\hat{P},\frac{\zeta}{2}\hat{P},0) = (-igT^{a_3}\gamma^{\mu_3})\frac{i(\not{k}_3+\frac{1+\zeta}{2}\hat{\not{P}})}{(k_3+\frac{1+\zeta}{2}\hat{P})^2}(-igT^{a_1}\gamma^{\mu_1})\frac{i(-\not{k}_2-\frac{1-\zeta}{2}\hat{\not{P}})}{(-k_2-\frac{1-\zeta}{2}\hat{P})^2}(-igT^{a_2}\gamma^{\mu_2})\,. \tag{45}$$

Similar as Sec. IV A, we choose a axial gauge so that $\hat{P}\cdot\epsilon(k_i) = 0$, then some terms in $\Pi(\hat{P},\frac{\zeta}{2}\hat{P},0)$ do not contribute because there is a $\hat{\not{P}}$ on its either side. Thus,

$$\begin{aligned}
\Pi(\hat{P},\frac{\zeta}{2}\hat{P},0) \sim& (-igT^{a_3}\gamma^{\mu_3})\frac{i\not{k}_3}{(k_3+\frac{1+\zeta}{2}\hat{P})^2}(-igT^{a_1}\gamma^{\mu_1})\frac{-i\not{k}_2}{(-k_2-\frac{1-\zeta}{2}\hat{P})^2}(-igT^{a_2}\gamma^{\mu_2}) \\
=& -ig^3\frac{4}{1-\zeta^2}T^{a_3}T^{a_1}T^{a_2}\gamma^{\mu_3}\frac{\not{k}_3}{(k_3+\hat{P})^2}\gamma^{\mu_1}\frac{-\not{k}_2}{(k_2+\hat{P})^2}\gamma^{\mu_2}\,,
\end{aligned} \tag{46}$$

and

$$\mathcal{A}_1 = \frac{-ig^3}{2(1-\zeta^2)P^{+2}} \mathrm{Tr}\,[T^{a_1}T^{a_2}T^{a_3}] \sum_{\lambda=1,2,3} \mathrm{Tr}\left[\hat{\Gamma}^\lambda(-\frac{\not{P}}{2}+\not{q}+m)\gamma^\alpha(\frac{\not{P}}{2}+\not{q}+m)\right] \times \mathrm{Tr}\left[\hat{\not{P}}\hat{\Gamma}^\lambda\hat{\not{P}}\gamma^{\mu_3}\frac{\not{k}_3}{(k_3+\hat{P})^2}\gamma^{\mu_1}\frac{-\not{k}_2}{(k_2+\hat{P})^2}\gamma^{\mu_2}\right] + O(M^2)\,, \tag{47}$$

Observing that $\lambda = 3$ does not contribute because there are odd number of Dirac matrixes in the last trace; $\lambda = 2$ does not contribute because $\mathrm{Tr}\left[\hat{\Gamma}^2(-\frac{\not{P}}{2}+\not{q}+m)\gamma^\alpha(\frac{\not{P}}{2}+\not{q}+m)\right] \propto \epsilon^{nPq\alpha}$ thus $\mathcal{A}_1$ is odd in in q. As a result, we get

$$\begin{aligned}
\mathcal{A}_1 &= \frac{-ig^3}{2(1-\zeta^2)P^{+2}} \mathrm{Tr}\,[T^{a_1}T^{a_2}T^{a_3}]\, \mathrm{Tr}\left[\not{n}(-\frac{\not{P}}{2}+\not{q}+m)\gamma^\alpha(\frac{\not{P}}{2}+\not{q}+m)\right] \\
&\quad \times \mathrm{Tr}\left[\hat{\not{P}}\not{n}\hat{\not{P}}\gamma^{\mu_3}\frac{\not{k}_3}{(k_3+\hat{P})^2}\gamma^{\mu_1}\frac{-\not{k}_2}{(k_2+\hat{P})^2}\gamma^{\mu_2}\right] + O(M^2) \\
&= \frac{-ig^3}{2(1-\zeta^2)P^{+}} \mathrm{Tr}\,[T^{a_1}T^{a_2}T^{a_3}]\, \mathrm{Tr}\left[\not{n}(-\frac{\not{P}}{2}+\not{q}+m)\gamma^\alpha(\frac{\not{P}}{2}+\not{q}+m)\right] \\
&\quad \times \left\{\mathrm{Tr}\left[\gamma^{\mu_3}\gamma^{\mu_1}\frac{\not{k}_2}{u}\gamma^{\mu_2}\right] + \mathrm{Tr}\left[\gamma^{\mu_3}\frac{\not{k}_3}{s}\gamma^{\mu_1}\gamma^{\mu_2}\right]\right\} + O(M^2)\,.
\end{aligned} \tag{48}$$

Comparing it with Eq. (30) and using similar argument, we find $\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 = O(M^2)$ and that $\frac{d\sigma}{dp_T^2} \propto M^4/p_T^8$ holds.

Combining the above two points, we know that, in large p_T limit, the denominator is S_3^{-2} and the numerator has a factor S_1^2. The rest numerator, as it is symmetric in s, t and u and has mass dimension of $[M]^8$, must be proportional to S_2^2 using the general form in Sec. II. Finally, we find that relativistic correction term is proportional to $\frac{S_1^2 S_2^2}{S_3^2}$, which is the same as term at LO in v^2. This behavior for next-to-leading order (NLO) relativistic correction has been found in Ref. [17], but the generalization to all order in this work is new.

We conclude this section by explaining the logic to prove $\frac{d\sigma}{dp_T^2} \propto M^4/p_T^8$. After the Fierz transformation, we in fact factorize the amplitude to soft parts (with scales of $O(M)$) and hard parts (with scales of $O(p_T)$). This factorized form is equivalent to double parton fragmentation formula in Ref. [6], which give contribution of $O(M^2)$ at cross section level. Factorized terms are then shown to vanish at large p_T limit. Therefore, remained terms can only give contribution at $O(M^4)$.

V. SUMMARY AND OUTLOOK

In heavy quarkonium physics, many processes involve three gluons, which result in s, t, u symmetric cross sections or decay widths. In this work we study general form of s, t, u symmetric polynomials, and find that they can be expressed as polynomials of S_1, S_2 and S_3 where the symmetry is manifest. For massless case the general form is summarized in Eq. (18), and for massive case the general form is summarized in Eq. (19). These general forms can be used to simplify expressions that are symmetric in s, t and u. Asymptotically, this method can reduce the length of original expression to one-sixth. Based on these general forms, one can also predict many interesting results by simple analysis. We give two examples regarding J/ψ hadron production in this work. In the first example we work within the color-singlet model at LO in v^2. By only arguing that the differential cross section has the behavior $\frac{d\sigma}{dp_T^2} \propto M^4/p_T^8$ in large p_T limit, namely $s,\ t,\ u \gg M^2$, we successfully reproduce the exact differential cross section up to four unknown constant numbers. In the second example, we consider relativistic corrections for color-singlet model in large p_T limit. By showing that, in large p_T limit, relativistic corrections do not change denominator and the behavior $\frac{d\sigma}{dp_T^2} \propto M^4/p_T^8$, we prove that differential cross section proportional to $\frac{S_1^2 S_2^2}{S_3^2}$ holds to all order in v^2. This proof not only explains the proportion relation at NLO in v^2 found in Ref. [17], but also generalizes it to all order.

Calculations of $O(v^2)$ and $O(\alpha_s)$ corrections to heavy-quarkonium production and decay observables usually yield very lengthy expressions. In view of that the s, t, u symmetry can give so many constraints, it is possible to use our systematic method to simplify these lengthy expressions and to exhibit their symmetry. It will be also interesting to study symmetry induced by four or even more gluons.

Proportion relations at NLO in v^2 are also found for color-octet channel [18]. However, since differential cross sections for color-octet channel have the behavior $\frac{d\sigma}{dp_T^2} \propto M^2/p_T^6$ or $\frac{d\sigma}{dp_T^2} \propto 1/p_T^4$, proportion relations for them can not be constrained by only s, t, u symmetry. We will study this problem in a forthcoming work [29].

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