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Lorentz-Covariant Four-Vector Formalism for Two-Measure Theory

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Abstract

In the conventional two-measure theory (TMT), the scalar density function Φ is taken to be $\Phi \equiv \epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}(\partial_\mu\varphi^a)(\partial_\nu\varphi^b)(\partial_\rho\varphi^c)(\partial_\sigma\varphi^d)$, where the indices $a,~b,~c,~d = 1,~2,~3,~4$ are internal space indices. It is more natural to replace the four scalars φ^a by a Lorentz-covariant four-vector φ^m with a local Lorentz index $m = (0),~(1),~(2),~(3)$. We entertain this possibility, and show that the newly-proposed lagrangian respects not only Lorentz covariance, but also global-scale invariance. The crucial equation $\partial_\mu L = 0$ in the conventional TMT also arises in our new formulation, as the φ^m-field equation.

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1. Introduction

In the conventional two-measure theory (TMT) [1][2], there are four scalars φ^a introduced in the scalar density function $\Phi \equiv \epsilon^{\mu\nu\rho\sigma}\epsilon_{abcd}\,(\partial_\mu\varphi^a)(\partial_\nu\varphi^b)(\partial_\rho\varphi^c)(\partial_\sigma\varphi^d)$, carrying the four internal-space indices $a,\ b,\ c,\ d = 1,\ 2,\ 3,\ 4$.

In the present paper, we replace the set of four scalar fields φ^a $(a = 1,\ 2,\ 3,\ 4)$ in the original TMT [1][2] by a four-vector φ^m, where $m = (0),\ (1),\ (2),\ (3)$ are local four-dimensional (4D) holonomy indices.[4] Accordingly, we introduce the Lorentz connection $\omega_\mu{}^{mn}$ for maintaining local Lorentz symmetry.

2. Total Action

Our field content is $(e_\mu{}^m, \varphi^m, \omega_\mu{}^{mn}, \phi)$, where $e_\mu{}^m$ is the vierbein in 4D, φ^m is the Lorentz-covariant four vector field of the utmost importance, while ϕ represents general matter fields.

Our total action is $I \equiv \int d^4x\,\mathcal{L}$ with the lagrangian

$$\begin{aligned}\mathcal{L} = &+\Phi L + eL' - \frac{1}{4}\,(1-2\alpha)\,e\,{}^*R^*_{\mu\nu mn}(\varphi^t)^2 P^{\mu m}P^{\nu n} L \\ &+ \alpha\,e\,{}^*R^*_{\mu\nu mn}\varphi^m\varphi_r P^{\mu r}P^{\nu n}L + \frac{1}{64}\,(1-4\alpha)\,e\,{}^*R^*_{\mu\nu mn}R^{\mu\nu mn}(\varphi^r)^2(\varphi^s)^2 L \quad , \qquad (2.1)\end{aligned}$$

where L is a general lagrangian (but *not* a lagrangian *density*) in terms of $(e_\mu{}^m, \omega_\mu{}^{mn}, \phi)$ *without* φ^m, and L' is another general lagrangian containing only $(e_\mu{}^m, \omega_\mu{}^{mn}, \phi)$, while α is an arbitrary real constant. The term eL' is needed for the total system to have 'two measures'. The Φ and $P_\mu{}^m$ are defined by[5]

$$\Phi \equiv +\,\frac{1}{24}\,\epsilon^{\mu\nu\rho\sigma}\epsilon_{mnrs}P_\mu{}^m P_\nu{}^n P_\rho{}^r P_\sigma{}^s = \det\,(P_\mu{}^m) \quad , \qquad (2.2\text{a})$$

$$P_\mu{}^m \equiv +\,\partial_\mu\varphi^m + \omega_\mu{}^{mn}\varphi_n \quad , \qquad (2.2\text{b})$$

while curvature tensors are defined by

$${}^*R^*_{\mu\nu}{}^{mn} \equiv +\,\frac{1}{2}\,e^{-1}\epsilon_{\mu\nu}{}^{\rho\sigma}\,R^*_{\rho\sigma}{}^{mn} \equiv +\frac{1}{4}\,e^{-1}\epsilon_{\mu\nu}{}^{\rho\sigma}\,\epsilon^{mn}{}_{rs}R_{\rho\sigma}{}^{rs} \quad , \qquad (2.3\text{a})$$

[4] We use the parentheses for local Lorentz indices: $(0),\ (1),(2),\ (3)$, in order to distinguish them from the curved ones $\mu = 0,\ 1,\ 2,\ 3$.

[5] Compared with the conventional TMT [1][2], we have the front factor $1/24$. This is because of the convenient expression $\Phi = \det\,(P_\mu{}^m)$.

$$ {}^*R_{\mu\nu}{}^{mn} \equiv + \frac{1}{2}\, e^{-1}\epsilon_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{mn} \ , \qquad R^*_{\mu\nu}{}^{mn} \equiv + \frac{1}{2}\, \epsilon^{mn}{}_{rs} R_{\mu\nu}{}^{rs} \ , \tag{2.3b} $$

$$ R_{\mu\nu}{}^{mn} \equiv + 2\partial_{[\mu}\omega_{\nu]}{}^{mn} + 2\omega_{[\mu}{}^{mt}\omega_{\nu]t}{}^{n} \ , \qquad R_\mu{}^m \equiv e_n{}^\nu R_{\mu\nu}{}^{mn} \ , \qquad R \equiv e_m{}^\mu R_\mu{}^m \ , \tag{2.3c} $$

$$ \omega_{mrs} \equiv + \frac{1}{2}\,(C_{mrs} - C_{msr} - C_{rsm}) \ , \qquad C_{\mu\nu}{}^m \equiv 2\partial_{[\mu} e_{\nu]}{}^m \ . \tag{2.3d} $$

In particular, ${}^*R_{\mu\nu}{}^{mn}$ (or $R^*_{\mu\nu}{}^{mn}$) represents the dual with respect to the first two indices $\mu\nu$ (or the last two indices mn) of $R_{\mu\nu}{}^{mn}$. Since Φ transforms as a scalar density, there should be *no* factor of $e \equiv \det(e_\mu{}^m)$ in front of the first term in (2.1). In our work, we are working with the 2nd-order formalism for $\omega_\mu{}^{mn}$.

Most importantly, the ordinary derivative $\partial_\mu\varphi^a$ in Φ in the conventional TMT [1][2] is replaced by the Lorentz covariant derivative $D_\mu\varphi^m \equiv P_\mu{}^m$ due to the Lorentz-covariant four-vector φ^m in our new formulation.

The fact that $\Phi = \det(P_\mu{}^m)$ in a structure parallel to $e \equiv \det(e_\mu{}^m)$ also makes our formulation more natural with the Lorentz-covariant four-vector φ^m instead of the four scalars φ^a in the original TMT formulations [1][2].

3. The φ^m-Field Equation

The φ^m- field equation out of our action (2.1) is the most crucial one:

$$ \frac{\delta\mathcal{L}}{\delta\varphi^m} = M_m{}^\mu\, \partial_\mu L \doteq 0 \ , \tag{3.1a} $$

$$ \begin{aligned} M_m{}^\mu \equiv & - e^{-1}\,\Phi\,(P^{-1})_m{}^\mu \\ & - \left(\alpha - \frac{1}{2}\right) e\ {}^*R^{*\mu\nu}{}_{ms}(\varphi^t)^2 P_\nu{}^s - \alpha\, e\ {}^*R^{*\mu\nu}{}_{rs}\varphi_m\varphi^r P_\nu{}^s + \alpha\, e\ {}^*R^{*\mu\nu}{}_{ms}\varphi^s\varphi_t P_\nu{}^t . \end{aligned} \tag{3.1b} $$

As will be explained in Appendix, we can multiply (3.1a) by the inverse matrix $(M^{-1})_\nu{}^m$, and eventually get

$$ \partial_\nu L \doteq 0 \qquad \Longrightarrow \qquad L = \text{const.} \equiv M \ , \tag{3.2} $$

as in the conventional TMT [1][2]. In other words, our Lorentz-covariant four-vector formulation also yield exactly the same condition $L = \text{const.}$ as the conventional TMT [1][2].

4. Concluding Remarks

In this brief report, we have studied the implications of using a Lorentz-covariant four-vector φ^m instead of the four scalars φ^a in the conventional TMT [1][2]. As is shown in the next Appendix, our system yields the important field equation $\partial_\mu L \doteq 0$ and therefore its solution $L \doteq$ const. breaking global scale symmetry just as in the conventional TMT [1][2].

In eqs. (A3) $\sim$ (A.9), we have confirmed that the curvature-linear term arising from the lagrangian term ΦL is cancelled by adding the $R\varphi^2 P^2 L$-terms in the lagrangian. However, the latter terms also generate new curvature-bilinear terms, that are cancelled by adding the $R^2\varphi^4 L$-terms in the lagrangian. These cancellations are highly non-trivial, necessitating particular lemmas for unexpected identities holding among various terms arising, as well as the original possible lagrangian terms.

Interestingly enough, our final lagrangian (2.1) also maintains global scale invariance under (A.11), as explained in Appendix. This indicates the non-trivial feature of our new formulation with the four-vector φ^m.

Here we have considered only the 2nd-order formalism for the Lorentz connection $\omega_\mu{}^{rs}$. However, this does *not* exclude the possibility of the 1st-order formalism. The price to be paid is the complication of the usual expression $\omega_{mrs} = (1/2)(C_{mrs} - C_{msr} - C_{rsm})$ by additional terms. It is interesting to see what effect it will have in the 1st-order formalism.

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Appendix: φ^m-Field Equation, Action Invariance, and Non-Trivial Lemmas

In this Appendix, we describe the detailed process of confirming action invariance, getting the field equation of φ^m and related non-trivial lemmas.

The φ^m-field equation represents the crucial ingredient in the system. If we blindly replace as $\partial_\mu \varphi^a \rightarrow D_\mu \varphi^m \equiv P_\mu{}^m$ in Φ in the conventional TMT [1][2], setting up the action $I_0 \equiv \int d^4x \, \mathcal{L}_0 \equiv \int d^4x \, \Phi L$, then a curvature-linear term arises:

$$\begin{aligned}\frac{\delta \mathcal{L}_0}{\delta \varphi^m} &= -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\epsilon_{mnrs}(D_{[\mu}P_{\nu]}{}^n)P_\rho{}^r P_\sigma{}^s L - \frac{1}{6}\epsilon^{\mu\nu\rho\sigma}\epsilon_{mnrs}P_\nu{}^n P_\rho{}^r P_\sigma{}^s \partial_\mu L \\ &= -\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{mnrs}R_{\mu\nu}{}^{nu}\varphi_u P_\rho{}^r P_\sigma{}^s L + 4e_m{}^{[\mu}e_n{}^\nu e_r{}^\rho e_s{}^{\sigma]}P_\nu{}^n P_\rho{}^r P_\sigma{}^s \partial_\mu L \\ &= -2e\,{}^*R^{*\rho\sigma rs}\varphi^m P_\rho{}^r P_\sigma{}^s L - 4e\,{}^*R^{*\rho\sigma mr}\varphi^s P_\rho{}^r P_\sigma{}^s L - \Phi(P^{-1})_m{}^\mu \partial_\mu L \doteq 0 \quad . \end{aligned} \qquad (A.1)$$

Here we have used the relationship $\Phi = \det(P_\mu{}^m)$, while $(P^{-1})_m{}^\mu P_\mu{}^n = \delta_m{}^n$.[6)] The last term is like the analogous term in the conventional TMT that yields the key equation $\partial_\mu L \doteq 0$ [1][2] even in the Lorentz-covariant formalism. However, the problem is in the first R-linear term in (A.1) that should be cancelled by a specific counter-term.

Such a counter-term can be produced by new lagrangian terms of the type $R\varphi^2 P^2 L$. However, such new terms themselves in turn generate other new terms of the type $R^2\varphi^3 L$, because of the term $R(DP)\varphi^2 L$ generated out of the partial integration of D_μ in $R\varphi^2[D(\delta\varphi)]PL$. To cancel these secondary new terms, we need additional new lagrangian terms of the type $R^2\varphi^4 L$. Fortunately, these two sorts of new terms turn out to be enough to cancel all unwanted terms with curvature tensors. The terms containing $\partial_\mu L$ do not pose any problem, because they serve only as 'correction' terms to the conventional TMT formulation [1][2] that also yields $\partial_\mu L \doteq 0$ as the φ^a-field equation.

After these considerations, the resulting candidate lagrangian is taken to be

$$\begin{aligned} \mathcal{L} = & + \Phi L + a_1\, e\, {}^*R^*_{\mu\nu mn} (\varphi^t)^2 P^{\mu m} P^{\nu n} L + a_2\, e\, {}^*R^*_{\mu\nu mn} \varphi^m \varphi_r P^{\mu r} P^{\nu n} L \\ & + b_1\, e\, {}^*R^*_{\mu\nu mn} R^{\mu\nu n u} (\varphi^r)^2 \varphi^m \varphi_u L + b_2\, e\, {}^*R^*_{\mu\nu mn} R^{\mu\nu mn} (\varphi^r)^2 (\varphi^s)^2 L \ , \end{aligned} \tag{A.2}$$

where a_1, a_2, b_1, and b_2 are unknown real coefficients. Even though the two terms with b_1 and b_2 look independent of each other, they are actually proportional to each other, as the lemma (A.8) will show. For this reason, we put $b_1 = 0$ for simplicity sake.

By varying φ^m in $\mathcal{L}$ in (A.2), we generate three different categories of terms: (i) $R\varphi^2 P^2$-terms, (ii) $R^2\varphi^3 L$-terms, and (iii) ∂L-terms. We now study these terms in turn:

(i) $R\varphi^2 P^2 L$-Terms: These terms arise from the ΦL, a_1 and a_2-terms. The cancellation condition we get is

$$+ 2a_1 - a_2 = -\frac{1}{2} \ . \tag{A.3}$$

(ii) $R^2\varphi^3 L$-Terms: There actually is only one term of this sort, due to the lemma (A.8) shown below. The coefficient of such terms are combined to yield another condition, *i.e.,*

$$+ \frac{1}{4} a_1 + \frac{1}{8} a_2 + 4b_2 = 0 \ . \tag{A.4}$$

---

6) We have implicitly assumed the existence of the inverse matrix P^{-1}, as a natural assumption.

(iii) ∂L-Terms: There arises no condition on the unknown coefficients a_1, a_2 and b_2, because they are supposed to result in $L = \text{const}$, as shown in (3.2) in a way parallel to the conventional TMT [1][2].

In order to reach (3.2), however, we have to confirm the existence of the inverse matrix $(M^{-1})_m{}^\mu$. The existence of the inverse matrix $(M^{-1})_m{}^\mu$ is confirmed as follows. If we regard all the R-dependent terms as 'correction terms' to the very first leading term $(M_0)_m{}^\mu \equiv -e^{-1}\Phi(P^{-1})_m{}^\mu$ which has its inverse matrix $(M_0^{-1})_\mu{}^m = -e\,\Phi^{-1}P_\mu{}^m$, then it is legitimate to assume the existence of the inverse matrix M^{-1} such that $(M^{-1})_\mu{}^m M_m{}^\nu = \delta_\mu{}^\nu$. To be more rigorous, suppose the matrix M is expressed as

$$M = M_0 + X = M_0\,(I + M_0^{-1}X) \equiv M_0\,(I+Y) \quad . \tag{A.5}$$

Here $Y \equiv M_0^{-1}X$, and M_0 is guaranteed have its inverse matrix M_0^{-1}. Then the inverse matrix of the total M is formally

$$M^{-1} = (I+Y)^{-1}\,M_0^{-1} = (I - Y + Y^2 - \cdots)\,M_0^{-1} \quad . \tag{A.6}$$

Hence, the multiplication of (3.1a) by $(M^{-1})_\nu{}^m$ yields the key equation $L = \text{const.}$ (3.2), as in the conventional TMT [1][2].

There are only two conditions (A.3) and (A.4) for three unknown coefficients a_1, a_2 and b_2, so that one degree of freedom is left over. We can choose a_2 to be arbitrary, and set $a_2 \equiv \alpha$ $(\alpha \in \mathbb{R})$, so that the solutions are

$$a_1 = \frac{1}{2}\,\alpha - \frac{1}{4} \quad , \quad a_2 = \alpha \quad , \quad b_2 = -\frac{1}{16}\,\alpha + \frac{1}{64} \qquad (\alpha \in \mathbb{R} : \ \text{arbitrary}) \quad . \tag{A.7}$$

Substituting these into (A.2), we get our aforementioned result (2.1). The simplest choice is to put $\alpha = 0$, so that only three simplest terms (1st, 2nd and 4-th terms) survive in (2.1).

Amongst the $R^2\varphi^4 L$-type lagrangian terms, there is an identity[7)]

$${}^*R^*_{\mu\nu mn} R^{\mu\nu nu} (\varphi^t)^2 \varphi^m \varphi_u \equiv -\frac{1}{4}\,{}^*R^*_{\mu\nu rs} R^{\mu\nu rs} (\varphi^t)^2 (\varphi^u)^2 \quad . \tag{A.8}$$

This identity gives the latter term in (A.8) as the only possible independent term of the $R^2\varphi^4 L$-type in the lagrangian.

[7)] Actually, this identity is the sufficient condition of the next lemma (A.9).

Similarly, among $R^2\varphi^3 L$-terms in $\delta\mathcal{L}/\delta\varphi^m$, there are two non-trivial identities

$$
{}^*\!R^*_{\mu\nu m n} R^{\mu\nu n u} (\varphi^t)^2 \varphi_u \equiv {}^*\!R^*_{\mu\nu t n} R^{\mu\nu n u} \varphi_m \varphi^t \varphi_u \equiv -\frac{1}{4}\, {}^*\!R^*_{\mu\nu r s} R^{\mu\nu r s} \varphi_m (\varphi^t)^2 \quad . \tag{A.9}
$$

The confirmation of (A.9) goes as follows:

$$
\begin{aligned}
[\,\text{LHS of (A.9)}\,] \equiv e\, {}^*\!R^*_{\mu\nu m n} R^{\mu\nu n u} (\varphi^t)^2 \varphi_u &= -\frac{1}{4}\, \epsilon_{m n r s} \epsilon^{n u v w} R^{\mu\nu r s}\, {}^*\!R^*_{\mu\nu v w} (\varphi^t)^2 \varphi_u \\
&= -\,\frac{3}{2}\, e\, R_{\mu\nu}{}^{r s}\, {}^*\!R^{*\mu\nu}{}_{[m r|} (\varphi^t)^2 \varphi_{|s]} \\
&= -\, e\, R_{\mu\nu}{}^{r s}\, {}^*\!R^{*\mu\nu}{}_{m r} (\varphi^t)^2 \varphi_s - \frac{1}{2}\, e\, R_{\mu\nu}{}^{r s}\, {}^*\!R^{*\mu\nu}{}_{r s} (\varphi^t)^2 \varphi_m \\
&= -\, e\, {}^*\!R^*_{\mu\nu m r} R^{\mu\nu r s} (\varphi^t)^2 \varphi_s - \frac{1}{2}\, e\, {}^*\!R^*{}_{\mu\nu r s} R^{\mu\nu r s} \varphi_m (\varphi^t)^2 \\
&= -\,[\,\text{LHS of (A.9)}\,] + 2\,[\,\text{RHS of (A.9)}\,] \quad .
\end{aligned} \tag{A.10}
$$

As desired, this yields $[\,\text{LHS of (A.9)}\,] = [\,\text{RHS of (A.9)}\,]$. Essentially, this proof follows from the duality status of the two R's.

Similar confirmation holds with the middle-hand side (MHS), *i.e.,* $[\,\text{MHS of (A.9)}\,] = [\,\text{RHS of (A.9)}\,]$, but since the pattern is the same, its confirmation is skipped here. The lemma (A.8) can be interpreted as a corollary of (A.9), if the latter is multiplied by φ^m. The identities in (A.9) drastically simply the computation for the $R^2\varphi^3 L$-terms arising in the φ-field equation.

We mention the global scale invariance of our action. Even though our new system with local Lorentz invariance is more sophisticated than the conventional TMT [1][2], our system still respects global scale invariance under

$$
\begin{aligned}
& e_\mu{}^m \rightarrow e^\Lambda e_\mu{}^m \;, \quad \varphi^m \rightarrow e^\Lambda \varphi^m \;, \quad e \rightarrow e^{4\Lambda} e \;, \quad P_\mu{}^m \rightarrow e^\Lambda P_\mu{}^m \;, \quad \Phi \rightarrow e^{4\Lambda}\Phi \;, \\
& R_{\mu\nu}{}^{mn} \rightarrow R_{\mu\nu}{}^{mn} \;, \quad {}^*\!R^*_{\mu\nu}{}^{mn} \rightarrow {}^*\!R^*_{\mu\nu}{}^{mn} \;, \quad L \rightarrow e^{-4\Lambda} L \;,
\end{aligned} \tag{A.11}
$$

where Λ is a finite real-number *global*-scale transformation parameter: $\partial_\mu \Lambda = 0$. The matter lagrangian L is required to transform such that the lagrangian term ΦL is invariant. We can re-express (A.11) in terms of scaling weights as $w(e_\mu{}^m) = +1$, $w(\varphi^m) = +1$, $w(e) = +4$, $w(\Phi) = +4$, $w(R_{\mu\nu}{}^{mn}) = 0$, $w({}^*\!R^*_{\mu\nu}{}^{mn}) = 0$, $w(L) = -4$. Compared with the original TMT [1][2], the difference in our transformation rule (A.11) is that φ^m transforms with a common factor $e^{\Lambda}\varphi^m$ instead of the matrix form $\Lambda^m{}_n\varphi^n$. This is because in our formulation the index ^{m} on both $e_\mu{}^m$ and φ^m transform in a parallel way under global scale transformations.

These lead to, *e.g.*, $w(P^{\mu m}) = w(g^{\mu\nu}P_\nu{}^m) = -2+1 = -1$. It is then straightforward to see that each term in (2.1) is invariant under (A.11), *e.g.*, $w[\, e\ {}^*R^*_{\mu\nu mn}R^{\mu\nu mn}(\varphi^r)^2(\varphi^s)^2L\,] = +4+0-2-2+1\times2+1\times2-4=0$. Note that the second, third and fourth terms in (2.1) all have the common factor e in front.

It is analogous to the conventional TMT [1][2] that our field equation $\partial_\mu L \doteq 0$ contains the solution $L=$ const. with the global-scale symmetry breaking, due to $w(L)=-4$. In other words, our formulation of TMT induces the breaking of global-scale invariance, as in the conventional TMT [1][2].

The sophisticated global scale-invariance structure of the new terms in (2.1) provides additional evidence for the non-trivial feature of our new formulation with the Lorentz-covariant four-vector φ^m.

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