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Phys. Rev. D **87**, 016005 — Published 9 January 2013

DOI: 10.1103/PhysRevD.87.016005

$B_s \to D_s K$ as a Probe of CPT Violation

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We discuss some possible signals of CPT violation in the B_s system that may be probed at the Large Hadron Collider (LHC). We show how one can construct combinations of observables coming from tagged and untagged decay rates of $B_s \to D_s^{\pm} K^{\mp}$ that can unambiguously differentiate between CPT violating and CPT conserving new physics (NP) models contributing in $B_s^0 - \bar{B}_s^0$ mixing. We choose this particular mode as an illustrative example for two reasons: (i) In the Standard Model, there is only one decay amplitude, so it is easier to untangle any new physics; (ii) B_s being a neutral meson, it is possible to unambiguously identify any sign of CPT violation that occurs only in mixing but not in decay. We define an observable which is useful to extract the CPT violating parameter in B_s decay, and also discuss how far the results are applicable even if CPT violation is present in both mixing and decay.

PACS numbers: 11.30.Er, 14.40.Nd

Keywords: CPT violation, B_s meson, LHCb

I. INTRODUCTION

The combined discrete symmetry CPT, taken in any order, is an exact symmetry of any axiomatic quantum field theory (QFT). CPT conservation is indeed supported by the experiments; all tests for CPT violation (CPTV) that have been done so far [1] have yielded null results, consistent with no CPTV, and very stringent limits on CPTV parameters have been obtained [2] in different systems. The only possible exception is the apparent mass difference between the top quark and its antiparticle as obtained by the CDF collaboration in Fermilab [3]:

$$m_t - m_{\bar{t}} = -3.3 \pm 1.7 \text{ GeV}\,, \tag{1}$$

but other experiments got results which are consistent with zero, and so is the world average [4]:

$$m_t - m_{\bar{t}} = [-0.44 \pm 0.46 \text{ (stat.)} \pm 0.27 \text{ (syst.)}] \text{ GeV}\,. \tag{2}$$

What, then, should be the motivation to investigate the possibility of CPTV particularly in the B system? There are three main reasons:

- Any symmetry which is supposed to be exact ought to be questioned. We may get a surprise, just like the discovery of CP violation. CPTV may very well be flavor-sensitive, and so the constraints obtained from the K system [5] may not be applicable to the B systems. There is still the possibility of a sizable CPTV in the B systems. If there is some tension between the data and the Standard Model (SM) expectations, we should ask whether this is due to CPTV, or a more canonical CPT conserving new physics (NP).
- For the bound systems like mesons, asymptotic states, whose existence is a prerequisite for the CPT theorem, are not uniquely defined [6]. Quarks and gluons are bound inside the hadrons and cannot be considered, in a true sense, asymptotic states.
- Some nonlocal and nonrenormalizable string-theoretic effects may appear at the Planck scale with a possible ramification at the weak scale through the effective Hamiltonian [7]. CPTV through such non-local interacting QFT does not necessarily lead to the violation of Lorentz symmetry [8].

Recently the issue of CPTV has received more attention due to the growing phenomenological importance of CPT violating scenarios in neutrino physics and in cosmology [9]. It is also necessary to find some observables that will clearly discriminate CPT violating signals from CPT conserving ones. A comprehensive study of CPTV in the neutral K meson system, with a formulation that is closely analogous to that in the B system, may be found in [10].

CPTV in the B systems, and its possible signatures, have been already investigated by several authors [11, 12]. It was shown that the lifetime difference of the two mass eigenstates, or the direct CP asymmetries and semileptonic observables, may be affected by such new physics. The experimental limits are set by both BaBar, who looked for diurnal variations of CP-violating observables [13], and Belle, who looked for lifetime difference of B_d mass eigenstates [14]. This makes it worthwhile to look for possible CPTV effects in the B_s system (by B_s we generically mean both B_s^0 and $\bar{B}_s^0$ mesons).

In this paper, we would like to investigate the signatures of CPT violation in the B_s system, both in $B_s^0 - \bar{B}_s^0$ mixing and in B_s decays. We would like to emphasize that this is a model-independent approach in the sense that we do not specify any definite model that might lead to CPT violation; in fact, as far as we know, all studies on CPT violation are based on some phenomenological Lagrangian to start with.

As an illustrative example, we consider the nonleptonic $B_s^0(\bar{B}_s^0) \to D_s^+ K^-$ and $B_s^0(\bar{B}_s^0) \to D_s^- K^+$ decays. The $\bar{B}_s^0$ decays are mediated by color-allowed tree-level transitions $b \to u\bar{c}s$ and $b \to c\bar{u}s$. These are single-amplitude processes in the SM, so that any non-trivial contribution beyond the SM expectations, like direct CP asymmetry, is a clear signal of NP. This set of channels is also of interest as in the SM, both the amplitudes are of same order, $\mathcal{O}(\lambda^3)$ in the standard Wolfenstein parametrization of the CKM matrix (so that the event rates are comparable), and same final states can be reached both from B_s^0 and $\bar{B}_s^0$. The importance of such modes to unveil any NP has already been emphasized; *e.g.*, see [15–18]. The decay was first observed by the CDF and the Belle collaborations [19, 20], and recently the LHCb collaboration has measured the branching ratio to be [21]

$$\mathrm{Br}(B_s \to D_s^\mp K^\pm) = (1.90 \pm 0.23) \times 10^{-4} \tag{3}$$

where the errors have been added in quadrature. We also note that flavor-specific NP in these channels is relatively unconstrained [22]. LHCb has also measured several time-dependent CP violating observables in $B_s \to D_s^\mp K^\pm$ using flavor-tagged and flavor-untagged observables [23].

Here we do a more general analysis considering both the CPT violating and CPT conserving NP contributions to $B_s^0 - \bar{B}_s^0$ mixing. We show how one can construct combinations of observables coming from tagged and untagged decay rates that can unambiguously differentiate between CPT violating and CPT conserving NP models. On the other hand, if there is some CPTV contribution only to B_s decays, it might be difficult to differentiate it from CPT conserving NP in this approach. We define an observable which is useful to extract the CPT violating parameter in decay.

We will consider both these cases separately: first, when CPTV (or CPT conserving NP) is present only in the operators responsible for decay but not in those responsible for the mixing; and second, when the same is present also in the $B_s^0 - \bar{B}_s^0$ mixing amplitude. As we will show explicitly, the extraction of CPTV in mixing is independent of the CPTV in decay and any other CPT conserving NP either in decay or mixing.

The first possibility of NP (including CPT violation) only in decay can arise if the NP operators are strongly flavor-dependent, like those in R-parity violating supersymmetry, or leptoquark models. As we are considering final states that can be accessed both from B_s^0 and $\bar{B}_s^0$, any such NP will necessarily contribute in $B_s^0 - \bar{B}_s^0$ mixing, in particular to its absorptive part, and will change the decay width difference $\Delta\Gamma_s$. Apart from the short-distance contributions to the absorptive part, there can be non-negligible long-distance effects too, coming from mesonic intermediate states [25]. However, the accuracy of the present data on $\Delta\Gamma_s$, the lifetime difference of two B_s mass eigenstates, is relatively weak. The most accurate result comes from the LHCb collaboration [26]: $\Delta\Gamma_s/\Gamma_s = 0.176 \pm 0.028$. Even the SM prediction [27] has a large uncertainty. Thus, as a first approximation, one can consider such NP effects only in decay and not in mixing, where it is in all probability subleading.

For the second case, one can construct several observables from the time-dependent tagged and untagged decay rates, and some of them are identically zero if there is no CPTV in mixing, irrespective of whether there is any CPTV in decay, or some CPT conserving NP.

The Belle Collaboration [14] places limits on the CPTV parameters in mixing, but no such limits exist for CPTV in decay. Also, the Belle limits are valid for the B_d system, but one can expect similar numbers for the B_s system too, even if CPTV is flavor-dependent. Like the experimental tests on CP-violation, various independent cross-checks on CPTV are also essential. Needless to say, one can play the same game with decays like $B_s \to D^0\phi$ and $B_s \to \bar{D}^0\phi$, and can form more observables (although not independent of the original ones) out of the CP-eigenstates of D^0 and $\bar{D}^0$ in the final state.

The paper is arranged as follows. In the next section, we outline the necessary formalism for CPTV in decay, vis-a-vis that for the SM as well as CPT conserving NP. We also construct observables that may indicate the presence of CPT violation (or any NP in general). In Section III, we do the same for CPT violation in $B_s^0 - \bar{B}_s^0$ mixing, including the construction of observables that can differentiate CPTV and CPT conserving NP. In Section IV, we summarize and conclude.

II. CPT VIOLATION IN DECAY

A. $B_s^0 - \bar{B}_s^0$ mixing and $B_s \to D_s^\pm K^\mp$ in the SM

The $B_s^0 - \bar{B}_s^0$ mixing is controlled by the off-diagonal term $H_{12} = M_{12} - (i/2)\Gamma_{12}$ of the 2×2 Hamiltonian matrix, with the mass difference between two mass eigenstates B_H and B_L given by (in the limit $|\Gamma_{12}| \ll |M_{12}|$)

$$\Delta M_s \equiv M_{sH} - M_{sL} \approx 2|M_{12}|\,, \tag{4}$$

and the width difference by

$$\Delta\Gamma_s \equiv \Gamma_{sL} - \Gamma_{sH} \approx 2|\Gamma_{12}|\cos\phi_s\,, \tag{5}$$

where $\phi_s \equiv \arg(-M_{12}/\Gamma_{12})$. CPT conservation ensures $H_{11} = H_{22}$.

The eigenstates are defined as

$$|B_{H(L)}\rangle = p|B_s^0\rangle + (-)q|\bar{B}_s^0\rangle\,, \tag{6}$$

where $|p|^2 + |q|^2 = 1$ is the normalization, and one defines

$$\alpha \equiv q/p = \exp(-2\beta_s) \tag{7}$$

where $2\beta_s$ is the mixing phase of the $B_s^0 - \bar{B}_s^0$ box diagram.

For the single-amplitude decays $B_s \to D_s^\pm K^\mp$, the amplitudes are of the form

$$\begin{aligned} A(B_s^0 \to D_s^+ K^-) &= T_1 e^{i\gamma}\,, \qquad A(B_s^0 \to D_s^- K^+) = T_2\,, \\ A(\bar{B}_s^0 \to D_s^+ K^-) &= T_2\,, \qquad A(\bar{B}_s^0 \to D_s^- K^+) = T_1 e^{-i\gamma}\,, \end{aligned} \tag{8}$$

where T_1 and T_2 are real amplitudes times the strong phase, which we parametrize as

$$\arg\left(\frac{T_1}{T_2}\right) = \Delta\,, \tag{9}$$

and $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, so that to a very good approximation, $V_{ub} \approx |V_{ub}|\exp(-i\gamma)$. The quantity $\xi_f \equiv \alpha\bar{A}_f/A_f$, where $A_f \equiv A(B_s^0 \to D_s^+ K^-)$ and $\bar{A}_f \equiv A(\bar{B}_s^0 \to D_s^+ K^-)$, carries a weak phase of $-(2\beta_s + \gamma)$.

Let us define, following [15],

$$\begin{aligned} \langle \mathrm{Br}(B_s \to D_s^+ K^-)\rangle &= \mathrm{Br}(B_s^0 \to D_s^+ K^-) + \mathrm{Br}(\bar{B}_s^0 \to D_s^+ K^-)\,, \\ \langle \mathrm{Br}(B_s \to D_s^- K^+)\rangle &= \mathrm{Br}(B_s^0 \to D_s^- K^+) + \mathrm{Br}(\bar{B}_s^0 \to D_s^- K^+)\,, \end{aligned} \tag{10}$$

so that these untagged rates are the same in the SM, even though a future measurement of the time-dependent branching fractions at the LHCb may show nonzero CP violation.

B. CPT violation in B_s decay

In order to take into account CPTV in decay, we parametrize various transition amplitudes for the decay $B_s \to D_s^\pm K^\mp$ as [28, 29]

$$\begin{aligned} A(B_s^0 \to D_s^+ K^-) &= T_1 e^{i\gamma}(1 - y_f)\,, \qquad A(B_s^0 \to D_s^- K^+) = T_2\left(1 + y_f^*\right)\,, \\ A(\bar{B}_s^0 \to D_s^+ K^-) &= T_2(1 - y_f)\,, \qquad A(\bar{B}_s^0 \to D_s^- K^+) = T_1 e^{-i\gamma}\left(1 + y_f^*\right)\,, \end{aligned} \tag{11}$$

where CPT violation (in decay) is parametrized by the complex parameter y_f, and y_f is real if T is conserved. The CPT violation is proportional to the difference $A(B_s^0 \to D_s^+ K^-)^* - A(\bar{B}_s^0 \to D_s^- K^+)$ or $A(\bar{B}_s^0 \to D_s^+ K^-)^* - A(B_s^0 \to D_s^- K^+)$.

We define the complete set of four relevant amplitudes, with $|f\rangle \equiv |D_s^+ K^-\rangle$ and $|\bar{f}\rangle \equiv |D_s^- K^+\rangle$,

$$\begin{aligned} A_f &= \langle f|H|B_s^0\rangle\,, \quad A_{\bar{f}} = \langle \bar{f}|H|B_s^0\rangle\,, \\ \bar{A}_f &= \langle f|H|\bar{B}_s^0\rangle\,, \quad \bar{A}_{\bar{f}} = \langle \bar{f}|H|\bar{B}_s^0\rangle\,, \end{aligned} \tag{12}$$

so that the ratios

$$\xi_f = \alpha \bar{A}_f/A_f\,, \quad \xi_{\bar{f}} = \alpha \bar{A}_{\bar{f}}/A_{\bar{f}}\,, \tag{13}$$

are independent of y_f; the CPTV effect in the decays cancels in the ratio. We also have $|\xi_f| = 1/|\xi_{\bar{f}}|$ and $\arg(\xi_{f(\bar{f})}) = -(2\beta_s + \gamma + (-)\Delta)$ where Δ is defined in eq. (9).

From Eq. (11) we get

$$\begin{aligned} |A(B_s^0 \to D_s^+ K^-)|^2 + |A(\bar{B}_s^0 \to D_s^+ K^-)|^2 &= \left(|T_1|^2 + |T_2|^2\right) |1 - y_f|^2\,, \\ |A(B_s^0 \to D_s^- K^+)|^2 + |A(\bar{B}_s^0 \to D_s^- K^+)|^2 &= \left(|T_1|^2 + |T_2|^2\right) \left|1 + y_f^*\right|^2\,. \end{aligned} \tag{14}$$

Thus we can define an asymmetry

$$A_{br}^{CPT} = \frac{\langle \mathrm{Br}(B_s \to D_s^+ K^-)\rangle - \langle \mathrm{Br}(B_s \to D_s^- K^+)\rangle}{\langle \mathrm{Br}(B_s \to D_s^+ K^-)\rangle + \langle \mathrm{Br}(B_s \to D_s^- K^+)\rangle} = -2\,\frac{\mathrm{Re}(y_f)}{1 + |y_f|^2} \approx -2\,\mathrm{Re}(y_f)\,,\ \text{for } |y_f|^2 \ll 1 \tag{15}$$

We have already seen that this asymmetry is zero in the SM. Using Eq. (15), the real part of the CPTV parameter y_f can be directly probed from the difference of the untagged rates (as the initial state B_s flavor is summed over) $\mathrm{Br}(B_s \to D_s^+ K^-)$ and $\mathrm{Br}(B_s \to D_s^- K^+)$.

One can have a rough idea of the LHCb reach in measuring $\mathrm{Re}(y_f)$. With 1 fb^{-1} of integrated luminosity, LHCb has obtained 1390 ± 98 events [23]. With full LHCb upgrade to an integrated luminosity of 50 fb^{-1}, total number of events should go up by a factor of about 200, as a twofold gain in the yield is expected when the LHC reaches $\sqrt{s} = 13$-14 TeV (as the cross section of $pp \to b\bar{b}X$ scales almost linearly with $\sqrt{s}$), and another twofold gain is expected in the trigger efficiency when the detector is upgraded. These 0.28 million events should be roughly equally divided between $D_s^+ K^-$ and $D_s^- K^+$. The advantage is that there is no need to tag the flavor of the initial B_s. The statistical fluctuation for each channel is about 375, and detection of CPT violation over such fluctuations results in a sensitivity of $375/140000 \approx 0.0027$ for $\mathrm{Re}(y_f)$. Note that LHCb already has a plan to measure CPT violation in the decay $B^0 \to J/\psi[\to \pi^\mp \mu^\pm \nu(\bar{\nu})]K^0$ [24]. However, in this estimate we have only concerned ourselves with the statistical reach; we leave it to the experimentalists to address the systematic errors.

Let us compare this to a case where there is no CPT violation, but some CPT conserving NP is present which contributes to either $b \to u\bar{c}s$ or $b \to c\bar{u}s$ transitions, or maybe both. If this NP leads to observable CP violating effects, we can write the various amplitudes for the $B_s \to D_s^\pm K^\mp$ decays as

$$\begin{aligned} &A(B_s^0 \to D_s^+ K^-) = T_1 e^{i\gamma}\left(1 + a\ e^{i(\theta - \gamma + \sigma)}\right)\,, \quad A(B_s^0 \to D_s^- K^+) = T_2\left(1 + a'\ e^{i(\theta' + \sigma')}\right)\,, \\ &A(\bar{B}_s^0 \to D_s^+ K^-) = T_2\left(1 + a'\ e^{-i(\theta' - \sigma')}\right)\,, \quad A(\bar{B}_s^0 \to D_s^- K^+) = T_1 e^{-i\gamma}\left(1 + a\ e^{-i(\theta - \gamma - \sigma)}\right)\,. \end{aligned} \tag{16}$$

The amplitudes, obviously, are related by CP conjugation. The NP is parametrized by the (relative) amplitudes a, a', the new weak phases θ, θ', and the new strong phase differences σ, σ'. Therefore, the asymmetry defined in Eq. 15 is given by

$$A_{br}^{NP} = -2\frac{a\,|T_1|^2 \sin(\theta - \gamma)\sin\sigma + a'\,|T_2|^2 \sin\theta' \sin\sigma'}{|T_1|^2\left(1 + a^2 + 2\,a\cos(\theta - \gamma)\cos\sigma\right) + |T_2|^2\left(1 + a'^2 + 2\,a'\cos\theta'\cos\sigma'\right)}. \tag{17}$$

Hence, a nonzero value of A_{br} could be due to either CPTV or CPT conserving NP (which, perhaps, is flavor-dependent, and definitely not of the minimal flavor violation type). As both the decays are color-allowed, one can even invoke the color-transparency argument [30] to claim that all strong phases are small; but CPTV effects are not expected to be large either.

Eq. (15) is in general true for all decays which are either (i) single-amplitude in the SM, be it tree or penguin, or (ii) multi-amplitude in the SM but with one amplitude highly dominant over the others. Single-amplitude decays are preferred simply because any nonzero asymmetry as in Eqs.~(15) or (17) can be unambiguously correlated with NP. The same observable A_{br}^{CPT} can be defined for charged B decays, or even D and K decays. However, in all cases, CPT conserving (but necessarily CP violating) NP can always mimic the asymmetry, unless there are strong motivations for the corresponding amplitudes to be highly subdominant, or the strong phase difference between the two amplitudes to be zero or vanishingly small.

On the other hand, if there is CPT violation in mixing too, this formalism does not hold, because the definition of the mass eigenstates also contains CPT violating parameters (see later). In that case, we suggest using single-amplitude charged B meson decay modes, like $B^+ \to D^0 K^+$ and $B^+ \to \overline{D}^0 K^+$.

If there is no other CPT conserving NP, but the $B_s^0 - \bar{B}_s^0$ mixing matrix has CPTV built in, the asymmetry is still nonzero, as the individual branching fractions are functions of the CPTV parameter δ (see below) in the mixing matrix [12].

III. CPT VIOLATION IN MIXING

This subsection closely follows the formulation developed in [12], but let us quote some relevant expressions for completeness. CPT violation in the Hamiltonian matrix is introduced through the complex parameter δ:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}}\,, \tag{18}$$

so that the Hamiltonian matrix looks like

$$\mathcal{H} = \left[\begin{pmatrix} M_0 - \mathrm{Re}(\delta') & M_{12} \\ M_{12}^* & M_0 + \mathrm{Re}(\delta') \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_0 + 2\mathrm{Im}(\delta') & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 - 2\mathrm{Im}(\delta') \end{pmatrix} \right]\,, \tag{19}$$

where δ' is defined by

$$\delta = \frac{2\delta'}{\sqrt{H_{12}H_{21}}}\,. \tag{20}$$

One could even relax the assumption of $H_{21} = H_{12}^*$. However, there are two points that one must note. First, the effect of expressing $H_{12} = h_{12} + \bar{\delta}$, $H_{21} = h_{12}^* - \bar{\delta}$ appears as $\bar{\delta}^2$ in $\sqrt{H_{12}H_{21}}$, the relevant expression in Eq. (18), and can be neglected if we assume $\bar{\delta}$ to be small. The second point, which is more important, is that CPT conservation constrains only the diagonal elements and puts no constraint whatsoever on the off-diagonal elements. It has been shown in [10] that $H_{12} \neq H_{21}^*$ leads to T violation, and only $H_{11} \neq H_{22}$ leads to unambiguous CPT violation. Thus, we will focus on the parametrization used in Eqs. (18) and (19) to discuss the effects of CPT violation.

In the review on CPT violation in [1], the authors have used a formalism which is close to ours. While their treatment is for the K_S-K_L pair, this can be generalized to any neutral meson system. The mass eigenstates are defined as

$$|K_S(K_L)\rangle = \frac{1}{\sqrt{2(1+|\epsilon_{s(L)}|^2)}} \left[(1+\epsilon_{S(L)})|K^0\rangle + (1-\epsilon_{S(L)})|\overline{K}^0\rangle \right] \tag{21}$$

where

$$\begin{aligned} \epsilon_{S(L)} &= \frac{-i\mathrm{Im}(M_{12}) - \frac{1}{2}\mathrm{Im}(\Gamma_{12}) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22}) \right]}{M_L - M_S + i(\Gamma_S - \Gamma_L)/2} \\ &\equiv \epsilon \pm \tilde{\delta}\,. \end{aligned} \tag{22}$$

Note that $\tilde{\delta}$ and δ are not the same, but related; both parametrize CPT violation. On the other hand, $\epsilon_{S(L)}$ is not truely a CPT conserving quantity, as the expression contains the mass and width differences of the two eigenstates, and both depend on the CPT violating parameter δ that we have used here.

The Belle collaboration [14] recently put stringent limits on the real and imaginary parts of δ,

$$\mathrm{Re}(\delta_d) = (-3.8 \pm 9.9) \times 10^{-2}\,, \quad \mathrm{Im}(\delta_d) = (1.14 \pm 0.93) \times 10^{-2}\,, \tag{23}$$

where we have added the errors in quadrature, and used the straightforward translation valid for small δ, *viz.*, $\delta = -2z$ (the subscript emphasizes that these results are for the B_d system). The CPT violating parameter z is defined as

$$|B_{L(H)}\rangle = p\sqrt{1-(+)z}|B^0\rangle + (-)q\sqrt{1+(-)z}|\overline{B}^0\rangle\,. \tag{24}$$

We can see that within the error bars data are consistent with no CPTV case i.e $\mathrm{Re}(\delta_d) = \mathrm{Im}(\delta_d) = 0$. However, more precise measurements are important and essential. In any case it is safe to assume $|\delta| \ll 1$, even for the B_s system. In ΔM_s and $\Delta\Gamma_s$ the CPT-violating effects are quadratic in δ and hence negligible.

We can write

$$|B_H\rangle = p_1|B_s^0\rangle + q_1|\bar{B}_s^0\rangle\,, \quad |B_L\rangle = p_2|B_s^0\rangle - q_2|\bar{B}_s^0\rangle\,. \tag{25}$$

with the normalization conditions $|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1$, so that with CPT violation, $p_1 \neq p_2$ and $q_1 \neq q_2$. The time evolutions of B_H and B_L are controlled by $\lambda_1 \equiv m_1 - i\Gamma_1/2$ and $\lambda_2 \equiv m_2 - i\Gamma_2/2$ respectively. We also use

$$\Delta M_s = m_1 - m_2\,, \quad \Delta\Gamma_s = \Gamma_2 - \Gamma_1\,. \tag{26}$$

Let us define,

$$y = \sqrt{1 + \frac{\delta^2}{4}}\,; \quad \eta_1 \equiv \frac{q_1}{p_1} = \left(y + \frac{\delta}{2}\right)\alpha\,; \quad \eta_2 \equiv \frac{q_2}{p_2} = \left(y - \frac{\delta}{2}\right)\alpha\,; \quad \omega = \frac{\eta_1}{\eta_2}\,, \tag{27}$$

where $\alpha = \sqrt{H_{21}/H_{12}}$. For $|\delta| \ll 1$, we can approximate y with unity.

The time-dependent flavor eigenstates are given by

$$\begin{aligned} |B_s^0(t)\rangle &= h_+(t)|B_s^0\rangle + \eta_1 h_-(t)|\bar{B}_s^0\rangle \\ |\bar{B}_s^0(t)\rangle &= \frac{h_-(t)}{\eta_2}|B_s^0\rangle + \bar{h}_+(t)|\bar{B}_s^0\rangle\,, \end{aligned} \tag{28}$$

where

$$\begin{aligned} h_-(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right)\,, \\ h_+(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}\right)\,, \\ \bar{h}_+(t) &= \frac{1}{(1+\omega)}\left(\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}\right)\,. \end{aligned} \tag{29}$$

and we refer the reader to [12] for detailed expressions. Note that in the absence of CPTV, $\eta_1 = \eta_2$, $\omega = 1$, and hence $h_+(t) = \bar{h}_+(t)$. In the limit $|\delta| \ll 1$, $\omega \approx 1 + \delta$.

With our convention of $|f\rangle \equiv |D_s^+ K^-\rangle$ and $|\bar{f}\rangle \equiv |D_s^- K^+\rangle$, where both the states are directly accessible to B_s^0 and $\bar{B}_s^0$, the time dependent decay rates are [12]

$$\begin{aligned} \Gamma(B_s^0(t) \to f) &= \left[|h_+(t)|^2 + |\xi_{f_1}|^2|h_-(t)|^2 + 2\mathrm{Re}\left(\xi_{f_1} h_-(t) h_+^*(t)\right)\right]|A_f|^2\,, \\ \Gamma(\bar{B}_s^0(t) \to f) &= \left[|h_-(t)|^2 + |\xi_{f_2}|^2|\bar{h}_+(t)|^2 + 2\mathrm{Re}\left(\xi_{f_2} \bar{h}_+(t) h_-^*(t)\right)\right]\left|\frac{A_f}{\eta_2}\right|^2\,, \\ \Gamma(B_s^0(t) \to \bar{f}) &= \left[|h_+(t)|^2 + |\xi'_{f_1}|^2|h_-(t)|^2 + 2\mathrm{Re}\left(\xi'_{f_1} h_-(t) h_+^*(t)\right)\right]|A_{\bar{f}}|^2\,, \\ \Gamma(\bar{B}_s^0(t) \to \bar{f}) &= \left[|h_-(t)|^2 + |\xi'_{f_2}|^2|\bar{h}_+(t)|^2 + 2\mathrm{Re}\left(\xi'_{f_2} \bar{h}_+(t) h_-^*(t)\right)\right]\left|\frac{A_{\bar{f}}}{\eta_2}\right|^2\,, \end{aligned} \tag{30}$$

where,

$$\begin{aligned} \xi_{f_1} &= \eta_1 \frac{\bar{A}_f}{A_f} = \left(1 + \frac{\delta}{2}\right)\xi_f\,, \quad \xi_{f_2} = \eta_2 \frac{\bar{A}_f}{A_f} = \left(1 - \frac{\delta}{2}\right)\xi_f\,, \\ \xi'_{f_1} &= \eta_1 \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} = \left(1 + \frac{\delta}{2}\right)\xi_{\bar{f}}\,, \quad \xi'_{f_2} = \eta_2 \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} = \left(1 - \frac{\delta}{2}\right)\xi_{\bar{f}}\,. \end{aligned} \tag{31}$$

Dropping terms $\mathcal{O}(\delta^2)$ or higher, we get the following expressions for the tagged and untagged time-dependent decay rates:

$$\begin{aligned} \Gamma(B_s^0(t) \to f) - \Gamma(\bar{B}_s^0(t) \to f) &= [P_1 \sinh(\Delta\Gamma_s t/2) + Q_1 \cosh(\Delta\Gamma_s t/2) \\ &\quad + R_1 \cos(\Delta M_s t) + S_1 \sin(\Delta M_s t)]\, e^{-\Gamma_s t}|A_f|^2\,, \\ \Gamma(B_s^0(t) \to f) + \Gamma(\bar{B}_s^0(t) \to f) &= [P_2 \sinh(\Delta\Gamma_s t/2) + Q_2 \cosh(\Delta\Gamma_s t/2) \\ &\quad + R_2 \cos(\Delta M_s t) + S_2 \sin(\Delta M_s t)]\, e^{-\Gamma_s t}|A_f|^2\,, \end{aligned} \tag{32}$$

with

$$
\begin{aligned}
P_1 &= -\frac{1}{2}\, Re(\delta)\left(1+|\xi_f|^2\right)\,, \\
Q_1 &= -|\xi_f|\,\cos(\gamma+2\beta_s+\Delta)\,\mathrm{Re}(\delta)\,, \\
R_1 &= 1-|\xi_f|^2+|\xi_f|\,\cos(\gamma+2\beta_s+\Delta)\mathrm{Re}(\delta)\,, \\
S_1 &= 2\,|\xi_f|\,\sin(\gamma+2\beta_s+\Delta)-\frac{1}{2}\,\mathrm{Im}(\delta)\left(1+|\xi_f|^2\right)\,, \\
P_2 &= 2\,|\xi_f|\,\cos(\gamma+2\beta_s+\Delta)-\frac{1}{2}\,\mathrm{Re}(\delta)\left(1-|\xi_f|^2\right)\,, \\
Q_2 &= 1+|\xi_f|^2-|\xi_f|\,\sin(\gamma+2\beta_s+\Delta)\mathrm{Im}(\delta)\,, \\
R_2 &= |\xi_f|\,\sin(\gamma+2\beta_s+\Delta)Im(\delta)\,, \\
S_2 &= -\frac{1}{2}\,\mathrm{Im}(\delta)\left(1-|\xi_f|^2\right)\,.
\end{aligned}
\tag{33}
$$

It is clear from Eq. (33) that CPT violating effects in decay will not affect the determination of these 8 coefficients. Whatever the effects are, they will be lumped in the overall normalization $|A_f|^2$ and will not appear in the coefficients of the trigonometric and hyperbolic functions.

All the 8 coefficients can theoretically be extracted from a fit to the time-dependent decay rates, but admittedly the coefficients of the hyperbolic functions are harder to extract and need more statistics. The coefficients P_1 - S_1 are to be extracted from the tagged measurements, and P_2 - S_2 from untagged measurements. Note that whether or not any CPT-conserving NP is present, absence of CPT violation definitely means $\delta = 0$, so $P_1 = Q_1 = R_2 = S_2 = 0$. If any of these four observables are found to be nonzero, that is a sure signal of CPT violation. (While P_1 and S_2 depend only on δ, Q_1 and R_2 also have an implicit dependence on the $B_s^0 - \bar{B}_s^0$ mixing phase $2\beta_s$, which might depend on CPT conserving NP effects.) Therefore, if CPT is conserved, the tagged measurements are sensitive only to the trigonometric functions, and the untagged measurements only to the hyperbolic functions, but we urge our experimental colleagues to perform a complete fit.

If at least P_1 or S_2 be nonzero (maybe with nonzero Q_1 and R_2), one gets

$$
\mathrm{Im}(\delta) = -\frac{2S_2}{R_1+Q_1}\,, \qquad \mathrm{Re}(\delta) = -\frac{2P_1}{R_2+Q_2}, \tag{34}
$$

which is theoretically clean, *i.e.* free from hadronic uncertainties. The overall normalization can be extracted from the CP averaged branching fractions.

Even if the experiment is not sensitive enough to extract unambiguously nonzero values of P_1, Q_1, R_2, or S_2, one can still find signals of CPTV, from the fact that P_2, Q_2, R_1, and S_1 contain CPTV terms over and above CPT conserving but CP violating terms. For example, one can extract the following analogous quantities from the tagged and untagged $B_s \to \bar{f}$ decays:

$$
\begin{aligned}
\bar{P}_2 &= 2\,|\xi_f|\,\cos(\gamma+2\beta_s-\Delta)+\frac{1}{2}\,\mathrm{Re}(\delta)\left(1-|\xi_f|^2\right)\,, \\
\bar{Q}_2 &= 1+|\xi_f|^2-|\xi_f|\,\sin(\gamma+2\beta_s-\Delta)\mathrm{Im}(\delta)\,, \\
\bar{R}_1 &= -1+|\xi_f|^2+|\xi_f|\,\cos(\gamma+2\beta_s-\Delta)\mathrm{Re}(\delta)\,, \\
\bar{S}_1 &= 2\,|\xi_f|\,\sin(\gamma+2\beta_s-\Delta)-\frac{1}{2}\,\mathrm{Im}(\delta)\left(1+|\xi_f|^2\right)\,.
\end{aligned}
\tag{35}
$$

It is easy to derive Eq. (35) from Eq. (33). First, note that the relevant expressions contain $|A_{\bar{f}}|$ and $\xi_{\bar{f}}$. Eq. (35) follows when one substitutes $|\xi_{\bar{f}}| = 1/|\xi_f|$ and $|A_{\bar{f}}|^2/|\xi_f|^2 = |A_f|^2$. However, the strong phase changes sign because of the definitions of ξ_f and $\xi_{\bar{f}}$.

Therefore, from Eqs. (33) and (35) we can define observables which are only sensitive to the CPTV effect independent of any other NP effects in mixing,

$$
\frac{R_1+\bar{R}_1}{P_2+\bar{P}_2} = \frac{\mathrm{Re}(\delta)}{2}\,, \qquad \frac{Q_2-\bar{Q}_2}{S_1-\bar{S}_1} = \frac{\mathrm{Im}(\delta)}{2}\,. \tag{36}
$$

From Eq. (36) we note that it is possible to probe the CPTV parameter δ even in the presence of any other generic NP in mixing or decays (which modifies $2\beta_s$); the NP effects in mixing are cancelled in the ratio. In addition we note that the strong phase is also exactly cancelled in the ratio, hence the measurement of δ is free from hadronic uncertainties.

LHCb performs the decay profile fit assuming CPT invariance [23], so it is not easy to predict the reach for the new CPT violating parameters, or even the CPT conserving ones. For this we need a full fit, assuming the possibility of CPT violation. Still, one can try to have an estimate of the reach. As there exists no measurement on the CPT violating parameters, let us use the first relation of Eq. (36). The parameter R_1 (called C in [23]) has an error of about 56% right now; if the data sample increases by a factor of 200, this might come down to 4%. The same is true for $\bar{R}_1$, which should be measured independently (the central value, in the absence of CPT violation, should be equal and opposite to that of R_1). Thus the total uncertainty, added in quadrature, should be about 6%. Similarly, the uncertainty in the denominator should be about 6%, and is to be added in quadrature with the numerator. Thus, $\mathrm{Re}(\delta) \geq 0.16$ should be measurable using this relationship. Of course, we expect a much better reach with a full 4-parameter fit to each decay profile.

We reiterate that even if CPTV is present in decay, the conclusion that a nonzero value of any one of the four observables P_1, Q_1, R_2, or S_2 indicates CPTV in mixing remains valid. Consider the expressions for the tagged and untagged decay rates, Eq. (32). With enough satistics, one gets the coefficients of the trigonometric and the hyperbolic functions, as well as the overall normalization $|A_f|^2$. If CPTV is present in decay, the expression for $|A_f|^2$ will change and be a function of y_f, but the eight coefficients of Eq. (32) will remain the same.

The same method is applicable to decays like $B_s \to D_0\phi$, with $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ transitions.

IV. SUMMARY AND CONCLUSIONS

While the effects of CPT violation are severely constrained for systems with first and/or second generation fermions, the B systems, in particular B_s, are relatively less constrained. This opens up the possibility of a CPT violating action that is flavor-dependent. As a typical example of the effects of CPT violation, we consider the decays $B_s^0, \bar{B}_s^0 \to D_s^\pm K^\mp$. These decays are excellent probes of any NP; in the SM, they are single-amplitude processes, and both $B_s^0 \to D_s^+ K^-$ and $B_s^0 \to D_s^- K^+$ amplitudes are of the same order in Wolfenstein parametrization. Thus, the number of events for both $D_s^+ K^-$ and $D_s^- K^+$, summing over parents B_s^0 and $\bar{B}_s^0$, should be the same in the SM. We show how this asymmetry becomes nonzero if there is CPT violation in the decay.

At the same time, we see that if there is some NP that conserves CPT but comes with different strong and weak phases from the corresponding SM amplitude, the asymmetry is again nonzero. So, while this asymmetry serves as an excellent indicator of any NP, it might be either CPT conserving (but necessarily CP violating) or CPT violating, and further checks are necessary.

The situation is far better if there is CPT violation in mixing. The best way to put CPTV in mixing is to make the diagonal terms of the 2×2 mixing Hamiltonian unequal. With this, the CPTV parameter enters the definition of the mass eigenstates, and through that, to various time-dependent decay rates. With sufficient statistics, one can extract the coefficients of the trigonometric and hyperbolic terms of both tagged and untagged time-dependent rates. We find that there are four coefficients which are zero not only in the SM but also any extension with CPT conservation, so any nonzero value for any of them is a definite indication for CPT violation. There are several ways to extract these coefficients, and LHCb should have enough statistics to be able to measure them with sufficient precision. The argument goes through even if CPTV is present in both decay and mixing; this is because different sets of observables are extracted for the two different cases.

Acknowledgments

AK was supported by CSIR, Government of India, and also by the DRS programme of the UGC, Government of India. The work of AS was supported in part by the US DOE contract no. DE-AC02-98CH10886 (BNL). We acknowledge helpful discussions with Tim Gershon (in particular about the LHCb reach), Vladimir Gligorov, and Tomasz Skwarnicki.

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