E_{6} inspired supersymmetric models with exact custodial symmetry

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E_6 inspired SUSY models with exact custodial symmetry

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Abstract

The breakdown of E_6 gauge symmetry at high energies may lead to supersymmetric (SUSY) models based on the Standard Model (SM) gauge group together with extra $U(1)_\psi$ and $U(1)_\chi$ gauge symmetries. To ensure anomaly cancellation the particle content of these E_6 inspired models involves extra exotic states that generically give rise to non-diagonal flavour transitions and rapid proton decay. We argue that a single discrete $\tilde{Z}_2^H$ symmetry can be used to forbid tree-level flavor-changing transitions, as well as the most dangerous baryon and lepton number violating operators. We present $5D$ and $6D$ orbifold GUT constructions that lead to the E_6 inspired SUSY models of this type. The breakdown of $U(1)_\psi$ and $U(1)_\chi$ gauge symmetries that preserves E_6 matter parity assignment guarantees that ordinary quarks and leptons and their superpartners, as well as the exotic states which originate from 27 representations of E_6 survive to low energies. These E_6 inspired models contain two dark-matter candidates and must also include additional TeV scale vectorlike lepton or vectorlike down type quark states to render the lightest exotic quark unstable. We examine gauge coupling unification in these models and discuss their implications for collider phenomenology and cosmology.

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1 Introduction

E_6 inspired models are well motivated extensions of the Standard Model (SM). Indeed, supersymmetric (SUSY) models based on the E_6 gauge symmetry or its subgroup can originate from the ten–dimensional heterotic superstring theory [1]. Within this framework gauge and gravitational anomaly cancellation was found to occur for the gauge groups $SO(32)$ or $E_8 \times E_8'$. However only $E_8 \times E_8'$ can contain the SM since it allows for chiral fermions while $SO(32)$ does not. Compactification of the extra dimensions results in the breakdown of E_8 up to E_6 or one of its subgroups in the observable sector [2]. The remaining E_8' couples to the usual matter representations of the E_6 only by virtue of gravitational interactions and comprises a hidden sector that is thought to be responsible for the spontaneous breakdown of local SUSY (supergravity). At low energies the hidden sector decouples from the observable sector of quarks and leptons, the gauge and Higgs bosons and their superpartners. Its only manifest effect is a set of soft SUSY breaking terms which spoil the degeneracy between bosons and fermions within one supermultiplet [3]. The scale of soft SUSY breaking terms is set by the gravitino mass, $m_{3/2}$. In the simplest SUSY extensions of the SM these terms also determine the electroweak (EW) scale. A large mass hierarchy between $m_{3/2}$ and Planck scale can be caused by the non–perturbative effects in the hidden sector that may trigger the breakdown of supergravity (SUGRA) [4].

Since E_6 is a rank - 6 group the breakdown of E_6 symmetry may result in low energy models based on rank - 5 or rank - 6 gauge groups, with one or two additional $U(1)$ gauge group factors in comparison to the SM. Indeed, E_6 contains the maximal subgroup $SO(10)\times U(1)_\psi$ while $SO(10)$ can be decomposed in terms of the $SU(5)\times U(1)_\chi$ subgroup [5]–[6]. By means of the Hosotani mechanism [7] E_6 can be broken directly to

$$E_6 \to SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$$

which has rank–6. This rank–6 model may be reduced further to an effective rank–5 model with only one extra gauge symmetry $U(1)'$ which is a linear combination of $U(1)_\chi$ and $U(1)_\psi$:

$$U(1)' = U(1)_\chi \cos\theta + U(1)_\psi \sin\theta\,. \tag{1}$$

In the models based on rank - 6 or rank - 5 subgroups of E_6 the anomalies are automatically cancelled if the low energy particle spectrum consists of a complete representations of E_6. Consequently, in E_6-inspired SUSY models one is forced to augment the minimal particle spectrum by a number of exotics which, together with ordinary quarks and leptons, form complete fundamental 27 representations of E_6. Thus we will assume that the particle content of these models includes at least three fundamental representations of E_6 at low energies. These multiplets decompose under the $SU(5)\times U(1)_\psi \times U(1)_\chi$ subgroup of E_6 as follows:

$$\begin{aligned} 27_i \;\to\; & \left(10,\, \frac{1}{\sqrt{24}},\, -\frac{1}{\sqrt{40}}\right)_i + \left(5^*,\, \frac{1}{\sqrt{24}},\, \frac{3}{\sqrt{40}}\right)_i + \left(5^*,\, -\frac{2}{\sqrt{24}},\, -\frac{2}{\sqrt{40}}\right)_i \\ + \;& \left(5,\, -\frac{2}{\sqrt{24}},\, \frac{2}{\sqrt{40}}\right)_i + \left(1,\, \frac{4}{\sqrt{24}},\, 0\right)_i + \left(1,\, \frac{1}{\sqrt{24}},\, -\frac{5}{\sqrt{40}}\right)_i . \end{aligned} \tag{2}$$

The first, second and third quantities in brackets are the $SU(5)$ representation and extra $U(1)_\psi$ and $U(1)_\chi$ charges respectively, while i is a family index that runs from 1 to 3. An ordinary SM family, which contains the doublets of left–handed quarks Q_i and leptons L_i,

right-handed up– and down–quarks (u_i^c and d_i^c) as well as right–handed charged leptons (e_i^c), is assigned to $\left(10,\, \frac{1}{\sqrt{24}},\, -\frac{1}{\sqrt{40}}\right)_i + \left(5^*,\, \frac{1}{\sqrt{24}},\, \frac{3}{\sqrt{40}}\right)_i$. Right-handed neutrinos N_i^c are associated with the last term in Eq. (2), $\left(1,\, \frac{1}{\sqrt{24}},\, -\frac{5}{\sqrt{40}}\right)_i$. The next-to-last term, $\left(1,\, \frac{4}{\sqrt{24}},\, 0\right)_i$, represents new SM-singlet fields S_i, with non-zero $U(1)_\psi$ charges that therefore survive down to the EW scale. The pair of $SU(2)_W$–doublets (H_i^d and H_i^u) that are contained in $\left(5^*,\, -\frac{2}{\sqrt{24}},\, -\frac{2}{\sqrt{40}}\right)_i$ and $\left(5,\, -\frac{2}{\sqrt{24}},\, \frac{2}{\sqrt{40}}\right)_i$ have the quantum numbers of Higgs doublets. They form either Higgs or Inert Higgs $SU(2)_W$ multiplets. [2] Other components of these $SU(5)$ multiplets form colour triplets of exotic quarks $\overline{D}_i$ and D_i with electric charges $+1/3$ and $-1/3$ respectively. These exotic quark states carry a $B-L$ charge $\left(\pm\frac{2}{3}\right)$ twice larger than that of ordinary ones. In phenomenologically viable E_6 inspired models they can be either diquarks or leptoquarks.

The presence of the Z' bosons associated with extra $U(1)$ gauge symmetries and exotic matter in the low-energy spectrum stimulated the extensive studies of the E_6 inspired SUSY models over the years [5], [8]. Recently, the latest Tevatron and early LHC Z' mass limits in these models have been discussed in [9] while different aspects of phenomenology of exotic quarks and squarks have been considered in [10]. Also the implications of the E_6 inspired SUSY models have been studied for EW symmetry breaking (EWSB) [11]–[14], neutrino physics [15]–[16], leptogenesis [17]–[18], EW baryogenesis [19], muon anomalous magnetic moment [20], electric dipole moment of electron [21] and tau lepton [22], lepton flavour violating processes like $\mu \to e\gamma$ [23] and CP-violation in the Higgs sector [24]. The neutralino sector in E_6 inspired SUSY models was analysed previously in [13], [21]–[23], [25]–[29]. Such models have also been proposed as the solution to the tachyon problems of anomaly mediated SUSY breaking, via $U(1)'$ D-term contributions [30], and used in combination with a generation symmetry to construct a model explaining fermion mass hierarchy and mixing [31]. An important feature of E_6 inspired SUSY models is that the mass of the lightest Higgs particle can be substantially larger in these models than in the minimal supersymmetric standard model (MSSM) and next-to-minimal supersymmetric standard model (NMSSM) [14], [32]–[34]. The Higgs sector in these models was examined recently in [29], [32], [35].

Within the class of rank - 5 E_6 inspired SUSY models, there is a unique choice of Abelian $U(1)_N$ gauge symmetry that allows zero charges for right-handed neutrinos and thus a high scale see-saw mechanism. This corresponds to $\theta = \arctan\sqrt{15}$. Only in this Exceptional Supersymmetric Standard Model (E_6SSM) [32]–[33] right–handed neutrinos may be superheavy, shedding light on the origin of the mass hierarchy in the lepton sector and providing a mechanism for the generation of the baryon asymmetry in the Universe via leptogenesis [17]–[18]. Indeed, the heavy Majorana right-handed neutrinos may decay into final states with lepton number $L = \pm 1$, thereby creating a lepton asymmetry in the early universe. Since in the E_6SSM the Yukawa couplings of the new exotic particles are not constrained by neutrino oscillation data, substantial values of the CP–asymmetries can be induced even for a relatively small mass of the lightest right–handed neutrino ($M_1 \sim 10^6\,\mathrm{GeV}$) so that successful thermal leptogenesis may be achieved without encoun-

[2] We use the terminology "Inert Higgs" to denote Higgs–like doublets that do not develop VEVs.

tering a gravitino problem [18].

Supersymmetric models with an additional $U(1)_N$ gauge symmetry have been studied in [16] in the context of non–standard neutrino models with extra singlets, in [25] from the point of view of $Z - Z'$ mixing, in [13] and [25]–[26] where the neutralino sector was explored, in [13], [36] where the renormalisation group (RG) flow of couplings was examined and in [12]–[14] where EWSB was studied. The presence of a Z' boson and of exotic quarks predicted by the Exceptional SUSY model provides spectacular new physics signals at the LHC which were analysed in [32]–[34], [37]. The presence of light exotic particles in the E_6SSM spectrum also lead to the nonstandard decays of the SM–like Higgs boson that were discussed in details in [38]. Recently the particle spectrum and collider signatures associated with it were studied within the constrained version of the E_6SSM [39].

Although the presence of TeV scale exotic matter in E_6 inspired SUSY models gives rise to specatucular collider signatures, it also causes some serious problems. In particular, light exotic states generically lead to non–diagonal flavour transitions and rapid proton decay. To suppress flavour changing processes as well as baryon and lepton number violating operators one can impose a set of discrete symmetries. For example, one can impose an approximate Z_2^H symmetry, under which all superfields except one pair of H_i^d and H_i^u (say $H_d \equiv H_3^d$ and $H_u \equiv H_3^u$) and one SM-type singlet field ($S \equiv S_3$) are odd [32]–[33]. When all Z_2^H symmetry violating couplings are small this discrete symmetry allows to suppress flavour changing processes. If the Lagrangian of the E_6 inspired SUSY models is invariant with respect to either a Z_2^L symmetry, under which all superfields except leptons are even (Model I), or a Z_2^B discrete symmetry that implies that exotic quark and lepton superfields are odd whereas the others remain even (Model II), then the most dangerous baryon and lepton number violating operators get forbidden and proton is sufficiently longlived [32]–[33]. The symmetries Z_2^H, Z_2^L and Z_2^B obviously do not commute with E_6 because different components of fundamental representations of E_6 transform differently under these symmetries.

The necessity of introducing multiple discrete symmetries to ameliorate phenomenological problems that generically arise due to the presence of low mass exotics is an undesirable feature of these models. In this paper we consider rank - 6 E_6 inspired SUSY models in which a *single* discrete $\tilde{Z}_2^H$ symmetry serves to simultaneously forbid tree–level flavor–changing transitions and the most dangerous baryon and lepton number violating operators. We consider models where the $U(1)_\psi$ and $U(1)_\chi$ gauge symmetries are spontaneously broken at some intermediate scale so that the matter parity,

$$Z_2^M = (-1)^{3(B-L)} \,, \tag{3}$$

is preserved. As a consequence the low-energy spectrum of the models will include *two* stable weakly interacting particles that potentially contribute to the dark matter density of our Universe. The invariance of the Lagrangian with respect to Z_2^M and $\tilde{Z}_2^H$ symmetries leads to unusual collider signatures associated with exotic states that originate from 27–plets. These signatures have not been studied in details before. In addition to the exotic matter multiplets that stem from the fundamental 27 representations of E_6 the considered models predict the existence of a set of vector-like supermultiplets. In particular the low-energy spectrum of the models involves either a doublet of vector-like leptons or a triplet of vector-like down type quarks. If these extra states are relatively light, they will manifest themselves at the LHC in the near future.

It is worth noting that in the E_6 inspired SUSY models considered here different

superfields, that are expected to originate from the same E_6 supermultiplet, transform differently under the transformations of the $\tilde{Z}_2^H$ symmetry. Therefore one can naively think that this may be inconsistent with Grand Unified theories (GUTs) based on the E_6 gauge group. In this paper we argue that the orbifolding in higher–dimensional E_6 theories might split 27-plets so that the low energy limit of these theories is described by the SUSY models with exact custodial $\tilde{Z}_2^H$ symmetry mentioned above.

The layout of this paper is as follows. In Section 2 we specify the rank–6 E_6 inspired SUSY models with exact custodial symmetry. In Section 3 we present five–dimensional ($5D$) and six–dimensional ($6D$) orbifold GUTs that lead to the rank–6 E_6 inspired SUSY models that we propose. In Sections 4 and 5 the RG flow of gauge couplings and implications for collider phenomenology and cosmology are discussed. Our results are summarized in Section 6.

2 E_6 inspired SUSY models with exact custodial $\tilde{Z}_2^H$ symmetry

In our analysis we concentrate on the rank–6 E_6 inspired SUSY models with two extra $U(1)$ gauge symmetries — $U(1)_\chi$ and $U(1)_\psi$. In other words we assume that near the GUT or string scale E_6 or its subgroup is broken down to $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$. In the next section we argue that this breakdown can be achieved within orbifold GUT models. We also allow three copies of 27-plets to survive to low energies so that anomalies get cancelled generation by generation within each complete 27_i representation of E_6. In E_6 models the renormalisable part of the superpotential comes from the $27 \times 27 \times 27$ decomposition of the E_6 fundamental representation and can be written as

$$\begin{aligned}
W_{E_6} &= W_0 + W_1 + W_2, \\
W_0 &= \lambda_{ijk} S_i (H_j^d H_k^u) + \kappa_{ijk} S_i (D_j \overline{D}_k) + h^N_{ijk} N_i^c (H_j^u L_k) + h^U_{ijk} u_i^c (H_j^u Q_k) + \\
&\quad + h^D_{ijk} d_i^c (H_j^d Q_k) + h^E_{ijk} e_i^c (H_j^d L_k)\,, \qquad (4) \\
W_1 &= g^Q_{ijk} D_i (Q_j Q_k) + g^q_{ijk} \overline{D}_i d_j^c u_k^c\,, \\
W_2 &= g^N_{ijk} N_i^c D_j d_k^c + g^E_{ijk} e_i^c D_j u_k^c + g^D_{ijk} (Q_i L_j) \overline{D}_k\,.
\end{aligned}$$

Here the summation over repeated family indexes ($i, j, k = 1, 2, 3$) is implied. In the considered models $B-L$ number is conserved automatically since the corresponding global symmetry $U(1)_{B-L}$ is a linear superposition of $U(1)_Y$ and $U(1)_\chi$. At the same time if terms in W_1 and W_2 are simultaneously present in the superpotential then baryon and lepton numbers are violated. In other words one cannot define the baryon and lepton numbers of the exotic quarks D_i and $\overline{D}_i$ so that the complete Lagrangian is invariant separately under $U(1)_B$ and $U(1)_L$ global symmetries. In this case the Yukawa interactions in W_1 and W_2 give rise to rapid proton decay.

Another problem is associated with the presence of three families of H_i^u and H_i^d. All these Higgs–like doublets can couple to ordinary quarks and charged leptons of different generations resulting in the phenomenologically unwanted flavor changing transitions. For example, non–diagonal flavor interactions contribute to the amplitude of $K^0 - \overline{K}^0$ oscillations and give rise to new channels of muon decay like $\mu \to e^- e^+ e^-$. In order to

avoid the appearance of flavor changing neutral currents (FCNCs) at the tree level and forbid the most dangerous baryon and lepton number violating operators one can try to impose a single $\tilde{Z}_2^H$ discrete symmetry. One should note that the imposition of additional discrete symmetry to stabilize the proton is a generic feature of many phenomenologically viable SUSY models.

In our model building strategy we use $SU(5)$ SUSY GUT as a guideline. Indeed, the low–energy spectrum of the MSSM, in addition to the complete $SU(5)$ multiplets, contains an extra pair of doublets from 5 and $\overline{5}$ fundamental representations, that play a role of the Higgs fields which break EW symmetry. In the MSSM the potentially dangerous operators, that lead to the rapid proton decay, are forbidden by the matter parity Z_2^M under which Higgs doublets are even while all matter superfields, that fill in complete $SU(5)$ representations, are odd. Following this inspirational example we augment three 27-plets of E_6 by a number of components M_l and $\overline{M}_l$ from extra $27'_l$ and $\overline{27'}_l$ below the GUT scale. Because additional pairs of multiplets M_l and $\overline{M}_l$ have opposite $U(1)_Y$, $U(1)_\psi$ and $U(1)_\chi$ charges their contributions to the anomalies get cancelled identically. As in the case of the MSSM we allow the set of multiplets M_l to be used for the breakdown of gauge symmetry. If the corresponding set includes $H^u \equiv H_u$, $H^d \equiv H_d$, S and $N^c \equiv N_H^c$ then the $SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$ symmetry can be broken down to $U(1)_{em}$ associated with electromagnetism. The VEVs of S and N^c break $U(1)_\psi$ and $U(1)_\chi$ entirely while the $SU(2)_W \times U(1)_Y$ symmetry remains intact. When the neutral components of H_u and H_d acquire non–zero VEVs then $SU(2)_W \times U(1)_Y$ symmetry gets broken to $U(1)_{em}$ and the masses of all quarks and charged leptons are generated.

As in the case of the MSSM we assume that all multiplets M_l are even under $\tilde{Z}_2^H$ symmetry while three copies of the complete fundamental representations of E_6 are odd. This forbids couplings in the superpotential that come from $27_i \times 27_j \times 27_k$. On the other hand the $\tilde{Z}_2^H$ symmetry allows the Yukawa interactions that stem from $27'_l \times 27'_m \times 27'_n$, and $27'_l \times 27_i \times 27_k$ The multiplets M_l have to be even under $\tilde{Z}_2^H$ symmetry because some of them are expected to get VEVs. Otherwise the VEVs of the corresponding fields lead to the breakdown of the discrete $\tilde{Z}_2^H$ symmetry giving rise to the baryon and lepton number violating operators in general. If the set of multiplets M_l includes only one pair of doublets H_d and H_u the $\tilde{Z}_2^H$ symmetry defined above permits to suppress unwanted FCNC processes at the tree level since down-type quarks and charged leptons couple to just one Higgs doublet H_d, whereas the up-type quarks couple to H_u only.

The superfields $\overline{M}_l$ can be either odd or even under this $\tilde{Z}_2^H$ symmetry. Depending on whether these fields are even or odd under $\tilde{Z}_2^H$ a subset of terms in the most general renormalizable superpotential can be written as

$$\begin{aligned} W_{\rm total} = Y'_{lmn} 27'_l 27'_m 27'_n + Y_{lij} 27'_l 27_i 27_j + \tilde{Y}_{lmn} \overline{27'}_l \overline{27'}_m \overline{27'}_n + \\ + \mu'_{il} 27_i \overline{27'}_l + \tilde{\mu}'_{ml} 27'_m \overline{27'}_l \dots , \end{aligned} \tag{5}$$

where Y'_{lmn} and Y_{lij} are Yukawa couplings and μ'_{il} and $\tilde{\mu}'_{ml}$ are mass parameters. Also one should keep in mind that only M_l and $\overline{M}_l$ components of $27'_l$ and $\overline{27'}_l$ appear below the GUT scale. If $\overline{M}_l$ is odd under $\tilde{Z}_2^H$ symmetry then the term $\tilde{\mu}'_{ml} 27'_m \overline{27'}_l$ and $\tilde{Y}_{lmn} \overline{27'}_l \overline{27'}_m \overline{27'}_n$ are forbidden while μ'_{il} can have non-zero values. When $\overline{M}_l$ is even μ'_{il} vanish whereas $\tilde{\mu}'_{ml} 27'_m \overline{27'}_l$ and $\tilde{Y}_{lmn} \overline{27'}_l \overline{27'}_m \overline{27'}_n$ are allowed by $\tilde{Z}_2^H$ symmetry. In general mass parameters μ'_{il} and $\tilde{\mu}'_{ml}$ are expected to be of the order of GUT scale. In order to allow some of the $\overline{M}_l$ multiplets to survive to low energies we assume that the corresponding mass terms are forbidden at high energies and get induced at some intermediate scale which is much lower than M_X.

The VEVs of the superfields N_H^c and $\overline{N}_H^c$ (that originate from $27'_N$ and $\overline{27'}_N$) can be used not only for the breakdown of $U(1)_\psi$ and $U(1)_\chi$ gauge symmetries, but also to generate Majorana masses for the right–handed neutrinos that can be induced through interactions

$$\Delta W_N = \frac{\varkappa_{ij}}{M_{Pl}}(27_i\,\overline{27'}_N)(27_j\,\overline{27'}_N)\,. \tag{6}$$

The non–renormalizable operators (6) give rise to the right–handed neutrino masses which are substantially lower than the VEVs of N_H^c and $\overline{N}_H^c$. Because the observed pattern of the left–handed neutrino masses and mixings can be naturally reproduced by means of seesaw mechanism if the right–handed neutrinos are superheavy, the N_H^c and $\overline{N}_H^c$ are expected to acquire VEVs $< N_H^c > \simeq < \overline{N}_H^c > \lesssim M_X$. This implies that $U(1)_\psi \times U(1)_\chi$ symmetry is broken down to $U(1)_N$ near the GUT scale, where $U(1)_N$ symmetry is a linear superposition of $U(1)_\psi$ and $U(1)_\chi$, i.e.

$$U(1)_N = \frac{1}{4}U(1)_\chi + \frac{\sqrt{15}}{4}U(1)_\psi\,, \tag{7}$$

under which right-handed neutrinos have zero charges. Since N_H^c and $\overline{N}_H^c$ acquire VEVs both supermultiplets must be even under $\tilde{Z}_2^H$ symmetry.

At the same time the VEVs of N_H^c and $\overline{N}_H^c$ may break $U(1)_{B-L}$ symmetry. In particular, as follows from Eq. (4) the VEV of N_H^c can induce the bilinear terms $M_{ij}^L(H_i^u L_j)$ and $M_{ij}^B(D_i d_j^c)$ in the superpotential. Although such breakdown of gauge symmetry might be possible the extra particles tend to be rather heavy in the considered case and thus irrelevant for collider phenomenology. Therefore we shall assume further that the couplings of N_H^c to 27_i are forbidden. This, for example, can be achieved by imposing an extra discrete symmetry Z_n. Although this symmetry can forbid the interactions of N_H^c with three complete 27_i representations of E_6 it should allow non–renormalizable interactions (6) that induce the large Majorana masses for right-handed neutrinos. These requirements are fulfilled if Lagrangian is invariant under Z_2 symmetry transformations $N_H^c \to -N_H^c$ and $\overline{N}_H^c \to -\overline{N}_H^c$. Alternatively, one can impose Z_n symmetry ($n > 2$) under which only N_H^c transforms. The invariance of the Lagrangian with respect to Z_n symmetry ($n > 2$) under which only N_H^c transforms implies that the mass term $\mu_H N_H^c \overline{N}_H^c$ in the superpotential (5) is forbidden. On the other hand this symmetry allows non–renormalizable term in the superpotential

$$\Delta W_{N_H^c} = \varkappa\,\frac{(N_H^c\overline{N}_H^c)^n}{M_{Pl}^{2n-3}}\,,. \tag{8}$$

In this case N_H^c and $\overline{N}_H^c$ can develop VEVs along the D–flat direction so that

$$< N_H^c > \simeq < \overline{N}_H^c > \sim M_{Pl}\cdot\left[\frac{1}{\varkappa}\frac{M_S}{M_{Pl}}\right]^{\frac{1}{2n-2}}\,, \tag{9}$$

where M_S is a low–energy supersymmetry breaking scale. This mechanism permits to generate $< N_H^c > \gtrsim 10^{14}\,\mathrm{GeV}$ resulting in right-handed neutrino masses of order of

$$\varkappa_{ij}M_{Pl}\cdot\left[\frac{1}{\varkappa}\frac{M_S}{M_{Pl}}\right]^{\frac{1}{n-1}} \gtrsim 10^{11}\,\mathrm{GeV}\,.$$

	27_i	27_i	$27'_{H_u}$ $(27'_{H_d})$	$27'_S$	$\overline{27'}_{H_u}$ $(\overline{27'}_{H_d})$	$\overline{27'}_S$	$27'_N$ $(\overline{27'}_N)$	$27'_L$ $(\overline{27'}_L)$	$27'_d$ $(\overline{27'}_d)$
	$Q_i, u^c_i, d^c_i,$ L_i, e^c_i, N^c_i	$\overline{D}_i, D_i,$ H^d_i, H^u_i, S_i	H_u (H_d)	S	$\overline{H}_u$ $(\overline{H}_d)$	$\overline{S}$	N^c_H $(\overline{N}^c_H)$	L_4 $(\overline{L}_4)$	d^c_4 $(\overline{d^c}_4)$
$\tilde{Z}_2^H$	$-$	$-$	$+$	$+$	$-$	$\pm$	$+$	$+$	$+$
Z_2^M	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$
Z_2^E	$+$	$-$	$+$	$+$	$-$	$\pm$	$-$	$-$	$-$

Table 1: Transformation properties of different components of E_6 multiplets under $\tilde{Z}_2^H$, Z_2^M and Z_2^E discrete symmetries.

The mechanism of the gauge symmetry breaking discussed above ensures that the low–energy effective Lagrangian is automatically invariant under the matter parity Z_2^M. Such spontaneous breakdown of the $U(1)_\psi \times U(1)_\chi$ gauge symmetry can occur because Z_2^M is a discrete subgroup of $U(1)_\psi$ and $U(1)_\chi$. This follows from the $U(1)_\psi$ and $U(1)_\chi$ charge assignments presented in Eq. (2). Thus in the considered case the VEVs of N^c_H and $\overline{N}^c_H$ break $U(1)_\psi \times U(1)_\chi$ gauge symmetry down to $U(1)_N \times Z_2^M$. As a consequence the low–energy effective Lagrangian is invariant under both Z_2^M and $\tilde{Z}_2^H$ discrete symmetries. Moreover the $\tilde{Z}_2^H$ symmetry is a product of

$$\tilde{Z}_2^H = Z_2^M \times Z_2^E \,, \tag{10}$$

where Z_2^E is associated with most of the exotic states. In other words all exotic quarks and squarks, Inert Higgs and Higgsino multiplets as well as SM singlet and singlino states that do not get VEV are odd under Z_2^E symmetry. The transformation properties of different components of 27_i, $27'_l$ and $\overline{27'}_l$ multiplets under the $\tilde{Z}_2^H$, Z_2^M and Z_2^E symmetries are summarized in Table 1. Since the Lagrangian of the considered E_6 inspired models is invariant under Z_2^M and $\tilde{Z}_2^H$ symmetries it is also invariant under the transformations of Z_2^E symmetry. Because Z_2^E is conserved the lightest exotic state, which is odd under this symmetry, is absolutely stable and contributes to the relic density of dark matter.

It is also well known that in SUSY models the lightest supersymmetric particle (LSP), i.e. the lightest R–parity odd particle ($Z_2^R = (-1)^{3(B-L)+2s}$), must be stable. If in the considered models the lightest exotic state (i.e. state with $Z_2^E = -1$) has even R–parity then the lightest R–parity odd state cannot decay as usual. When the lightest exotic state is R–parity odd particle either the lightest R–parity even exotic state or the next-to-lightest R–parity odd state with $Z_2^E = +1$ must be absolutely stable. Thus the considered E_6 inspired SUSY models contain at least two dark-matter candidates.

The residual extra $U(1)_N$ gauge symmetry gets broken by the VEV of the SM–singlet superfield S (and possibly $\overline{S}$). The VEV of the field S induces the mass of the Z' associated with $U(1)_N$ symmetry as well as the masses of all exotic quarks and inert Higgsinos. If S acquires VEV of order $10 - 100\,\mathrm{TeV}$ (or even lower) the lightest exotic particles can be produced at the LHC. This is the most interesting scenario that we are going to focus on here. In some cases the superfield $\overline{S}$ may also acquire non–zero VEV breaking $U(1)_N$ symmetry as we will discuss later. If this is a case then $\overline{S}$ should be even under the $\tilde{Z}_2^H$ symmetry. Otherwise the superfield $\overline{S}$ can be $\tilde{Z}_2^H$ odd.

The above consideration indicate that the set of multiplets M_l has to contain at least H_u, H_d, S and N^c_H in order to guarantee the appropriate breakdown of the gauge symmetry in the rank–6 E_6 inspired SUSY models. However if the set of $\tilde{Z}_2^H$ even supermultiplets M_l

involve only H_u, H_d, S and N_H^c then the lightest exotic quarks are extremely long–lived particles. Indeed, in the considered case the $\tilde{Z}_2^H$ symmetry forbids all Yukawa interactions in W_1 and W_2 that allow the lightest exotic quarks to decay. Moreover the Lagrangian of such model is invariant not only with respect to $U(1)_L$ and $U(1)_B$ but also under $U(1)_D$ symmetry transformations

$$D \to e^{i\alpha} D\,, \qquad\qquad \overline{D} \to e^{-i\alpha} \overline{D}\,. \tag{11}$$

The $U(1)_D$ invariance ensures that the lightest exotic quark is very long–lived. The $U(1)_L$, $U(1)_B$ and $U(1)_D$ global symmetries are expected to be broken by a set of non–renormalizable operators which are suppressed by inverse power of the GUT scale M_X or M_{Pl}. These operators give rise to the decays of the exotic quarks but do not lead to the rapid proton decay. Since the extended gauge symmetry in the considered rank–6 E_6 inspired SUSY models forbids any dimension five operators that break $U(1)_D$ global symmetry the lifetime of the lightest exotic quarks is expected to be of order of

$$\tau_D \gtrsim M_X^4/\mu_D^5\,, \tag{12}$$

where μ_D is the mass of the lightest exotic quark. When $\mu_D \simeq$ TeV the lifetime of the lightest exotic quarks $\tau_D \gtrsim 10^{49}\,\mathrm{GeV}^{-1} \sim 10^{17}$ years, i.e. considerably larger than the age of the Universe.

The long–lived exotic quarks would have been copiously produced during the very early epochs of the Big Bang. Those lightest exotic quarks which survive annihilation would subsequently have been confined in heavy hadrons which would annihilate further. The remaining heavy hadrons originating from the Big Bang should be present in terrestrial matter. There are very strong upper limits on the abundances of nuclear isotopes which contain such stable relics in the mass range from 1 GeV to 10 TeV. Different experiments set limits on their relative concentrations from 10^{-15} to 10^{-30} per nucleon [40]. At the same time various theoretical estimations [41] show that if remnant particles would exist in nature today their concentration is expected to be at the level of 10^{-10} per nucleon. Therefore E_6 inspired models with very long–lived exotic quarks are ruled out.

To ensure that the lightest exotic quarks decay within a reasonable time the set of $\tilde{Z}_2^H$ even supermultiplets M_l needs to be supplemented by some components of 27-plet that carry $SU(3)_C$ color or lepton number. In this context we consider two scenarios that lead to different collider signatures associated with the exotic quarks. In the simplest case (**scenario A**) the set of $\tilde{Z}_2^H$ even supermultiplets M_l involves lepton superfields L_4 and/or e_4^c that survive to low energies. This implies that $\overline{D}_i$ and D_i can interact with leptons and quarks only while the couplings of these exotic quarks to a pair of quarks are forbidden by the postulated $\tilde{Z}_2^H$ symmetry. Then baryon number is conserved and exotic quarks are leptoquarks.

In this paper we restrict our consideration to the E_6 inspired SUSY models that lead to the approximate unification of the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ gauge couplings at some high energy scale M_X. This requirement implies that in the one–loop approximation the gauge coupling unification is expected to be almost exact. On the other hand it is well known that the one–loop gauge coupling unification in SUSY models remains intact if the MSSM particle content is supplemented by the complete representations of $SU(5)$ (see for example [42]). Thus we require that the extra matter beyond the MSSM fill in complete $SU(5)$ representations. In the **scenario A** this requirement can be fulfilled if $\overline{H}_u$ and $\overline{H}_d$ are odd under the $\tilde{Z}_2^H$ symmetry while $\overline{L}_4$ is $\tilde{Z}_2^H$ even supermultiplet. Then

$\overline{H}_u$ and $\overline{H}_d$ from the $\overline{27'}_l$ can get combined with the superposition of the corresponding components from 27_i so that the resulting vectorlike states gain masses of order of M_X. The supermultiplets L_4 and $\overline{L}_4$ are also expected to form vectorlike states. However these states are required to be light enough to ensure that the lightest exotic quarks decay sufficiently fast[3]. The appropriate mass term $\mu_L L_4 \overline{L}_4$ in the superpotential can be induced within SUGRA models just after the breakdown of local SUSY if the Kähler potential contains an extra term $(Z_L(L_4\overline{L}_4) + h.c)$[43].

The presence of the bosonic and fermionic components of $\overline{S}$ at low energies is not constrained by the unification of the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ gauge couplings since $\overline{S}$ is the SM singlet superfield. If $\overline{S}$ is odd under the $\tilde{Z}_2^H$ symmetry then it can get combined with the superposition of the appropriate components of 27_i. The corresponding vectorlike states may be either superheavy ($\sim M_X$) or gain TeV scale masses. When $\overline{S}$ is $\tilde{Z}_2^H$ even superfield then its scalar component is expected to acquire a non-zero VEV breaking $U(1)_N$ gauge symmetry.

Thus **scenario A** implies that in the simplest case the low energy matter content of the considered E_6 inspired SUSY models involves:

$$\begin{aligned} 3\left[(Q_i,\, u_i^c,\, d_i^c,\, L_i,\, e_i^c,\, N_i^c)\right] + 3(D_i,\, \bar{D}_i) + 2(S_\alpha) + 2(H_\alpha^u) + 2(H_\alpha^d) \\ + L_4 + \overline{L}_4 + N_H^c + \overline{N}_H^c + S + H_u + H_d\,, \end{aligned} \tag{13}$$

where the right–handed neutrinos N_i^c are expected to gain masses at some intermediate scale, while the remaining matter survives down to the EW scale. In Eq. (13) $\alpha = 1, 2$ and $i = 1, 2, 3$. Integrating out N_i^c, N_H^c and $\overline{N}_H^c$ as well as neglecting all suppressed non-renormalisable interactions one gets an explicit expression for the superpotential in the considered case

$$\begin{aligned} W_A = \lambda S(H_u H_d) + \lambda_{\alpha\beta} S(H_\alpha^d H_\beta^u) + \kappa_{ij} S(D_i \overline{D}_j) + \tilde{f}_{\alpha\beta} S_\alpha (H_\beta^d H_u) + f_{\alpha\beta} S_\alpha (H_d H_\beta^u) \\ + g_{ij}^D (Q_i L_4)\overline{D}_j + h_{i\alpha}^E e_i^c (H_\alpha^d L_4) + \mu_L L_4 \overline{L}_4 + W_{MSSM}(\mu = 0)\,. \end{aligned} \tag{14}$$

A second scenario, that allows the lightest exotic quarks to decay within a reasonable time and prevents rapid proton decay, is realized when the set of multiplets M_l together with H_u, H_d, S and N_H^c contains an extra d_4^c superfield (instead of L_4) from $27'_d$. If the $\tilde{Z}_2^H$ even supermultiplet d_4^c survives to low energies then exotic quarks are allowed to have non-zero Yukawa couplings with pair of quarks which permit their decays. They can also interact with d_4^c and right-handed neutrinos. However if Majorana right-handed neutrinos are very heavy ($\sim M_X$) then the interactions of exotic quarks with leptons are extremely suppressed. As a consequence in this **scenario B** $\overline{D}_i$ and D_i manifest themselves in the Yukawa interactions as superfields with baryon number $\left(\pm\frac{2}{3}\right)$.

Although in the **scenario B** the baryon and lepton number violating operators are expected to be suppressed by inverse powers of the masses of the right–handed neutrinos they can still lead to the rapid proton decay. The Yukawa interactions of the $\tilde{Z}_2^H$ even superfield d_4^c with other supermultiplets of ordinary and exotic matter can be written in the following form

$$\Delta W_{d_4^c} = h_{ik}^D d_4^c (H_i^d Q_k) + g_{ij}^q \overline{D}_i d_4^c u_j^c + g_{ij}^N N_i^c D_j d_4^c\,. \tag{15}$$

[3]Note that the superfields e_4^c and $\overline{e^c}_4$ are not allowed to survive to low energies because they spoil the one–loop gauge coupling unification.

Integrating out Majorana right-handed neutrinos one obtains in the leading approximation

$$\Delta W_{d_4^c} \to h^D_{ik} d^c_4 (H^d_i Q_k) + g^q_{ij} \overline{D}_i d^c_4 u^c_j + \frac{\tilde{\varkappa}_{ij}}{M_N} (L_i H_u)(D_j d^c_4)\,, \tag{16}$$

where M_N is an effective seesaw scale which is determined by the masses and couplings of N^c_i and $\tilde{\varkappa}_{ij} \sim g^N_{ij}$. In the considered case the baryon and lepton number violation takes place only when all three terms in Eqs. (15)–(16) are present in the superpotential. If $g^N_{ij} = 0$ ($\tilde{\varkappa}_{ij} = 0$) or $g^q_{ij} = 0$ the baryon and lepton number conservation requires exotic quarks to be either diquarks or leptoquarks respectively. When h^D_{ik} vanish the conservation of the baryon and lepton numbers implies that the superfields D_i, $\overline{D}_i$ and d^c_4 have the following $U(1)_L$ and $U(1)_B$ charges $B_D = -B_{\overline{D}} = -B_{d^c_4} = -1/6$ and $L_D = -L_{\overline{D}} = L_{d^c_4} = -1/2$. This consideration indicates that in the case when all three terms are present in Eqs. (15)–(16) the $U(1)_L$ and $U(1)_B$ global symmetries can not be preserved. It means that in the leading approximation the proton decay rate is caused by all three types of the corresponding Yukawa couplings and has to go to zero when the Yukawa couplings of at least one type of Yukawa interactions vanish. In practice, the proton lifetime is determined by the one–loop box diagram that leads to the dimension seven operator

$$\mathcal{L}_p \simeq \left(\frac{c_{ijkl}}{M^2_S}\right)\left(\frac{\langle H_u\rangle}{M_N}\right)\left[\epsilon_{\alpha\beta\gamma}\overline{u^c}_{\alpha i} d_{\beta j}\overline{\nu}_k d_{\gamma l}\right], \tag{17}$$

where $\langle H_u\rangle = v_2/\sqrt{2}$ and $c_{ijkl} \propto \tilde{\varkappa}\, g^q\, (h^D)^2$. In Eq. (17) Greek indices denote the color degrees of freedom while $SU(2)$ indices are suppressed. Here we assume that all particles propagating in the loop have masses of the order of M_S. For $M_N \gtrsim 10^{11}\,\mathrm{GeV}$ and $h^D_{ik} \sim g^q_{ij} \sim g^N_{ij}$ the appropriate suppression of the proton decay rate can be achieved if the corresponding Yukawa couplings are less than 10^{-5}.

Once again, the requirement of the approximate unification of the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ gauge couplings constrains the low energy matter content in the **scenario B**. The concept of gauge coupling unification implies that the perturbation theory method provides an adequate description of the RG flow of gauge couplings up to the GUT scale M_X at least. The requirement of the validity of perturbation theory up to the scale M_X sets stringent constraint on the number of extra $SU(2)_W$ and $SU(3)_C$ supermultiplets that can survive to low energies in addition to three complete fundamental representations of E_6. For example, the applicability of perturbation theory up to the high energies permits only one extra pair of $SU(3)_C$ triplet superfields to have mass of the order of TeV scale. The same requirement limits the number of pairs of $SU(2)_W$ doublets to two.

Because in the **scenario B** the $\tilde{Z}^H_2$ even supermultiplets d^c_4 and $\overline{d^c}_4$ are expected to form vectorlike states which have to have TeV scale masses the limit caused by the validity of perturbation theory up to the scale M_X is saturated. Then in order to ensure that the extra matter beyond the MSSM fill in complete $SU(5)$ representations $\overline{H}_u$ and $\overline{H}_d$ should survive to the TeV scale as well. As before we assume that these supermultiplets are odd under the $\tilde{Z}^H_2$ symmetry so that they can get combined with the superposition of the corresponding components from 27_i at low energies forming vectorlike states. Again the superfield $\overline{S}$ may or may not survive to the TeV scale. It can be either even or odd under the $\tilde{Z}^H_2$ symmetry. If $\overline{S}$ is $\tilde{Z}^H_2$ even, it should survive to low energies and its scalar component is expected to get a VEV.

Following the above discussion the low energy matter content in the simplest case of

the **scenario B** may be summarized as:

$$\begin{aligned} 3\,[(Q_i,\,u_i^c,\,d_i^c,\,L_i,\,e_i^c,\,N_i^c)] + 3(D_i,\,\bar{D}_i) + 3(H_i^u) + 3(H_i^d) + 2(S_\alpha) \\ +d_4^c + \overline{d^c}_4 + N_H^c + \overline{N}^c_H + H_u + \overline{H}_u + H_d + \overline{H}_d + S\,. \end{aligned} \tag{18}$$

All states in Eq. (18) are expected to be considerably lighter than the GUT scale M_X. Assuming that N_i^c, N_H^c and $\overline{N}^c_H$ gain intermediate scale masses the renormalizable part of the TeV scale superpotential associated with the **scenario B** can be written as

$$\begin{aligned} W_B = \lambda S(H_u H_d) + \lambda_{ij} S(H_i^d H_j^u) + \kappa_{ij} S(D_i \overline{D}_j) + \tilde{f}_{\alpha i} S_\alpha (H_i^d H_u) + f_{\alpha i} S_\alpha (H_d H_i^u) \\ +g_{ij}^q \overline{D}_i d_4^c u_j^c + h_{ij}^D d_4^c (H_i^d Q_j) + \mu_d d_4^c \overline{d^c}_4 + \mu_i^u H_i^u \overline{H}_u + \mu_i^d H_i^d \overline{H}_d + W_{MSSM}(\mu = 0)\,. \end{aligned} \tag{19}$$

The superpotential (19) contains a set of the TeV scale mass parameters, i.e. μ_d, μ_i^u, μ_i^d. These are introduced to avoid massless fermionic states associated with d_4^c, $\overline{d^c}_4$, $\overline{H}_u$ and $\overline{H}_d$ supermultiplets and can be induced after the breakdown of local SUSY as it has been discussed earlier. On the other hand the superpotential (19) also contains the Yukawa couplings g_{ij}^q and h_{ij}^D which are expected to be small in order to avoid rapid proton decay. The appropriate suppression of the corresponding Yukawa couplings and mass parameters μ_d, μ_i^u and μ_i^d can be achieved if the Lagrangian of the E_6 inspired model is invariant under the discrete Z_k symmetry which gets broken spontaneously at the intermediate scale. As an example one can consider the model with extra SM singlet superfield Φ which transforms under the discrete Z_k symmetry. For concreteness here we assume that at high energies the Lagrangian of the model is invariant under the Z_6 symmetry transformations

$$\Phi \to \omega\,\Phi\,, \qquad d_4^c \to \omega^5\,d_4^c\,, \qquad \overline{d^c}_4 \to \omega^3\,\overline{d^c}_4\,, \qquad \overline{H}_u \to \omega^2\,\overline{H}_u\,, \qquad \overline{H}_d \to \omega^2\,\overline{H}_d\,, \tag{20}$$

where $\omega = e^{i\pi/3}$. Then the part of the superpotential that depends on the d_4^c, $\overline{d^c}_4$, $\overline{H}_u$, $\overline{H}_d$ and Φ takes the form

$$\begin{aligned} \Delta W_{Z_6} &= \frac{\Phi}{M_{Pl}}\left[\sigma_{ij} d_4^c (H_i^d Q_j) + \tilde{\sigma}_{ij} \overline{D}_i d_4^c u_j^c + \hat{\sigma}_{ij} N_i^c D_j d_4^c\right] \\ &+ \frac{\Phi^4}{M_{Pl}^3}\left[\eta_d d_4^c \overline{d^c}_4 + \eta_i^u H_i^u \overline{H}_u + \eta_i^d H_i^d \overline{H}_d\right] + \sigma \frac{\Phi^6}{M_{Pl}^3} + \dots\,. \end{aligned} \tag{21}$$

At the intermediate scale the imposed Z_6 symmetry may be broken spontaneously by the VEV of the superfield Φ

$$< \Phi > \sim \left[\frac{M_S}{M_{Pl}}\right]^{1/4} M_{Pl} \simeq 10^{14}\,\mathrm{GeV} \tag{22}$$

inducing bilinear mass terms in the superpotential and small Yukawa couplings of the d_4^c supermultiplet to other superfields. The corresponding Yukawa couplings and mass parameters are given by [4]

$$\mu_d \sim \mu_i^u \sim \mu_i^d \sim \frac{< \Phi^4 >}{M_{Pl}^3} \simeq M_S\,, \qquad h_{ik}^D \sim g_{ij}^q \sim g_{ij}^N \lesssim \frac{< \Phi >}{M_{Pl}} \sim 10^{-4}\,. \tag{23}$$

Although **scenarios A** and **B** discussed in this section allow us to suppress baryon and lepton number violating operators and non-diagonal flavor transitions they have at

[4] The same mechanism can be used for the generation of the mass term $\mu_L L_4 \overline{L}_4$ in the scenario A.

least one drawback. Both scenarios imply that a number of incomplete E_6 multiplets survive below the scale M_X. In fact, the number of incomplete E_6 multiplets tends to be larger than the number of generations. Therefore the origin and mechanism resulting in the incomplete E_6 representations requires further justification. The splitting of GUT multiplets can be naturally achieved in the framework of orbifold GUTs. In the next section we present $5D$ and $6D$ orbifold GUT models that can lead to the **scenarios A** and **B** just below the GUT scale.

3 5D and 6D orbifold GUT models

The structure of the E_6 inspired SUSY models discussed in the previous section, its gauge group and field content, points towards an underlying GUT model based on the E_6 or its subgroup. The breaking of these GUT groups down to the $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$ is in general rather involved and requires often large Higgs representations. In particular, the splitting of GUT multiplets (like doublet-triplet splitting within SU(5) GUT) requires either fine–tuning of parameters or additional, sophisticated mechanisms [44]–[45].

Higher–dimensional theories offer new possibilities to describe gauge symmetry breaking. A simple and elegant scheme is provided by orbifold compactifications which have been considered for SUSY GUT models in five dimensions [46]–[55] and six dimensions [54]–[59]. These models apply ideas that first appeared in string–motivated work [60]: the gauge symmetry is broken by identifications imposed on the gauge fields under the spacetime symmetries of an orbifold. In these models many good properties of GUT's like gauge coupling unification and charge quantization are maintained while some unsatisfactory properties of the conventional breaking mechanism, like doublet-triplet splitting, are avoided. Recently, orbifold compactifications of the heterotic string have been constructed which can account for the SM in four dimensions and which have five–dimensional or six–dimensional GUT structures as intermediate step very similar to orbifold GUT models [61]. Hence, orbifold compactifications provide an attractive starting point for attempts to embed the SM into higher dimensional string theories.

3.1 $SU(5) \times U(1)_\chi \times U(1)_\psi$ model in five dimensions

The simplest GUT group which unifies the gauge interactions of the SM is $SU(5)$ [62]. Therefore we first analyze the higher dimensional SUSY GUT model based on the $SU(5) \times U(1)_\chi \times U(1)_\psi$ gauge group which is a rank–6 subgroup of E_6. For simplicity we consider a single compact extra dimension S^1, $y(= x_5)$, and assume a fixed radius with size given by the GUT scale ($R \sim 1/M_X$). The orbifold S^1/Z_2 is obtained by dividing the circle S^1 with a Z_2 transformation which acts on S^1 according to $y \to -y$. The components of the $SU(5)$ supermultiplets that propagate in 5 dimensions transform under the specified Z_2 action as $\Phi(x_\mu, -y) = P\Phi(x_\mu, y)$, where P acts on each component of the $SU(5)$ representation Φ, making some components positive and some components negative, i.e. $P = (+, +, ...-, -, ...)$. The Lagrangian should be invariant under the Z_2 transformations[5]. The Z_2 transformation can be regarded as equivalence relation that allows to reduce the circle S^1 to the interval $y \in [0, \pi R]$.

[5]It is worth to point out that the Z_2 invariance of the Lagrangian does not require that $P = \pm I$, where I is the unit matrix. In general, matrix P should satisfy the condition $P^2 = I$.

Here we consider a 5-dimensional space-time factorized into a product of the ordinary $4D$ Minkowski space time M^4 and the orbifold $S^1/(Z_2 \times Z_2')$. The orbifold $S^1/(Z_2 \times Z_2')$ is obtained by dividing S^1/Z_2 with another Z_2 transformation, denoted by Z_2', which acts as $y' \to -y'$, with $y' \equiv y - \pi R/2$. Each reflection symmetry, $y \to -y$ and $y' \to -y'$, has its own orbifold parity, P and P', which are defined by

$$\begin{aligned} \Phi(x,y) \to \Phi(x,-y) = P\Phi(x_\mu, y)\,, \\ \Phi(x,y') \to \Phi(x,-y') = P'\Phi(x_\mu, y') \end{aligned} \tag{24}$$

where $\Phi(x,y)$ is an $SU(5)$ multiplet field living in the $5D$ bulk, while P and P' are matrix representations of the two Z_2 operator actions which have eigenvalues ± 1. All interactions must be invariant under $Z_2 \times Z_2'$ symmetry.

Each reflection also introduces special points, O and O', located at $y = 0$ and $y = \pi R/2 \equiv \ell$ which are fixed points of the transformations. The equivalences associated with the two reflection symmetries allow to work with the theory obtained by truncating to the physically irreducible interval $y \in [0, \ell]$ with the two $4D$ walls (branes) placed at the fixed points $y = 0$ and $y = \ell$. These are only two inequivalent branes (the branes at $y = \pi R$ and $y = -\pi R/2$ are identified with those at $y = 0$ and $y = \pi R/2$, respectively). Thus physical space reduces to the interval $[0, \ell]$ with a length of $\pi R/2$.

Denoting the $5D$ bulk field with $(P,\, P') = (\pm 1, \pm 1)$ by $\phi_{\pm\pm}$ one obtains the following Fourier expansions [46]–[49]:

$$\phi_{++}(x,y) = \sum_{n=0}^{\infty} \frac{1}{\sqrt{2^{\delta_{n,0}}}} \sqrt{\frac{4}{\pi R}} \phi_{++}^{(2n)}(x) \cos\frac{2ny}{R}\,, \tag{25}$$

$$\phi_{+-}(x,y) = \sum_{n=0}^{\infty} \sqrt{\frac{4}{\pi R}} \phi_{+-}^{(2n+1)}(x) \cos\frac{(2n+1)y}{R}\,, \tag{26}$$

$$\phi_{-+}(x,y) = \sum_{n=0}^{\infty} \sqrt{\frac{4}{\pi R}} \phi_{-+}^{(2n+1)}(x) \sin\frac{(2n+1)y}{R}\,, \tag{27}$$

$$\phi_{--}(x,y) = \sum_{n=0}^{\infty} \sqrt{\frac{4}{\pi R}} \phi_{--}^{(2n+2)}(x) \sin\frac{(2n+2)y}{R}\,, \tag{28}$$

where n is a non–negative integer. From the $4D$ perspective the Fourier component fields $\phi_{++}^{(2n)}(x)$, $\phi_{+-}^{(2n+1)}(x)$, $\phi_{-+}^{(2n+1)}(x)$ and $\phi_{--}^{(2n+2)}(x)$ acquire masses $2n/R$, $(2n+1)/R$, $(2n+1)/R$ and $(2n+2)/R$ upon compactification. Note that only $\phi_{++}(x,y)$ and $\phi_{+-}(x,y)$ can exist on the $y = 0$ brane. The fields $\phi_{++}(x,y)$ and $\phi_{-+}(x,y)$ are non–vanishing on the $y = \pi R/2$ brane, whereas the field $\phi_{--}(x,y)$ vanishes on both branes. Only $\phi_{++}(x,y)$ fields have zero–modes. Since full $SU(5)$ $5D$ multiplets $\Phi_i(x,y)$ can, in general, contain components with even and odd parities, P and P', the matter content of the massless sector can be smaller than that of the full $5D$ multiplet. Unless all components of $\Phi(x,y)$ have common parities, the gauge symmetry reduction occurs upon compactification.

As in the case of the simplest orbifold GUT scenarios [46]–[49] we start from the model with the minimal SUSY in $5D$ (with 8 real supercharges, corresponding to $N = 2$

in 4D). We assume that the vector supermultiplets associated with the $SU(5)$, $U(1)_\chi$ and $U(1)_\psi$ interactions exist in the bulk $M^4 \times S^1/(Z_2 \times Z_2')$. The $5D$ gauge supermultiplets contain vector bosons A_M ($M = 0, 1, 2, 3, 5$) and gauginos. The $5D$ gaugino is composed of two $4D$ Weyl fermions with opposite $4D$ chirality, λ and λ'. In addition $5D$ vector supermultiplets have to involve real scalars σ to ensure that the numbers of bosonic and fermionic degrees of freedom are equal. Thus $5D$ gauge supermultiplets can be decomposed into vector supermultiplets V with physical components (A_μ, λ) and chiral multiplets Σ with components $\left((\sigma + iA_5)/\sqrt{2}, \lambda'\right)$ under $N = 1$ supersymmetry in $4D$. These two $N = 1$ supermultiplets also form $N = 2$ vector supermultiplet in $4D$.

In addition to the $5D$ vector supermultiplets we assume the presence of other $SU(5)$ representations as well as $SU(5)$ singlet superfields that carry non–zero $U(1)_\chi$ and $U(1)_\psi$ charges in the $5D$ bulk. The corresponding representations also contain $5D$ fermions. Since each $5D$ fermion state is composed of two $4D$ Weyl fermions, ψ and ψ^c, SUSY implies that each $5D$ supermultiplet includes two complex scalars ϕ and ϕ^c as well. The states ϕ, ψ, ϕ^c and ψ^c form one $4D$ $N = 2$ hypermultiplet that consists of two $4D$ $N = 1$ chiral multiplets, $\hat{\Phi} \equiv (\phi,\, \psi)$ and $\hat{\Phi}^c \equiv (\phi^c,\, \psi^c)$, transforming as conjugate representations with each other under the gauge group.

Taking into account that the derivative ∂_5 is odd under the reflection Z_2 one can show that the 5D SUSY Lagrangian is invariant under the following transformations [46]

$$\begin{aligned}
A_\mu(x,y) &\to A_\mu(x,-y) = PA_\mu(x,y)P^{-1}\,,\\
A_5(x,y) &\to A_5(x,-y) = -PA_5(x,y)P^{-1}\,,\\
\sigma(x,y) &\to \sigma(x,-y) = -P\sigma(x,y)P^{-1}\,,\\
\lambda(x,y) &\to \lambda(x,-y) = P\lambda(x,y)P^{-1}\,,\\
\lambda'(x,y) &\to \lambda'(x,-y) = -P\lambda'(x,y)P^{-1}\,,\\
\phi_i(x,y) &\to \phi_i(x,-y) = P\phi_i(x,y)\,,\\
\psi_i(x,y) &\to \psi_i(x,-y) = P\psi_i(x,y)\,,\\
\phi_i^c(x,y) &\to \phi_i^c(x,-y) = -P\phi_i^c(x,y)\,,\\
\psi_i^c(x,y) &\to \psi_i^c(x,-y) = -P\psi_i^c(x,y)\,,
\end{aligned} \tag{29}$$

where index i represents different $SU(5)$ supermultiplets that exist in the bulk $M^4 \times S^1/(Z_2 \times Z_2')$. In the case of $SU(5)$ the components of the corresponding $N = 2$ vector supermultiplet in Eq. (29) are given by $V(x,y) = V^A(x,y)T^A$ and $\Sigma(x,y) = \Sigma^A(x,y)T^A$, where T^A is the set of the $SU(5)$ generators ($A = 1, 2, ..., 24$). The transformations in Eq. (29) are associated with the Z_2 reflection symmetry. By replacing y and P by y' and P' in Eq. (29) one obtains Z_2' transformations. Note that mass terms for ϕ_i, ψ_i, ϕ_i^c and ψ_i^c are allowed by $N = 2$ SUSY but these terms are not compatible with the P and P' parity assignments as follows from Eq. (29). Therefore the zero–modes of these fields do not receive a bulk mass contribution.

It is convenient to choose the matrix representation of the parity assignment P, expressed in the fundamental representation of $SU(5)$, to be $P = \mathrm{diag}(+1, +1, +1, +1, +1)$ so that $V^A(x,-y)T^A = V^A(x,y)T^A$. This boundary condition does not break $SU(5)$ on the O brane at $y = 0$. However $4D$ $N = 2$ supersymmetry gets broken by this parity assignment to $4D$ $N = 1$ SUSY. This can be seen explicitly by examining the masses of the Kaluza–Klein (KK) towers of the fields. Indeed, according to the parity assignment P only A_μ, λ, ϕ and ψ are allowed to have zero–modes whereas other components of the

$N = 2$ vector supermultiplet (σ, λ') and $N = 2$ hypermultiplets $(\phi_i^c,\ \psi_i^c)$ with odd parity P do not possess massless modes. For the $SU(5)$ gauge symmetry to provide an understanding of the quark and lepton quantum numbers, the three families of 27_i representations of E_6 should reside on the O brane where the $SU(5) \times U(1)_\chi \times U(1)_\psi$ gauge symmetry and $N = 1$ SUSY remains intact. Then at low energies all ordinary quarks and leptons have to fill in complete $SU(5)$ multiplets.

The $5D$ $SU(5)$ gauge symmetry is reduced to $4D$ $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge symmetry by choosing $P' = \mathrm{diag}(-1, -1, -1, +1, +1)$ acting on the fundamental representation of $SU(5)$. This boundary condition breaks not only $SU(5)$ but also $4D$ $N = 2$ SUSY to $4D$ $N = 1$ SUSY on the O' brane at $y = \ell$. The parity assignment associated with the Z_2' reflection symmetry leads to the two types of the $SU(5)$ gauge generators T^a and $T^{\hat{a}}$. All generators of the SM gauge group satisfy the condition

$$P'\, T^a\, P' = T^a\,. \tag{30}$$

Therefore the corresponding gauge fields $A_\mu^a(x, y)$ and gauginos $\lambda^a(x, y)$ are even under the reflections Z_2 and Z_2' whereas $\sigma^a(x, y)$ and $\lambda'^a(x, y)$ are odd. As a consequence the KK expansions of vector bosons $A_\mu^a(x, y)$ and gauginos $\lambda^a(x, y)$ contain massless zero modes $A_\mu^{a(0)}(x)$ and $\lambda^{a(0)}(x)$ corresponding to the unbroken gauge symmetry of the SM. These zero modes form $4D$ $N = 1$ vector supermultiplets. The KK modes $A_5^{a(2n)}(x)$ are swallowed by $A_\mu^{a(2n)}(x)$ resulting in the formation of vector boson state with mass $2n/R$. The KK gaugino modes $\lambda^{a(2n)}(x)$ and $\lambda'^{a(2n)}(x)$ form $4D$ fermion state with mass $2n/R$. The KK scalar mode $\sigma^{a(2n)}(x)$ also gains mass $2n/R$.

The other gauge generators $T^{\hat{a}}$ of $SU(5)$ obey the relationship

$$P'\, T^{\hat{a}}\, P' = -T^{\hat{a}}\,, \tag{31}$$

which implies that $A_\mu^{\hat{a}}(x, y)$ and $\lambda^{\hat{a}}(x, y)$ are odd under the Z_2' symmetry while $\sigma^{\hat{a}}(x, y)$ and $\lambda'^{\hat{a}}(x, y)$ are even. This means that all components of the $5D$ vector supermultiplet associated with the broken $SU(5)$ generators $T^{\hat{a}}$ are odd either under the reflection Z_2 or Z_2' so that their KK expansions does not possess massless modes. The Z_2 and Z_2' parity assignments for all components of the $5D$ bulk vector supermultiplets are shown in Table 2. The KK modes $A_\mu^{\hat{a}(2n+1)}(x)$, $A_5^{\hat{a}(2n+1)}(x)$, $\sigma^{\hat{a}(2n+1)}(x)$, $\lambda^{\hat{a}(2n+1)}(x)$ and $\lambda'^{\hat{a}(2n+1)}(x)$ form vector boson, scalar and fermion states with masses $(2n + 1)/R$.

$5D$ fields	$SU(3)_C \times SU(2)_W$ quantum numbers	$Z_2 \times Z_2'$ parity	Mass
$A_\mu^a,\ \lambda^a$	$(8,1) + (1,3) + (1,1)$	$(+,+)$	$2n/R$
$A_\mu^{\hat{a}},\ \lambda^{\hat{a}}$	$(3,2) + (\bar{3},2)$	$(+,-)$	$(2n+1)/R$
$A_5^a,\ \sigma^a,\ \lambda'^a$	$(8,1) + (1,3) + (1,1)$	$(-,-)$	$(2n+2)/R$
$A_5^{\hat{a}},\ \sigma^{\hat{a}},\ \lambda'^{\hat{a}}$	$(3,2) + (\bar{3},2)$	$(-,+)$	$(2n+1)/R$
$A_\mu^\chi,\ \lambda_\chi$	$(1,1)$	$(+,+)$	$2n/R$
$A_5^\chi,\ \sigma_\chi,\ \lambda'_\chi$	$(1,1)$	$(-,-)$	$(2n+2)/R$
$A_\mu^\psi,\ \lambda_\psi$	$(1,1)$	$(+,+)$	$2n/R$
$A_5^\psi,\ \sigma_\psi,\ \lambda'_\psi$	$(1,1)$	$(-,-)$	$(2n+2)/R$

Table 2: Parity assignments and KK masses of fields in the $5D$ bulk vector supermultiplets associated with the $SU(5)$, $U(1)_\psi$ and $U(1)_\chi$ gauge interactions.

At the fixed point O' the gauge transformations generated by $T^{\hat{a}}$ as well as the corresponding components of the $5D$ $SU(5)$ vector supermultiplet vanish. At the same time at an arbitrary point in the bulk all generators of the $SU(5)$ gauge group are operative. Thus orbifold procedure leads to a local explicit breaking of $SU(5)$ at the fixed point O' due to the non–trivial orbifold quantum numbers of the gauge parameters.

The Z_2 and Z_2' parity assignments for the components of the $U(1)_\psi$ and $U(1)_\chi$ bulk vector supermultiplets are such that the KK expansions of vector bosons $A_\mu^\chi(x,y)$ and $A_\mu^\psi(x,y)$ as well as the corresponding gaugino states $\lambda_\chi(x,y)$ and $\lambda_\psi(x,y)$ contain massless zero modes $A_\mu^{\chi(0)}(x)$, $A_\mu^{\psi(0)}(x)$, $\lambda_\chi^{(0)}(x)$ and $\lambda_\psi^{(0)}(x)$ associated with the unbroken $U(1)_\psi$ and $U(1)_\chi$ gauge symmetries (see Table 2). Other KK modes form vector boson, scalar and fermion states with masses $(2n+2)/R$ similar to the ones that appear in the case of unbroken generators T^a of $SU(5)$.

As in the simplest orbifold GUT scenarios [46]–[48] we assume that all incomplete $SU(5)$ supermultiplets which are even under the custodial symmetry (the matter parity Z_2^M in the case of the MSSM and the $\tilde{Z}_2^H$ symmetry in the case of the E$_6$SSM) originate from the $5D$ bulk supermultiplets. In order to ensure that H_u and $\overline{H}_u$ as well as H_d and $\overline{H}_d$ survive below the scale $M_X \sim 1/R$ we include two pairs of the $5D$ $SU(5)$ bulk supermultiplets $\Phi_{H_u} + \Phi_{\overline{H}_u}$ and $\Phi_{H_d} + \Phi_{\overline{H}_d}$ that decompose as follows

$$\Phi_{H_u} = \Phi_{\overline{H}_u} = \left(5,\, -\frac{2}{\sqrt{24}},\, \frac{2}{\sqrt{40}}\right), \qquad \Phi_{H_d} = \Phi_{\overline{H}_d} = \left(5,\, \frac{2}{\sqrt{24}},\, \frac{2}{\sqrt{40}}\right), \tag{32}$$

where first, second and third quantities in brackets are the $SU(5)$ representation, extra $U(1)_\psi$ and $U(1)_\chi$ charges respectively. The multiplets Φ_{H_u} and $\Phi_{\overline{H}_u}$ as well as Φ_{H_d} and $\Phi_{\overline{H}_d}$ transform differently under Z_2 and Z_2' (see Table 3). Since P' does not commute with $SU(5)$ each $5D$ 5–plet is divided into four pieces associated with different $N=1$ chiral supermultiplets:

$$5 = (3,1,-1/3) + (1,2,1/2) + (\bar{3},1,1/3) + (1,2,-1/2)\,. \tag{33}$$

In Eq. (33) first and second quantities in brackets are $SU(3)_C$ and $SU(2)_W$ quantum numbers whereas the third quantity is $U(1)_Y$ charge. As one can see from Table 3 chiral supermultiplets in Eq. (33) have different P and P' parity assignments that result in different KK mode structures. These parity assignments are such that the orbifold projection accomplishes doublet–triplet splitting, in the sense that only one doublet superfield in Eq. (33) has zero mode while the KK expansions of another doublet, triplet and antitriplet superfields do not possess massless modes. Thus only H_u, $\overline{H}_u$, H_d and $\overline{H}_d$ may survive to low–energies.

The $4D$ superfields N_H^c, $\overline{N}_H^c$, S and $\overline{S}$ can stem from the $5D$ SM singlet superfields that carry $U(1)_\psi$ and $U(1)_\chi$ charges

$$\Phi_S = \Phi_{\overline{S}} = \left(1,\, \frac{4}{\sqrt{24}},\, 0\right), \qquad \Phi_{N_H^c} = \Phi_{\overline{N}_H^c} = \left(1,\, \frac{1}{\sqrt{24}},\, -\frac{5}{\sqrt{40}}\right). \tag{34}$$

According to Eq. (29) only either ϕ_i and ψ_i or ϕ_i^c and ψ_i^c can have massless modes. Different parity assignments of Φ_S and $\Phi_{\overline{S}}$ as well as $\Phi_{N_H^c}$ and $\Phi_{\overline{N}_H^c}$ allow to project out different components of these superfields so that only $4D$ superfields N_H^c, $\overline{N}_H^c$, S and $\overline{S}$ may be light (see Table 3).

Finally, the particle spectrum below the scale M_X should be supplemented by either L_4 and $\overline{L}_4$ or d_4^c and $\overline{d^c}_4$ (but not both) to allow the lightest exotic quarks to decay. These

$5D$ fields	$SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$ quantum numbers	$Z_2 \times Z_2'$ parity	Mass
$\Phi_{H_u} + \Phi_{\overline{H}_u}$	$(3,1,-1/3,-2,2) + (\bar{3},1,1/3,2,-2)$	$(+,-)$	$(2n+1)/R$
	$(1,2,1/2,-2,2) + (1,2,-1/2,2,-2)$	$(+,+)$	$2n/R$
	$(\bar{3},1,1/3,2,-2) + (3,1,-1/3,-2,2)$	$(-,+)$	$(2n+1)/R$
	$(1,2,-1/2,2,-2) + (1,2,1/2,-2,2)$	$(-,-)$	$(2n+2)/R$
$\Phi_{H_d} + \Phi_{\overline{H}_d}$	$(3,1,-1/3,2,2) + (\bar{3},1,1/3,-2,-2)$	$(-,+)$	$(2n+1)/R$
	$(1,2,1/2,2,2) + (1,2,-1/2,-2,-2)$	$(-,-)$	$(2n+2)/R$
	$(\bar{3},1,1/3,-2,-2) + (3,1,-1/3,2,2)$	$(+,-)$	$(2n+1)/R$
	$(1,2,-1/2,-2,-2) + (1,2,1/2,2,2)$	$(+,+)$	$2n/R$
$\Phi_S + \Phi_{\overline{S}}$	$(1,1,0,4,0) + (1,1,0,-4,0)$	$(+,+)$	$2n/R$
	$(1,1,0,-4,0) + (1,1,0,4,0)$	$(-,-)$	$(2n+2)/R$
$\Phi_{N_H^c} + \Phi_{\overline{N}_H^c}$	$(1,1,0,1,-5) + (1,1,0,-1,5)$	$(+,+)$	$2n/R$
	$(1,1,0,-1,5) + (1,1,0,1,-5)$	$(-,-)$	$(2n+2)/R$
$\Phi_{L_4} + \Phi_{\overline{L}_4}$	$(3,1,-1/3,-1,-3) + (\bar{3},1,1/3,1,3)$	$(-,+)$	$(2n+1)/R$
	$(1,2,1/2,-1,-3) + (1,2,-1/2,1,3)$	$(-,-)$	$(2n+2)/R$
	$(\bar{3},1,1/3,1,3) + (3,1,-1/3,-1,-3)$	$(+,-)$	$(2n+1)/R$
	$(1,2,-1/2,1,3) + (1,2,1/2,-1,-3)$	$(+,+)$	$2n/R$
$\Phi_{d_4^c} + \Phi_{\overline{d^c}_4}$	$(3,1,-1/3,-1,-3) + (\bar{3},1,1/3,1,3)$	$(-,-)$	$(2n+2)/R$
	$(1,2,1/2,-1,-3) + (1,2,-1/2,1,3)$	$(-,+)$	$(2n+1)/R$
	$(\bar{3},1,1/3,1,3) + (3,1,-1/3,-1,-3)$	$(+,+)$	$2n/R$
	$(1,2,-1/2,1,3) + (1,2,1/2,-1,-3)$	$(+,-)$	$(2n+1)/R$

Table 3: Parity assignments and KK masses of fields in the $4D$ chiral supermultiplets resulting from the $5D$ bulk supermultiplets Φ_{H_u}, $\Phi_{\overline{H}_u}$, Φ_{H_d}, $\Phi_{\overline{H}_d}$ Φ_S, $\Phi_{\overline{S}}$, $\Phi_{N_H^c}$, $\Phi_{\overline{N}_H^c}$, Φ_{L_4}, $\Phi_{\overline{L}_4}$ $\Phi_{d_4^c}$ and $\Phi_{\overline{d^c}_4}$.

$4D$ $N=1$ chiral superfields can come from either Φ_{L_4} and $\Phi_{\overline{L}_4}$ or $\Phi_{d_4^c}$ and $\Phi_{\overline{d^c}_4}$ which are $5D$ $SU(5)$ bulk supermultiplets with quantum numbers

$$\Phi_{L_4} = \Phi_{\overline{L}_4} = \Phi_{d_4^c} = \Phi_{\overline{d^c}_4} = \left(5, -\frac{1}{\sqrt{24}}, -\frac{3}{\sqrt{40}}\right) . \tag{35}$$

Again parity assignments guarantee that only two $4D$ doublet superfields L_4 and $\overline{L}_4$ from Φ_{L_4} and $\Phi_{\overline{L}_4}$ can survive to low–energies whereas the other $SU(2)_W$ doublet, color triplet and antitriplet partners do not have zero modes. Using the freedom to flip the overall action of the P' parity on the $SU(5)$ multiplets by a sign relative to $\Phi_{L_4} + \Phi_{\overline{L}_4}$ one can get the KK spectrum in which only triplet or antitriplet components of $SU(5)$ fundamental supermultiplets possess massless modes. From Table 3 one can see that this freedom is used in the case $\Phi_{d_4^c}$ and $\Phi_{\overline{d^c}_4}$ supermultiplets. Due to the different structure of the KK spectrum only $4D$ triplet or antitriplet superfields, $\overline{d^c}_4$ and d_4^c, from $\Phi_{\overline{d^c}_4}$ and $\Phi_{d_4^c}$ are allowed to be light.

Since the three families of 27_i representations of E_6 are located on the O brane, where the $SU(5) \times U(1)_\chi \times U(1)_\psi$ gauge symmetry remains intact, the Yukawa interactions of quarks and leptons are necessarily $SU(5)$ symmetric. In general the $SU(5)$ invariance yields the prediction for the first and second generation fermion mass ratios $m_s/m_d = m_\mu/m_e$, which is in conflict with the data. In $4D$ GUTs acceptable mass relations can be obtained using higher dimensional operators and relatively large representations which acquire VEVs breaking $SU(5)$ or $SO(10)$ [45], [63]. In the case of the

simplest 5D orbifold GUTs there are no $SU(5)$ breaking VEVs. Nevertheless in this case one can introduce two additional $5D$ bulk supermultiplets with quantum numbers given by Eq. (35) that transform under Z_2 and Z_2' as either Φ_{L_4} and $\Phi_{\overline{L}_4}$ or $\Phi_{d_4^c}$ and $\Phi_{\overline{d^c}_4}$. Furthermore we assume that these bulk supermultiplets are odd under $\tilde{Z}_2^H$ symmetry which is defined on the O brane. Hence the zero modes of these extra $5D$ supermultiplets, which are either weak doublets (L_5 and $\overline{L}_5$) or $SU(3)_C$ triplet and antitriplet ($\overline{d^c}_5$ and d_5^c), can mix with quark or lepton superfields from 27_i spoiling the $SU(5)$ relations between the down type quark and charged lepton masses. Indeed, suppose that zero modes are weak doublet superfields L_5 and $\overline{L}_5$. Then $\overline{L}_5$ can get combined with the superposition of lepton doublet superfields from 27_i so that the resulting vectorlike states gain masses slightly below M_X. The remaining three families of lepton doublets, that survive to low energies, are superpositions of the corresponding components from 27_i and L_5 while three generations of down type quarks stem from 27_i completely. As a consequence the $SU(5)$ relations between the down type quark and charged lepton masses may get spoiled entirely if the Yukawa couplings of L_5 to Higgs doublet H_d are relatively large ($\sim 0.01 - 0.1$).

$5D$ fields	$SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$ quantum numbers	$Z_2 \times Z_2'$ parity	Mass
$\Phi_{e^c} + \Phi_{\overline{e^c}}$	$(\bar{3}, 1, -2/3, 1, -1) + (3, 1, 2/3, -1, 1)$	$(+,+)$	$2n/R$
	$(3, 2, 1/6, 1, -1) + (\bar{3}, 2, -1/6, -1, 1)$	$(+,-)$	$(2n+1)/R$
	$(1, 1, 1, 1, -1) + (1, 1, -1, -1, 1)$	$(+,+)$	$2n/R$
	$(3, 1, 2/3, -1, 1) + (\bar{3}, 1, -2/3, 1, -1)$	$(-,-)$	$(2n+2)/R$
	$(\bar{3}, 2, -1/6, -1, 1) + (3, 2, 1/6, 1, -1)$	$(-,+)$	$(2n+1)/R$
	$(1, 1, -1, -1, 1) + (1, 1, 1, 1, -1)$	$(-,-)$	$(2n+2)/R$

Table 4: The (Z_2, Z_2') transformation properties and KK masses of $4D$ chiral supermultiplets that stem from $SU(5)$ bulk supermultiplets Φ_{e^c} and $\Phi_{\overline{e^c}}$.

Although the discussed specific realization of the mechanism which allows to obtain the realistic pattern of fermion masses is the simplest one it is worth to consider another very attractive possibility. Instead of two additional $5D$ $SU(5)$ fundamental supermultiplets one can include two larger representations of $SU(5)$ that decompose under $SU(5) \times U(1)_\chi \times U(1)_\psi$ as follows:

$$\Phi_{e^c} = \Phi_{\overline{e^c}} = \left(10,\, \frac{1}{\sqrt{24}},\, -\frac{1}{\sqrt{40}}\right) . \tag{36}$$

As before we assume that Φ_{e^c} and $\Phi_{\overline{e^c}}$ supermultiplets are odd under $\tilde{Z}_2^H$ symmetry. Due to P and P' parity assignments each $SU(5)$ bulk decuplet is divided into six pieces associated with different $N = 1$ chiral supermultiplets:

$$10 = (\bar{3}, 1, -2/3) + (3, 2, 1/6) + (1, 1, 1) + (3, 1, 2/3) + (\bar{3}, 2, -1/6) + (1, 1, -1)\,, \tag{37}$$

where quantities in brackets are $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ quantum numbers. The Z_2 and Z_2' parity assignments and mass spectrum for all components of the $5D$ decuplets are given in Table 4. These parity assignments guarantee that only two $4D$ $SU(2)_W$ singlet superfields (e_5^c and $\overline{e^c}_5$) as well as $4D$ triplet and antitriplet supermultiplets (u_5^c and $\overline{u^c}_5$) from Φ_{e^c} and $\Phi_{\overline{e^c}}$ can survive below scale $M_X \sim 1/R$. Again $\overline{e^c}_5$ and $\overline{u^c}_5$ can get combined with the superposition of the appropriate components of 27_i forming vectorlike

states which may have masses slightly below M_X. At the same time e_5^c can mix with the corresponding components of 27_i spoiling the $SU(5)$ relations between the masses of the down type quarks and charged leptons. It is worth noting that together bulk supermultiplets (32), (34), (35) and (36) form two complete 27 representations of E_6. This simplifies the structure of bulk supermultiplets making the considered $5D$ orbifold GUT model more elegant.

For the consistency of the considered model it is crucial that all anomalies get cancelled. In $5D$ theories no bulk anomalies exist. Nevertheless orbifold compactification may lead to anomalies at orbifold fixpoints [64]–[65]. At the fixed point brane anomaly reduces to the anomaly of the unbroken subgroup of the original group, i.e. $SU(5) \times U(1)_\chi \times U(1)_\psi$ on the O brane and $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\chi \times U(1)_\psi$ on the O' brane. It was also shown that the sum of the contributions to the $4D$ anomalies at the fixpoint equals to the sum of the contributions of the zero modes localized at the corresponding brane [64]–[65]. In this context it is worth to emphasize that the contributions of three families of 27_i representations of E_6, which reside on the O brane, to the anomalies associated with this fixpoint get cancelled automatically. Moreover from Tables 3 and 4 one can see that the P and P' parity assignments are chosen so that the zero modes of the bulk fields localized at the O and O' branes always form pairs of $N = 1$ supermultiplets with opposite quantum numbers. Such choice of parity assignments guarantees that the contributions of zero modes of the bulk superfields to the brane anomalies are cancelled as well.

Another important issue for any GUT model is proton stability which was discussed in the context of $5D$ orbifold GUT models in [47], [51]–[52]. In orbifold GUT models the dimension five operators, which are caused by an exchange of the color triplet Higgsino multiplets and give rise to proton decay in ordinary GUTs, do not get induced. Indeed, in the considered class of models colored Higgsinos acquire mass via the KK mode expansion of operators $\psi_i \partial_5 \psi_i^c$ that leads to the Dirac mass terms of the form $\psi_i^{(2n+1)} \psi_i^{c(2n+1)}$. Since $\psi_i^{c(2n+1)}$ do not couple directly to the quarks (squarks) and sleptons (leptons) the dimension five operators are not generated. It turns out that the absence of tree-level amplitudes caused by the colored Higgsino exchange which result in proton decay is deeply entangled with the orbifold construction and continuous global $U(1)_R$ symmetry that $5D$ bulk Lagrangian possesses [47]. Although the dimension five operators discussed above do not get induced within orbifold GUT models one must also suppress the brane interactions $[QQQL]_F$ and $[u^c u^c d^c e^c]_F$ that may be already present on the O brane as non–renormalizable interactions. Such operators can give a substantial contribution to the proton decay rate if the fundamental scale of gravity is close to the GUT scale. In the $5D$ orbifold GUT model considered here these dangerous operators are forbidden by $U(1)_\chi$ and $U(1)_\psi$ gauge symmetries. Nevertheless proton decay is mediated by dimension six operators induced by the leptoquark gauge bosons [66].

Finally, one should mention that in the $5D$ orbifold GUT models gauge couplings of the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ interactions do not exactly unify at the scale $M_X \sim 1/R$ where $SU(5)$ gauge symmetry gets broken. The reason for this is that the symmetry of the model on the GUT–breaking brane O' remains limited to the SM gauge group. In particular, on this brane there are brane–localized $4D$ kinetic terms for the SM gauge fields with $SU(5)$–violating coefficients $1/g_{O'i}^2$. The part of the $5D$ effective SUSY Lagrangian that contains kinetic terms for the SM gauge fields can be written as follows

$$\mathcal{L}_{eff} = \int d^2\theta \left(\frac{1}{g_5^2} + \frac{1}{2g_O^2} \left\{ \delta(y) + \delta(y - \pi R) \right\} \right) \mathrm{Tr}\, \mathcal{W}^\alpha \mathcal{W}_\alpha \tag{38}$$

$$+\sum_i \int d^2\theta \frac{1}{2g^2_{O'i}} \Big\{ \delta(y-\frac{\pi}{2}R) + \delta(y+\frac{\pi}{2}R) \Big\} \mathrm{Tr}\, \mathcal{W}^\alpha_i \mathcal{W}^i_\alpha + \mathrm{h.c.},$$

where $\mathcal{W}^i_\alpha$ $(i=1,2,3)$ are the supersymmetric gauge field strengths of the $U(1)_Y$, $SU(2)_W$ and $SU(3)_C$ gauge interactions on the O' brane, and $\mathcal{W}_\alpha$ is the $SU(5)$ gauge field strength on the O brane and in the bulk [6]. Integrating over y one obtains zero–mode $4D$ SM gauge couplings at the scale $M_X \sim 1/R$

$$\frac{1}{g^2_i(M_X)} = \frac{2\pi R}{g^2_5} + \frac{1}{g^2_O} + \frac{1}{g^2_{O'i}}\,. \tag{39}$$

Since $SU(5)$–violating coefficients $1/g^2_{O'i}$ may differ from each other substantially the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ gauge couplings $g^2_i(M_X)$ are not identical. However if in the $5D$ model the bulk and brane gauge couplings have almost equal strength then after integrating out y the zero–mode gauge couplings are dominated by the bulk contributions because of the spread of the wavefunction of the zero–mode gauge bosons. In other words the $SU(5)$–violating brane kinetic terms are dominated by the bulk contributions when the linear extent of the 5th dimension is sufficiently large. Because the bulk contributions to the gauge couplings (39) are necessarily $SU(5)$ symmetric, a $4D$ observer sees an approximate unification of the SM gauge couplings. The gauge coupling unification within $5D$ orbifold GUT models was discussed in [52]–[53].

As one can see from Eqs. (38)–(39) the discrepancy between $g^2_i(M_X)$ is determined by the $SU(5)$–violating gauge kinetic terms on the O' brane. This discrepancy is small when $g^2_i(M_X)$ are relatively small whereas $g^2_{O'i}$ are large ($g^2_{O'i} \sim 4\pi$). On the other hand one can expect that the relative contribution of the $SU(5)$–violating brane corrections to $g^2_i(M_X)$ becomes more sizable in the case when the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ gauge couplings are large at the scale M_X.

3.2 E_6 orbifold GUT model in six dimensions

Having discussed in detail the simplest 5D orbifold GUT model, that may lead at low energies to the gauge group and field content of the E_6 inspired SUSY model specified in section 2, we next study E_6 gauge theory in $6D$ with $N=1$ supersymmetry. We consider the compactification on a torus T^2 with two fixed radii R_5 and R_6 so that two extra dimensions $y(=x_5)$ and $z(=x_6)$ are compact, i.e. $y \in (-\pi R_5, \pi R_5]$ and $z \in (-\pi R_6, \pi R_6]$. The physical region associated with the compactification on the orbifold T^2/Z_2 is a pillow with the four fixed points of the Z_2 transformations ($y \to -y$, $z \to -z$) as corners. The orbifold T^2/Z_2 has the following fixpoints $(0,0)$, $(\pi R_5, 0)$, $(0, \pi R_6)$ and $(\pi R_5, \pi R_6)$.

Here we discuss E_6 gauge theory in $6D$ compactified on the orbifold $T^2/(Z_2 \times Z^I_2 \times Z^{II}_2)$. The Z_2, Z^I_2 and Z^{II}_2 symmetries are reflections. The Z_2 transformations are defined as before, i.e. $y \to -y$, $z \to -z$. The Z^I_2 reflection symmetry transformations act as $y' \to -y'$, $z \to -z$ with $y' = y - \pi R_5/2$. The reflection Z^{II}_2 corresponds to $y \to -y$, $z' \to -z'$ where $z' = z - \pi R_6/2$. The Z^I_2 and Z^{II}_2 reflection symmetries introduce additional fixed points. As in the case of $5D$ orbifold GUT models extra reflection symmetries lead to the reduction of the physical region which is again limited by the appropriate fixed points. The Z_2, Z^I_2 and Z^{II}_2 reflection symmetries allow to work with the theory obtained by

[6] Note the O' brane contribution vanish for $\mathcal{W}_\alpha$ associated with the leptoquark gauge bosons which are odd under Z'_2.

truncating to the physically irreducible space in which $y \in [0, \pi R_5/2]$ and $z \in [0, \pi R_6/2]$ with the four $4D$ walls (branes) located at its corners.

Again, we assume that the considered orbifold GUT model contains a set of E_6 bulk supermultiplets and another set of $N = 1$ superfields which are confined on one of the branes. The set of superfields that propagate in the bulk $M^4 \times T^2/(Z_2 \times Z_2^I \times Z_2^{II})$ includes E_6 gauge supermultiplet and a few 27–plets. As before all quark and lepton superfields are expected to be confined on one brane.

The E_6 gauge supermultiplet that exist in the bulk must involve vector bosons A_M ($M = 0, 1, 2, 3, 5, 6$) and $6D$ Weyl fermions (gauginos) which are composed of two $4D$ Weyl fermions, λ and λ'. These fields can be conveniently grouped into vector and chiral multiplets of the $N = 1$ supersymmetry in $4D$, i.e.

$$V = (A_\mu, \lambda)\,, \qquad\qquad \Sigma = \left((A_5 + iA_6)/\sqrt{2}, \lambda'\right), \tag{40}$$

where V, A_M, λ and λ' are matrices in the adjoint representation of E_6. Two $N = 1$ supermultiplets (40) form $N = 2$ vector supermultiplet in $4D$. The bulk $27'$ supermultiplets also include $6D$ Weyl fermion states (that involve two $4D$ Weyl fermions, ψ_i and ψ_i^c) together with two complex scalars ϕ_i and ϕ_i^c. The fields ψ_i, ψ_i^c, ϕ_i and ϕ_i^c compose $4D$ $N = 2$ hypermultiplet containing two $4D$ $N = 1$ chiral superfields: $\hat{\Phi}_i = (\phi_i,\, \psi_i)$ and its conjugate $\hat{\Phi}_i^c = (\phi_i^c,\, \psi_i^c)$ with opposite quantum numbers. Thus each bulk $27'$ supermultiplet involves two $4D$ $N = 1$ supermultiplets $27'$ and $\overline{27'}$.

To ensure the consistency of the construction the Lagrangian of the considered orbifold GUT model has to be invariant under Z_2, Z_2^I and Z_2^{II} symmetries. As in the case of 5D orbifold GUT models each reflection symmetry, Z_2, Z_2^I and Z_2^{II}, has its own orbifold parity, P, P_I and P_{II}. The components $\hat{\Phi}$ and $\hat{\Phi}^c$ of the bulk $27'$ supermultiplet Φ transform under Z_2, Z_2^I and Z_2^{II} as follows

$$\begin{array}{ll}
\hat{\Phi}(x, -y, -z) = P\hat{\Phi}(x, y, z)\,, & \hat{\Phi}^c(x, -y, -z) = -P\hat{\Phi}^c(x, y, z)\,, \\
\hat{\Phi}(x, -y', -z) = P_I\hat{\Phi}(x, y', z)\,, & \hat{\Phi}^c(x, -y', -z) = -P_I\hat{\Phi}^c(x, y', z)\,, \\
\hat{\Phi}(x, -y, -z') = P_{II}\hat{\Phi}(x, y, z')\,, & \hat{\Phi}^c(x, -y, -z') = -P_{II}\hat{\Phi}^c(x, y, z')\,,
\end{array} \tag{41}$$

where P, P_I and P_{II} are diagonal matrices with eigenvalues ± 1 that act on each component of the fundamental representation of E_6.

It is convenient to specify the matrix representation of the orbifold parity assignments in terms of the E_6 weights α_j and gauge shifts, Δ, Δ_I and Δ_{II}, associated with Z_2, Z_2^I and Z_2^{II}. The diagonal elements of the matrices P, P_I and P_{II} can be presented in the following form [55]

$$\begin{array}{c}
(P)_{jj} = \sigma \exp\{2\pi i \Delta\alpha_j\}\,, \qquad (P_I)_{jj} = \sigma_I \exp\{2\pi i \Delta_I\alpha_j\}\,, \\
(P_{II})_{jj} = \sigma_{II} \exp\{2\pi i \Delta_{II}\alpha_j\}\,,
\end{array} \tag{42}$$

where σ, σ_I and σ_{II} are parities of the bulk $27'$ supermultiplet, i.e. $\sigma, \sigma_I, \sigma_{II} \in \{+, -\}$. The particle assignments of the weights in the fundamental representation of E_6 are well known (see, for example [55]). Here we choose the following gauge shifts

$$\begin{array}{c}
\Delta = \left(\frac{1}{2},\, \frac{1}{2},\, 0,\, \frac{1}{2},\, \frac{1}{2},\, 0\right), \qquad \Delta_I = \left(\frac{1}{2},\, \frac{1}{2},\, \frac{1}{2},\, \frac{1}{2},\, \frac{1}{2},\, 0\right), \\
\Delta_{II} = \left(\frac{1}{2},\, \frac{1}{2},\, 0,\, 0,\, \frac{1}{2},\, 0\right),
\end{array} \tag{43}$$

	Q	u^c	e^c	L	d^c	N^c	S	H^u	D	H^d	$\overline{D}$
Z_2	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
Z_2^I	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$-$	$+$	$-$	$+$
Z_2^{II}	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$+$	$+$	$-$	$-$

Table 5: Orbifold parity assignments in the bulk $27'$ supermultiplet with $\sigma = \sigma_I = \sigma_{II} = +1$.

that correspond to the orbifold parity assignments shown in Table 5.

The components V and Σ of the E_6 gauge supermultiplet transform under Z_2, Z_2^I and Z_2^{II} as follows

$$\begin{array}{ll} V(x,-y,-z) = PV(x,y,z)P^{-1}\,, & \Sigma(x,-y,-z) = -P\Sigma(x,y,z)P^{-1}\,, \\ V(x,-y',-z) = P_I V(x,y',z)P_I^{-1}\,, & \Sigma(x,-y',-z) = -P_I\Sigma(x,y',z)P_I^{-1}\,, \\ V(x,-y,-z') = P_{II} V(x,y,z')P_{II}^{-1}\,, & \Sigma(x,-y,-z') = -P_{II}\Sigma(x,y,z')P_{II}^{-1}\,, \end{array} \qquad (44)$$

where $V(x,y,z) = V^A(x,y,z)T^A$ and $\Sigma(x,y,z) = \Sigma^A(x,y,z)T^A$ while T^A is the set of generators of the E_6 group. The boundary conditions given by Eqs. (41) and (44) break $4D$ $N = 2$ supersymmetry because different components of the $N = 2$ supermultiplets transform differently under Z_2, Z_2^I and Z_2^{II} reflection symmetries. Moreover since P, P_I and P_{II} are not unit matrices the E_6 gauge symmetry also gets broken by these parity assignments.

The P parity assignment indicates that on the O brane at $y = z = 0$ associated with the Z_2 reflection symmetry the E_6 gauge group is broken down to $SO(10) \times U(1)_\psi$ subgroup. Indeed, according to Table 5 the $SO(10)$ representations that compose bulk $27'$ supermultiplet ($27 \to 16{+}10{+}1$) transform differently under Z_2 symmetry, i.e. $16 \to -16$, $10 \to 10$ and $1 \to 1$. Since the considered symmetry breaking mechanism preserves the rank of the group the unbroken subgroup at the fixed point O should be $SO(10) \times U(1)_\psi$.

On the brane O_I located at the fixed point $y = \pi R_5/2$, $z = 0$ and associated with the Z_2^I symmetry the E_6 gauge symmetry is broken to $SU(6) \times SU(2)_W$. Again this follows from the P_I parity assignment in the bulk $27'$ supermultiplet. The fundamental representation of E_6 decomposes under the $SU(6) \times SU(2)_W$ as follows:

$$27 \to (\overline{15},\, 1) + (6,\, 2)\,,$$

where the first and second quantities in brackets are the $SU(6)$ and $SU(2)_W$ representations respectively. The multiplet $(6,\, 2)$ is formed by all $SU(2)_W$ doublets which are contained in 27–plet. From Table 5 one can see that all $SU(2)_W$ doublet components of the $27'$ supermultiplet transform differently under the Z_2^I reflection symmetry as compared with other components of this supermultiplet which form $(\overline{15},\, 1)$.

The E_6 gauge symmetry is also broken on the brane O_{II} placed at the fixed point $y = 0$, $z = \pi R_6/2$ of the Z_2^{II} symmetry transformations. The P_{II} parity assignment is such that 16 components of the $27'$ are odd whereas $10 + 1$ components are even or viceversa. This implies that E_6 group gets broken down to its $SO(10)' \times U(1)'$ subgroup. It is worth to emphasize here that $SO(10)$ and $SO(10)'$ are not the same $SO(10)$ subgroups of E_6. In particular, from Table 5 one can see that the 16-plets of $SO(10)$ and $SO(10)'$ are formed by different components of the fundamental representation of E_6. The $U(1)_\psi$ and $U(1)'$ charge assignments should be also different.

In addition to the three branes mentioned above there is a fourth brane located at the corner $O_{III} = (\pi R_5/2, \pi R_6/2)$ of the physically irreducible space. The Z_2^{III} reflection

symmetry associated with this brane is obtained by combining the three symmetries Z_2, Z_2^I and Z_2^{II} defined above. As a consequence the corresponding parity assignment $P_{III} = P\,P_I\,P_{II}$. Combining three parity assignments P, P_I and P_{II} it is easy to see that on the brane O_{III} the unbroken subgroup is $SO(10)'' \times \tilde{U}(1)$.

The unbroken gauge group of the effective $4D$ theory is given by the intersection of the E_6 subgroups at the fixed points. Since P and P_{II} commute with $SU(5)$ the intersection of the E_6 subgroups $SO(10) \times U(1)_\psi$ and $SO(10)' \times U(1)'$ is $SU(5) \times U(1)_\chi \times U(1)_\psi$. The intersection of $SU(6) \times SU(2)_W$ and $SU(5) \times U(1)_\chi \times U(1)_\psi$ gives the SM gauge group with two additional $U(1)$ factors, $U(1)_\psi$ and $U(1)_\chi$.

The mode expansion for the $6D$ bulk fields $\phi(x,y,z)$ with any combinations of parities reads [58]:

$$\phi_{+++}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{2^{\delta_{n,0}\delta_{m,0}}\pi\sqrt{R_5 R_6}} \phi_{+++}^{(2n,2m)}(x) \cos\left(\frac{2ny}{R_5} + \frac{2mz}{R_6}\right), \tag{45}$$

$$\phi_{+-+}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{\pi\sqrt{R_5 R_6}} \phi_{+-+}^{(2n+1,2m)}(x) \cos\left(\frac{(2n+1)y}{R_5} + \frac{2mz}{R_6}\right), \tag{46}$$

$$\phi_{++-}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{\pi\sqrt{R_5 R_6}} \phi_{++-}^{(2n,2m+1)}(x) \cos\left(\frac{2ny}{R_5} + \frac{(2m+1)z}{R_6}\right), \tag{47}$$

$$\phi_{+--}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{\pi\sqrt{R_5 R_6}} \phi_{+--}^{(2n+1,2m+1)}(x) \cos\left(\frac{(2n+1)y}{R_5} + \frac{(2m+1)z}{R_6}\right), \tag{48}$$

$$\phi_{-++}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{\pi\sqrt{R_5 R_6}} \phi_{-++}^{(2n+1,2m+1)}(x) \sin\left(\frac{(2n+1)y}{R_5} + \frac{(2m+1)z}{R_6}\right), \tag{49}$$

$$\phi_{--+}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{\pi\sqrt{R_5 R_6}} \phi_{--+}^{(2n,2m+1)}(x) \sin\left(\frac{2ny}{R_5} + \frac{(2m+1)z}{R_6}\right), \tag{50}$$

$$\phi_{-+-}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{\pi\sqrt{R_5 R_6}} \phi_{--+}^{(2n+1,2m)}(x) \sin\left(\frac{(2n+1)y}{R_5} + \frac{2mz}{R_6}\right), \tag{51}$$

$$\phi_{---}(x,y,z) = \sum_{n,m}^{\infty} \frac{1}{\pi\sqrt{R_5 R_6}} \phi_{---}^{(2n,2m)}(x) \sin\left(\frac{2ny}{R_5} + \frac{2mz}{R_6}\right), \tag{52}$$

where n and m are non–negative integers. As follows from Eqs. (45)–(52) each bosonic and fermionic KK mode $\phi^{(k,\ell)}(x)$ is characterized by two integer numbers and from the $4D$ perspective acquires mass $\sqrt{\left(\frac{k}{R_5}\right)^2 + \left(\frac{\ell}{R_5}\right)^2}$ upon compactification. Only

fields for which all parities are positive have zero modes, i.e. modes with $k = 0$ and $\ell = 0$. Such modes form $4D$ $N = 1$ massless vector multiplet of the unbroken $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$ subgroup of E_6. The corresponding $6D$ bulk fields are non–vanishing on all branes. All other KK modes of the bulk gauge fields combine to massive states. In particular, one linear combination of $A_5^{a(k,\ell)}(x)$ and $A_6^{a(k,\ell)}(x)$ play the role of the Nambu–Goldstone boson, i.e. it is swallowed by $A_\mu^{a(k,\ell)}(x)$ leading to the formation of the $4D$ vector boson state with mass $\sqrt{\left(\frac{k}{R_5}\right)^2 + \left(\frac{\ell}{R_5}\right)^2}$. Thus the mass generation of the vector boson states is analogous to the Higgs mechanism. The orthogonal superposition of $A_5^{a(k,\ell)}(x)$ and $A_6^{a(k,\ell)}(x)$ compose a scalar state with the same mass. The KK gaugino modes $\lambda^{a(k,\ell)}(x)$ and $\lambda'^{a(k,\ell)}(x)$ form $4D$ fermion state which is degenerate with the corresponding vector and scalar states.

As before we assume that all incomplete E_6 supermultiplets in the E$_6$SSM, which are even under the $\tilde{Z}_2^H$ symmetry, stem from the $6D$ bulk superfields. Hereafter we also require that the three complete families of 27_i representations of E_6 are located on the O brane where E_6 gauge group is broken down to $SO(10) \times U(1)_\psi$. The $4D$ superfields H_u and $\overline{H}_u$ can originate from the bulk $27'$–plets Φ'_{H_u} and $\Phi'_{\overline{H}_u}$ that decompose as follows

$$\Phi'_{H_u} = (27,\, +,\, -,\, +)\,, \qquad \Phi'_{\overline{H}_u} = (27,\, -,\, +,\, -)\,, \tag{53}$$

where first, second, third and fourth quantities in brackets are the E_6 representation as well as σ, σ_I and σ_{II} associated with this representation respectively. The parities of these bulk $27'$–plets are chosen so that H_u and $\overline{H}_u$ components of the $N = 1$ chiral superfields $\hat{\Phi}'_{H_u}$ and $\hat{\Phi}'^c_{\overline{H}_u}$ have positive parities with respect to Z_2, Z_2^I and Z_2^{II} reflection symmetries (see Table 5). In this context it is essential to keep in mind that the invariance of the $6D$ action requires that the parities of the $4D$ chiral supermultiplets $\hat{\Phi}'_{\overline{H}_u}$ and $\hat{\Phi}'^c_{\overline{H}_u}$ are opposite. Since the parities of H_u and $\overline{H}_u$ are positive the KK expansions of the bulk $27'$–plets Φ'_{H_u} and $\Phi'_{\overline{H}_u}$ contain zero modes that form $N = 1$ chiral superfields with quantum numbers of H_u and $\overline{H}_u$.

The $SU(2)_W$ doublet chiral superfields H_u and $\overline{H}_u$ are not the only supermultiplets from Φ'_{H_u} and $\Phi'_{\overline{H}_u}$ that may survive below the scale $M_X \sim 1/R$. Indeed, the parity assignments in Eq. (53) indicate that the $\overline{u}^c$ and $\overline{e}^c$ components of the $\hat{\Phi}'^c_{H_u}$ as well as u^c and e^c components of the $\hat{\Phi}'_{\overline{H}_u}$ also have positive parities with respect to Z_2, Z_2^I and Z_2^{II} symmetries. It means that the KK mode structures of the bulk supermultiplets Φ'_{H_u} and $\Phi'_{\overline{H}_u}$ involve zero modes that correspond to $N = 1$ chiral superfields u^c, e^c, $\overline{u}^c$ and $\overline{e}^c$. Because the E_6 gauge symmetry is broken down to the $SO(10) \times U(1)_\psi$ subgroup on the O brane the zero modes that come from the same bulk $27'$–plet but belong to different $SO(10)$ representations are not required to have the same transformation properties under the custodial $\tilde{Z}_2^H$ symmetry. This permits us to assume that $4D$ chiral superfields u^c, e^c, $\overline{u}^c$ and $\overline{e}^c$ are odd under the $\tilde{Z}_2^H$ symmetry. Then these supermultiplets are expected to mix with the appropriate components from other 27–plets forming vectorlike states with masses slightly below M_X and spoiling the $SO(10)$ relations between the Yukawa couplings of quarks and leptons to H_u and H_d as it is discussed in the previous subsection.

The $4D$ superfields H_d and $\overline{H}_d$ can originate from another pair of bulk $27'$–plets

$$\Phi'_{H_d} = (27,\, +,\, -,\, -)\,, \qquad \Phi'_{\overline{H}_d} = (27,\, -,\, +,\, +)\,. \tag{54}$$

Using the orbifold parity assignments presented in Table 5 it is easy to check that all parities of H_d and $\overline{H}_d$ components of the $N = 1$ superfields $\hat{\Phi}'_{H_d}$ and $\hat{\Phi}'^c_{\overline{H}_d}$ are positive

so that the KK expansions of $6D$ superfields Φ'_{H_u} and $\Phi'_{\overline{H}_u}$ contain the appropriate zero modes. On the other hand one can also find that $\overline{d}^c$ and $\overline{N}^c$ components of the $\hat{\Phi}'^c_{H_d}$ as well as d^c and N^c components of the $\hat{\Phi}'_{\overline{H}_d}$ also have positive parities with respect to Z_2, Z_2^I and Z_2^{II} reflection symmetries. Therefore the particle content below the scale M_X includes bosonic and fermionic states from $N = 1$ chiral supermultiplets d^c, N^c, $\overline{d}^c$ and $\overline{N}^c$ as well. The scalar components of the $4D$ superfields N^c and $\overline{N}^c$ can be used to break $U(1)_\psi$ and $U(1)_\chi$ down to $U(1)_N \times Z_2^M$. Because of this the supermultiplets d^c, N^c, $\overline{d}^c$ and $\overline{N}^c$ are expected to be even under the $\tilde{Z}_2^H$ symmetry and therefore can not mix with the components of 27_i localised on the O brane. The large VEVs of N^c and $\overline{N}^c$ ($\lesssim M_X$) can give rise to the masses of the bosonic and fermionic components of N^c and $\overline{N}^c$ as well as d^c and $\overline{d}^c$ which are just slightly below M_X.

In order to achieve the appropriate breakdown of the $SU(2)_W \times U(1)_Y \times U(1)_N$ gauge symmetry at low energies the particle spectrum below the scale M_X should be supplemented by the $4D$ chiral superfields S and $\overline{S}$ which are even under the $\tilde{Z}_2^H$ symmetry. The corresponding zero modes can come from the pair of bulk $27'$–plets

$$\Phi'_S = (27,\, +,\, +,\, -)\,, \qquad \Phi'_{\overline{S}} = (27,\, -,\, -,\, +)\ . \tag{55}$$

The S and $\overline{S}$ components of the $N = 1$ superfields $\hat{\Phi}'_S$ and $\hat{\Phi}'^c_{\overline{S}}$ have positive orbifold parities. The $\overline{D}$ component of $\hat{\Phi}'_S$ and the companion component from the $\hat{\Phi}'^c_{\overline{S}}$ superfield have also positive parities with respect to Z_2, Z_2^I and Z_2^{II} symmetries. It is convenient to assume that the states associated with these exotic quark supermultiplets are odd under the $\tilde{Z}_2^H$ symmetry so that the corresponding zero modes can mix with the appropriate components of the 27–plets localised on the O brane leading to the formation of the vectorlike states with masses slightly below M_X and spoiling the $SO(10)$ relations between the Yukawa couplings of S to the inert Higgs and exotic quark states. In addition to the components of $\hat{\Phi}'_S$ and $\hat{\Phi}'^c_{\overline{S}}$ mentioned above the orbifold parities of $\overline{L}$ and L components of $\hat{\Phi}'^c_S$ and $\hat{\Phi}'_{\overline{S}}$ are positive. If the zero modes associated with these components survive to low energies and the corresponding $N = 1$ supermultiplets are even under the $\tilde{Z}_2^H$ symmetry then the Yukawa couplings of these superfields to Q_i and $\overline{D}_k$ allow the lightest exotic quarks to decay like in the case of Scenario A.

The discussion above indicate that the simplest $6D$ orbifold GUT model based on the E_6 gauge group, which may lead at low energies to the gauge group and field content of the Scenario A specified in section 2, include six bulk $27'$–plets. The consistency of this orbifold GUT model requires the absence of anomalies. In the 6D orbifold models there are two types of anomalies: $4D$ anomalies [67] intrinsic to the fixed points and bulk anomalies [65], [68]–[69] which are induced by box diagrams with four gauge currents. For the $6D$ orbifold GUT model to be consistent it is necessary that both the fixed point and the bulk anomalies must cancel. The contributions of the anomalous box diagrams with four gauge currents to the $6D$ bulk anomalies are determined by the trace of four generators of gauge group. This trace contains nonfactorizable part and part which can be reduced to the product of traces of two generators. The nonfactorizable part is associated with the irreducible gauge anomaly while the factorized contribution corresponds to what is known as reducible anomaly. The reducible anomalies can be canceled by the Green–Schwarz mechanism [70]. For the consistency the chiral field content of the $6D$ orbifold model must lead to the cancellation of the irreducible anomalies which is normally highly restrictive requirement [71]. However $6D$ orbifold GUT models based on the E_6 gauge group do not have irreducible bulk anomaly [68]–[69]. Moreover using the results obtained in [69]

one can show that the reducible gauge anomaly gets cancelled if the field content of the $6D$ orbifold model involves six bulk $27'$–plets. The $4D$ anomalies at the fixpoints get also cancelled within the $6D$ orbifold GUT model discussed above. Indeed, the contributions of 27_i supermultiplets, that reside on the O brane, to the anomalies vanish. Since the orbifold parity assignments are such that the KK modes of the bulk $27'$ superfields localized at the fixpoints always form pairs of $N = 1$ supermultiplets with opposite quantum numbers the contributions of the bulk $27'$–plets to the $4D$ fixed point anomalies are cancelled automatically as well.

Phenomenological viability of the $5D$ and $6D$ orbifold GUT models considered in this section requires the adequate suppression of the baryon and lepton number violating operators which can be induced at the scale M_X giving rise to proton decay. As it was mentioned before the dimension five operators, that lead to the proton decay, are forbidden by the gauge symmetry in these models. However baryon and lepton number violating operators, which are mediated by the exchange of the leptoquark gauge bosons, are enhanced compared to the usual $4D$ case due to the presence of KK towers of such states. The proton decay rate in the $6D$ orbifold GUT models based on the $SO(10)$ gauge group was studied in [59] where it was shown that in order to satisfy the experimental lower limit on the proton lifetime the scale M_X should be larger than $9 \cdot 10^{15}\,\text{GeV}$. This restriction on the scale M_X can be used in the case of the E_6 inspired SUSY models as well. However the analysis of the RG flow of the gauge couplings, which we are going to consider next, indicates that the value of $g_i^2(M_X)$ in these models are 3-5 times larger than in the MSSM. This implies that the lower bound on the scale M_X in the considered E_6 inspired models is expected to be $1.5 - 2 \cdot 10^{16}\,\text{GeV}$. It is worth noting here again that the simplest $5D$ and $6D$ orbifold GUT models discussed in this section do not lead to the exact gauge coupling unification at the scale M_X due to the brane contributions to the gauge couplings. The relative contribution of these brane corrections is expected to become more sizable with increasing $g_i^2(M_X)$ as it was discussed before. The gauge coupling unification in the $6D$ orbifold GUT models was considered in [57].

4 RG flow of gauge couplings in the E_6SSM

In this section we discuss the RG flow of the SM gauge couplings $g_i(t)$ above the EW scale. The running of these couplings between M_X and M_Z is described by a system of renormalisation group equations (RGEs). To simplify our analysis we assume that $U(1)_\psi \times U(1)_\chi$ gauge symmetry is broken down to $U(1)_N \times Z_2^M$ near the scale M_X. This permits us to restrict our consideration to the analysis of the RG flow of four diagonal gauge couplings $g_3(t)$, $g_2(t)$, $g_1(t)$ and $g_1'(t)$ which correspond to $SU(3)_C$, $SU(2)_W$, $U(1)_Y$ and $U(1)_N$ gauge interactions respectively. Besides the evolution of these gauge couplings is affected by a kinetic term mixing. The mixing effect can be concealed in the interaction between the $U(1)_N$ gauge field and matter fields that can be parametrized in terms of off–diagonal gauge coupling g_{11} (see [11], [32], [72]). In this framework the RG equations can be written as follows:

$$\frac{dG}{dt} = G \times B\,, \qquad \frac{dg_2}{dt} = \frac{\beta_2 g_2^3}{(4\pi)^2}\,, \qquad \frac{dg_3}{dt} = \frac{\beta_3 g_3^3}{(4\pi)^2}\,, \tag{56}$$

where $t = \ln(q/M_Z)$, q is a renormalisation scale while B and G are 2×2 matrices

$$G = \begin{pmatrix} g_1 & g_{11} \\ 0 & g_1' \end{pmatrix}, \qquad B = \frac{1}{(4\pi)^2} \begin{pmatrix} \beta_1 g_1^2 & 2g_1 g_1' \beta_{11} + 2 g_1 g_{11} \beta_1 \\ 0 & g_1'^2 \beta_1' + 2 g_1' g_{11} \beta_{11} + g_{11}^2 \beta_1 \end{pmatrix}. \tag{57}$$

In Eqs. (56)–(57) β_i and β_{11} are beta functions.

Here we examine the RG flow of gauge couplings in the two–loop approximation. In general the two–loop diagonal β_i and off–diagonal β_{11} beta functions may be presented as a sum of one–loop and two–loop contributions. However the previous analysis performed in [36] revealed that an off–diagonal gauge coupling g_{11} being set to zero at the scale M_X remains very small at any other scale below M_X. Since it seems to be rather natural to assume that just after the breakdown of the E_6 symmetry there is no mixing in the gauge kinetic part of the Lagrangian between the field strengths associated with the $U(1)_Y$ and $U(1)_N$ gauge interactions g_{11} tends to be substantially smaller than the diagonal gauge couplings. Because of this we can neglect two–loop corrections to the off–diagonal beta function β_{11}. In the case of scenario A the one–loop off–diagonal beta function is given by $\beta_{11} = -\dfrac{\sqrt{6}}{5}$ while in the scenario B $\beta_{11} = \dfrac{3\sqrt{6}}{10}$.

In the scenario A the two–loop diagonal beta functions β_i are given by:

$$\begin{aligned}
\beta_3 &= -9 + 3N_g + \frac{1}{16\pi^2}\Bigg[g_3^2(-54 + 34N_g) + 3N_g\, g_2^2 + N_g\, g_1^2 \\
&\quad + N_g\, g_1'^2 - 4h_t^2 - 4h_b^2 - 2\Sigma_\kappa \Bigg], \\
\beta_2 &= -5 + 3N_g + \frac{1}{16\pi^2}\Bigg[8N_g g_3^2 + (-17 + 21N_g) g_2^2 + \left(\frac{3}{5} + N_g\right) g_1^2 \\
&\quad + \left(\frac{2}{5} + N_g\right) g_1'^2 - 6h_t^2 - 6h_b^2 - 2h_\tau^2 - 2\Sigma_\lambda \Bigg], \\
\beta_1 &= \frac{3}{5} + 3N_g + \frac{1}{16\pi^2}\Bigg[8N_g g_3^2 + \left(\frac{9}{5} + 3N_g\right) g_2^2 + \left(\frac{9}{25} + 3N_g\right) g_1^2 \\
&\quad + \left(\frac{6}{25} + N_g\right) g_1'^2 - \frac{26}{5} h_t^2 - \frac{14}{5} h_b^2 - \frac{18}{5} h_\tau^2 - \frac{6}{5}\Sigma_\lambda - \frac{4}{5}\Sigma_\kappa \Bigg], \\
\beta_1' &= \frac{2}{5} + 3N_g + \frac{5}{4}n + \frac{1}{16\pi^2}\Bigg[8N_g g_3^2 + \left(\frac{6}{5} + 3N_g\right) g_2^2 + \left(\frac{6}{25} + N_g\right) g_1^2 \\
&\quad + \left(\frac{4}{25} + 3N_g + \frac{25}{8}n\right) g_1'^2 - \frac{9}{5} h_t^2 - \frac{21}{5} h_b^2 - \frac{7}{5} h_\tau^2 - \frac{19}{5}\Sigma_\lambda - \frac{57}{10}\Sigma_\kappa \Bigg], \\
\Sigma_\lambda &= \lambda_1^2 + \lambda_2^2 + \lambda^2, \qquad\qquad \Sigma_\kappa = \kappa_1^2 + \kappa_2^2 + \kappa_3^2,
\end{aligned} \tag{58}$$

where N_g is a number of generations forming complete E_6 fundamental representations that the considered model involves at low energies, i.e. $N_g = 3$, whereas n is a number of S and $\overline{S}$ supermultiplets from $27'_S$ and $\overline{27'}_S$ that survive to low energies (i.e. $n = 0$ or 1). Here we assume that the structure of the Yukawa interactions appearing in the superpotential (14) is relatively simple, i.e. $\lambda_{\alpha\beta} = \lambda_\alpha \delta_{\alpha\beta}$, and $\kappa_{ij} = \kappa_i \delta_{ij}$ while $\tilde{f}_{\alpha\beta}$, $f_{\alpha\beta}$, g^D_{ij} and $h^E_{i\alpha}$ are small and can therefore be ignored (i, j = 1, 2, 3 and α, β = 1, 2). We have also neglected all Yukawa couplings that may be associated with the presence

of extra S and $\overline{S}$ supermultiplets at low energies. In Eqs. (58) h_t, h_b and h_τ are top quark, b-quark and τ–lepton Yukawa couplings respectively. In the limit of $n = 0$ the RG equations (58) coincide with the ones presented in [36].

In the scenario B the two–loop diagonal beta functions β_i can be written in the following form:

$$\begin{aligned}
\beta_3 &= -8 + 3N_g + \frac{1}{16\pi^2}\Biggl[g_3^2\left(-\frac{128}{3} + 34N_g\right) + 3N_g\,g_2^2 + \left(N_g + \frac{4}{15}\right)g_1^2 \\
&\quad + \left(N_g + \frac{2}{5}\right)g_1'^2 - 4h_t^2 - 4h_b^2 - 2\Sigma_\kappa\Biggr], \\
\beta_2 &= -4 + 3N_g + \frac{1}{16\pi^2}\Biggl[8N_g g_3^2 + (-10 + 21N_g)g_2^2 + \left(\frac{6}{5} + N_g\right)g_1^2 \\
&\quad + \left(\frac{13}{10} + N_g\right)g_1'^2 - 6h_t^2 - 6h_b^2 - 2h_\tau^2 - 2\tilde{\Sigma}_\lambda\Biggr], \\
\beta_1 &= \frac{8}{5} + 3N_g + \frac{1}{16\pi^2}\Biggl[\left(8N_g + \frac{32}{15}\right)g_3^2 + \left(\frac{18}{5} + 3N_g\right)g_2^2 + \left(\frac{62}{75} + 3N_g\right)g_1^2 \\
&\quad + \left(\frac{47}{50} + N_g\right)g_1'^2 - \frac{26}{5}h_t^2 - \frac{14}{5}h_b^2 - \frac{18}{5}h_\tau^2 - \frac{6}{5}\tilde{\Sigma}_\lambda - \frac{4}{5}\Sigma_\kappa\Biggr], \qquad (59) \\
\beta_1' &= \frac{19}{10} + 3N_g + \frac{5}{4}n + \frac{1}{16\pi^2}\Biggl[\left(8N_g + \frac{16}{5}\right)g_3^2 + \left(\frac{39}{10} + 3N_g\right)g_2^2 \\
&\quad + \left(\frac{47}{50} + N_g\right)g_1^2 + \left(\frac{121}{100} + 3N_g + \frac{25}{8}n\right)g_1'^2 \\
&\quad - \frac{9}{5}h_t^2 - \frac{21}{5}h_b^2 - \frac{7}{5}h_\tau^2 - \frac{19}{5}\tilde{\Sigma}_\lambda - \frac{57}{10}\Sigma_\kappa\Biggr],
\end{aligned}$$

where $\tilde{\Sigma}_\lambda = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda^2$. As before we assume relatively simple structure of the Yukawa interactions in the superpotential (19), i.e. $\lambda_{ij} = \lambda_i\delta_{ij}$, $\kappa_{ij} = \kappa_i\delta_{ij}$, and ignore $\tilde{f}_{\alpha i}$, $f_{\alpha i}$, g^q_{ij}, h^D_{ij} as well as all Yukawa couplings of extra S and $\overline{S}$ supermultiplets.

As one can see from Eqs. (58)–(59) $N_g = 3$ is the critical value for the one–loop beta function of the strong interactions in the case of scenario A. Indeed, in the one–loop approximation the $SU(3)_C$ gauge coupling is equal to zero in this case. In the scenario B the one–loop contribution to β_3 remains rather small ($b_3 = 1$). Because of this any reliable analysis of the RG flow of gauge couplings requires the inclusion of two–loop corrections to the diagonal beta functions.

One can obtain an approximate solution of the two–loop RGEs presented above (see [73]). At high energies this solution for the SM gauge couplings can be written as

$$\frac{1}{\alpha_i(t)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi}t - \frac{C_i}{12\pi} - \Theta_i(t) + \frac{b_i - b_i^{SM}}{2\pi}\ln\frac{T_i}{M_Z}, \qquad (60)$$

where $\alpha_i(t) = \dfrac{g_i^2(t)}{(4\pi)}$, b_i and b_i^{SM} are the coefficients of the one–loop beta functions in the E$_6$SSM and SM respectively, the third term in the right–hand side of Eq. (60) is the

$\overline{MS} \to \overline{DR}$ conversion factor with $C_1 = 0$, $C_2 = 2$, $C_3 = 3$ [74], while

$$\Theta_i(t) = \frac{1}{2\pi}\int_0^t (\beta_i - b_i) d\tau\,, \qquad\qquad T_i = \prod_{k=1}^{N}\Big(m_k\Big)^{\frac{\Delta b_i^k}{b_i - b_i^{SM}}}\,. \tag{61}$$

In Eq. (61) m_k and Δb_i^k are masses and one–loop contributions to the beta functions due to new particles appearing in the E$_6$SSM. For the calculation of $\Theta_i(t)$ the solutions of the one–loop RGEs are normally used. In Eqs. (60)–(61) only leading one–loop threshold effects are taken into account.

Using the approximate solution of the two–loop RGEs in Eqs. (60)–(61) one can establish the relationships between the values of the gauge couplings at low energies and GUT scale. Then by using the expressions describing the RG flow of $\alpha_1(t)$ and $\alpha_2(t)$ it is rather easy to find the scale M_X where $\alpha_1(M_X) = \alpha_2(M_X) = \alpha_0$ and the value of the overall gauge coupling α_0 at this scale. Substituting M_X and α_0 into the solution of the RGE for the strong gauge coupling one finds the value of $\alpha_3(M_Z)$ for which exact gauge coupling unification occurs (see [75]):

$$\begin{aligned}
\frac{1}{\alpha_3(M_Z)} &= \frac{1}{b_1 - b_2}\left[\frac{b_1 - b_3}{\alpha_2(M_Z)} - \frac{b_2 - b_3}{\alpha_1(M_Z)}\right] - \frac{1}{28\pi} + \Theta_s + \frac{19}{28\pi}\ln\frac{T_S}{M_Z}\,,\\
\Theta_s &= \left(\frac{b_2 - b_3}{b_1 - b_2}\Theta_1 - \frac{b_1 - b_3}{b_1 - b_2}\Theta_2 + \Theta_3\right), \qquad \Theta_i = \Theta_i(M_X)\,.
\end{aligned} \tag{62}$$

The combined threshold scale T_S, that appears in Eq. (62), can be expressed in terms of the effective threshold scales T_1, T_2 and T_3. The expression for T_S is model–dependent. In the scenario A T_S is given by

$$\begin{aligned}
T_S &= \frac{T_2^{172/19}}{T_1^{55/19} T_3^{98/19}}\,,\\
T_1 &= \tilde{M}_1^{5/11}\mu_L^{4/55} m_L^{2/55}\left(\prod\nolimits_{i=1,2,3} m_{\tilde{D}_i}^{4/165}\mu_{D_i}^{8/165}\right)\left(\prod\nolimits_{\alpha=1,2} m_{H_\alpha}^{2/55}\mu_{\tilde{H}_\alpha}^{4/55}\right),\\
T_2 &= \tilde{M}_2^{25/43}\mu_L^{4/43} m_L^{2/43}\left(\prod_{\alpha=1,2} m_{H_\alpha}^{2/43}\mu_{\tilde{H}_\alpha}^{4/43}\right),\\
T_3 &= \tilde{M}_3^{4/7}\left(\prod_{i=1,2,3} m_{\tilde{D}_i}^{1/21}\mu_{D_i}^{2/21}\right),
\end{aligned} \tag{63}$$

where μ_{D_i} and $m_{\tilde{D}_i}$ are the masses of exotic quarks and their superpartners, m_{H_α} and $\mu_{\tilde{H}_\alpha}$ are the masses of Inert Higgs and Inert Higgsino fields, m_L and μ_L are the masses of the scalar and fermion components of L_4 and $\overline{L}_4$ while $\tilde{M}_1$, $\tilde{M}_2$ and $\tilde{M}_3$ are the effective threshold scales in the MSSM

$$\begin{aligned}
\tilde{M}_1 &= \mu^{4/25} m_A^{1/25}\left(\prod_{i=1,2,3} m_{\tilde{Q}_i}^{1/75} m_{\tilde{d}_i}^{2/75} m_{\tilde{u}_i}^{8/75} m_{\tilde{L}_i}^{1/25} m_{\tilde{e}_i}^{2/25}\right),\\
\tilde{M}_2 &= M_{\tilde{W}}^{8/25}\mu^{4/25} m_A^{1/25}\left(\prod_{i=1,2,3} m_{\tilde{Q}_i}^{3/25} m_{\tilde{L}_i}^{1/25}\right),\\
\tilde{M}_3 &= M_{\tilde{g}}^{1/2}\left(\prod_{i=1,2,3} m_{\tilde{Q}_i}^{1/12} m_{\tilde{u}_i}^{1/24} m_{\tilde{d}_i}^{1/24}\right).
\end{aligned} \tag{64}$$

In Eqs. (64) $M_{\tilde{g}}$ and $M_{\tilde{W}}$ are masses of gluinos and winos (superpartners of $SU(2)_W$ gauge bosons), μ and m_A are effective μ–term and masses of heavy Higgs states respectively; $m_{\tilde{u}_i}$, $m_{\tilde{d}_i}$ and $m_{\tilde{Q}_i}$ are the masses of the right–handed and left–handed squarks and $m_{\tilde{L}_i}$ and $m_{\tilde{e}_i}$ are the masses of the left–handed and right–handed sleptons.

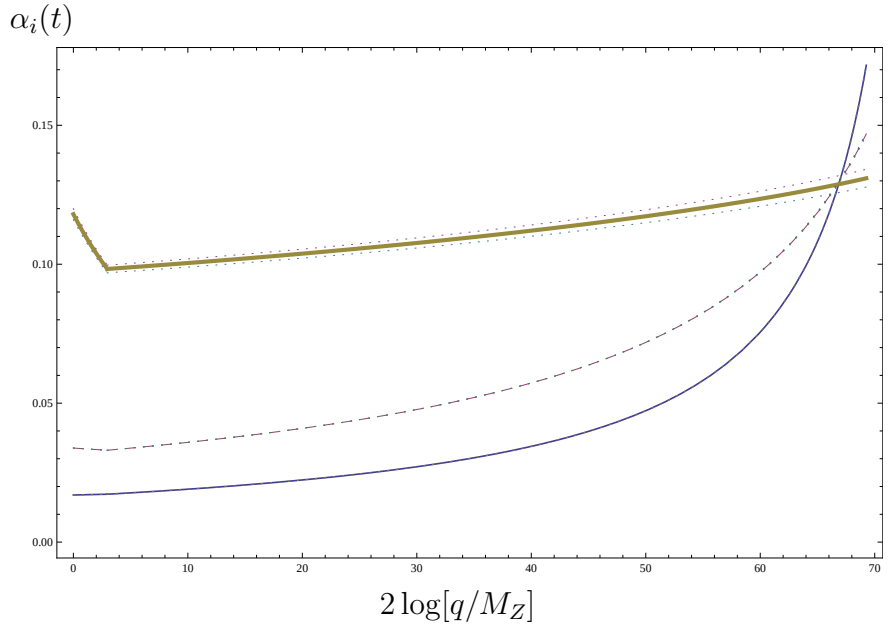
In the case of scenario B we find

$$\begin{aligned}
\tilde{T}_S &= \frac{\tilde{T}_2^{196/19}}{\tilde{T}_1^{65/19}\tilde{T}_3^{112/19}}\,,\\
\tilde{T}_1 &= \tilde{M}_1^{5/13}\mu_{d_4}^{8/195}m_{d_4}^{4/195}\mu_{H_u}^{4/65}m_{H_u}^{2/65}\mu_{H_d}^{4/65}m_{H_d}^{2/65}\left(\prod_{i=1,2,3} m_{\tilde{D}_i}^{4/195}\mu_{D_i}^{8/195}\right)\left(\prod_{\alpha=1,2} m_{H_\alpha}^{2/65}\mu_{\tilde{H}_\alpha}^{4/65}\right),\\
\tilde{T}_2 &= \tilde{M}_2^{25/49}\mu_{H_u}^{4/49}m_{H_u}^{2/49}\mu_{H_d}^{4/49}m_{H_d}^{2/49}\left(\prod_{\alpha=1,2} m_{H_\alpha}^{2/49}\mu_{\tilde{H}_\alpha}^{4/49}\right),\\
\tilde{T}_3 &= \tilde{M}_3^{1/2}\mu_{d_4}^{1/12}m_{d_4}^{1/24}\left(\prod_{i=1,2,3} m_{\tilde{D}_i}^{1/24}\mu_{D_i}^{1/12}\right),
\end{aligned} \tag{65}$$

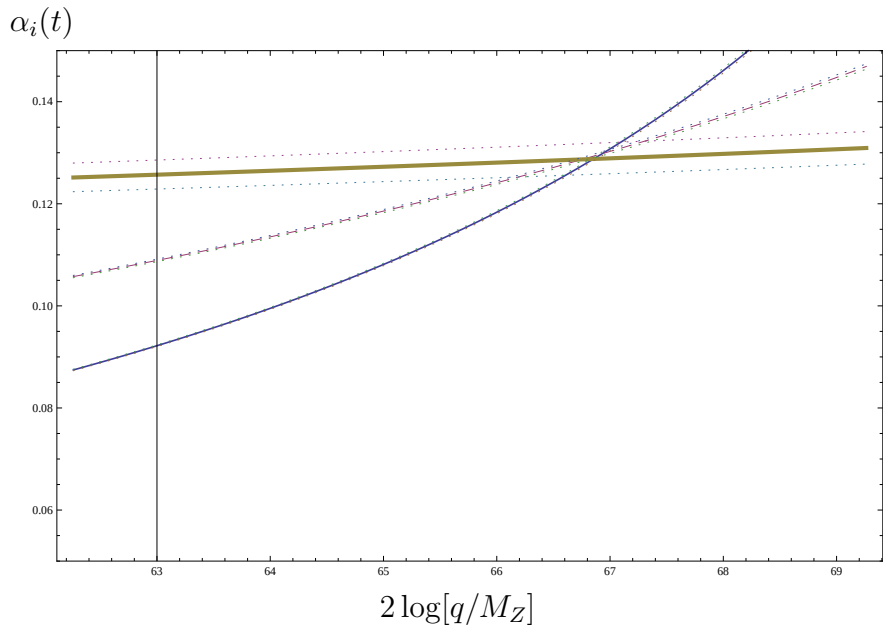
where μ_{d_4}, μ_{H_u} and μ_{H_d} are the masses of the fermionic components of d_4^c and $\overline{d^c}_4$, H_i^u and $\overline{H}_u$ as well as H_i^d and $\overline{H}_d$, that form vector-like states at low energies, whereas m_{d_4}, m_{H_u} and m_{H_d} are the masses of the scalar components of the corresponding supermultiplets.

In general the effective threshold scales derived above can be quite different. Since our purpose is to establish the range of the values of T_S and $\tilde{T}_S$ that leads to the unification of gauge couplings we shall set these effective threshold scales equal to each other. Then from Eqs. (63) and (65) it follows that $T_1 = T_2 = T_3 = T_S$ and $\tilde{T}_1 = \tilde{T}_2 = \tilde{T}_3 = \tilde{T}_S$. The results of our numerical studies of the two–loop RG flow of gauge couplings in the case of scenarios A and B are summarized in Figs. 4 and 4 respectively. We use the two–loop SM beta functions to describe the running of gauge couplings between M_Z and $T_1 = T_2 = T_3 = T_S$ (or $\tilde{T}_1 = \tilde{T}_2 = \tilde{T}_3 = \tilde{T}_S$), then we apply the two–loop RGEs of the E$_6$SSM to compute the flow of $g_i(t)$ from T_S (or $\tilde{T}_S$) to M_X which is equal to $3 \cdot 10^{16}\,\mathrm{GeV}$ in the case of the E$_6$SSM. The low energy values of g_1' and g_{11} are chosen so that all four diagonal gauge couplings are approximately equal near the GUT scale and $g_{11} = 0$ at this scale. For the calculation of the evolution of Yukawa couplings a set of one–loop RGEs is used. The corresponding one–loop RG equations are specified in [32].

In Fig. 4 we fix the effective threshold scale to be equal to 400 GeV. In Fig. 1a we plot the running of the gauge couplings from M_Z to M_X assuming that the low energy matter content involves three 27-plets of E_6 as well as L_4, $\overline{L}_4$, S and $\overline{S}$ supermultiplets. Fig. 1b shows a blow–up of the crucial region in the vicinity of the GUT scale. Dotted lines show the interval of variations of gauge couplings caused by $1\,\sigma$ deviations of $\alpha_3(M_Z)$ around its average value, i.e. $\alpha_3(M_Z) \simeq 0.118 \pm 0.002$. The results of the numerical analysis presented in Fig. 4 demonstrate that in the scenario A almost exact unification of the SM gauge couplings can be achieved for $\alpha_3(M_Z) = 0.118$ and $\tilde{T}_S = 400\,\mathrm{GeV}$. With increasing (decreasing) the effective threshold scale the value of $\alpha_3(M_Z)$, at which exact gauge coupling unification takes place, becomes lower (greater). Thus in this case the gauge coupling unification can be achieved for any phenomenologically reasonable value of $\alpha_3(M_Z)$, consistent with the central measured low energy value, unlike in the MSSM where it is rather problematic to get the exact unification of gauge couplings [73], [76]–[77]. Indeed, it is well known that in order to achieve gauge coupling unification in the MSSM with $\alpha_s(M_Z) \simeq 0.118$, the combined threshold scale, which is given by [73], [75],



(a)



(b)

Figure 1: Two–loop RG flow of gauge couplings in the Scenario A: *(a)* RG flow of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings from M_Z to M_X for $T_S = 400\,\text{GeV}$ and $n_S = 1$; *(b)* running of SM gauge couplings in the vicinity of M_X for $T_S = 400\,\text{GeV}$ and $n_S = 1$. Thick, dashed and solid lines correspond to the running of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings respectively. We used $\tan\beta = 10$, $\alpha_s(M_Z) = 0.118$, $\alpha(M_Z) = 1/127.9$, $\sin^2\theta_W = 0.231$ and $\kappa_1(T_S) = \kappa_2(T_S) = \kappa_3(T_S) = \lambda_1(T_S) = \lambda_2(T_S) = \lambda_3(T_S) = g_1^{'}(T_S)$. The dotted lines represent the uncertainty in $\alpha_i(t)$ caused by the variation of the strong gauge coupling from 0.116 to 0.120 at the EW scale.

[77]–[78]

$$\tilde{M}_S = \frac{\tilde{M}_2^{100/19}}{\tilde{M}_1^{25/19}\tilde{M}_3^{56/19}} \simeq \mu/6\,, \tag{66}$$

must be around $\tilde{M}_S \approx 1\,\mathrm{TeV}$. However the correct pattern of EW symmetry breaking requires μ to lie within the $1-2\,\mathrm{TeV}$ range which implies $\tilde{M}_S < 200-300\,\mathrm{GeV}$, so that, ignoring the effects of high energy threshold corrections, the exact gauge coupling unification in the MSSM requires significantly higher values of $\alpha_3(M_Z)$, well above the experimentally measured central value [73], [75], [77]–[79]. It was argued that it is possible to get the unification of gauge couplings in the minimal SUSY model for $\alpha_3(M_Z) \simeq 0.123$ [80].

On the other hand in the case of scenario A the combined threshold scale T_S can be substantially larger than in the MSSM. This can be seen directly from the explicit expression for T_S. Combining Eqs. (63) we find

$$T_S \;=\; \tilde{M}_S \cdot \left(\frac{\mu_L^{12/19} m_L^{6/19}}{\mu_{D_3}^{12/19} m_{\tilde{D}_3}^{6/19}}\right)\left(\prod_{\alpha=1,2} \frac{m_{H_\alpha}^{6/19}\mu_{\tilde{H}_\alpha}^{12/19}}{m_{\tilde{D}_\alpha}^{6/19}\mu_{D_\alpha}^{12/19}}\right). \tag{67}$$

From Eq. (67) it is obvious that T_S is determined by the masses of the scalar and fermion components of L_4 and $\overline{L}_4$. The term $\mu_L L_4 \overline{L}_4$ in the superpotential (14) is not involved in the process of EW symmetry breaking. As a consequence the parameter μ_L remains arbitrary[7]. In particular, since the corresponding mass term is not suppressed by the E_6 symmetry the components of the doublet superfields L_4 and $\overline{L}_4$ may be much heavier than the masses of all exotic states resulting in the large combined threshold scale T_S that lies in a few hundred GeV range even when scale $\tilde{M}_S$ is relatively low. The large range of variation of T_S allows to achieve the exact unification of gauge couplings in the scenario A for any value of $\alpha_3(M_Z)$ which is in agreement with current data.

It is worth noting here that, in principle, one could naively expect that large two–loop corrections to the diagonal beta functions would spoil the unification of the SM gauge couplings entirely in the considered case. Indeed, in the scenario A these corrections affect the RG flow of gauge couplings much more strongly than in the case of the MSSM because at any intermediate scale the values of the gauge couplings in the E$_6$SSM are substantially larger as compared to the ones in the MSSM. Nevertheless the results of our analysis discussed above are not as surprising as they may first appear. The analysis of the RG flow of the SM gauge couplings performed in [36] revealed that the two–loop corrections to $\alpha_i(M_X)$ are a few times bigger in the E$_6$SSM than in the MSSM. At the same time due to the remarkable cancellation of different two–loop corrections the absolute value of Θ_s is more than three times smaller in the E$_6$SSM as compared with the MSSM. This cancellation is caused by the structure of the two–loop corrections to the diagonal beta functions in the considered model. As a result, the prediction for the value of $\alpha_3(M_Z)$ at which exact gauge coupling unification takes place is considerably lower in the E$_6$SSM than in the MSSM.

The only difference between the E$_6$SSM scenario, which was studied in [36], and scenario A discussed above is in the possible presence of extra S and $\overline{S}$ supermultiplets at low energies. From Eqs. (58) it follows that these supermultiplets do not contribute to the diagonal beta functions of the SM gauge couplings. Our analysis of the RG flow of $g_i(t)$ reveals that the evolution of the SM gauge couplings does not change much when the low

[7]When μ_L is considerably larger than the SUSY breaking scale $m_L \simeq \mu_L$.

energy particle spectrum is supplemented by the bosonic and fermionic components that originate from the extra S and $\overline{S}$ chiral superfields. This explains why our results are so similar to those previously obtained in [36].

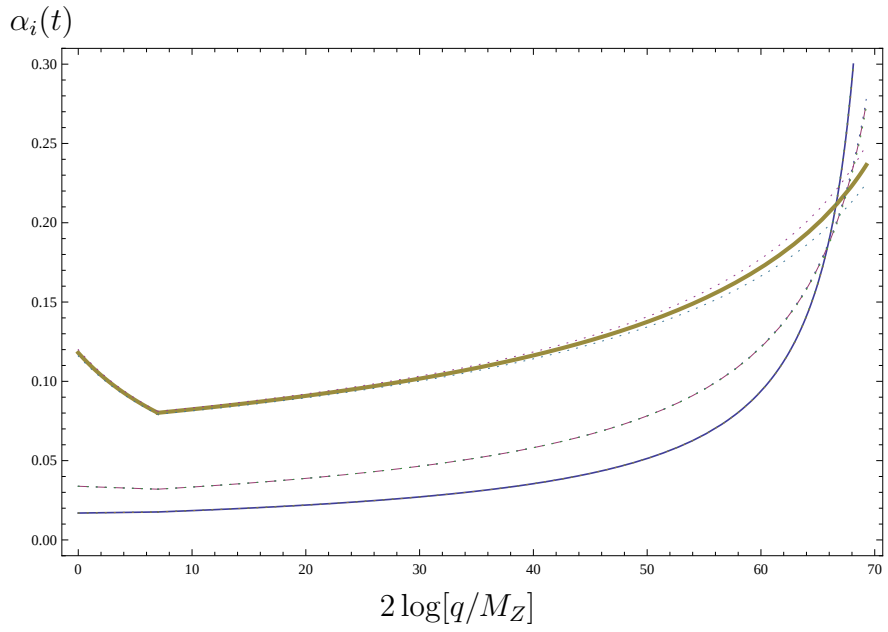
It is also worthwhile to point out that at high energies the uncertainty in $\alpha_3(t)$ caused by the variations of $\alpha_3(M_Z)$ is much bigger in the E$_6$SSM than in the MSSM. This is because in the E$_6$SSM the strong gauge coupling grows slightly with increasing renormalisation scale whereas in the MSSM it decreases at high energies. This implies that the uncertainty in the high energy value of $\alpha_3(t)$ in the E$_6$SSM is approximately equal to the low energy uncertainty in $\alpha_3(t)$ while in the MSSM the interval of variations of $\alpha_3(t)$ near the scale M_X shrinks drastically. The relatively large uncertainty in $\alpha_3(M_X)$ in the E$_6$SSM, compared to the MSSM, allows one to achieve exact unification of gauge couplings for values of $\alpha_3(M_Z)$ which are within one standard deviation of its measured central value.

The RG flow of the SM gauge couplings changes substantially in the case of scenario B as can be seen from Figs. 4. As before we assume that the effective threshold scales are equal, i.e. $\tilde{T}_1 = \tilde{T}_2 = \tilde{T}_3 = \tilde{T}_S$. Our numerical analysis reveals that the evolution of $\alpha_i(t)$ depends very strongly on $\tilde{T}_S$. When $\tilde{T}_S \lesssim 1\,\text{TeV}$ the gauge couplings become rather large near the GUT scale, i.e. $\alpha_i(M_X) \sim 1$, where as before we set $M_X \simeq 3 \cdot 10^{16}\,\text{GeV}$. For so large values of $\alpha_i(t)$ the perturbation theory method becomes inapplicable. Therefore in our analysis we consider the range of scales $\tilde{T}_S$ which are much higher than 1 TeV. In Figs. 4 we set the threshold scale $\tilde{T}_S$ to be equal to 3 TeV. As one can see from these figures for $\tilde{T}_S = 3\,\text{TeV}$ the values of $\alpha_i(M_X)$ are about 0.2 that still allows us to use the perturbation theory up to the scale M_X.

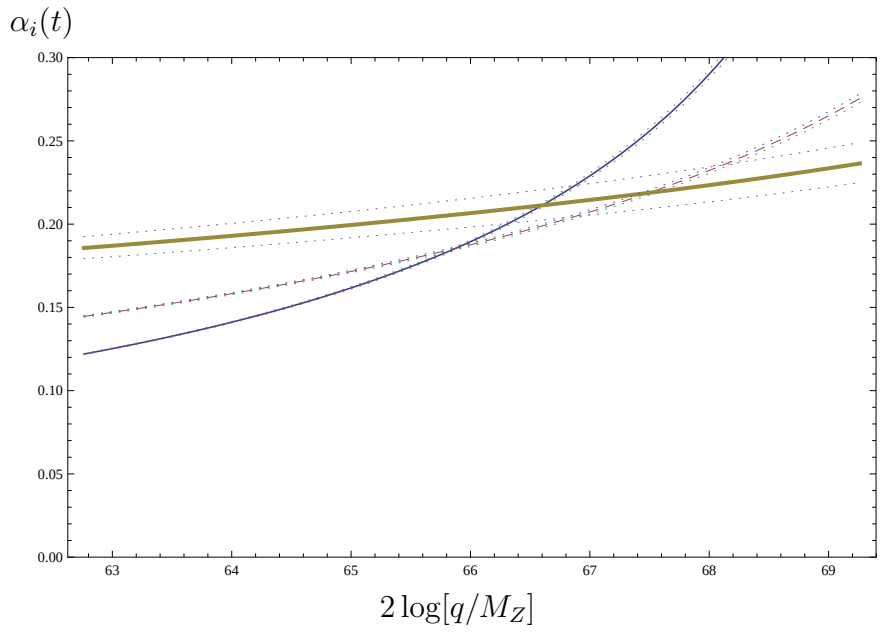
The effective threshold scale that we consider in our analysis $\tilde{T}_S$ is in the multi TeV range. At first glance, it is not clear if so large values of $\tilde{T}_i$ and $\tilde{T}_S$ can be obtained for a reasonable set of parameters. In particular, to satisfy naturalness requirements the third generation sfermions as well as neutralino and chargino states which are superparners of the SM gauge bosons and Higgs fields are expected to have masses below 1 TeV. Because of this in the MSSM naturalness arguments constrain the combined threshold scale $\tilde{M}_S$ to be lower than $200 - 300\,\text{GeV}$ as it was mentioned above. In the case of scenario B the analytical expression for the threshold scale $\tilde{T}_S$ can be obtained by combining Eqs. (65) that gives

$$\tilde{T}_S \;=\; \tilde{M}_S \cdot \left(\frac{\mu_{H_u}^{12/19} m_{H_u}^{6/19} \mu_{H_d}^{12/19} m_{H_d}^{6/19}}{\mu_{d_4}^{12/19} m_{d_4}^{6/19} \mu_{D_3}^{12/19} m_{\tilde{D}_3}^{6/19}}\right) \left(\prod_{\alpha=1,2} \frac{m_{H_\alpha}^{6/19} \mu_{\tilde{H}_\alpha}^{12/19}}{m_{\tilde{D}_\alpha}^{6/19} \mu_{D_\alpha}^{12/19}}\right). \tag{68}$$

Eq. (68) indicates that the combined threshold scale $\tilde{T}_S$ tends to be very large if, for example, $\mu_{H_u} \simeq m_{H_u} \simeq \mu_{H_d} \simeq m_{H_d}$ are considerably larger than the masses of the scalar and fermion components of d_4^c and $\overline{d^c}_4$ as well as the masses of all exotic states. In this case $\tilde{T}_S$ can be as large as 10 TeV even when $\tilde{M}_S$ lies in a few hundred GeV range and $\mu_{H_u} \simeq m_{H_u} \simeq \mu_{H_d} \simeq m_{H_d} \lesssim 10\,\text{TeV}$. This can be achieved if the components of d_4^c and $\overline{d^c}_4$ and some of the exotic quark and squark states have masses below 1 TeV. The effective threshold scales $\tilde{T}_1$, $\tilde{T}_2$ and $\tilde{T}_3$ can be also as large as a few TeV if the scalar superpartners of the first and second generation fermions and some of the exotic states have masses above 10 TeV. Naturalness does not require these states to be light and, in fact, allowing them to be heavy ameliorates SUSY flavor and CP problems. As a consequence the several TeV threshold scales $\tilde{T}_1$, $\tilde{T}_2$, $\tilde{T}_3$ and $\tilde{T}_S$ can naturally emerge in the scenario B.



(a)



(b)

Figure 2: Two–loop RG flow of gauge couplings in the Scenario B: *(a)* evolution of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ couplings from the EW scale to the GUT scale for $\tilde{T}_S = 3\,\text{TeV}$ and $n_S = 0$; *(b)* running of SM gauge couplings near the scale M_X for $\tilde{T}_S = 3\,\text{TeV}$ and $n_S = 0$. The parameters and notations are the same as in Fig. 4.

In Fig. 2a we show the running of the SM gauge couplings from the EW scale to high energies. We assume that in this case the low energy matter content includes three 27-plets of E_6 as well as d^c_4, $\overline{d^c}_4$, H_u, $\overline{H}_u$ H_d and $\overline{H}_d$ supermultiplets. Fig. 2b shows the same RG flow of the SM gauge couplings but just around the scale where the values of $\alpha_i(t)$ become rather close. Again dotted lines in Figs. 2a and 2b represent the changes of the evolution of the SM gauge couplings induced by the variations of $\alpha_3(M_Z)$ within $1\,\sigma$ around its average value.

From Figs. 2a and 2b one can see that the interval of variations of $\alpha_3(t)$ enlarges with increasing renormalisation scale. The growth of the uncertainty in the high energy value of $\alpha_3(t)$ is caused by the raise of this coupling itself. As follows from Figs. 4 and 4 in the scenario B the SM gauge couplings grow faster with increasing renormalisation scale than in the case of scenario A. This happens because the one–loop beta functions of these couplings are larger in the scenario B as compared to the ones in the scenario A. As a consequence the interval of variations of $\alpha_3(t)$ at high energies is also a bit bigger in the former than in the latter. However as one can see from Figs. 2a and 2b this does not facilitate the gauge coupling unification in scenario B. In fact, these figures demonstrate that large two–loop corrections spoil the unification of gauge couplings in this case. Indeed, in the one–loop approximation Eq. (62) leads to the same prediction for $\alpha_3(M_Z)$ in the scenarios A and B because extra matter in these scenarios form complete $SU(5)$ representations which contribute equally to the one–loop beta functions of the $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ interactions so that the differences of the coefficients of the one–loop beta functions $b_i - b_j$ remain intact. At the same time the contributions of two–loop corrections to $\alpha_i(M_X)$ (Θ_i) and $\alpha_3(M_Z)$ (Θ_s) are different in these cases. Our numerical analysis reveals that for $\tilde{T}_S \simeq 3\,\mathrm{TeV}$ the exact gauge coupling unification can be achieved in the scenario B only if the value of $\alpha_3(M_Z)$ is around 0.112. For higher scale T_S the exact unification of $\alpha_i(t)$ requires even smaller values of $\alpha_3(M_Z)$ which are disfavoured by the recent fit to experimental data. The lower scales $T_S \lesssim 3\,\mathrm{TeV}$ lead to the larger values of $\alpha_i(M_X)$ making questionable the validity of our calculations.

As before extra S and $\overline{S}$ superfields, that may survive to low energies, do not contribute to the diagonal beta functions of the SM gauge couplings and, therefore, do not change much the RG flow of $\alpha_i(t)$. As a result the value of $\alpha_3(M_Z)$ at which exact gauge coupling unification takes place does not change much as well after the inclusion of the bosonic and fermionic components of these supermultiplets. Thus it seems to be rather difficult to reconcile the unification of gauge couplings with present data in the Scenario B. Nevertheless the values of $\alpha_i(M_X)$ are not so much different from each other. From Fig. 2b it follows that the relative discrepancy of $\alpha_i(M_X)$ is about 10% . This brings us back to the orbifold GUT framework which was discussed in the previous section. As it has been already mentioned orbifold GUTs do not imply the exact gauge coupling unification near the scale M_X, which is associated with the size of compact extra dimensions, due to the brane contributions to the gauge couplings (see Eq. (39)). Since one can expect that these brane corrections become more sizable when $\alpha_i(M_X)$ are large, the relative discrepancy of 10% between $\alpha_i(M_X)$ should not be probably considered as a big problem in the case of scenario B.

5 Phenomenological implications

We now consider cosmological implications and collider signatures of the E_6 inspired SUSY models discussed above. The phenomenological implications of these models are determined by the structure of the particle spectrum that can vary substantially depending on the choice of the parameters. For example, the masses of the Z' boson, exotic quarks, Inert Higgsinos and Inert singlinos are set by the VEVs of the Higgs fields. In this section we primarily focus on the simplest case when only H_u, H_d and S acquire non–zero VEVs breaking $SU(2)_W \times U(1)_Y \times U(1)_N$ symmetry to $U(1)_{em}$ associated with electromagnetism. Assuming that $f_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$ are sufficiently small the masses of the exotic quarks, Inert Higgsino states and Z' boson are given by

$$\mu_{D_i} = \frac{\kappa_i}{\sqrt{2}}\, s\,, \qquad \mu_{H_\alpha} = \frac{\lambda_\alpha}{\sqrt{2}}\, s\,, \qquad M_{Z'} \simeq g_1^{'} \tilde{Q}_S s\,, \tag{69}$$

where s is a VEV of the field S, i.e. $\langle S \rangle = s/\sqrt{2}$. Here without loss of generality we set $\kappa_{ij} = \kappa_i \delta_{ij}$ and $\lambda_{\alpha\beta} = \lambda_\alpha \delta_{\alpha\beta}$. Since μ_{D_i}, μ_{H_α} and $M_{Z'}$ are determined by s, that remains a free parameter, the Z' boson mass and the masses of exotic quarks and Inert Higgsinos cannot be predicted. Because recent measurements from the LHC experiments exclude E_6 inspired Z' with masses lower than $2 - 2.15\,\text{TeV}$ [81] the singlet field S must acquire a large VEV ($s \gtrsim 5.5 - 6\,\text{TeV}$) to induce sufficiently large $M_{Z'}$. The couplings κ_i should be also large enough to ensure that the exotic fermions are sufficiently heavy to avoiding conflict with direct particle searches at present and former accelerators. However the exotic fermions (quarks and Inert Higgsinos) can be relatively light in the E$_6$SSM. This happens, for example, when the Yukawa couplings of the exotic particles have hierarchical structure similar to the one observed in the ordinary quark and lepton sectors. Then Z' mass lie beyond 10 TeV and the only manifestation of the considered models may be the presence of light exotic quark and/or Inert Higgsino states in the particle spectrum.

Since the qualitative pattern of the particle spectrum and associated collider signatures are so sensitive to the parameter choice it is worth to discuss first the robust predictions that the considered models have. It is well known that SUSY models predict that the mass of the lightest Higgs particle is limited from above. The E$_6$SSM is not an exception. In the simplest case when only H_u, H_d and S develop the VEVs, so that $\langle H_d \rangle = \frac{v_1}{\sqrt{2}}$, $\langle H_u \rangle = \frac{v_2}{\sqrt{2}}$ and $\langle S \rangle = \frac{s}{\sqrt{2}}$, the Higgs sector involves ten degrees of freedom. However four of them are massless Goldstone modes which are swallowed by the $W^\pm$, Z and Z' gauge bosons that gain non-zero masses. If CP–invariance is preserved the other degrees of freedom form two charged, one CP–odd and three CP-even Higgs states. When the SUSY breaking scale is considerably larger than the EW scale, the mass matrix of the CP-even Higgs sector has a hierarchical structure and can be diagonalised using the perturbation theory [82]-[83]. In this case the mass of one CP–even Higgs particle is always very close to the Z' boson mass $M_{Z'}$. The masses of another CP–even, the CP–odd and the charged Higgs states are almost degenerate. When $\lambda \gtrsim g_1'$, the qualitative pattern of the Higgs spectrum is rather similar to the one which arises in the PQ symmetric NMSSM [83]-[84]. In the considered limit the heaviest CP–even, CP–odd and charged states are almost degenerate and lie beyond the TeV range [32]. Finally, like in the MSSM and NMSSM, one of the CP–even Higgs bosons is always light irrespective of the SUSY breaking scale. However, in contrast with the MSSM, the lightest Higgs boson in the E$_6$SSM can be heavier than

$110 - 120\,\mathrm{GeV}$ even at tree level. In the two–loop approximation the lightest Higgs boson mass does not exceed $150 - 155\,\mathrm{GeV}$ [32].

5.1 Dark matter

The structure of the Yukawa interactions in the E_6SSM leads to another important prediction. Using the method proposed in [85] one can argue that there are theoretical upper bounds on the masses of the lightest and second lightest inert neutralino states [38]. To simplify the analysis we assume that the fermion components of the supermultiplets $\overline{S}$, $\overline{H}_u$ and $\overline{H}_d$, which may survive below the scale M_X, get combined with the corresponding superpositions of the fermion components of the superfields S_i, H_i^u and H_i^d resulting in a set of heavy vectorlike states. Furthermore we also assume that these vectorlike states completely decouple so that the particle spectrum below the TeV scale contains only two generations of inert Higgsinos ($\tilde{H}_\alpha^u$ and $\tilde{H}_\alpha^d$) and two generations of inert singlinos $\tilde{S}_\alpha$. The Yukawa interactions of these superfields are described by the superpotential

$$W_{IH} = \lambda_{\alpha\beta} S (H_\alpha^d H_\beta^u) + f_{\alpha\beta} S_\alpha (H_d H_\beta^u) + \tilde{f}_{\alpha\beta} S_\alpha (H_\beta^d H_u)\,, \tag{70}$$

where $\alpha, \beta = 1, 2$.

Thus below the TeV scale the inert neutralino states are linear superposition of the inert singlino states ($\tilde{S}_1$, $\tilde{S}_2$) and neutral components of inert Higgsinos ($\tilde{H}_1^{d0}$, $\tilde{H}_2^{d0}$, $\tilde{H}_1^{u0}$, $\tilde{H}_2^{u0}$). The charged components of the inert Higgsinos ($\tilde{H}_2^{u+}$, $\tilde{H}_1^{u+}$, $\tilde{H}_2^{d-}$, $\tilde{H}_1^{d-}$), form inert chargino sector. In order to avoid the LEP lower limit on the masses of inert charginos the couplings $\lambda_{\alpha\beta}$ and s must be chosen so that all inert chargino states are heavier than $100\,\mathrm{GeV}$. In addition, the requirement of the validity of perturbation theory up to the GUT scale constrains the allowed range of Yukawa couplings $\lambda_{\alpha\beta}$, $f_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$. The restrictions specified above set very stringent limits on the masses of two lightest inert neutralinos. The analysis performed in [38] indicates that the lightest and second lightest inert neutralinos ($\tilde{H}_1^0$ and $\tilde{H}_2^0$) are typically lighter than $60 - 65\,\mathrm{GeV}$. These neutralinos are predominantly inert singlinos so that they can have rather small couplings to the Z–boson. Therefore any possible signal which these neutralinos could give rise to at LEP would be extremely suppressed. On the other hand the couplings of χ_1^0 and χ_2^0 to the lightest CP–even Higgs boson h_1 are proportional to the mass$/\sqrt{v_1^2 + v_2^2}$ in the leading approximation [38]. As a consequence the couplings of two lightest inert neutralino to the lightest Higgs state are always large if the corresponding states have appreciable masses.

The discussion above indicates that the lightest and second lightest inert neutralinos tend to be the lightest states which are odd under the Z_2^E symmetry. It is worth to remind here that in the considered E_6 inspired SUSY models $U(1)_\psi \times U(1)_\chi$ gauge symmetry is broken down to $U(1)_N \times Z_2^M$ where $Z_2^M = (-1)^{3(B-L)}$ is the so–called matter parity which is a discrete subgroup of $U(1)_\psi$ and $U(1)_\chi$. Since the low–energy effective Lagrangian is invariant under both Z_2^M and $\tilde{Z}_2^H$ symmetries and $\tilde{Z}_2^H = Z_2^M \times Z_2^E$ (see Table 1), the Z_2^E symmetry is also conserved. This means that the lightest exotic state, which is odd under the Z_2^E symmetry, is absolutely stable and contributes to the relic density of dark matter.

Because the lightest inert neutralino is also the lightest R–parity odd state either the lightest R–parity even exotic state or the lightest R–parity odd state with $Z_2^E = +1$ must be absolutely stable. When $f_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$ are large enough ($f_{\alpha\beta} \sim \tilde{f}_{\alpha\beta} \sim 0.5$) the large mixing in the inert Higgs sector may lead to the lightest CP–even (or CP–odd) inert Higgs state with mass of the order of the EW scale. The corresponding exotic state is R–parity even neutral particle. If it is substantially lighter than the lightest ordinary

neutralino state χ_1^0 and the decay of χ_1^0 into the lightest inert neutralino and the lightest inert Higgs scalar (pseudoscalar) is kinematically allowed then this lightest inert Higgs scalar (pseudoscalar) is absolutely stable and may result in considerable contribution to the relic dark matter density.

Although the possibility mentioned above looks very attractive a substantial fine-tuning is normally required to make the lightest inert Higgs scalar (pseudoscalar) lighter than χ_1^0. Most commonly χ_1^0 is considerably lighter than the lightest inert Higgs scalar (pseudoscalar) so that the lightest CP–even (CP–odd) inert Higgs state can decay into χ_1^0 and the lightest inert neutralino state. In other words, in the considered E_6 inspired SUSY models the lightest R–parity odd state with $Z_2^E = +1$, i.e. χ_1^0, tend to be substantially lighter than the R–parity even exotic states. As a result the lightest neutralino state χ_1^0 is a natural candidate for a cold component of dark matter in these models.

In the neutralino sector of the E$_6$SSM there are two extra neutralinos besides the four MSSM ones. One of them is an extra gaugino $\tilde{B}'$ coming from the Z' vector supermultiplet. The other one is an additional singlino $\tilde{S}$ which is a fermion component of the SM singlet superfield S. Extra neutralinos form two eigenstates $(\tilde{B}' \pm \tilde{S})/\sqrt{2}$ with masses around $M_{Z'}$ [32]. Since LHC experiments set very stringent lower bound on the mass of the Z' boson extra neutralino eigenstates tend to be the heaviest ones and decouple. The mixing between these heavy neutralino states and other gauginos and Higgsinos is very small. Therefore the lightest neutralino states in the E$_6$SSM, that determine the composition of χ_1^0 and as a consequence its contribution to the relic dark matter density, become almost indistinguishable from the ones in the MSSM. This means that in the E$_6$SSM, like in the MSSM, the lightest neutralino χ_1^0 can give a substantial contribution to the relic density which is in agreement with the measured abundance of cold dark matter $\Omega_{\rm CDM}h^2 = 0.1099 \pm 0.0062$ [86].

In the E$_6$SSM the lightest inert neutralino can account for all or some of the observed cold dark matter relic density if χ_1^0 has mass close to half the Z mass. In this case the lightest inert neutralino states annihilate mainly through an s–channel Z–boson, via its Inert Higgsino doublet components which couple to the Z–boson [38], [87]. When $|m_{\tilde{H}_1^0}| \ll M_Z$ the lightest inert neutralino states are almost inert singlinos and the couplings of $\tilde{H}_1^0$ to gauge bosons, Higgs states, quarks (squarks) and leptons (sleptons) are quite small leading to a relatively small annihilation cross section for $\tilde{H}_1^0\tilde{H}_1^0 \to$ SM particles. Since the dark matter number density is inversely proportional to the annihilation cross section at the freeze-out temperature the lightest inert neutralino state with mass $|m_{\tilde{H}_{1,2}^0}| \ll M_Z$ gives rise to a relic density which is typically much larger than its measured value[8].

Because the scenarios with $|m_{\tilde{H}_{1,2}^0}| \sim M_Z/2$ imply that the couplings of $\tilde{H}_1^0$ and $\tilde{H}_2^0$ to the lightest Higgs boson are much larger than the b–quark Yukawa coupling the lightest Higgs state decays more than 95% of the time into $\tilde{H}_1^0$ and $\tilde{H}_2^0$ in these cases while the total branching ratio into SM particles varies from 2% to 4% [38]. At the same time the LHC production cross section of the lightest Higgs state in the considered E_6 inspired SUSY models is almost the same as in the MSSM. Therefore the evidence for the Higgs boson recently presented by ATLAS [90] and CMS [91] indicates that the corresponding scenarios are basically ruled out.

[8]When $f_{\alpha\beta}$, $\tilde{f}_{\alpha\beta} \to 0$ the masses of $\tilde{H}_1^0$ and $\tilde{H}_2^0$ tend to zero and inert singlino states essentially decouple from the rest of the spectrum. In this limit the lightest non-decoupled Inert neutralino may be rather stable and can play the role of dark matter [88]. The presence of very light neutral fermions in the particle spectrum might have interesting implications for the neutrino physics (see, for example [89]).

In this context one should point out another class of scenarios that might have interesting cosmological implications. Let us consider a limit when $f_{\alpha\beta} \sim \tilde{f}_{\alpha\beta} \sim 10^{-5}$. So small values of the Yukawa couplings $f_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$ result in extremely light inert neutralino states $\tilde{H}_1^0$ and $\tilde{H}_2^0$ which are basically inert singlinos. These states have masses about 1 eV. Since $\tilde{H}_1^0$ and $\tilde{H}_2^0$ are so light and absolutely stable they form hot dark matter in the Universe[9]. These inert neutralinos have negligible couplings to Z boson and would not have been observed at earlier collider experiments. These states also do not change the branching ratios of the Z boson and Higgs decays. Moreover if Z' boson is sufficiently heavy the presence of such light Inert neutralinos does not affect Big Bang Nucleosynthesis [88]. When the masses of $\tilde{H}_1^0$ and $\tilde{H}_2^0$ are about 1 eV these states give only a very minor contribution to the dark matter density while the lightest neutralino may account for all or some of the observed dark matter density. In this case one can expect that the lifetime of the next-to-lightest exotic state (for example, inert chargino) is given by

$$\tau_{NLES} \sim \frac{8\pi^2}{f^2 M_{NLES}}\,, \tag{71}$$

where $f_{\alpha\beta} \sim \tilde{f}_{\alpha\beta} \sim f$ and M_{NLES} is the mass of the next-to-lightest exotic state. Assuming that $M_{NLES} \sim 1\,\text{TeV}$ we get $\tau_{NLES} \sim 10^{-15}\,s$. With increasing $f_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$ the masses of the lightest inert neutralino states grow and their contribution to the relic density of dark matter becomes larger. This may lead to some interesting cosmological implications. The detailed study of these implications is beyond the scope of this paper and will be considered elsewhere.

5.2 LHC signatures

We can now turn to the possible collider signatures of the E_6 inspired SUSY models with exact custodial $\tilde{Z}_2^H$ symmetry. The presence of Z' boson and exotic multiplets of matter in the particle spectrum is a very peculiar feature that may permit to distinguish the considered E_6 inspired SUSY models from the MSSM or NMSSM. Although the masses of the Z' boson and exotic states cannot be predicted there are serious reasons to believe that the corresponding particles should be relatively light. Indeed, in the simplest scenario the VEVs of H_u, H_d and S are determined by the corresponding soft scalar masses. Since naturalness arguments favor SUSY models with $O(1\,\text{TeV})$ soft SUSY breaking terms the VEV s is expected to be of the order of $1 - 10\,\text{TeV}$. On the other hand the requirement of the validity of perturbation theory up to the GUT scale sets stringent upper bounds on the low–energy values of the Yukawa couplings κ_i and λ_α whereas the gauge coupling unification implies that $g_1^{'}(q) \simeq g_1(q)$. As a consequence the Z' boson and exotic states are expected to have masses below $10\,\text{TeV}$ [10].

Collider experiments and precision EW tests set stringent limits on the mass of the Z' boson and $Z - Z'$ mixing. The direct searches at the Fermilab Tevatron ($p\overline{p} \to Z' \to l^+ l^-$) exclude Z', which is associated with $U(1)_N$, with mass below $892\,\text{GeV}$ [9] [11]. Recently ATLAS and CMS experiments ruled out E_6 inspired Z' with masses lower than $2 - 2.15\,\text{TeV}$

[9] In the context of E_6 inspired SUSY models warm dark matter was recently discussed in [92].

[10] Note that the effective μ–term ($\mu = \frac{\lambda}{\sqrt{2}} s$) can be substantially smaller than the masses of the exotic quark and inert Higgsino states because coupling λ can be considerably smaller than the Yukawa couplings λ_α and κ_i that determine the masses of these states. Indeed, in the considered E_6 inspired models there is no any relation between λ and other Yukawa couplings (see discussion in Section 2).

[11] Slightly weaker lower bound on the mass of the Z'_N boson was obtained in [93].

[81]. The analysis performed in [94] revealed that Z' boson in the E_6 inspired models can be discovered at the LHC if its mass is less than $4 - 4.5\,\text{TeV}$. The determination of its couplings should be possible if $M_{Z'} \lesssim 2 - 2.5\,\text{TeV}$ [95]. The precision EW tests bound the $Z - Z'$ mixing angle to be around $[-1.5,\, 0.7] \times 10^{-3}$ [96]. Possible Z' decay channels in E_6 inspired supersymmetric models were studied in [9], [28]. The potential influence of gauge kinetic mixing on Z' production at the 7 TEV LHC was considered in [97].

The production of a TeV scale exotic states will also provide spectacular LHC signals. Several experiments at LEP, HERA, Tevatron and LHC have searched for colored objects that decay into either a pair of quarks or quark and lepton. But most searches focus on exotic color states, i.e leptoquarks or diquarks, have integer–spin. So they are either scalars or vectors. These colored objects can be coupled directly to either a pair of quarks or to quark and lepton. Moreover it is usually assumed that leptoquarks and diquarks have appreciable couplings to the quarks and leptons of the first generation. The most stringent constraints on the the masses of leptoquarks come from the nonobservation of these exotic color states at the ATLAS and CMS experiments. Recently ATLAS collaboration ruled out first and second generation scalar leptoquarks (i.e. leptoquarks that couple to the first and second generation fermions respectively) with masses below $600 - 700\,\text{GeV}$ [98]. The CMS collaboration excluded first and second generation scalar leptoquarks which are lighter than $640 - 840\,\text{GeV}$ [99]. The experimental lower bounds on the masses of dijet resonances (in particular, diquarks) tend to be considerably higher (see, for example, [100]).

However the LHC lower bounds on the masses of exotic quarks mentioned above are not directly applicable in the case of the E_6 inspired SUSY models considered here. Since Z_2^E symmetry is conserved every interaction vertex contains an even number of exotic states. As a consequence each exotic particle must eventually decay into a final state that contains at least one lightest Inert neutralino (or an odd number of the lightest Inert neutralinos). Since stable lightest Inert neutralinos cannot be detected directly each exotic state should result in the missing energy and transverse momentum in the final state. The Z_2^E symmetry conservation also implies that in collider experiments exotic particles can only be created in pairs.

In this context let us consider the production and sequential decays of the lightest exotic quarks at the LHC first. Because D and $\overline{D}$ states are odd under the Z_2^E symmetry they can be only pair produced via strong interactions. In the scenario A the lifetime and decay modes of the lightest exotic quarks are determined by the operators $g_{ij}^D (Q_i L_4)\overline{D}_j$ and $h_{i\alpha}^E e_i^c (H_\alpha^d L_4)$ in the superpotential (14). These operators ensure that the lightest exotic quarks decay into

$$D \to u_i(d_i) + \ell(\nu) + E_T^{\text{miss}} + X\,,$$

where ℓ is either electron or muon. Here X may contain extra charged leptons that can originate from the decays of intermediate states (like Inert chargino or Inert neutralino). Since lightest exotic quarks are pair produced these states may lead to a substantial enhancement of the cross section $pp \to jj\ell^+\ell^- + E_T^{\text{miss}} + X$ if they are relatively light. In the scenario B the decays of the lightest exotic quarks are induced by the operators $g_{ij}^q \overline{D}_i d_4^c u_j^c$ and $h_{ij}^D d_4^c (H_i^d Q_j)$. As a consequence the lightest diquarks decay into

$$D \to u_i^c + d_j^c + E_T^{\text{miss}} + X\,,$$

where X again can contain charged leptons that may come from the decays of intermediate

states. In this case the presence of light D-fermions in the particle spectrum could result in an appreciable enhancement of the cross section $pp \to jjjj + E_T^{\rm miss} + X$.

In general exotic squarks are expected to be substantially heavier than the exotic quarks because their masses are determined by the soft SUSY breaking terms. Nevertheless the exotic squark associated with the heavy exotic quark maybe relatively light. Indeed, as in the case of the superpartners of the top quark in the MSSM, the large mass of the heaviest exotic quark in the E_6SSM gives rise to the large mixing in the corresponding exotic squark sector that may result in the large mass splitting between the appropriate mass eigenstates. As a consequence the lightest exotic squark can have mass in TeV range. Moreover, in principle, the lightest exotic squark can be even lighter than the lightest exotic quark. If this is a case then the decays of the lightest exotic squark are induced by the same operators which give rise to the decays of the lightest exotic quarks when all exotic squarks are heavy. Therefore the decay patterns of the lightest exotic color states are rather similar in both cases. In other words when exotic squark is the lightest exotic color state in the particle spectrum it decays into either

$$\tilde{D} \to u_i(d_i) + \ell(\nu) + E_T^{\rm miss} + X\,,$$

if exotic squark is a scalar leptoquark or

$$\tilde{D} \to u_i^c + d_j^c + E_T^{\rm miss} + X\,,$$

if it is a scalar diquark. Due to the Z_2^E symmetry conservation $E_T^{\rm miss}$ should always contain contribution associated with the lightest exotic particle. However since the lightest exotic squark is R–parity even state whereas the lightest Inert neutralino is R–parity odd particle the final state in the decay of $\tilde{D}$ should also involve the lightest neutralino to ensure that R–parity is conserved. Again, X may contain charged leptons that can stem from the decays of intermediate states. Because the Z_2^E symmetry conservation implies that the lightest exotic squarks can be only pair produced in the considered case the presence of light $\tilde{D}$ is expected to lead to an appreciable enhancement of the cross section of either $pp \to jj\ell^+\ell^- + E_T^{\rm miss} + X$ if $\tilde{D}$ is scalar leptoquark or $pp \to jjjj + E_T^{\rm miss} + X$ if $\tilde{D}$ is scalar diquark.

Thus one can see that in both scenarios when the lightest exotic color state is either D-fermion or $\tilde{D}$-scalar the collider signatures associated with these new states are rather similar. Moreover since the decays of the lightest exotic color particles lead to the missing energy and transverse momentum in the final state it might be rather problematic to distinguish the corresponding signatures from the ones which are associated with the MSSM. For example, the pair production of gluinos at the LHC should also result in the enhancement of the cross section of $pp \to jjjj + E_T^{\rm miss} + X$. In this context the presence of additional charged leptons in X can play an important role leading to characteristic signatures such as $\ell^+\ell^-$ pairs together with large missing energy in the final state. The situation also becomes a bit more promising if one assumes that the Yukawa couplings of the exotic particles have hierarchical structure similar to the one observed in the ordinary quark and lepton sectors. In this case all states which are odd under the Z_2^E symmetry couple to the third generation fermions and sfermions mainly[12]. As a consequence the presence of the relatively light exotic color states should give rise to the enhancement of the cross section of either $pp \to t\bar{t}\ell^+\ell^- + E_T^{\rm miss} + X$ or $pp \to t\bar{t}b\bar{b} + E_T^{\rm miss} + X$.

[12]This possibility was discussed at length in [32]–[34], [39].

Here it is worthwhile to point out that the collider signatues associated with the light scalar leptoquarks or diquarks in the considered E_6 inspired SUSY models are very different from the commonly established ones which have been thoroughly studied. For instance, it is expected that scalar diquarks may be produced singly at the LHC and decay into quark–quark without missing energy in the final state. The scalar leptoquarks can be only pair produced at the LHC but it is commonly assumed that these states decay into quark–lepton without missing energy as well. On the other hand in the E_6 inspired SUSY models considered here the Z_2^E symmetry conservation necessarily leads to the missing energy and transverse momentum in the corresponding final state.

The presence of relatively light exotic quark and squark can substantially modify the collider signatures associated with the production and decay of gluinos [13]. Indeed, if all squarks except the lightest exotic squark are rather heavy and the decay of the gluino into exotic quark and squark are kinematically allowed then the gluino pair production at the LHC results in $D\bar{D}\tilde{D}\tilde{\bar{D}}$ in the corresponding final state. The sequential decays of exotic quarks and squarks give rise to the enhancement of either $pp \to 4\,\ell + 4\,j + E_T^{\rm miss} + X$ if exotic color states are leptoquarks or $pp \to 8\,j + E_T^{\rm miss} + X$ if exotic color states are diquarks, modulo of course effects of QCD radiation and jet merging. The modification of the gluino collider signatures discussed above might be possible only if there are non–zero flavor-off-diagonal couplings θ^g_{ij} of gluino to D_i and $\tilde{D}_j$ $(i \neq j)$. This is a necessary condition because the lightest exotic squark is normally associated with the heaviest exotic quark. Rough estimates indicate that the corresponding modification of the gluino collider signatures can occur even when the gluino flavour-off-diagonal couplings θ^g_{ij} are relatively small, i.e. $\theta^g_{ij} \gtrsim 0.01$.

If gluino is heavier than the lightest exotic color state, but is substantially lighter than the second lightest exotic color state than the branching ratios of the nonstandard gluino decays mentioned above are suppressed. In this case the second lightest exotic color state can decay mostly into the lightest exotic color state and gluino if the corresponding decay channel is kinematically allowed. This happens when the lightest exotic color state is exotic D-fermion while the second lightest exotic color state is $\tilde{D}$-scalar or vice versa.

Other possible manifestations of the E_6 inspired SUSY models considered here are related to the presence of vectorlike states d_4^c and $\overline{d^c}_4$ as well as L_4 and $\overline{L}_4$. In the case of scenario B the fermionic components of the supermultiplets d_4^c and $\overline{d^c}_4$ can have mass below the TeV scale. One of the superpartners of this vectorlike quark state may be also relatively light due to the mixing in the corresponding squark sector. If these quark and/or squark states are light they can be pair produced at the LHC via strong interactions. Since the superfields d_4^c and $\overline{d^c}_4$ are odd under the Z_2^E symmetry the decays of the corresponding quarks (d_4) and squarks $(\tilde{d}_4)$ must always lead to the missing energy in the final state. In the limit when the lightest exotic color states include d_4 and/or $\tilde{d}_4$ whereas all other exotic states and sparticles are much heavier, the operators $h^D_{ij} d^c_4 (H^d_i Q_j)$ give rise to the following decay modes of d_4 and $\tilde{d}_4$

$$d_4 \to q_i + E_T^{\rm miss} + X\,, \qquad\qquad \tilde{d}_4 \to d_i + E_T^{\rm miss} + X\,,$$

where q_i can be either up-type or down-type quark while X may contain charged leptons which can appear as a result of the decays of intermediate states. As in the case of exotic squark the final state in the decay of d_4 should contain the lightest neutralino and the lightest Inert neutralino to ensure the conservation of R–parity and Z_2^E symmetry.

[13] Novel gluino decays in the E_6 inspired models were recently considered in [101]

Again due to the Z_2^E symmetry conservation d_4 and $\tilde{d}_4$ can be only pair produced at the LHC resulting in an enhancement of $pp \to jj + E_T^{\rm miss} + X$. If d_4 and $\tilde{d}_4$ couple predominantly to the third generation fermions and sfermions then the pair production of these quarks/squarks should lead to the presence of two heavy quarks in the final state. As before these collider signatures do not permit to distinguish easily the considered E_6 inspired SUSY models from other supersymmetric models. For example, squark pair production at the LHC can also lead to two jets and missing energy in the final state. Again, the presence of additional charged leptons in X can lead to the signatures that may help to distinguish the considered E_6 inspired SUSY models from the simplest SUSY extensions of the SM.

In the case of scenario A the fermionic components of the supermultiplets L_4 and $\overline{L}_4$ as well as one of the superpartners of this vectorlike state may have masses below the TeV scale. If all other exotic states and sparticles are rather heavy the corresponding bosonic ($\tilde{L}_4$) and fermionic (L_4) states can be produced at the LHC via weak interactions only. Because of this their production cross section is relatively small. In the considered limit the decays of L_4 and/or $\tilde{L}_4$ are induced by the operators $h^E_{i\alpha} e^c_i (H^d_\alpha L_4)$. As a consequence the decays of L_4 and/or $\tilde{L}_4$ always lead to either τ–lepton or electron/muon as well as missing energy in the final state. In the case of $\tilde{L}_4$ decays the missing energy in the final state can be associated with only one lightest Inert neutralino whereas the final state of the L_4 decays must contain at least one lightest Inert neutralino and one lightest ordinary neutralino to ensure the conservation of R–parity and Z_2^E symmetry. More efficiently L_4 and/or $\tilde{L}_4$ can be produced through the decays of the lightest exotic color states (i.e. D and/or $\tilde{D}$) if these states are relatively light and the corresponding decay channels are kinematically allowed.

The Inert Higgs bosons and/or Inert neutralino and chargino states, which are predominantly Inert Higgsinos, can be also light or heavy depending on their free parameters. Indeed, as follows from Eq. (69) the lightest Inert Higgsinos may be light if the corresponding Yukawa coupling λ_α is rather small. On the other hand if at least one coupling λ_α is large it can induce a large mixing in the Inert Higgs sector that may lead to relatively light Inert Higgs boson states. Since Inert Higgs and Higgsino states do not couple to quarks directly at the LHC the corresponding states can be produced in pairs via off–shell W and Z–bosons. Therefore their production cross section remains relatively small even when these states have masses below the TeV scale. The lightest Inert Higgs and Higgsino states are expected to decay via virtual lightest Higgs, Z and W exchange. The conservation of R–parity and Z_2^E symmetry implies that the final state in the decay of Inert Higgsino involves at least one lightest Inert neutralino while the final state in the decay of Inert Higgs state should contain at least one lightest ordinary neutralino and one lightest Inert neutralino.

As it was mentioned in the beginning of this subsection in the simplest scenario, when only H_u, H_d and S acquire VEVs at low energies, there are serious reasons to believe that the Z' boson and all exotic states from three complete 27_i representations of E_6 have masses below 10 TeV. However the situation may change dramatically when $\tilde{Z}_2^H$ even superfield $\overline{S}$ survive to low energies. In order to demonstrate this, let us consider a simple toy model, where $U(1)_N$ gauge symmetry is broken by VEVs of a pair of SM singlet superfields S and $\overline{S}$. Assuming that the superpotential of the considered model involves bilinear term $\mu_S S\overline{S}$ the part of the tree–level scalar potential, which depends on

the scalar components of the superfields S and $\overline{S}$ only, can be written as

$$V_S = (m_S^2 + \mu_S^2)|S|^2 + (m_{\overline{S}}^2 + \mu_S^2)|\overline{S}|^2 + (B_S \mu_S S\overline{S} + h.c.) + \frac{Q_S^2 g_1^{'2}}{2}\left(|S|^2 - |\overline{S}|^2\right)^2 , \quad (72)$$

where m_S^2, $m_{\overline{S}}^2$ and B_S are soft SUSY breaking parameters and Q_S is a $U(1)_N$ charge of the SM singlet superfields S. The last term in Eq. (72), which is the $U(1)_N$ D–term contribution to the scalar potential, forces the minimum of the corresponding potential to be along the D–flat direction $\langle S\rangle = \langle\overline{S}\rangle$. Indeed, in the limit $\langle S\rangle = \langle\overline{S}\rangle$ the quartic terms in the potential (72) vanish. In the considered case the scalar potential (72) remains positive definite only if $(m_S^2 + m_{\overline{S}}^2 + 2\mu_S^2 - 2|B_S\mu_S|) > 0$. Otherwise physical vacuum becomes unstable, i.e. $\langle S\rangle = \langle\overline{S}\rangle \to \infty$.

The scalar potential can be easily stabilized the if bilinear term $\mu_S S\overline{S}$ in the superpotential is replaced by

$$W_S = \lambda_0 \tilde{\phi} S\overline{S} + f(\tilde{\phi}) , \quad (73)$$

where $\tilde{\phi}$ is $\tilde{Z}_2^H$ even superfield that does not participate in the $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$ gauge interactions. When λ_0 is small (i.e. $\lambda_0 \ll 0.1$) the $U(1)_N$ D–term contribution to the scalar potential still forces the minimum of the scalar potential to be along the nearly D–flat direction if $m_S^2 + m_{\overline{S}}^2 < 0$. This condition can be satisfied because sufficiently large values of κ_i affect the evolution of m_S^2 rather strongly resulting in negative values of m_S^2 at low energies [39]. If $m_S^2 + m_{\overline{S}}^2 < 0$ and λ_0 is small then the scalar components of the superfields $\tilde{\phi}$, S and $\overline{S}$ acquire very large VEVs, i.e.

$$\langle\tilde{\phi}\rangle \sim \langle S\rangle \simeq \langle\overline{S}\rangle \sim M_{SUSY}/\lambda_0 , \quad (74)$$

where M_{SUSY} is a supersymmetry breaking scale. If $\lambda_0 \simeq 10^{-3} - 10^{-4}$ the VEVs of the SM singlet superfields S and $\overline{S}$ are of the order of $10^3 - 10^4$ TeV even when $M_{SUSY} \sim 1$ TeV. So large VEV of the superfield S may give rise to the extremely heavy spectrum of exotic particles and Z'. This can lead to the MSSM type of particle spectrum at the TeV scale.

Nevertheless even in this case the broken $U(1)_N$ symmetry leaves its imprint on the MSSM sfermion mass spectrum. Since $m_S^2 \neq m_{\overline{S}}^2$ the VEVs of the SM singlet superfields S and $\overline{S}$ deviates from the D–flat direction

$$Q_S^2 g_1^{'2}\left(\langle S\rangle^2 - \langle\overline{S}\rangle^2\right) \simeq m_{\overline{S}}^2 - m_S^2 . \quad (75)$$

As a consequence all sfermions receive an additional contribution to the mass that come from the $U(1)_N$ D–term quartic interactions in the scalar potential [102]. This contribution Δ_i is proportional to the $U(1)_N$ charge of the corresponding sfermion Q_i, i.e.

$$\Delta_i = \frac{g_1^{'2}}{2}\left(Q_1 v_1^2 + Q_2 v_2^2 + 2Q_S\left(\langle S\rangle^2 - \langle\overline{S}\rangle^2\right)\right) Q_i = M_0^2 \sqrt{40}\, Q_i , \quad (76)$$

where Q_1 and Q_2 are the $U(1)_N$ charges of H_d and H_u. Thus for the superpartners of the

first and second generation quarks and leptons one finds

$$
\begin{aligned}
m^2_{\tilde{d}_{L\,i}} &\simeq m^2_{Q_i} + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta + M_0^2\,, \\
m^2_{\tilde{u}_{L\,i}} &\simeq m^2_{Q_i} + \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta + M_0^2\,, \\
m^2_{\tilde{u}_{R\,i}} &\simeq m^2_{u^c_i} + \frac{2}{3} M_Z^2 \sin^2\theta_W \cos 2\beta + M_0^2\,, \\
m^2_{\tilde{d}_{R\,i}} &\simeq m^2_{d^c_i} - \frac{1}{3} M_Z^2 \sin^2\theta_W \cos 2\beta + 2M_0^2\,, \\
m^2_{\tilde{e}_{L\,i}} &\simeq m^2_{L_i} + \left(-\frac{1}{2} + \sin^2\theta_W\right) M_Z^2 \cos 2\beta + 2M_0^2\,, \\
m^2_{\tilde{\nu}_i} &\simeq m^2_{L_i} + \frac{1}{2} M_Z^2 \cos 2\beta + 2M_0^2\,, \\
m^2_{\tilde{e}_{R\,i}} &\simeq m^2_{e^c_i} - M_Z^2 \sin^2\theta_W \cos 2\beta + M_0^2\,.
\end{aligned}
$$

6 Conclusions

In this paper we have considered the E_6 inspired SUSY models in which a single discrete $\tilde{Z}_2^H$ symmetry forbids the tree–level flavor–changing transitions and baryon number violating operators. We assumed that the breakdown of E_6 symmetry or its subgroup lead to the rank–6 SUSY models below the GUT scale M_X. These models are based on the Standard Model (SM) gauge group together with extra $U(1)_\psi$ and $U(1)_\chi$ gauge symmetries. We also allow three copies of 27_i representations of E_6 to survive below the scale M_X so that anomalies get canceled generation by generation. If extra exotic states from 27_i–plets survive to low energies they give rise to tree–level non–diagonal flavor transitions and rapid proton decay. In order to suppress baryon number violating operators one can impose $\tilde{Z}_2^H$ discrete symmetry. We assumed that all matter superfields, that fill in complete 27_i representations of E_6, are odd under this discrete symmetry. Thus $\tilde{Z}_2^H$ symmetry is defined analogously to the matter parity Z_2^M in the simplest $SU(5)$ SUSY GUTs, that lead to the low–energy spectrum of the MSSM.

In addition to three complete fundamental representations of E_6 we further assumed the presence of of M_l and $\overline{M}_l$ supermultiplets from the incomplete $27'_l$ and $\overline{27'}_l$ representation just below the GUT scale. Because multiplets M_l and $\overline{M}_l$ have opposite $U(1)_Y$, $U(1)_\psi$ and $U(1)_\chi$ charges their contributions to the anomalies get cancelled identically. As in the MSSM we allowed the set of multiplets M_l to be used for the breakdown of gauge symmetry and therefore assumed that all multiplets M_l are even under $\tilde{Z}_2^H$ symmetry. In order to ensure that the $SU(2)_W \times U(1)_Y \times U(1)_\psi \times U(1)_\chi$ symmetry is broken down to $U(1)_{em}$ associated with the electromagnetism the set of multiplets M_l should involve H_u, H_d, S and N^c_H.

We argued that $U(1)_\psi \times U(1)_\chi$ gauge symmetry can be broken by the VEVs of N^c_H and $\overline{N}^c_H$ down to $U(1)_N \times Z_2^M$ because matter parity is a discrete subgroup of $U(1)_\psi$ and $U(1)_\chi$. Such breakdown of $U(1)_\psi$ and $U(1)_\chi$ gauge symmetries guarantees that the exotic states which originate from 27_i representations of E_6 as well as ordinary quark and lepton states survive to low energies. On the other hand the large VEVs of N^c_H and $\overline{N}^c_H$ can induce the large Majorana masses for right-handed neutrinos allowing them to be used for the see–saw mechanism. For this reason we assumed that the $U(1)_\psi \times U(1)_\chi$ symmetry is broken down to $U(1)_N \times Z_2^M$ just below the GUT scale.

The $\tilde{Z}_2^H$ symmetry allows the Yukawa interactions in the superpotential that originate

from $27'_l \times 27'_m \times 27'_n$ and $27'_l \times 27_i \times 27_k$. Since the set of multiplets M_l contains only one pair of doublets H_d and H_u the $\tilde{Z}_2^H$ symmetry defined above forbids not only the most dangerous baryon and lepton number violating operators but also unwanted FCNC processes at the tree level. Nevertheless if the set of $\tilde{Z}_2^H$ even supermultiplets M_l involve only H_u, H_d, S and N_H^c then the lightest exotic quarks are extremely long–lived particles because $\tilde{Z}_2^H$ symmetry forbids all Yukawa interactions in the superpotential that allow the lightest exotic quarks to decay. Since models with stable charged exotic particles are ruled out by different terrestrial experiments the set of supermultiplets M_l in the phenomenologically viable E_6 inspired SUSY models should be supplemented by some components of 27-plet that carry $SU(3)_C$ colour or lepton number.

In this work we required that extra matter beyond the MSSM fill in complete $SU(5)$ representations because in this case the gauge coupling unification remains almost exact in the one–loop approximation. As a consequence we restricted our consideration to two scenarios that result in different collider signatures associated with the exotic quarks. In the scenario A the set of $\tilde{Z}_2^H$ even supermultiplets M_l involves lepton superfields L_4. To ensure the unification of gauge couplings we assumed that $\overline{H}_u$ and $\overline{H}_d$ are odd under the $\tilde{Z}_2^H$ symmetry whereas supermultiplet $\overline{L}_4$ is even. Then $\overline{H}_u$ and $\overline{H}_d$ from the $\overline{27'}_l$ get combined with the superposition of the corresponding components from 27_i so that the resulting vectorlike states gain masses of order of M_X. In contrast, L_4 and $\overline{L}_4$ should form vectorlike states at low energies facilitating the decays of exotic quarks. The superfield $\overline{S}$ can be either odd or even under the $\tilde{Z}_2^H$ symmetry. The bosonic and fermionic components of $\overline{S}$ may or may not survive to low energies. In the scenario A the exotic quarks are leptoquarks.

Another scenario, that permits the lightest exotic quarks to decay within a reasonable time, implies that the set of multiplets M_l together with H_u, H_d, S and N_H^c contains extra d_4^c supermultiplet. Because in this scenario B the $\tilde{Z}_2^H$ even supermultiplets d_4^c and $\overline{d^c}_4$ give rise to the decays of the lightest exotic color states they are expected to form vectorlike states with the TeV scale masses. Then to ensure that the extra matter beyond the MSSM fill in complete $SU(5)$ representations $\overline{H}_u$ and $\overline{H}_d$ should survive to the TeV scale as well. Again we assumed that $\overline{H}_u$ and $\overline{H}_d$ are odd under the $\tilde{Z}_2^H$ symmetry so that they can get combined with the superposition of the corresponding components from 27_i forming vectorlike states at low energies. As in the case of scenario A the superfield $\overline{S}$ can be either even or odd under the $\tilde{Z}_2^H$ symmetry and may or may not survive to the TeV scale. In the scenario B the exotic quarks manifest themselves in the Yukawa interactions as superfields with baryon number $\left(\pm\frac{2}{3}\right)$.

The gauge group and field content of the E_6 inspired SUSY model discussed here can originate from the 5D and 6D orbifold GUT models in which the splitting of GUT multiplets can be naturally achieved. In particular, we studied $SU(5) \times U(1)_\chi \times U(1)_\psi$ SUSY GUT model in 5D compactified on the orbifold $S^1/(Z_2 \times Z_2')$. At low energies this model may lead to the scenarios A and B. We also considered E_6 gauge theory in $6D$ compactified on the orbifold $T^2/(Z_2 \times Z_2^I \times Z_2^{II})$ that can lead to the scenario A at low energies. In these orbifold GUT models all anomalies get cancelled and GUT relations between Yukawa couplings get spoiled. The adequate suppression of the operators, that give rise to proton decay, can be also achieved if the GUT scale $M_X \sim 1/R$ is larger than $1.5 - 2 \cdot 10^{16}\,\mathrm{GeV}$.

We examined the RG flow of gauge couplings from M_Z to M_X in the case of scenarios A and B using both analytical and numerical techniques. We derived the corresponding two–

loop RG equations and studied the running of the gauge couplings with and without extra S and $\overline{S}$ superfields at the TeV scale. In the scenario A the gauge coupling unification can be achieved for any phenomenologically reasonable value of $\alpha_3(M_Z)$ consistent with the central measured low energy value. This was already established in the case of the SUSY model with extra $U(1)_N$ gauge symmetry and low energy matter content that involves three 27-plets of E_6 as well as L_4 and $\overline{L}_4$ [36]. Our analysis here revealed that the evolution of the SM gauge couplings does not change much when the low energy particle spectrum is supplemented by the S and $\overline{S}$ chiral superfields. Thus this is not so surprising that the unification of the SM gauge couplings can be so easily achieved even in this case. In the scenario B large two–loop corrections spoil the unification of gauge couplings. Indeed, in this case the exact gauge coupling unification can be achieved only if $\alpha_3(M_Z) \lesssim 0.112$. As before the inclusion of extra S and $\overline{S}$ superfields does not change much the RG flow of $\alpha_i(t)$ and therefore does not improve gauge coupling unification. However the relative discrepancy of $\alpha_i(M_X)$ is about 10% . At the same time orbifold GUT framework does not imply the exact gauge coupling unification near the scale $M_X \sim 1/R$ because of the brane contributions to the gauge couplings. Therefore relative discrepancy of 10% between $\alpha_i(M_X)$ should not be probably considered as a big problem.

Finally we also discussed the cosmological implications and collider signatures of the E_6 inspired SUSY models discussed above. As it was mentioned the low–energy effective Lagrangian of these models is invariant under both Z_2^M and $\tilde{Z}_2^H$ symmetries. Since $\tilde{Z}_2^H = Z_2^M \times Z_2^E$ the Z_2^E symmetry associated with exotic states is also conserved. As a result the lightest exotic state, which is odd under the Z_2^E symmetry, must be stable. In the scenarios A and B the lightest and second lightest inert neutralinos tend to be the lightest exotic states in the particle spectrum. On the other hand the Z_2^M symmetry conservation implies that R–parity is conserved. Because the lightest inert neutralino $\tilde{H}_1^0$ is also the lightest R–parity odd state either the lightest R–parity even exotic state or the lightest R–parity odd state with $Z_2^E = +1$ must be absolutely stable. Most commonly the second stable state is the lightest ordinary neutralino χ_1^0 ($Z_2^E = +1$). Both stable states are natural dark matter candidates in the considered E_6 inspired SUSY models.

When $|m_{\tilde{H}_1^0}| \ll M_Z$ the lightest inert neutralino is predominantly inert singlino and its couplings to the gauge bosons, Higgs states, quarks and leptons are very small resulting in too small annihilation cross section for $\tilde{H}_1^0\tilde{H}_1^0 \to$ SM particles. As a consequence the cold dark matter density is much larger than its measured value. In principle, $\tilde{H}_1^0$ could account for all or some of the observed cold dark matter density if it had mass close to half the Z mass. In this case the lightest inert neutralino states annihilate mainly through an s–channel Z–boson. However the usual SM-like Higgs boson decays more than 95% of the time into either $\tilde{H}_1^0$ or $\tilde{H}_2^0$ in these cases while the total branching ratio into SM particles is suppressed. Because of this the corresponding scenarios are basically ruled out nowadays. The simplest phenomenologically viable scenarios imply that the lightest and second lightest inert neutralinos are extremely light. For example, these states can have masses about 1 eV. The lightest and second lightest inert neutralinos with masses about 1 eV form hot dark matter in the Universe but give only a very minor contribution to the dark matter density while the lightest ordinary neutralino may account for all or some of the observed dark matter density.

The presence of two types of dark matter is a very peculiar feature that affect the collider signatures of the considered E_6 inspired SUSY models. The most spectacular LHC signals associated with these models may come from the TeV scale exotic color states and Z'. The production of the Z' boson, that corresponds to the $U(1)_N$ gauge

symmetry, should lead to unmistakable signal $pp \to Z' \to l^+l^-$ at the LHC. The Z_2^E symmetry conservation implies that in collider experiments exotic particles can only be created in pairs. Moreover each exotic particle has to decay into a final state that contains at least one lightest inert neutralino resulting in the missing energy. Because of this the lightest exotic color state, that can be either D-fermion or $\tilde{D}$-scalar, decay into either $u_i(d_i) + \ell(\nu) + E_T^{\rm miss} + X$ if exotic quark (squark) is leptoquark or $u_i^c + d_j^c + E_T^{\rm miss} + X$ if exotic quark (squark) is diquark. The Z_2^E symmetry conservation requires that $E_T^{\rm miss}$ should always contain contribution associated with the lightest inert neutralino. Since the lightest exotic squark is R–parity even state while the lightest inert neutralino is R–parity odd particle the final state in the decay of $\tilde{D}$ should also involve the lightest ordinary neutralino to ensure R–parity conservation. Thus the pair production of the lightest exotic color state is expected to lead to a substantial enhancement of the cross section of either $pp \to jj\ell^+\ell^- + E_T^{\rm miss} + X$ or $pp \to jjjj + E_T^{\rm miss} + X$. If the Yukawa couplings of the exotic particles have hierarchical structure similar to the one observed in the ordinary quark and lepton sectors then all states which are odd under the Z_2^E symmetry couple to the third generation fermions and sfermions mainly. As a result the TeV scale exotic color states should give rise to the enhancement of the cross section of either $pp \to t\bar{t}\ell^+\ell^- + E_T^{\rm miss} + X$ or $pp \to t\bar{t}b\bar{b} + E_T^{\rm miss} + X$.

Our consideration indicates that $\tilde{D}$-scalars in the considered E_6 inspired SUSY models lead to rather unusual collider signatures. Indeed, it is commonly expected that scalar diquarks decay into quark–quark without missing energy in the final state while the scalar leptoquarks decay into quark–lepton without missing energy as well. In the models considered here the Z_2^E symmetry conservation necessarily leads to the missing energy in the corresponding final states. In addition relatively light exotic quark and squark can modify the collider signatures associated with gluinos if the decay of the gluino into exotic quark and squark is kinematically allowed. In this case gluino pair production at the LHC may result in $D\bar{D}\tilde{D}\tilde{\bar{D}}$ in the final state. The sequential decays of D-fermions and $\tilde{D}$-scalars give rise to the enhancement of either $pp \to 4\,\ell + 4\,j + E_T^{\rm miss} + X$ or $pp \to 8\,j + E_T^{\rm miss} + X$.

In the scenario B the fermionic components of the supermultiplets d_4^c and $\overline{d^c}_4$ that form vectorlike quark state as well as their superpartner may have TeV scale masses. Then these quark and/or squark states can be pair produced at the LHC via strong interactions and decay into $q_i + E_T^{\rm miss} + X$ where q_i can be either up-type or down-type quark. This may lead to an enhancement of $pp \to jj + E_T^{\rm miss} + X$.

The discovery of Z' and new exotic particles predicted by the E_6 inspired SUSY models considered here will open a new era in elementary particle physics. This would not only represent a revolution in particle physics, but would also point towards an underlying E_6 gauge structure at high energies.

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