Precision electroweak constraints on the N=3 Lee-Wick standard model

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Precision Electroweak Constraints on the $N = 3$ Lee-Wick Standard Model

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The Lee-Wick (LW) formulation of higher-derivative theories can be extended from one in which the extra degrees of freedom are represented as a single heavy, negative-norm partner for each known particle ($N = 2$), to one in which a second, positive-norm partner appears ($N = 3$). We explore the extent to which the presence of these additional states in a LW Standard Model affect precision electroweak observables, and find that they tend either to have a marginal effect (*e.g.*, quark partners on T), or a substantial beneficial effect (*e.g.*, Higgs partners on the $Zb\bar{b}$ couplings). We find that precision constraints allow LW partners to exist in broad regions of mass parameter space accessible at the LHC, making LW theories a viable beyond-Standard Model candidate.

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I. INTRODUCTION

If the particle of mass 126 GeV recently discovered [1, 2] at the Large Hadron Collider (LHC) turns out (as is widely expected) to be the Higgs scalar, then particle physics will have at last undeniably moved into the beyond-Standard Model (BSM) era. The theoretical difficulties of a universe in which the Standard Model (SM) is the ultimate theory of particle physics are well known: In addition to requiring three complete generations of fermions, and ignoring gravity but nevertheless incorporating three distinct fundamental interactions, the SM suffers from the famous hierarchy problem of a scalar particle whose renormalized mass lies quite close to the scale of electroweak symmetry breaking, rather than being driven to GUT- or Planck-scale values by the exigencies of regularizing a quadratic divergence. The most popular BSM remedies for the hierarchy problem are also well known: Low-scale supersymmetry (SUSY), large extra spacetime dimensions, and little Higgs models. As the LHC continues to generate vast amounts of new experimental data, the constraints of phenomenological viability are pushing each approach into ever smaller regions of its respective parameter space. The moment of truth for many BSM models is rapidly approaching.

The same can be said for a less well-studied approach, the Lee-Wick Standard Model (LWSM) of Grinstein, O'Connell, and Wise [3]. Inspired by the Lee and Wick (LW) program [4] of performing renormalization by promoting the spurious Pauli-Villars regulator to the status of a full, dynamical, negative-norm field, Ref. [3] showed that introducing LW partners for SM particles with the same gauge couplings eliminates quadratic divergences in loop calculations. The cancellation between positive- and negative-norm states in loops resembles the cancellation between fermions and bosons in SUSY, while the fact that the particle and its LW partner share the same statistics but carry an opposite type of parity is reminiscent of the bottom of a tower of Kaluza-Klein excitations in extra-dimension models.

The latter analogy becomes more apparent when one realizes that LW models need not terminate with a single partner. As shown in Ref. [3], the LW Lagrangian is equivalent to a particular higher-derivative (HD) theory; in particular, it is one in which 4-derivative bosonic and 3-derivative fermionic interaction terms appear, and the full HD field consists of both the conventional field and its LW partner. Of course, not just any HD Lagrangian produces an equivalent LW theory; only those that produce propagator poles at real mass values are valid for the purpose. Labeling theories by N, the number of poles in the HD field propagator, the conventional single-pole theory is labeled as $N = 1$, and the original LW theory is labeled as $N = 2$, but in principle nothing prevents the construction of $N \geq 3$ theories [5]. In such theories, one can show that the partner states alternate in norm as their mass parameters increase. The cancellation of quadratic divergences requires the participation of all N states through delicate sum rules among their couplings that seem conspiratorial at the level of the LW theory, but merely reflect the improved power counting of the equivalent HD theory.

While not as thoroughly studied as other BSM approaches, the original LWSM approach [3] has nevertheless inspired research leading to numerous publications in several different areas, including early universe models, quantum gravity, thermodynamics, and formal studies of field theory. The last of these deserves special mention because negative-norm states in field theory are peculiar objects. As has been known for decades [6], the apparent violation of unitarity induced by such states can be traded for the imposition of future boundary conditions that introduce causality violation at microscopic levels. To date, no logical argument precludes the existence of such exotic behavior, and the existence of microcausality violation can only be bounded experimentally by measurements at successively higher energy scales.

For the purposes of this paper, we avoid such thorny issues and adopt instead the pragmatic viewpoint that LW theories (or their HD equivalents) should merely be treated as effective theories good to scales of at least 14 TeV, the upper limit of physics to be probed at the LHC in the near future. The question of the viability of LWSM variants then relies upon whether the new states can be produced and observed directly, and for what mass ranges they satisfy the stringent experimental constraints imposed by electroweak precision tests (EWPT). Both of these questions have been studied in some detail in the original $N = 2$ LWSM; in the case of direct production, Refs. [7, 8] find that $N = 2$ LW gauge bosons, for example, can readily be produced at the LHC, but may be difficult to distinguish from novel states from other scenarios such as extra-dimension models. Precision observables in the $N = 2$ theory, on the other hand, have been examined in a succession of improvements [9–12] (by scanning the LW parameter space in [9]; by including only LW masses for the fields most important for the hierarchy problem [10]; by using not just oblique parameters S, T, but also the "post-LEP" parameters W, Y [11]; by including bounds from the $Zb\bar{b}$ direct correction [12]), with the consensus conclusion that LW gauge boson masses must be well over 2 TeV, and in such cases, the LW fermion masses must be substantially higher (perhaps as much as 10 TeV). If all LW masses are comparable, then the lower bound on this scale is typically ~ 7 TeV. The LW Higgs partners, on the other hand, appear to be much less tightly constrained and produce milder constraints on collider phenomenology [13–16].

In comparison, only one collider physics study of the $N = 3$ LWSM has thus far appeared [17], a paper by the present authors generalizing the study of W boson production in Ref. [7], and showing not only that such bosons can readily be produced, but also that their mass spectrum generates a signature likely unique among known BSM models. The next logical step is, of course, a study of EWPT in the $N = 3$ LWSM, which is the purpose of this paper.

On general principles, one naturally expects the $N = 3$ LWSM to allow for less stringent lower bounds on new particle masses compared to the $N = 2$ model, making for earlier discovery potential at the LHC. Of course, simply by adding new degrees of freedom to the theory (extending from $N = 2$ to $N = 3$) and then fitting to EWPT, one expects the bounds to relax; however, in LW models, one might expect the effect to be more pronounced because the negative-norm states and the new positive-norm states can produce a substantial numerical cancellation just between themselves (although the SM state must also be included in order to cancel the quadratic divergences). Since the $N = 2$ LWSM may be thought of as an $N = 3$ model in which the masses of the negative-norm states are fixed and the masses of the additional positive-norm states are taken to infinity, one expects a substantial relaxation of tension in EWPT compared to the $N = 2$ LWSM when the positive-norm masses are adjusted to lie not excessively higher than the negative-norm masses. In detailed fits, we find that this reasoning holds up to scrutiny in the scalar sector, while the addition of $N = 3$ fermions generates much more nuanced changes, sometimes even moving in the same direction as the $N = 2$ contribution. After a detailed analysis, one finds that a large parameter space of LHC-accessible masses remains open to LW partner states, making the $N = 3$ LWSM phenomenologically viable and attractive.

This paper is organized as follows. In Sec. II we review the formalism of the $N = 3$ LWSM. Section III defines the oblique EWPT parameters used in the fits, while Sec. IV considers an important non-oblique EWPT variable, the $Zb_L\bar{b}_L$ coupling. In Sec. V we analyze the effects of EWPT and present bounds on the $N = 3$ LWSM particle masses. Section VI offers discussion and concluding remarks.

II. REVIEW OF THE $N = 3$ LEE-WICK STANDARD MODEL

A Lee-Wick theory of degree N for a given field $\hat{\phi}$ is a particular higher-derivative theory in which the original Lagrangian with a canonical kinetic energy term is augmented by the addition of terms containing up to $2N$ additional covariant derivatives. Such a Lagrangian may be re-expressed in terms of an equivalent auxiliary field formalism in which $\hat{\phi}$ is a linear combination of N fields $\phi^{(1),(2),\ldots,(N)}$ that alternate in the sign of their quantum-mechanical norm. As shown in Ref. [5] and summarized in this section, this construction can be implemented independently for fields $\hat{\phi}$ that are real or complex scalars, fermions, or gauge fields. In particular, no obvious theory constraint fixes the mass parameters that appear with each additional pair of derivatives acting upon each field, so that one may consider scenarios, for example, in which only some of the SM particles have one LW partner, some have two, and some have none.

In the $N = 2$ LW theory, the opposite-sign norms are incorporated by the fields corresponding to particles and their partners that appear in the Lagrangian with a relative sign, *i.e.*, $\hat{\phi} = \phi^{(1)} - \phi^{(2)}$. For any integer $N > 2$, the origin of the equivalence between the LW theory and its HD form is imposed by means of a set of fixed parameters $\eta_{1,2,\ldots,N}$. For $N = 3$ they read [5]

$$\eta_1 \equiv \frac{\Lambda^4}{(m_2^2 - m_1^2)(m_3^2 - m_1^2)}\,, \qquad (2.1)$$

$$\eta_2 \equiv \frac{\Lambda^4}{(m_1^2 - m_2^2)(m_3^2 - m_2^2)}\,, \qquad (2.2)$$

$$\eta_3 \equiv \frac{\Lambda^4}{(m_1^2 - m_3^2)(m_2^2 - m_3^2)}\,, \qquad (2.3)$$

where $m_1 < m_2 < m_3$ are the masses of the original state and its two LW partners, and $\Lambda^4 \equiv m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2$. The parameters satisfy a variety of sum rules,

$$\sum_{i=1}^{3} m_i^{2n}\, \eta_i = 0 \quad (n = 0, 1), \qquad (2.4)$$

$$\sum_{i=1}^{3} m_i^{2n}\, \eta_i = \Lambda^4 \quad (n = 2), \qquad (2.5)$$

$$m_1^2 m_2^2 \eta_3 + m_2^2 m_3^2 \eta_1 + m_3^2 m_1^2 \eta_2 = \Lambda^4\,. \qquad (2.6)$$

that provide the means by which cancellations of quadratic loop divergences are guaranteed. They appear in slightly different permutations in fields of different spin.

A. Neutral Scalar Fields

Upon writing

$$\hat{\phi} = \sqrt{\eta_1}\,\phi^{(1)} - \sqrt{-\eta_2}\,\phi^{(2)} + \sqrt{\eta_3}\,\phi^{(3)}\,, \tag{2.7}$$

an $N = 3$ HD Lagrangian of the general form

$$\mathcal{L}_{\rm HD}^{N=3} = -\frac{1}{2}\hat{\phi}\,\Box\,\hat{\phi} - \frac{1}{2M_1^2}\hat{\phi}\,\Box^2\hat{\phi} - \frac{1}{2M_2^4}\hat{\phi}\,\Box^3\hat{\phi} - \frac{1}{2}m_\phi^2\hat{\phi}^2 + \mathcal{L}_{\rm int}(\hat{\phi}) \tag{2.8}$$

is equivalent at the quantum level to the LW Lagrangian (note the alternation of norm):

$$\mathcal{L}_{\rm LW}^{N=3} = -\frac{1}{2}\phi^{(1)}(\Box + m_1^2)\,\phi^{(1)} + \frac{1}{2}\phi^{(2)}(\Box + m_2^2)\,\phi^{(2)} - \frac{1}{2}\phi^{(3)}(\Box + m_3^2)\,\phi^{(3)} + \mathcal{L}_{\rm int}(\hat{\phi})\,, \tag{2.9}$$

provided one identifies

$$m_\phi^2 = (m_1^2 m_2^2 m_3^2)/\Lambda^4\,, \tag{2.10}$$
$$M_1^2 = \Lambda^4/(m_1^2 + m_2^2 + m_3^2)\,, \tag{2.11}$$
$$M_2^2 = \Lambda^2\,. \tag{2.12}$$

B. Yang-Mills Fields

The analogue to Eq. (2.7) reads

$$\hat{A}^\mu = A_1^\mu - \sqrt{\frac{-\eta_2}{\eta_1}}A_2^\mu + \sqrt{\frac{\eta_3}{\eta_1}}A_3^\mu\,, \tag{2.13}$$

with m_1 set to zero to guarantee the masslessness of the gauge field A_1^μ. One defines the field strength and covariant derivative acting upon an adjoint representation field X in the usual way:

$$\hat{F}^{\mu\nu} \equiv \partial^\mu \hat{A}^\nu - \partial^\nu \hat{A}^\mu - ig\,[\hat{A}^\mu, \hat{A}^\nu]\,, \tag{2.14}$$
$$\hat{D}^\mu X \equiv \partial^\mu X - ig\,[\hat{A}^\mu, X]\,. \tag{2.15}$$

Then the $N = 3$ HD Lagrangian,

$$\mathcal{L}_{\rm HD}^{N=3} = -\frac{1}{2}\,{\rm Tr}\,\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - \left(\frac{1}{m_2^2}+\frac{1}{m_3^2}\right){\rm Tr}\hat{F}_{\mu\nu}\hat{D}^\mu\hat{D}_\alpha\hat{F}^{\alpha\nu} - \frac{1}{m_2^2 m_3^2}\,{\rm Tr}\hat{F}_{\mu\nu}\hat{D}^\mu\hat{D}_\alpha\hat{D}^{[\alpha}\hat{D}_\beta\hat{F}^{\beta\nu]}\,, \tag{2.16}$$

where the superscript brackets indicate antisymmetrization of just the first and last indices (α and ν here), is equivalent to the LW Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm LW}^{N=3} \;=\; & -\frac{1}{2}\,{\rm Tr}\,F_1^{\mu\nu}F_{1\mu\nu} + \frac{1}{2}\,{\rm Tr}(D_\mu A_{2\nu} - D_\nu A_{2\mu})^2 - \frac{1}{2}\,{\rm Tr}(D_\mu A_{3\nu} - D_\nu A_{3\mu})^2 \\ & - m_2^2\,{\rm Tr}A_2^\mu A_{2\mu} + m_3^2\,{\rm Tr}A_3^\mu A_{3\mu}\,,\end{aligned} \tag{2.17}$$

which includes all of the kinetic and mass terms, plus more involved but still fairly compact expressions for cubic and quartic terms given explicitly in Ref. [5]. The alternation of norm is again apparent.

C. Chiral Fermion Fields

Chiral fermions are only slightly more complicated because their LW partners have explicit LW Dirac mass partners. For a conventional left-handed Weyl fermion field ϕ_L, the analogue of Eq. (2.7) reads

$$\hat{\phi}_L = \phi_L^{(1)} - \sqrt{\frac{-\eta_2}{\eta_1}}\,\phi_L^{(2)} + \sqrt{\frac{\eta_3}{\eta_1}}\,\phi_L^{(3)}\,, \tag{2.18}$$

and the LW partner fields $\phi_L^{(2),(3)}$ possess their own chiral partners $\phi_R^{(2),(3)}$ that arise from the process of converting the HD Lagrangian into an equivalent LW form. Defining then for each LW partner the combined field $\phi \equiv \phi_L + \phi_R$ and noting that $m_1 = 0$, the HD form reads

$$\mathcal{L}_{\rm HD}^{N=3} = \frac{1}{m_2^2 m_3^2} \overline{\hat{\phi}}_L \left[(i\hat{\not{D}})^2 - m_2^2 \right] \left[(i\hat{\not{D}})^2 - m_3^2 \right] i\hat{\not{D}} \hat{\phi}_L \, , \tag{2.19}$$

where $\hat{\not{D}}$ includes both the gauge bosons and their LW partners. The equivalent LW Lagrangian then reads

$$\mathcal{L}_{\rm LW}^{N=3} = \overline{\phi}_L^{(1)} i\hat{\not{D}} \phi_L^{(1)} - \overline{\phi}^{(2)} (i\hat{\not{D}} - m_2) \phi^{(2)} + \overline{\phi}^{(3)} (i\hat{\not{D}} - m_3) \phi^{(3)} \, . \tag{2.20}$$

In the case of a fundamental right-handed Weyl field ϕ_R contained in a HD Lagrangian field $\hat{\phi}_R$, the definitions proceed exactly as above, with the substitution $L \leftrightarrow R$. However, one should note that the R chiral partners induced in the $\hat{\phi}_L$ construction are distinct fields from those appearing directly in the definition $\hat{\phi}_R$, and vice versa for L chiral partners.

The original paper [3] adopts the notation of placing a prime on fields that appear not through HD superfields but rather through their Dirac mass terms[1]; for example, in the third generation, the SM fields t_L, b_L transforming under SU(2)×U(1) as $(\mathbf{2}, +\frac{1}{6})$ are joined by $N = 2$ LW partners $\tilde{t}_L$, $\tilde{b}_L$, and the latter have Dirac mass partners (mass parameter M_q) $\tilde{t}'_R$, $\tilde{b}'_R$, respectively, all of which transform as $(\mathbf{2}, +\frac{1}{6})$. The SM fields t_R and b_R, transforming as $(\mathbf{1}, +\frac{2}{3})$ and $(\mathbf{1}, -\frac{1}{3})$, respectively, have $N = 2$ LW partners $\tilde{t}_R$, $\tilde{b}_R$, which in turn have Dirac mass partners $\tilde{t}'_L$ (mass M_t), $\tilde{b}'_L$ (mass M_b), respectively. For $N > 2$, we retain the prime convention of [3], replace the tildes with superscripts $(2), (3), \ldots$, and attach corresponding subscripts to the masses (*e.g.*, M_{q2}, M_{b3}). For purposes of numerical analysis, the fields are more conveniently collected [10] by flavor and chirality, rather than by SU(2)×U(1) quantum numbers. In the $N = 3$ case,

$$\begin{aligned} T_{L,R}^T &\equiv \left(t_{L,R}^{(1)}, \, t_{L,R}^{(2)}, \, t_{L,R}^{'\,(2)}, \, t_{L,R}^{(3)}, \, t_{L,R}^{'\,(3)} \right) , \\ B_{L,R}^T &\equiv \left(b_{L,R}^{(1)}, \, b_{L,R}^{(2)}, \, b_{L,R}^{'\,(2)}, \, b_{L,R}^{(3)}, \, b_{L,R}^{'\,(3)} \right) . \end{aligned} \tag{2.21}$$

D. Complex Scalar Fields

The generalization of the real scalar field ϕ to a complex scalar multiplet H transforming in the fundamental representation of a non-Abelian gauge group requires only the promotion of ordinary derivatives to covariant ones. The analogue of Eq. (2.7) reads

$$\hat{H} = \sqrt{\eta_1} H^{(1)} - \sqrt{-\eta_2} H^{(2)} + \sqrt{\eta_3} H^{(3)} \, , \tag{2.22}$$

and relates the HD form,

$$\mathcal{L}_{\rm HD}^{N=3} = \hat{D}_\mu \hat{H}^\dagger \hat{D}^\mu \hat{H} - m_H^2 \hat{H}^\dagger \hat{H} - \frac{1}{M_1^2} \hat{H}^\dagger (\hat{D}_\mu \hat{D}^\mu)^2 \hat{H} - \frac{1}{M_2^4} \hat{H}^\dagger (\hat{D}_\mu \hat{D}^\mu)^3 \hat{H} + \mathcal{L}_{\rm int}(\hat{H}) \, , \tag{2.23}$$

to the equivalent LW form

$$\begin{aligned} \mathcal{L}_{\rm LW}^{N=3} \;=\; & -H^{(1)\dagger} (\hat{D}_\mu \hat{D}^\mu + m_1^2) H^{(1)} + H^{(2)\dagger} (\hat{D}_\mu \hat{D}^\mu + m_2^2) H^{(2)} - H^{(3)\dagger} (\hat{D}_\mu \hat{D}^\mu + m_3^2) H^{(3)} \\ & + \mathcal{L}_{\rm int}(\hat{H}) \, , \end{aligned} \tag{2.24}$$

with the mass parameters related as in Eqs. (2.10)–(2.12), with $m_\phi \to m_H$.

In the particular case of the SM Higgs multiplet, $m_1 = 0$, and the lightest scalar obtains mass only through spontaneous symmetry breaking with vacuum expectation value v. Writing

$$\mathcal{L}_{\rm HD}^{N=3} \;=\; \mathcal{L}_{\rm HD}^{N=3}(m_H^2 = 0) + \tilde{\mathcal{L}}_{\rm int}(\hat{H}) \, , \tag{2.25}$$

$$-\tilde{\mathcal{L}}_{\rm int}(\hat{H}) \;\equiv\; \frac{\lambda}{4} \left(\hat{H}^\dagger \hat{H} - \frac{v^2}{2} \right)^2 , \tag{2.26}$$

[1] In contrast, Ref. [12] uses primes exclusively for the right-handed HD superfields and Dirac mass partners of its component fields.

the equivalent LW Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\rm LW}^{N=3} &= \hat{D}_\mu H^{(1)\dagger}\hat{D}^\mu H^{(1)} - \hat{D}_\mu H^{(2)\dagger}\hat{D}^\mu H^{(2)} + \hat{D}_\mu H^{(3)\dagger}\hat{D}^\mu H^{(3)} \\ &\quad + m_2^2 H^{(2)\dagger}H^{(2)} - m_3^2 H^{(3)\dagger}H^{(3)} + \tilde{\mathcal{L}}_{\rm int}(\hat{H})\,. \end{aligned} \tag{2.27}$$

In unitary gauge,

$$H^{(1)} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h_1)\end{pmatrix}, \quad H^{(2)} = \begin{pmatrix} ih_2^+ \\ \frac{1}{\sqrt{2}}(h_2+iP_2)\end{pmatrix}, \quad H^{(3)} = \begin{pmatrix} ih_3^+ \\ \frac{1}{\sqrt{2}}(h_3+iP_3)\end{pmatrix}, \tag{2.28}$$

where the fields h_i, P_i, and h_i^+ denote the scalar, pseudoscalar, and charged Higgs components, respectively, the mass terms in Eq. (2.27) read

$$\begin{aligned}\mathcal{L}_{\rm mass}^{N=3} &= \frac{1}{2}m_2^2\,(2h_2^- h_2^+ + h_2^2 + P_2^2) - \frac{1}{2}m_3^2\,(2h_3^- h_3^+ + h_3^2 + P_3^2) \\ &\quad -\frac{1}{2}m^2(h_1 - \sqrt{-\eta_2}h_2 + \sqrt{\eta_3}h_3)^2\,, \end{aligned} \tag{2.29}$$

with $m^2 = \lambda v^2/2$. The pseudoscalar and charged scalar fields therefore have mass eigenvalues $m_{2,3}$, while the neutral scalar fields are mixed. The mass eigenvectors h^0 in the mixed sector are obtained by a symplectic transformation S that preserves the relative signs of the kinetic terms via a metric $\eta = {\rm diag}(+,-,+)$ but diagonalizes the mass matrix $\mathcal{M}$ in $h^\dagger \mathcal{M}\eta h$:

$$h^0 = S^{-1}h\,, \qquad S^\dagger \eta S = \eta\,, \tag{2.30}$$

so that

$$\mathcal{M}_0\eta = S^\dagger \mathcal{M}\eta S\,. \tag{2.31}$$

In the $N=2$ case [3], the elements of S consist of $\sinh\phi$ and $\cosh\phi$ of a single "Euler angle" ϕ. For higher N, S is similarly expressible as the symplectic analogue to a multidimensional Euler rotation matrix. In any case, the transformation S for any given mixing matrix $\mathcal{M}$ is easily found numerically.

E. Fermion Mass Diagonalization

Since the Yukawa couplings appear as

$$\mathcal{L}_{\rm Yuk} = -y_t\,\bar{\hat{q}}_L \hat{H}\hat{b}_R - y_b\,\bar{\hat{q}}_L(\epsilon\hat{H}^\dagger)\,\hat{t}_R + {\rm H.c.}\,, \tag{2.32}$$

where $\epsilon \equiv i\sigma_2$, the fermion mass terms may be expressed in terms of the ratios of η's appearing in Eq. (2.18). In the case of t quarks for $N=3$, one may abbreviate $m_t \equiv y_t v/\sqrt{2}$ and:

$$\begin{aligned} \cosh\phi_q &= \frac{M_{q3}}{\sqrt{M_{q3}^2 - M_{q2}^2}}\,, &\qquad \sinh\phi_q &= \frac{M_{q2}}{\sqrt{M_{q3}^2 - M_{q2}^2}}\,, \\ \cosh\phi_t &= \frac{M_{t3}}{\sqrt{M_{t3}^2 - M_{t2}^2}}\,, &\qquad \sinh\phi_t &= \frac{M_{t2}}{\sqrt{M_{t3}^2 - M_{t2}^2}}\,, \end{aligned} \tag{2.33}$$

which give mass terms, using the notation of Eq. (2.21), of the form

$$\mathcal{L}_{t\,\rm mass}^{\rm N=3} = -\overline{T}_L \eta \mathcal{M}_t^\dagger T_R + {\rm H.c.}\,, \tag{2.34}$$

where

$$\mathcal{M}_t^{N=3}\eta = \begin{pmatrix} m_t & -m_t\cosh\phi_q & 0 & m_t\sinh\phi_q & 0 \\ -m_t\cosh\phi_t & m_t\cosh\phi_q\,\cosh\phi_t & -M_{t2} & -m_t\sinh\phi_q\,\cosh\phi_t & 0 \\ 0 & -M_{q2} & 0 & 0 & 0 \\ m_t\sinh\phi_t & -m_t\cosh\phi_q\,\sinh\phi_t & 0 & m_t\sinh\phi_q\,\sinh\phi_t & +M_{t3} \\ 0 & 0 & 0 & +M_{q3} & 0 \end{pmatrix}, \tag{2.35}$$

where the metric $\eta = \mathrm{diag}(+,-,-,+,+)$ reflects the norms of the component states, and thus also appears in the corresponding kinetic terms. The diagonalization of the mass matrix to a form $\mathcal{M}_{t0}$ with positive eigenvalues therefore requires independent transformation matrices $S^t_{L,R}$ for each quark flavor (here, t) satisfying the constraints

$$S_L^\dagger \eta S_L = \eta \, , \quad S_R^\dagger \eta S_R = \eta \, , \quad \mathcal{M}_0 \eta = S_R^\dagger \mathcal{M} \eta S_L \, , \tag{2.36}$$

so that the mass eigenstates are obtained as

$$T^0_{L,R} = (S^t_{L,R})^{-1} T_{L,R} \, , \tag{2.37}$$

and similarly for the B sector. Obtaining numerical solutions for $S^t_{L,R}$ is most efficiently accomplished by converting this system into an equivalent eigenvalue problem [16].

III. BOUNDS ON OBLIQUE PARAMETERS

A. Formalism and Tree-Level Contributions

Bounds on BSM physics are typically expressed in terms of oblique (flavor-universal, arising from gauge boson vacuum polarization loops) and direct (flavor-specific, arising from vertex, box, *etc.*, corrections) parameters [18]. The best-known oblique electroweak observables are the dimensionless Peskin-Takeuchi (PT) parameters [19] S, T, U, which represent all independent finite combinations obtained from differences of the vacuum polarization functions and their first derivatives. As better data (particularly from LEP2) became available in the 1990s, probing the oblique corrections to second-derivative order became possible; Barbieri *et al.* [20] developed a complete set of such "post-LEP" parameters, $\hat{S}$, $\hat{T}$, $\hat{U}$ (the PT parameters with different normalizations[2]), V, W, X, Y, and Z. Just as Ref. [19] argued that U is numerically small, Ref. [20] argued that V, X, and Z can be neglected in EWPT, leaving only $\hat{S}$, $\hat{T}$, W, and Y as the important independent oblique parameters. As argued in Ref. [20], the post-LEP parameters are essential for describing EWPT in all "universal" models, defined as those in which deviations from the SM appear only in gauge boson self-energy contributions, and are coupled to the light fermion currents in the usual $gJ{\cdot}A$ manner; as shown in Ref. [11], the ($N = 2$) LWSM is of this type.

The primitive electroweak parameters are obtained in Ref. [20] as:

$$\frac{1}{g'^2} \equiv \Pi'_{\hat{B}\hat{B}}(0) \, , \quad \frac{1}{g^2} \equiv \Pi'_{\hat{W}^+\hat{W}^-}(0) \, , \tag{3.1}$$

$$\frac{1}{\sqrt{2}G_F} = -4\Pi_{\hat{W}^+\hat{W}^-}(0) = v^2 \, . \tag{3.2}$$

In the tree-level SM, these just give the usual parameters $g' = g_1$, $g = g_2$ and v; however, these relations persist in the LWSM as well. The reciprocal powers of coupling constant arise from the choice of a noncanonical normalization of the field strengths [20] designed to give a convenient separation of g', g, and v in Eqs. (3.1)–(3.2). From Eq. (2.16) one quickly extracts for the $N = 3$ model [where, *e.g.*, $M_1^{(3)}$ indicates the 3rd LW partner mass for the U(1) SM gauge group]:

$$\begin{aligned} \Pi_{\hat{W}^+\hat{W}^-}(q^2) = \Pi_{\hat{W}^3\hat{W}^3}(q^2) &= \frac{q^2}{g_2^2} - \frac{(q^2)^2}{g_2^2}\left[\frac{1}{M_2^{(2)\,2}} + \frac{1}{M_2^{(3)\,2}}\right] - \frac{v^2}{4} \, , \\ \Pi_{\hat{W}^3\hat{B}}(q^2) &= \frac{v^2}{4} \, , \\ \Pi_{\hat{B}\hat{B}}(q^2) &= \frac{q^2}{g_1^2} - \frac{(q^2)^2}{g_1^2}\left[\frac{1}{M_1^{(2)\,2}} + \frac{1}{M_1^{(3)\,2}}\right] - \frac{v^2}{4} \, , \end{aligned} \tag{3.3}$$

[2] Note that the vacuum polarization functions $\Pi(q^2)$ of Ref. [20] are opposite in sign to those as defined in Ref. [19].

from which one sees that the relations $g' = g_1$, $g = g_2$, and Eq. (3.2) are preserved. In addition, one can easily compute the tree-level oblique electroweak parameters as done for the $N = 2$ model in Ref. [11]:

$$\hat{S} \equiv g^2 \, \Pi'_{\hat{W}^3\hat{B}}(0) = 0 \,, \tag{3.4}$$

$$\hat{T} \equiv \frac{g^2}{m_W^2} \left[\Pi_{\hat{W}^3\hat{W}^3}(0) - \Pi_{\hat{W}^+\hat{W}^-}(0)\right] = 0 \,, \tag{3.5}$$

$$W \equiv \frac{1}{2} g^2 m_W^2 \, \Pi''_{\hat{W}^3\hat{W}^3}(0) = -m_W^2 \left[\frac{1}{M_2^{(1)\,2}} + \frac{1}{M_2^{(2)\,2}}\right] , \tag{3.6}$$

$$Y \equiv \frac{1}{2} g'^{\,2} m_W^2 \, \Pi''_{\hat{B}\hat{B}}(0) = -m_W^2 \left[\frac{1}{M_1^{(2)\,2}} + \frac{1}{M_1^{(3)\,2}}\right] . \tag{3.7}$$

Here, the first equality in each equation defines the corresponding post-LEP parameter [20]. The absence of tree-level contributions to $\hat{S}$ and $\hat{T}$ was first noted in Ref. [11]. Moreover, Ref. [12] noted that the scheme defining Eq. (3.7) precludes fermionic one-loop corrections to Y, while W (which is defined in terms of $\Pi_{\hat{W}^3\hat{W}^3}$ rather than $\Pi_{\hat{W}^+\hat{W}^-}$, even when loop corrections are included) was found to have fermionic one-loop corrections that are numerically small compared to the tree-level value given in Eq. (3.6). At this level of analysis, one therefore only needs to compute one-loop contributions to $\hat{S}$ and $\hat{T}$, as was done for the $N = 2$ LWSM in Ref. [12].

B. Fermion Loop Contributions

After the tree-level contributions, the most important contributions to the oblique parameters (indeed, the leading ones for $\hat{S}$ and $\hat{T}$) arise from one-loop diagrams of the t and b quarks, as depicted in Fig. 1.

Consider the one-loop fermionic contributions to the self-energy connecting generic gauge bosons $\hat{A}$ and $\hat{B}$ (the latter not to be confused with the actual $\hat{B}$ field in the Standard Model). To do so, we begin with mass-diagonalized fermion fields labeled by i, j, and write the interaction Lagrangian:

$$\mathcal{L} = \bar{\Psi}_i^0 \gamma^\mu [\hat{A}_\mu (A_{ij}^{L,\Psi} P_L + A_{ij}^{R,\Psi} P_R) + \hat{B}_\mu (B_{ij}^{L,\Psi} P_L + B_{ij}^{R,\Psi} P_R)] \Psi_j^0 \,. \tag{3.8}$$

The fermionic mass eigenstate fields $(\Psi_i^0)^T$ are defined by combining Eqs. (2.21) and (2.37). The coupling matrices are the charges in mass basis, *e.g.*, $A_{ij}^{L,\Psi} = S_L^{\Psi\,\dagger} Q_{A,L}^{\Psi} \eta S_L^{\Psi}$. Here, Q_A^Ψ is the matrix of fermion charges under the gauge group A, and the superscript Ψ may refer to a single flavor (as for γ, Z^0) or a specific flavor transition (as for $W^\pm$). The right-handed coupling matrices are obtained by exchanging $L \leftrightarrow R$.

In accord with the noncanonical normalization of fields inherited by the polarization functions in Eqs. (3.1)–(3.2), the fermionic one-loop contribution to the self-energy contains no gauge coupling constants, and is expressed as:

$$\begin{aligned}\Pi_{AB}(q^2) = {} & \frac{C}{8\pi^2} \\ & \times \sum_{\Psi=T,B} \sum_{i,j} \eta_{ii}\eta_{jj} \left[(A_{ij}^{L,\Psi} B_{ji}^{L,\Psi} + A_{ij}^{R,\Psi} B_{ji}^{R,\Psi}) I_1(q^2) + (A_{ij}^{L,\Psi} B_{ji}^{R,\Psi} + A_{ij}^{R,\Psi} B_{ji}^{L,\Psi}) I_2(q^2) m_i m_j \right] ,\end{aligned} \tag{3.9}$$

where C is a color factor ($= N_c$ for quarks coupling to colorless gauge bosons). Defining $\Delta \equiv -q^2 x(1-x) + m_i^2 x + m_j^2(1-x)$ for the usual two-propagator factor, and using primes to indicate q^2 derivatives and subscript 0 to indicate a function evaluated at $q^2 = 0$ so that $\Delta_0 = m_i^2 x + m_j^2(1-x)$, $\Delta_0' = -x(1-x)$, and $\Delta_0'' = 0$, the integrals are defined as follows:

$$I_1(q^2) \equiv \int_0^1 dx \;\; (2\Delta - \Delta_0) \, \ln(\Delta/M^2) \,, \tag{3.10}$$

$$I_2(q^2) \equiv -\int_0^1 dx \;\; \ln(\Delta/M^2) \,. \tag{3.11}$$

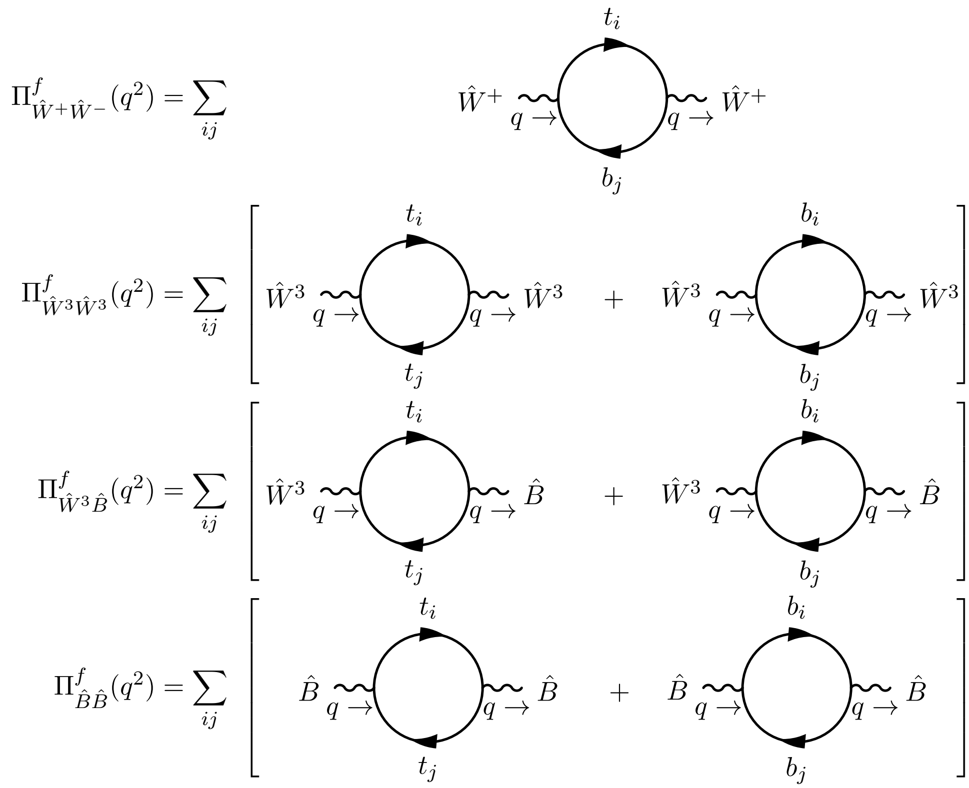

FIG. 1: Fermion vacuum polarization Feynman diagrams that provide the dominant contributions to the electroweak precision observables $\hat{S}$ and $\hat{T}$.

One then obtains the moments of the integrals relevant to the oblique parameters:

$$I_{10} = \int_0^1 dx\ \Delta_0 \ln(\Delta_0/M^2)\,, \tag{3.12}$$

$$I_{20} = -\int_0^1 dx\ \ln(\Delta_0/M^2)\,, \tag{3.13}$$

$$I'_{10} = \int_0^1 dx\ \Delta'_0[1 + 2\ln(\Delta_0/M^2)]\,, \tag{3.14}$$

$$I'_{20} = -\int_0^1 dx\ \Delta'_0/\Delta_0\,, \tag{3.15}$$

$$I''_{10} = 3\int_0^1 dx\ (\Delta'_0)^2/\Delta_0\,, \tag{3.16}$$

$$I''_{20} = \int_0^1 dx\ (\Delta'_0/\Delta_0)^2\,. \tag{3.17}$$

The factor M^2 contains the parameter of the logarithmic divergence and various subtraction constants associated with the regularization procedure. Of course, M^2 must cancel from the complete expressions for the oblique parameters, since they are observables. The individual integrals are straightforward and give:

$$
\begin{aligned}
I_{10} &= -\frac{1}{4}(m_i^2+m_j^2)+\frac{1}{2}\frac{m_i^4\ln(m_i^2/M^2)-m_j^4\ln(m_j^2/M^2)}{m_i^2-m_j^2}\,, \\
&\to m_i^2\ln\frac{m_i^2}{M^2}\,, \quad m_j\to m_i\,; \qquad (3.18)\\
I_{20} &= 1-\frac{m_i^2\ln(m_i^2/M^2)-m_j^2\ln(m_j^2/M^2)}{m_i^2-m_j^2}\,, \\
&\to -\ln\frac{m_i^2}{M^2}\,, \quad m_j\to m_i\,; \qquad (3.19)\\
I'_{10} &= -\frac{1}{3}\left\{\frac{m_i^4(m_i^2-3m_j^2)}{(m_i^2-m_j^2)^3}\ln\left(\frac{m_i^2}{M^2}\right)-\frac{m_j^4(m_j^2-3m_i^2)}{(m_i^2-m_j^2)^3}\ln\left(\frac{m_j^2}{M^2}\right)+\frac{m_i^4-8m_i^2m_j^2+m_j^4}{3(m_i^2-m_j^2)^2}\right\}, \\
&\to -\frac{1}{6}\left[1+2\ln\left(\frac{m_i^2}{M^2}\right)\right]\,, \quad m_j\to m_i\,; \qquad (3.20)\\
I'_{20} &= -\frac{(m_im_j)^2}{(m_i^2-m_j^2)^3}\ln\left(\frac{m_i^2}{m_j^2}\right)+\frac{m_i^2+m_j^2}{2(m_i^2-m_j^2)^2}\,, \\
&\to \frac{1}{6m_i^2}\,, \quad m_j\to m_i\,; \qquad (3.21)\\
I''_{10} &= \frac{3(m_im_j)^4}{(m_i^2-m_j^2)^5}\ln\left(\frac{m_i^2}{m_j^2}\right)+\frac{(m_i^2+m_j^2)(m_j^2-8m_i^2m_j^2+m_i^4)}{4(m_i^2-m_j^2)^4}\,, \\
&\to \frac{1}{10m_i^2}\,, \quad m_j\to m_i\,; \qquad (3.22)\\
I''_{20} &= -\frac{2(m_im_j)^2(m_i^2+m_j^2)}{(m_i^2-m_j^2)^5}\ln\left(\frac{m_i^2}{m_j^2}\right)+\frac{m_i^4+10m_i^2m_j^2+m_j^4}{3(m_i^2-m_j^2)^4}\,, \\
&\to \frac{1}{30m_i^4}\,, \quad m_j\to m_i\,. \qquad (3.23)
\end{aligned}
$$

These expressions are inserted into Eq. (3.9) to produce the full results for the fermionic one-loop contributions; however, the $S_{L,R}$ matrices enter the couplings A, B (and both $S_{L,R}$ are required [Eq. (2.36)] to produce the fermion mass eigenvalues). While analytic expansions for $S_{L,R}$ appear in the literature [9, 13], in practice we perform the calculations numerically and therefore do not present the full cumbersome expressions for the oblique parameters.

IV. CONSTRAINTS FROM THE $Zb_L\bar{b}_L$ COUPLING

One of the more interesting direct electroweak precision observables in terms of the tension between the experimental measurement and its SM prediction is the $Zb_L\bar{b}_L$ coupling. As noted long ago [21], its leading contribution in the gaugeless limit [*i.e.*, ignoring effects suppressed by $(m_{Z^0}/m_t)^2$] is most easily obtained by computing the triangle loop diagram of Fig. 2, in which a Goldstone boson ϕ^0 (the one eaten by the Z^0) of momentum p splits into a $t\bar{t}$ pair, which subsequently (via exchange of a charged scalar) decays to $b_L\bar{b}_L$. The invariant amplitude for this triangle loop diagram in the $p\to 0$ limit can be parametrized as

$$
i\mathcal{M} = -\frac{2}{v}(\delta g_L^{b\bar{b}})\not{p}P_L\,. \qquad (4.1)
$$

The coupling $g_L^{b\bar{b}}$ is derived from a combination of the $Z^0\to b\bar{b}$ branching fraction R_b and its forward-backward asymmetry A_b; an indication of its sensitivity to small changes in both is given in Ref. [22]:

$$
\begin{aligned}
\delta g_L^{b\bar{b}} &\equiv g_L^{b\bar{b},\,\mathrm{exp}}-g_L^{b\bar{b},\,\mathrm{SM}} \\
&= -1.731\,\delta R_b-0.1502\,\delta A_b\,, \qquad (4.2)
\end{aligned}
$$

where the normalization has been adjusted [*i.e.*, removing the $e/(\sin\theta_W\cos\theta_W)$ coefficient] to match that used elsewhere in this section. Its most recent experimental value $g_L^{b\bar{b},\,\mathrm{exp}} = -0.4182(15)$ has not changed since the

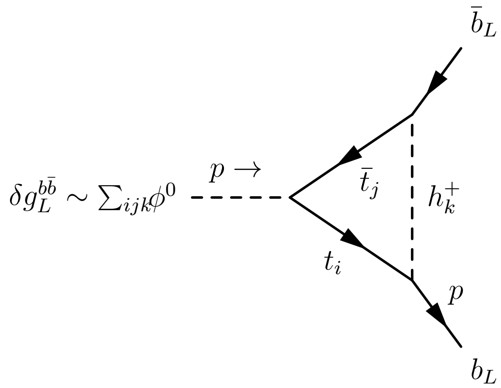

FIG. 2: Dominant diagram contributing to the $Zb_L\bar{b}_L$ coupling. ϕ^0 is the Goldstone boson eaten by the Z^0, and indices i,j,k denote mass eigenstates. The coupling is defined in the limit $p \rightarrow 0$.

combined LEP/SLD 2005 analysis [23]. The SM value $g_L^{b\bar{b},\,\mathrm{SM}} = -0.42114^{+45}_{-24}$ from [23] gives $\delta g_L^{b\bar{b}} = +2.94(157)\cdot 10^{-3}$, meaning that the SM value was $\approx 2\sigma$ low, thus strongly disfavoring any new physics contribution with $\delta g_L^{b\bar{b}} < 0$. The current Particle Data Group [24] values for R_b^{SM} and A_b^{SM}, however, lead [via Eq. (4.2)] to a somewhat relaxed bound,

$$\delta g_L^{b\bar{b}} = +2.69(157) \cdot 10^{-3} \,, \tag{4.3}$$

which we use in our analysis.

The effect of $N = 2$ LWSM states on $\delta g_L^{b\bar{b}}$ has been considered twice in the literature. The central result of Ref. [14] is that current precision bounds allow LW Higgs partner masses to be significantly lighter than other LW states. Therefore, [14] effectively compute $\delta g_L^{b\bar{b}}$ including only a LW Higgs partner in the triangle loop diagram, giving (in our normalization):

$$\delta g_L^{b\bar{b}} = -\frac{m_t^2}{16\pi^2 v^2}\left[\frac{R}{R-1} - \frac{R\ln R}{(R-1)^2}\right] , \tag{4.4}$$

where $R = (m_t/m_{h_2})^2$, so that $\delta g_L^{b\bar{b}} < 0$. $\delta g_R^{b\bar{b}}$ in the LWSM is driven by m_b and hence is numerically much smaller. Since $\delta g_L^{b\bar{b}}$ and δR_b are anti-correlated [Eq. (4.2)], and since δR_b is positive [23, 24], Ref. [14] then states that the LW Higgs partner contribution acts in the direction of reconciling the discrepancy, and concludes that $\delta g_L^{b\bar{b}}$ analysis gives no meaningful bound on the LW scalar mass. However, Eq. (4.2) shows that $\delta g_L^{b\bar{b}}$ also depends strongly upon δA_b, and the combined effect is to create the situation described above, in which new physics $\delta g_L^{b\bar{b}} < 0$ contributions are actually more difficult to accommodate. We take this additional effect into account in our analysis.

On the other hand, Ref. [12] uses the full $\delta g_L^{b\bar{b}}$ bound from [23, 24] described above, but includes only LW t-quark partners in the triangle diagram, thus producing the result

$$\delta g_L^{b\bar{b}} = -\frac{m_t^4}{32\pi^2 v^2 M_q^2}\left[5\ln\frac{M_q^2}{m_t^2} - \frac{49}{6}\right] , \tag{4.5}$$

at leading orders in m_t^2/M_q^2. The result of [12] obtained from this observable is the most stringent in their entire analysis, giving a lower bound of $M_q \gtrsim 4$ TeV. However, the LW correction (4.5) is a very shallow function of M_q (see their Fig. 8), and the small change in the SM value of $g_L^{b\bar{b}}$ described above is alone enough to push the bound back to about $M_q \gtrsim 1.2$ TeV. Obviously, the contribution from the LW Higgs partner must also be included in a global analysis, and since it is also negative (and indeed, turns out to be comparable in magnitude to the LW t contribution), all of the mass lower bounds in such a circumstance would be higher, but these multiple considerations should serve to illustrate that room exists in mass parameter space to accommodate interesting LWSM possibilities even in the $N = 2$ case.

Here, we examine the $N = 3$ LWSM contribution to $\delta g_L^{b\bar{b}}$; since the $N = 2$ effect was computed in Ref. [12], we closely follow the notation introduced there. The Yukawa Lagrangian

$$\mathcal{L}_{\rm Yuk} = -iy_t \sum_{i,j} \left\{ \frac{1}{\sqrt{2}} \hat{\phi}^0 \left[\alpha_{ij} \bar{t}_i P_R t_j - \alpha_{ji} \bar{t}_i P_L t_j \right] + \beta_{ij} \left[\hat{\phi}^- \bar{b}_i P_R t_j - \hat{\phi}^+ \bar{t}_j P_L b_i \right] \right\} \tag{4.6}$$

has couplings α and β closely related to the ones appearing in the mass matrix (2.35) with the Dirac mass parameters excluded. Specifically,

$$\begin{aligned} \alpha &\equiv (S_L^t)^\dagger \alpha_0 S_R^t \,, \\ \beta &\equiv (S_L^b)^\dagger \beta_0 S_R^t \,, \end{aligned} \tag{4.7}$$

where, for the example of the $N = 3$ case,

$$\alpha_0^{N=3} = \beta_0^{N=3} \equiv \begin{pmatrix} 1 & -\cosh\phi_q & 0 & \sinh\phi_q & 0 \\ -\cosh\phi_t & \cosh\phi_q \cosh\phi_t & 0 & -\sinh\phi_q \cosh\phi_t & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \sinh\phi_t & -\cosh\phi_q \sinh\phi_t & 0 & \sinh\phi_q \sinh\phi_t & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} . \tag{4.8}$$

The most important distinction between the expressions here and those in Ref. [12] is actually not the addition of the $N = 3$ fermion partners, but rather the presence of the entire HD scalar fields $\hat{\phi}^0$, $\hat{\phi}^\pm$ whose SM content is the set of Goldstone bosons, and that enter with the relative weights as in Eq. (2.22). As indicated in Eq. (2.28)–(2.29), the LW partners to these fields are physical, massive states that must be included in the calculation of $\delta g_L^{b\bar{b}}$ but were omitted in Ref. [12].

The basic result of the $\delta g_L^{b\bar{b}}$ calculation in Ref. [12] is that the LW t-quark partners in the loop tend to slightly exacerbate the tension with the measured value, thus forcing an even more stringent lower bound on the LW quark mass (4 TeV) than that obtained from $\hat{T}$. As pointed out in Ref. [14], however, the heavy $h_2^\pm$ can be much lighter ($\gtrsim 500$ GeV) and still satisfy all precision constraints. Noting first from Eq. (2.29) that the charged scalar masses do not mix, and recalling that the virtual scalar in the $\delta g_L^{b\bar{b}}$ diagram is charged, the extra signs in the $h_{2,3}^\pm$ propagators can be used to oppose the contribution from the original diagram with a virtual $\phi^\pm$, thus relieving much of the additional tension in $\delta g_L^{b\bar{b}}$. The full expression reads

$$\begin{aligned} \delta g_L^{b\bar{b}} \;=\; & \frac{1}{16\pi^2} \cdot \frac{y_t^3 v}{2\sqrt{2}} \left\{ \sum_i \eta_k \beta_{0i}^2 \alpha_{ii} \frac{m_{t_i}}{m_{t_i}^2 - m_{h_k}^2} \left[1 - \frac{m_{h_k}^2}{m_{t_i}^2 - m_{h_k}^2} \ln\left(\frac{m_{t_i}^2}{m_{h_k}^2} \right) \right] \right. \\ & + \sum_{i \neq j;\, k} (-1)^{i+j} \eta_k \beta_{0i} \beta_{0j} \alpha_{ji} m_{t_j} \left[\frac{-1}{m_{t_i}^2 - m_{t_j}^2} \cdot \frac{1}{2} \left(\frac{m_{t_i}^2}{m_{t_i}^2 - m_{h_k}^2} + \frac{m_{t_j}^2}{m_{t_j}^2 - m_{h_k}^2} \right) \right. \\ & + \frac{m_{t_i}^2}{2(m_{t_i}^2 - m_{t_j}^2)^2} \left(\frac{2m_{t_i}^2 - m_{t_j}^2}{m_{t_i}^2 - m_{h_k}^2} + \frac{m_{t_j}^2}{m_{t_j}^2 - m_{h_k}^2} \right) \ln\left(\frac{m_{t_i}^2}{m_{t_j}^2} \right) \\ & - \frac{m_{h_k}^2}{2(m_{t_i}^2 - m_{h_k}^2)(m_{t_j}^2 - m_{h_k}^2)} \left[\frac{2m_{t_i}^2 - m_{h_k}^2}{m_{t_i}^2 - m_{h_k}^2} \ln\left(\frac{m_{t_j}^2}{m_{h_k}^2} \right) - \frac{m_{h_k}^2}{m_{t_j}^2 - m_{h_k}^2} \ln\left(\frac{m_{t_i}^2}{m_{h_k}^2} \right) \right] \\ & \left. \left. - \frac{m_{h_k}^2}{2(m_{t_i}^2 - m_{t_j}^2)} \ln\left(\frac{m_{t_i}^2}{m_{t_j}^2} \right) \left(\frac{m_{t_i}^2}{(m_{t_i}^2 - m_{h_k}^2)^2} - \frac{m_{t_j}^2}{(m_{t_j}^2 - m_{h_k}^2)^2} \right) \right] \right\} . \end{aligned} \tag{4.9}$$

The coefficients η_k here are ones that appear in Eq. (2.22). This expression reduces, in the limits $m_{h_1} \to 0$ and $m_{h_{2,3}} \to \infty$, to Eq. (A6) of Ref. [12] [which, in turn, reduces to Eq. (4.5) in the further limit $m_t \ll m_{t_{2,3}}$]. Alternately, it reduces in the limit $m_{t_{2,3}}, m_{h_3} \to \infty$ to Eq. (4.4), as was used in Ref. [14].

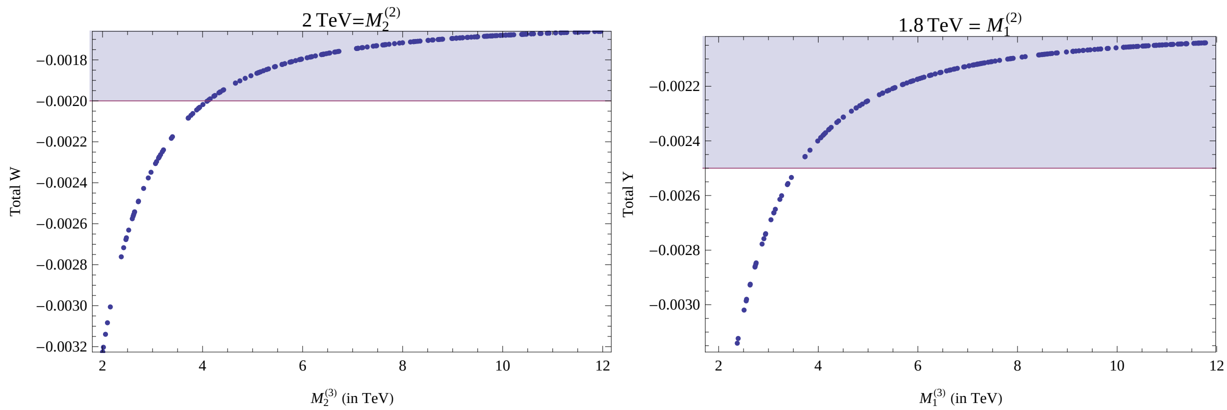

FIG. 3: Bounds on LW gauge boson mass partners from the oblique parameters W and Y. The shaded area (blue online) is experimentally allowed at 2σ.

V. ANALYSIS

We use the definitions of the post-LEP oblique parameters in Eqs. (3.4)–(3.7). As discussed above, the tree-level expressions for W and Y are sufficient for our analysis (and provide the most useful bounds on electroweak gauge boson partner masses), while the leading contributions to $\hat{S}$ and $\hat{T}$ arise from one-loop fermion effects. Since the sums in Eq. (3.9) include the SM quarks, their effects must be subtracted from the full result, giving $\hat{S}_{\rm new} \equiv \hat{S} - \hat{S}_{\rm SM}$ and $\hat{T}_{\rm new} \equiv \hat{T} - \hat{T}_{\rm SM}$. In our subsequent discussion, $\hat{S}$, $\hat{T}$ are understood to mean $\hat{S}_{\rm new}$, $\hat{T}_{\rm new}$, respectively. As a benchmark for the magnitude of new physics effects, one finds $\hat{S}_{\rm SM} = -1.98 \cdot 10^{-3}$, $\hat{T}_{\rm SM} = +9.25 \cdot 10^{-3}$.

As seen in Ref. [20], the measured values of the parameters $\hat{S}$, $\hat{T}$, W, and Y are all of order 10^{-3}, and they are correlated. However, for simplicity we use the values listed in Table 4 of [20] with 2σ uncertainties:

$$10^3\,\hat{S} \;=\; 0.0 \pm 2.6\,, \tag{5.1}$$
$$10^3\,\hat{T} \;=\; 0.1 \pm 1.8\,, \tag{5.2}$$
$$10^3\,W \;=\; -0.4 \pm 1.6\,, \tag{5.3}$$
$$10^3\,Y \;=\; 0.1 \pm 2.4\,. \tag{5.4}$$

To this list we add the bound on $\delta g_L^{b\bar{b}}$ in Eq. (4.3), which serves to constrain both LW fermion masses and scalar masses, as discussed in the previous section.

First note that the $N = 2$ and $N = 3$ gauge boson masses contribute at tree level in Eqs. (3.6)–(3.7) additively, and therefore the bounds that hold for the $N = 2$ theory (*e.g.*, $M_1^{(2)} = M_2^{(2)} \geq 2.4$ TeV according to Ref. [12]) are tightened by the addition of $N = 3$ partners. In Fig. 3 one sees that taking $M_2^{(2)} = 2$ TeV requires $M_2^{(3)} \gtrsim 4$ TeV, the latter likely outside the discovery range of the current LHC. In particular, the discovery scenario described in Ref. [17] of $M_2^{(2)} = 2.0$ TeV, $M_2^{(3)} = 2.5$ TeV is unlikely unless the bounds on W are not as stringent as given in Eq. (5.3). Likewise, for Y, Fig. 3 indicates $M_1^{(2)} = 1.8$ TeV is possible for $M_1^{(3)} \gtrsim 3.5$ TeV. If, however, the $N = 2$ and $N = 3$ masses are quasi-degenerate, universal values $\gtrsim 2.5$ TeV remain possible.

The constraints from $\hat{S}$ are much less restrictive. Unlike in other BSM scenarios where the addition of extra chiral fermions create insurmountable tension with the measured value of $\hat{S}$, the extra fermions in the LWSM are all vectorlike, and contribute to $\hat{S}$ only through diagonalization with the chiral fermion mass parameters arising through Yukawa couplings. Assuming for simplicity the degenerate case $M_{q2} = M_{t2} = M_{b2}$ studied in [12] and extending to $M_{q3} = M_{t3} = M_{b3}$, one finds no meaningful constraint on the fermion mass parameters M_{q2} or M_{q3}.

The bounds from $\hat{T}$ are much more interesting; they were found in [12] (Fig. 5) to require $M_{q2} \geq 1.5$ TeV in order for $\hat{T}$ to lie no more than 2σ below its measured central value, and provide one of the strongest constraints on LW quark partner masses. At the inception of this work, it was believed that the opposite signs of the $N = 2$ and $N = 3$ LW quark propagators would allow for a near-complete cancellation of their loop effects, essentially removing the $\hat{T}$ constraint as a significant bound on the quark partners if their masses were sufficiently close. However, the detailed result in fact requires much greater care in its analysis: While the $N = 2$ and $N = 3$ loops do indeed cancel to a large extent, the propagating fermions in the loops are the mass eigenstates. The act of mass diagonalization not only shifts mass eigenvalues of the heavy states slightly away from M_{q2} and M_{q3}, but also modifies the strength of

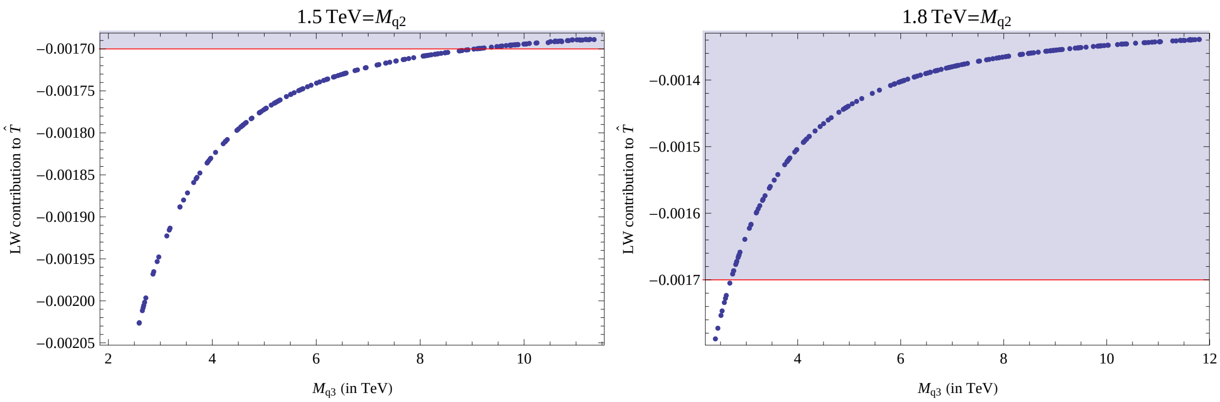

FIG. 4: Bounds on the oblique parameter $\hat{T}$ in two scenarios, $M_{q2} = 1.5$ TeV and 1.8 TeV. The shaded area (blue online) is experimentally allowed at 2σ.

the contribution of the $N = 1$ (SM) quarks to $\hat{T}$. The effect of this shift is pronounced due to the large size of the SM t Yukawa coupling; it actually serves to push the full value of $\hat{T}$ slightly further from its measured central value, thus forcing an allowable $N = 2$ LW mass M_{q2} to be slightly larger than before the addition of the $N = 3$ state. However, the effect is not extreme; from Fig. 4, one sees that $M_{q2} = 1.5$ TeV remains viable for $M_{q3} \gtrsim 9$ TeV, while increasing M_{q2} only slightly, to 1.8 TeV, allows M_{q3} to be $\lesssim 2.8$ TeV. The transition between extremely strong and extremely weak M_{q3} bounds occurs in a very narrow window of M_{q2} values.

Finally, consider constraints from $\delta g_L^{b\bar{b}}$, which in Ref. [12] provide the most stringent bounds on the quark partner masses, $M_{q2} \gtrsim 4$ TeV. However, as noted in the previous section, the bottom of the 2σ-allowed region has since moved slightly downward. Since $\delta g_L^{b\bar{b}}$ is a very shallow function of M_{q2}, this small change dramatically alters the bound to $M_{q2} \gtrsim 1.2$ TeV, as seen in the first inset of Fig. 5. The $N = 3$ theory is used in the second inset of Fig. 5, where one sees that raising M_{q2} only slightly (to 1.4 TeV) allows $M_{q3} \gtrsim 2.3$ TeV. On the other hand, if the LW quark masses are assumed sufficiently large to decouple, $\delta g_L^{b\bar{b}}$ provides a lower bound on the $N = 2$ LW scalar of $m_{h_2} \gtrsim 640$ GeV (first inset of Fig. 6), as would have been found in a more complete calculation (including not only R_b but also A_b bounds) by Ref. [14]. The fact that the $Zb\bar{b}$ vertex constrains the masses of heavy t's more strongly than those of the scalars appears to follow directly from Eq. (4.9) being $\propto y_t^3$ [and indeed, in certain limits such as in Eq. (4.5), $\propto y_t^4$], and because the act of mass diagonalization among the fermions allows $y_t v/\sqrt{2}$ to shift substantially from the physical t mass (as was noted in the analysis of $\hat{T}$). Since mass diagonalization does not mix the charged scalar parameters, including the $N = 3$ LW state leads to a dramatic cancellation: For example, in the second inset of Fig. 6 one sees that $m_{h_2} = 400$ GeV, $m_{h_3} \lesssim 850$ GeV satisfy the $\delta g_L^{b\bar{b}}$ constraint. In retrospect, the bounds on charged scalar masses in the $N = 2$ theory obtained by Ref. [14] from $B\bar{B}$ mixing and $b \to s\gamma$ now lead to weaker constraints ($m_{h_2} > 463$ GeV) than that from $\delta g_L^{b\bar{b}}$, and the former bounds moreover would also likely be significantly softened by the addition of an $N = 3$ charged scalar due to the cancellations described above. When both LW quarks and charged scalars are included, the bounds again become more constrained, but many interesting scenarios remain possible; for example, Fig. 7 shows that the combined set $M_{q2} = 2.5$ TeV, $M_{q3} = 4$ TeV, $m_{h_2} = 400$ GeV, $m_{h_3} = 600$ GeV satisfies the $\delta g_L^{b\bar{b}}$ constraint.

VI. DISCUSSION AND CONCLUSIONS

The Lee-Wick approach to extending the Standard Model provides a variety of interesting effects that can be tested experimentally. Since the couplings of the new particles equal those of the SM fields and only their masses remain as free parameters, one can obtain bounds on these masses from electroweak precision constraints. For such particles for which the masses are $\lesssim 3$ TeV, one can even hope to directly produce the particles at the current incarnation of the LHC. On the other hand, the LWSM was originally motivated by its potential to provide an alternate resolution to the hierarchy problem, which ideally requires fields with masses in the several hundred GeV range. In our calculations, we find that only the scalar partners to the Higgs can be so light, and therefore the LWSM does not offer an especially natural resolution of the hierarchy, although by construction all quadratic divergences in loop diagrams cancel.

Nevertheless, we find that the imposition of precision constraints on the $N = 3$ LWSM still allows masses for LW partner states to lie in large swathes of the parameter space directly accessible at the LHC, providing phenomenological

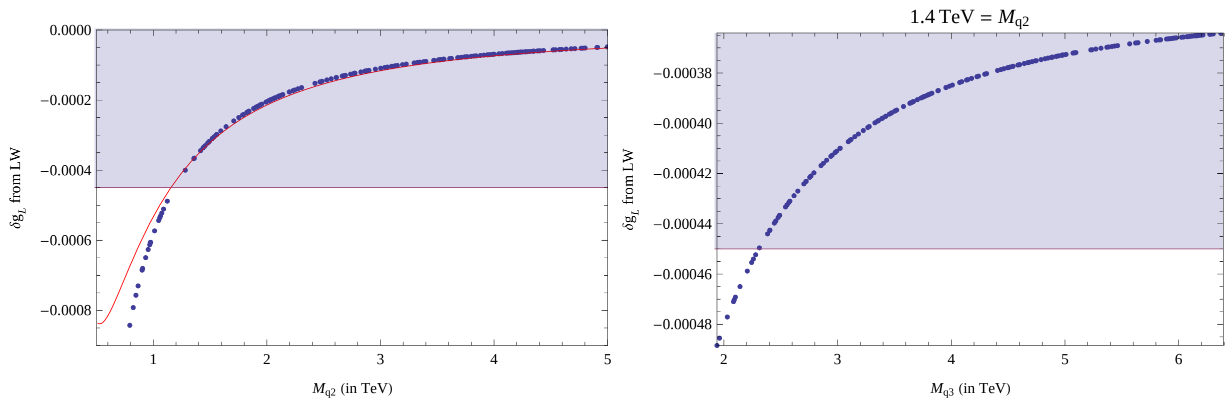

FIG. 5: Bounds on $\delta g_L^{b\bar{b}}$ with LW t quark partners. The first inset presents an updated calculation in the $N = 2$ theory, and the red line is the leading-order result Eq. (4.5). The second inset presents an $N = 3$ calculation in which M_{q2} is fixed and M_{q3} is allowed to vary. The shaded area (blue online) is experimentally allowed at 2σ.

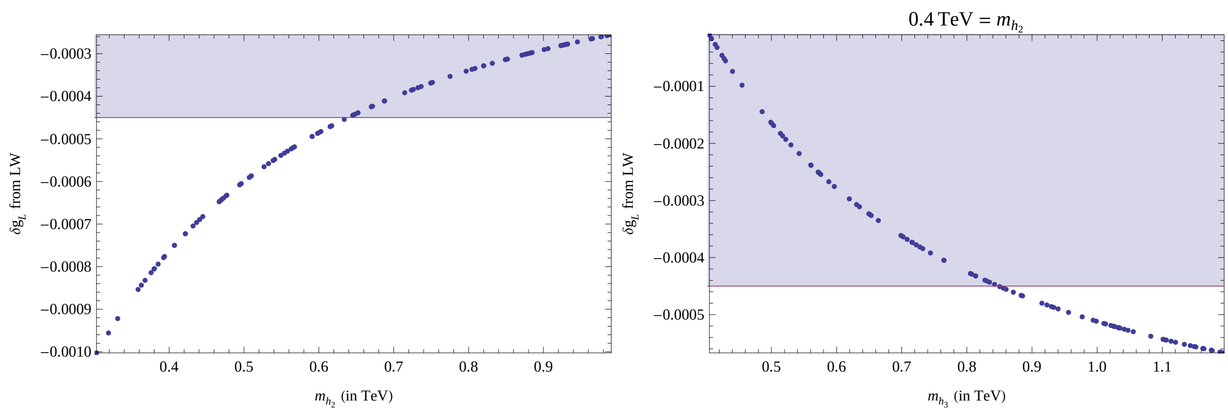

FIG. 6: Bounds on $\delta g_L^{b\bar{b}}$ with one (first inset, $N = 2$) and two (second inset, $N = 3$) charged scalar LW partners, of masses $m_{h_{2,3}}$. The shaded area (blue online) is experimentally allowed at 2σ.

significance to the LWSM. In particular, we have found that the post-LEP oblique parameters W and Y require the $N = 2$ partners of the W and B to be $\gtrsim 2.0$ and 1.8 TeV, respectively, and the $N = 3$ partners to be substantially heavier, or, by the same bound, they could be quasi-degenerate and all $\gtrsim 2.5$ TeV. The LW quark masses are constrained by custodial isospin ($\hat{T}$) and the $Zb\bar{b}$ coupling $g_L^{b\bar{b}}$ to be at least 1.5 TeV; one of the most interesting results of this work was the discovery that, as expected, the $N = 3$ quarks loops do cancel against the $N = 2$ loops, but this cancellation is largely nullified by the effects arising from the diagonalization of quark masses amongst the SM quarks and its LW partners. Even so, LW quark masses in the range $M_{q2} \gtrsim 1.8$ TeV remain viable if the $N = 3$ partner is somewhat heavier ($\gtrsim 2.8$ TeV). The least constrained masses, like in the original SM, appear to be in the scalar sector. From the $Zb\bar{b}$ coupling alone, values in the few hundred GeV range remain viable in the $N = 3$ theory due to the presence of a more complete cancellation between the $N = 2$ and $N = 3$ states, although a full analysis including $b \to s\gamma$ and $B\bar{B}$ mixing should be undertaken to obtain global constraints. Furthermore, the current LHC value for the $h^0 \to \gamma\gamma$ branching ratio [1, 2] shows a significant excess compared to the SM, while the $N = 2$ LWSM prediction differs from the SM value by only a few percent [13, 15]; the $N = 3$ LWSM, with a larger parameter space, offers the opportunity to produce a larger effect.

In summary, the LWSM is alive and well, particularly its $N = 3$ variant. Some of the gauge boson and fermion partners may be difficult to discern directly at the LHC, but the potential for direct discovery remains. The scalar sector, whose exploration is arguably the central business of the LHC, is the least constrained and therefore the most interesting from the immediate phenomenological point of view.

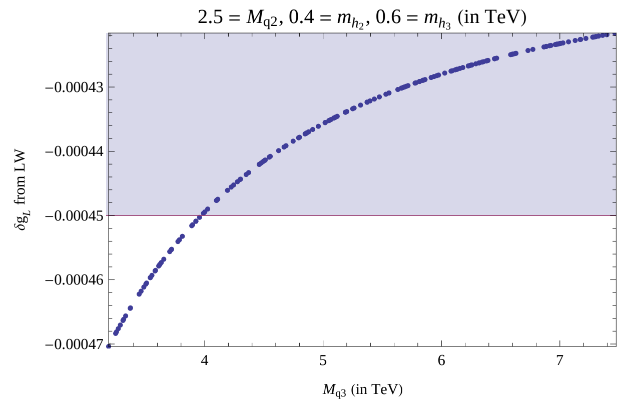

FIG. 7: Bounds on $\delta g_L^{b\bar{b}}$ in the $N = 3$ theory with both LW quark and charged scalar partners. The shaded area (blue online) is experimentally allowed at 2σ.

Acknowledgments

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